

Procyclical Finance: The Money View*

Ye Li[†]

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Abstract

Banks are important because agents hold their debts (“inside money”) as liquidity buffer. Banking crises are costly because the contraction of inside money supply compromises firms’ liquidity management and hurts investment. By highlighting the interaction between banks and firms in the money market, this paper offers a theory of procyclical inside money creation and the resulting instability. It sheds light on the cyclicity of bank leverage, and how it affects the frequency and duration of banking crises. Introducing outside money (government debt) to alleviate liquidity shortage can be counterproductive, because its competition with inside money destabilizes the banking sector.

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[†]The Ohio State University. E-mail: li.8935@osu.edu

1 Introduction

In the years leading up to the Great Recession, the financial sector grew rapidly, setting a favorable liquidity condition that stimulated the real economy. A booming real sector in turn fueled financial intermediaries' expansion and leverage. During the crisis, the spiral flipped. Much progress has been made in recent years to characterize crisis dynamics by incorporating intermediaries in macro models (e.g., He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)), yet a complete account of procyclical intermediation, and in particular, the run-up to crisis, remains a challenge.

This paper argues that at the heart of this procyclicality is intermediaries' role as money creators. The monetary aspect of financial intermediation is so ubiquitous that we often fail to notice. Intermediary debt (e.g., bank deposits) is a store of value, but more importantly, it supports trade by serving as a means of payment, or "inside money".¹ In an economy where agents' future income is not fully pledgeable, money facilitates spot transactions and resource reallocation. However, the money demand of the real sector feeds leverage to the financial sector, and thus, breeds instability.

I build a continuous-time model of macroeconomy that crystallizes this money view of financial intermediation. A key ingredient is the money demand from firms' liquidity management problem that is similar to Holmström and Tirole (1998). Banks supply money by issuing debt, and thus, build up leverage in the process. The dynamic interaction between money demand and supply generates a rich set of unique predictions, such as bank leverage cycle, money premium dynamics, endogenous risk accumulation in booms, stagnant recessions, and investment inefficiencies.

The idea that financial intermediaries affect the real economy through inside money supply goes back at least to the classic account of the Great Depression by Friedman and Schwartz (1963). One may argue that this money view is less relevant today given the active supply of *outside money* in the form of liquid government securities and central bank liabilities (Woodford (2010)). However, the model shows that competition between inside and outside money *destabilizes* the banking sector by amplifying its leverage cycle, making booms more fragile and prolonging crises. These results complement the literature on outside money as a means to financial *stability* (Greenwood,

¹The term "inside money" is from Gurley and Shaw (1960). From the private sector's perspective, fiat money and government securities are in positive supply ("outside money"), while bank deposits are in zero net supply ("inside money"). Both outside and inside money facilitate transactions. Lagos (2008) briefly reviews the concept.

Hanson, and Stein (2015); Krishnamurthy and Vissing-Jørgensen (2015); Woodford (2016)).

The model economy operates in continuous time. It has three types of agents: bankers, entrepreneurs (“firms”), and households who play a limited role. All agents are risk-neutral with the same time discount rate, and consume nonstorable generic goods produced by firms’ capital.

Firms can hold capital and bank deposits, and borrow from banks and households *as long as* they are not hit by liquidity shocks. Every instant, firms face a constant probability of liquidity shock, and in such an event, their production halts, and their capital can either grow – if further investment is made – or perish, if not. This investment is not pledgeable, so firms can only obtain goods (investment inputs) from others in spot transactions, using deposits as means of payment. Therefore, banks add value because their debts (deposits) facilitate trade and resource reallocation.

Bankers issue deposits that are short-term risk-free debts, and extend loans to firms that are backed by designated capital as collateral. Every instant, a random fraction of capital collateral is destroyed, and the corresponding loans default. The only aggregate shock is a Brownian motion that drives this stochastic destruction of capital. Money creation requires risk-taking, because at the margin, one more dollar of deposits is backed by one more dollar of risky loan. Therefore, inside money supply depends on bank equity as risk buffer. Bankers may raise equity subject to an issuance cost. This friction ties the supply of inside money to the current level of bank equity. When bad shocks leave more loans in default and deplete bank equity, the supply of inside money declines, which hurts firms’ liquidity management and investment.

The model has a Markov equilibrium with the ratio of bank equity to firm capital as state variable, which is intuitively the size of money suppliers relative to money demanders. Because banks have leveraged exposure to the capital destruction shock, this state variable rises following good shocks, and falls following bad shocks. It is also bounded by two endogenous boundaries: when the banking sector is small, bankers raise equity because the marginal value of equity reaches one plus the issuance cost (lower boundary); when the banking sector is large, the marginal value of equity falls to one, so bankers consume and pay out dividends to shareholders (upper boundary).²

²As in Phelan (2016) and Klimenko et al. (2016), banks issue equity in bad times, and pay out dividends in good times, which is consistent with the evidence in Baron (2014) and Adrian, Boyarchenko, and Shin (2015).

Bank leverage is procyclical.³ Good shocks increase bank equity, but banks issue even more debt to meet firms' procyclical money demand. Good shock means less loans default than expected. Banks hoard the windfall instead of paying it out because the issuance cost creates a wedge between the value of retained equity and one dollar (payout value). Thus, shock impact dissipates gradually. Expecting banks to be better capitalized and to issue more money going forward, firms foresee themselves to carry more money *in the future* that will then finance faster capital growth. Thereby, capital becomes more valuable, inducing firms to hold more money *now* in case the liquidity shock arrives the very next instant. In sum, firms' money demand exhibits intertemporal complementarity. Asset price (capital value) plays a key role here, feeding the expectation of future money market conditions into the current money demand of firms. By strengthening the procyclicality of money demand and bank leverage, endogenous asset price has a unique destabilizing effect that is distinct from the typical balance-sheet channel (e.g., Brunnermeier and Sannikov (2014)).

Downside risk accumulates through procyclical leverage.⁴ As banks become more levered, their equity is more sensitive to shocks. And, as the economy approaches bank payout boundary, high leverage only serves to amplify bad shocks, because good shocks cannot increase bank equity above the boundary without triggering payout. The longer booms last, the higher downside risk is.

Crises are stagnant. As the economy approaches bank issuance boundary, low bank leverage only serves to dampen good shocks, because bank equity never falls below the issuance boundary. Therefore, banks can only rebuild equity after a sequence of sufficiently large good shocks. The calibrated model predicts an eight-year recovery period, during which the economy is stuck with insufficient money supply that compromises firms' liquidity management. In sum, fragile booms and stagnant crises result from a combination of procyclical leverage and the asymmetric impact of shocks near the reflecting boundaries given by bank payout and equity issuance policies.

So far, we have focused on the procyclical quantity of inside money. The model also generates a countercyclical price of money, the *money premium*, which is a spread between the time discount rate and deposit rate. Borrowing at an interest rate lower than the time discount rate, banks

³The leverage here is the ratio of book asset to book equity, as will be clearly defined in the model.

⁴Intermediary leverage does not exhibit procyclicality in models that feature a static demand for intermediary debt and focus on the asset-side of intermediary balance sheet for channels of shock amplification (e.g. He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014); Phelan (2016); Klimenko et al. (2016)).

earn the money premium. Carrying low-yield deposits, firms pay the money premium, which is a cost of liquidity management, but they optimally do so in anticipation of transaction needs.

Another important theme of this paper is the financial stability implications of outside money. I model outside money as government debt that offers the same monetary service to firms as bank deposits. Its empirical counterparts include a broad range of liquid government securities, not just central bank liabilities.⁵ To highlight the competition between inside and outside money, I abstract away other fiscal distortions by assuming the debt issuance proceeds are paid to agents as lump-sum transfer and debt is repaid with lump-sum tax. Outside money decreases the money premium in *every* state of the world, which seems to indicate a more favorable condition for firms and more investments as a result. However, the impact of outside money depends on how banks respond.

By lowering the money premium, outside money increases banks' debt cost, and thereby, decreases the net interest margin. This profit crowding-out effect amplifies bank leverage cycle. In the states where banks are well capitalized and willing to take risks, they raise leverage even higher, so that on average, return on equity (i.e., net interest margin multiplied by leverage) is still high enough to justify the occasionally incurred cost of equity issuance. The crowding-out effect can also lengthen the crisis. With lower profit, banks become more reluctant to raise equity, which translates into a lower equity issuance boundary. And, with lower return on equity, it takes more time for banks to rebuild equity through retained earnings.

With booms being more fragile and crises more stagnant, the economy spends more time in states where banks are undercapitalized and inside money creation depressed. Unless outside money satiates firms' money demand, the economy still relies on banks as the *marginal* suppliers of money. Therefore, even if outside money increases the total money supply in *every* state of the world, by shifting the probability mass to relatively worse states, it can lead to a lower *average* money supply over the cycles, and thus, hurt resource reallocation and welfare in the long run.

Related literature. Financial intermediaries provide liquidity – the ease of transferring resources over time and between agents. They finance projects (supply credit) and issue securities that fa-

⁵The monetary service of government liabilities is an old theme (e.g., Patinkin (1965); Friedman (1969)). Recent contributions include Bansal and Coleman (1996), Bansal, Coleman, and Lundblad (2011), Krishnamurthy and Vissing-Jørgensen (2012), Greenwood, Hanson, and Stein (2015), Bolton and Huang (2016), and Nagel (2016).

facilitate trade (supply money). The cost of liquidity is zero in the frictionless world of Modigliani and Miller (1958), but in reality, we rely on intermediaries to supply money and credit, and due to their limited balance-sheet capacity, they earn a spread. This paper focuses on money supply. It advances an old tradition in macroeconomics by taking a corporate-finance approach, which emphasizes money as a store of value and a means of payment rather than a unit of account (the critical ingredient of models with nominal rigidities, e.g., Christiano, Eichenbaum, and Evans (2005)).

A recent literature has revived the money view of financial intermediation by emphasizing bank liabilities as stores of value and means of payment (Hart and Zingales (2014); Brunnermeier and Sannikov (2016); Donaldson, Piacentino, and Thakor (2016); Piazzesi and Schneider (2017); Quadrini (2017)).⁶ This paper takes this money view of banking to understand the dynamic interaction between firms' liquidity management and banks' choice of leverage, frequency and duration of crises, and how government debt may contribute to financial instability.

Because of the equity issuance cost, the model has a “balance-sheet channel”, through which shock impact is persistent (e.g., Bernanke and Gertler (1989)).⁷ Bank net worth is important, which is a feature shared with other models of balance-sheet channel.⁸ My model differs in two aspects. First, asset price (capital value) plays a role in shock amplification through the intertemporal complementarity of firms' money holdings, instead of the typical balance-sheet impact (e.g., Brunnermeier and Sannikov (2014)). Second, the demand for intermediary debt is dynamic, contributing to the procyclicality of bank leverage and endogenous risk accumulation. In contrast, many models have a static/passive demand for intermediary debt, so book leverage is countercyclical because equity is more responsive to shocks than assets (e.g., He and Krishnamurthy (2013)).

Models that produce procyclical leverage often focus on the impact of asset price variations on collateral or risk constraints (Brunnermeier and Pedersen (2009); Geanakoplos (2010); Adrian

⁶Several branches of literature provide microfoundations for bank debts serving as means of payment. Limited commitment (Kiyotaki and Moore (2002)) and imperfect record keeping (Kocherlakota (1998)) limits credit, so trades must engage in *quid pro quo*, involving a transaction medium. Banks overcome such problems and supply money (e.g., Kiyotaki and Moore (2000); Cavalcanti and Wallace (1999)). Ostroy and Starr (1990) and Williamson and Wright (2010) review monetary theories. Another approach relates resalability to information sensitivity. Banks create money by issuing safe claims that circulate in secondary markets (Gorton and Pennacchi (1990); Dang et al. (2014)).

⁷The issuance cost limits the sharing of aggregate risk between banks and the rest of the economy (Di Tella (2015)).

⁸Bank equity is a common measure of financial slackness, as it alleviates agency friction (Holmström and Tirole (1997); Diamond and Rajan (2000)) and facilitates collateralization (Rampini and Viswanathan (2017)).

and Boyarchenko (2012); Danielsson, Shin, and Zigrand (2012); Moreira and Savov (2014)). This paper offers a complementary explanation based on corporate money demand. More importantly, it unveils a feedback mechanism between the real and financial sectors that sets the stage for a formal analysis of the financial stability implications of government debt.

A recent literature documents a money premium that lowers the yield on government securities (e.g., Bansal and Coleman (1996); Krishnamurthy and Vissing-Jørgensen (2012); Nagel (2016)). Intermediaries earn the money premium by issuing money-like liabilities, such as asset-backed commercial paper (Sunderam (2015)), deposits (Drechsler, Savov, and Schnabl (2016)), and certificates of deposits (Kacperczyk, Pérignon, and Vuillemeys (2017)). Many have emphasized the risk of excessive leverage (Gorton (2010); Stein (2012)), and pointed out that increasing government debt stabilizes the economy by crowding out intermediary debt (Greenwood, Hanson, and Stein (2015); Krishnamurthy and Vissing-Jørgensen (2015); Woodford (2016)). Advancing this line of research, this paper highlights banks' *dynamic* balance-sheet management under issuance cost, the key ingredient that generates the destabilizing effects of outside money.⁹

As in Woodford (1990b) and Holmström and Tirole (1998), government debt facilitates investment by allowing entrepreneurs to transfer wealth across contingencies. This paper adds to this line of research by emphasizing government as intermediaries' competitor in liquidity supply. By crowding out intermediated liquidity, public liquidity can reduce the overall liquidity and welfare. After the financial crisis, governments increased their indebtedness and central banks expanded balance sheets dramatically in advanced economies, raising concerns such as moral hazard and excessive inflation (Fischer (2009)). This paper highlights a financial instability channel through which an expanding government balance sheet can be counterproductive.¹⁰

A key ingredient of the model is firms' money demand, which is motivated by studies on the enormous amount of corporate cash holdings (e.g., Bates, Kahle, and Stulz (2009); Eisfeldt and Muir (2016)).¹¹ This paper connects corporate cash holdings to bank leverage, and money demand

⁹Also highlighting the dilution cost of equity issuance, Bolton and Freixas (2000) analyze the effects of monetary policy on banks' profit and equity capital through changes in the lending spread instead of the money premium.

¹⁰The model omits other important channels through which the government may interact with the banking sector, such as monetary policy (Drechsler, Savov, and Schnabl (2017); Di Tella and Kurlat (2017)).

¹¹Firms' liquidity management problem in the model can be viewed as simplified version of He and Kondor (2016).

arises from firms' investment needs.¹² R&D is a typical form of investment that heavily relies on internal liquidity and exhibits procyclicality.¹³ A number of studies have shown that the rise of corporate cash holdings in the last few decades is largely driven by R&D-intensive firms (Begenau and Palazzo (2015); Graham and Leary (2015); Pinkowitz, Stulz, and Williamson (2016)). There is a rising literature on the demand for money-like or safe assets and its implications for financial instability. Theoretical studies often relegate the modeling of money demand side, for instance, by assuming money or safety in utility (Stein (2012); Caballero and Farhi (2017)). This paper takes a step forward, relating money demand to firms' liquidity management and showing its endogenous and unique dynamics is relevant for understanding the causes and consequences of financial crisis.

The remainder is organized as follows. Section 2 lays out key economic forces in a static setting. The continuous-time model is in Section 3. Section 4 adds government debt. Section 5 concludes. Appendices contain proofs, algorithm, and calibration details. Internet Appendix offers preliminary evidence, and discusses the setup of outside money and its broader implications.

2 Static Model: An Anatomy of Money Shortage

This section lays out the key economic forces in a two-period model ($t = 0, 1$). There are goods, capital, and three types of agents, households, bankers, and firms. Firms own capital that produces goods at $t = 1$, but some are hit by a liquidity shock before production and need further investment. To buffer the shock, firms carry deposits issued by bankers at $t = 0$ (inside money). Bankers back

By allowing capital to be traded, the model easily aggregates firms' liquidity demand (linear in firm wealth). Bolton, Chen, and Wang (2011) and (2013) study nonlinear liquidity demand of firms when capital is nontradable.

¹²Eisfeldt (2007) shows that liquidity demand from consumption smoothing cannot explain the liquidity premium on Treasury bills. Eisfeldt and Rampini (2009) show that liquidity premium rises when asynchronicity between cash flow and investment in the corporate sector becomes more severe, consistent with the theory of Holmström and Tirole (2001). Investment need is a key determinant of cash holdings (Denis and Sibilkov (2010); Duchin (2010)), especially for firms with less collateral (Almeida and Campello (2007)) and more R&D activities (Falato and Sim (2014)).

¹³The procyclicality of R&D expenditures, as measured by the NSF, has been documented by many studies, including Griliches (1990), Fatas (2000), and Comin and Gertler (2006). Using data from the NSF and Compustat, Barlevy (2007) finds a significant positive correlation between real GDP growth and the growth rate of R&D. Using French firm-level data, Aghion et al. (2012) show that the procyclicality of R&D investment (with respect to sales growth) is found among firms that are financially constrained. Fabrizio and Tsoimon (2014) find that R&D investments are more procyclical in industries with faster obsolescence. The setup of firms' liquidity shock is motivated by these findings.

deposits by loans extended to firms. Inside money supply depends on bankers' balance-sheet capacity (net worth). Insufficient supply compromises firms' liquidity management and investment.

2.1 Setup

Physical structure. All agents consume a non-storable, generic good, and have the same risk-neutral utility with discount rate ρ . At $t = 0$, there are K_0 units of capital endowed to a unit mass of homogeneous entrepreneurs (firms). One unit of capital produces α units of goods at $t = 1$, and it is only productive in the hands of entrepreneurs. Capital can be traded in a competitive market at $t = 0$, at price q_0^K . Let k_0 denote a firm's holdings of capital, so that $K_0 = \int_{s \in [0,1]} k_0(s) ds$. I use subscripts for time, and whenever necessary, superscripts for type ("B" for bankers, "H" for households, and "K" for firms who own capital). There is also a unit mass of homogeneous bankers. Each is endowed with e_0 units of goods, so their aggregate endowment is $E_0 = \int_{s \in [0,1]} e_0(s) ds$. The index "s" will be suppressed without loss of clarity. There are a unit mass of homogeneous households endowed with a large amount of goods per period. Households play a very limited role.

At the beginning of date 1 ($t = 1$), all firms experience a capital destruction shock, while some also experience a liquidity shock. The economy has one aggregate shock Z_1 , a binary random variable that takes value 1 or -1 with equal probability. After Z_1 is realized, firms lose a fraction $\pi(Z_1)$ of their capital. For simplicity, I assume that $\pi(Z_1) = \delta - \sigma Z_1$ ($\delta - \sigma \geq 0$ and $\delta + \sigma \leq 1$). Later we will see Z_1 makes bank equity essential for money creation. After capital loss, firms proceed to produce $\alpha [1 - \pi(Z_1)] k_0$ units of goods, if they not are hit by the liquidity shock.

Independent liquidity shocks hit firms with probability λ , and destroys *all* capital. In the spirit of Holmström and Tirole (1998) and (2001), firms must make further investment; otherwise, they can not produce anything, and thus, exit with zero terminal value. By investing i_1 units of goods per unit of capital, firms can create $F(i_1) k_0$ ($F'(\cdot) > 0$, $F''(\cdot) < 0$) units of new capital. Homogeneity in k_0 helps reduce the dimension of state variable later in the continuous-time dynamic analysis. I assume that after the investment, firms can revive their old capital, so the post-investment production is $\alpha [1 - \pi(Z_1) + F(i_1)] k_0$. Through investment, firms preserve the existing scale of production and grow. The first-best level of investment rate, i_{FB} , is defined by:

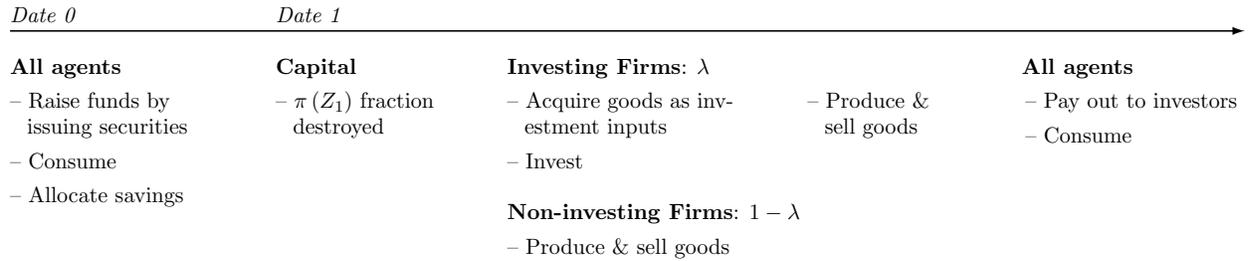


Figure 1: Timeline. This figure plots the timeline of static model. Agents set balance sheets at $t = 0$. Z_1 is realized at the beginning of $t = 1$. $\pi(Z_t)$ fraction of capital is destroyed. Then the liquidity shock hits λ fraction of firms. The rest produce. λ firms produce after investments. All agents consume and repay liability holders at the end of $t = 1$.

$$\alpha F'(i_{FB}) = 1, \tag{1}$$

Note that firms making investment at the beginning of $t = 1$ instead of $t = 0$. This backloaded specification gives rise to firms' liquidity demand later in the presence of financial constraints.

Last but not least, it is assumed that all securities issued by agents in this economy pay out *at the end of date 1*. This timing assumption is particularly relevant for defining what is liquidity from firms' perspective. Assets that firms carry to relax constraints on investment must be *resalable* in exchange for investment inputs *at the beginning of date 1*. Firms are not buy-and-hold investors. It will be shown that this resalability requirement relates the model setup to several strands of literature that study banks as issuers of inside money. Figure 1 shows how events unfold.

Liquidity demand. The model features three frictions, one that gives rise to firms' liquidity demand, and the other two limiting liquidity supply. The first friction is that investment has to be internally financed. In other words, the newly created capital is not pledgeable.¹⁴ As a result, firms need to carry liquidity (i.e., instruments that transfer wealth from $t = 0$ to $t = 1$). This is a common assumption used to model firms' liquidity demand.¹⁵ To achieve the first-best investment, a firm must have access to liquidity of at least $i_{FB}k_0$ when the λ shock hits.

The objective of this paper is to analyze the *endogenous* supply of liquidity, so I assume that goods cannot be stored (i.e., there is no exogenous storage technology). And, capital cannot

¹⁴This can be motivated by a typical moral hazard problem as in Holmström and Tirole (1998).

¹⁵See Froot, Scharfstein, and Stein (1993), Holmström and Tirole (1998) and (2001) in the micro literature, and Woodford (1990b), Kiyotaki and Moore (2005), and He and Kondor (2016) in the macro literature among others.

transfer wealth to a contingency where itself is destroyed without further spending. So firms must hold financial assets as liquidity buffer. While households receive endowments per period, it is assumed that they cannot sell claims on their future endowments because they can default with impunity; otherwise, there would be no liquidity shortage (as in Holmström and Tirole (1998)). Therefore, the focus is on the asset creation capacity of entrepreneurs themselves and bankers.

Liquidity supply. At date 0, what is a firm’s capacity to issue claims that pay out at date 1? I assume firms’ *endowed* capital is pledgeable.¹⁶ It is collateral that can be seized by investors when default happens.¹⁷ In an equilibrium where firms always carry liquidity and invest when hit by the λ shock, a fraction $[1 - \pi(Z_1)]$ of endowed capital is always preserved. Thus, a firm’s pledgeable value at $t = 1$ is $\alpha(1 - \delta)k_0$ in expectation, and $\alpha(1 - \delta - \sigma)k_0$ when $Z_1 = -1$.

Potentially, firms could hold securities issued by each other as liquidity buffer. If the aggregate pledgeable value *always* exceeds firms’ aggregate liquidity demand, i.e.,

$$\alpha(1 - \delta - \sigma)K_0 \geq i_{FB}K_0, \quad (2)$$

the economy achieves the first-best investment defined in Equation (1).¹⁸ Even better, as long as the liquidity shock is verifiable, firms’ liabilities can be pooled into a mutual fund that pays out to investing firms, so given this perfect risk-sharing, the first-best investment is achieved if

$$\alpha(1 - \delta - \sigma)K_0 \geq \lambda i_{FB}K_0, \text{ where } \lambda \in (0, 1). \quad (3)$$

In a similar setting, Holmström and Tirole (1998) study the question whether entrepreneurs’ supply of assets meets their own liquidity demand (i.e., Equation (2) and (3)), and emphasize the severity of liquidity shortage depends on aggregate shock (i.e., σ in my setting).

This paper departs from Holmström and Tirole (1998) by introducing the second friction: firms can hold liquidity only in the form of bank liabilities. Therefore, in the model, banks issue claims to firms that are in turn backed by banks’ holdings of firm liabilities (“loans”).

¹⁶New capital expected to be created at $t = 1$ is not pledgeable, in line with non-pledgeability of investment project.

¹⁷This reflects that mature capital can be relatively easily evaluated, verified, and seized by investors. Allowing capital created at date 1 to serve as collateral complicates the expressions but does not change the main results.

¹⁸Liquidity holdings cannot be pledged. Otherwise, pledgeable value is infinite: firms’ issuance of securities enlarge each other’s financing capacity, so more securities are issued. Holmström and Tirole (2011) make a related argument.

There are several reasons why firms hold *intermediated liquidity*. Entrepreneurs may simply lack the required expertise of asset management.¹⁹ And, cross holding is regulated in many countries and industries. This assumption is also motivated by strands of theoretical literature that study banks as inside money creators. Given the timing in Figure 1, entrepreneurs purchase goods as investment inputs by selling their liquidity holdings when the λ shock hits. In other words, firms carry liquidity as a means of payment. Kiyotaki and Moore (2000) model bankers as agents with superior ability to make multilateral commitment, i.e., to pay *whoever* holds their liabilities, so bank liabilities circulate as means of payment.²⁰ Taking a step further, money creation may require not only a special set of agents (i.e., bankers), but also a particular security design. In Gorton and Pennacchi (1990) and Dang et al. (2014), banks create money by issuing information-insensitive claims (safe debts) that do not suffer asymmetric information problem in secondary markets.

This paper takes the aforementioned literature as a starting point: firms are assumed to hold liquidity in the form of safe debt issued by banks (“deposits”).²¹ Let m_0 denote a firm’s deposit holdings *per unit of capital*. Investment at $t = 1$ is thus directly tied to deposits carried from $t = 0$:

$$i_1 k_0 \leq m_0 k_0. \quad (4)$$

Equation (4) resembles a money-in-advance constraint (e.g., Svensson (1985); Lucas and Stokey (1987)), except that what firms hold for transaction purposes is not fiat money, but bank debt, or “inside money.” Thus, banks add value to the economy by supplying deposits that can be held by firms to relax this “money-in-advance” constraint on investment. Linking firm cash holdings to bank debt is in line with evidence.²³ Pozsar (2014) shows that corporate treasury, as one of the major cash pools, feeds leverage to the financial sector in the run-up to the global financial crisis.

¹⁹There is a large literature on households’ limited participation in financial markets (Mankiw and Zeldes (1991); Basak and Cuoco (1998); He and Krishnamurthy (2013)), but firms’ portfolio choice is relatively less studied. Duchin et al. (2017) find firms hold risky securities, but what dominate are safe debts issued by intermediaries or governments.

²⁰In a richer setting with limited commitment and imperfect record keeping (Kocherlakota (1998)), credit is constrained, so trades must engage in *quid pro quo*, involving a transaction medium (Kiyotaki and Wright (1989)). Cavalcanti and Wallace (1999) show that bankers arise as issuers of inside money when their trading history is public knowledge. Ostroy and Starr (1990) and Williamson and Wright (2010) review the literature of monetary theories.

²¹The concavity of investment technology $F(\cdot)$ also implies that firms prefer safe assets as liquidity buffer.

²²To be consistent with the continuous-time expressions, deposits’ interest payments are ignored in Equation (4).

²³Based on Financial Accounts of the United States, Figure 1 in Online Appendix shows 80% of liquidity holdings of nonfinancial corporate businesses are in financial intermediaries’ debt, with the rest dominated by Treasury securities.

Inside money creation capacity. To impose more structure on the analysis, I assume loans take a particular contractual form: each dollar of loan extended at $t = 0$ is backed by a designated capital as collateral, and is repaid with interest rate R_0 at the end of date 1 if the collateral is intact. Thus, a fraction $\pi(Z_1)$ of loans default as their collateral is destroyed. The return to a diversified loan portfolio is $[1 - \pi(Z_1)](1 + R_0)$. To mimic the corresponding continuous-time expressions, I approximate it with $1 + R_0 - \pi(Z_1)$, ignoring $\pi(Z_1)R_0$, product of two percentages. This setup is similar to Klimenko et al. (2016).

Because of the aggregate shock, banks' safe debt capacity depends on their equity cushion. Let r_0 denote the deposit rate, and x_0 denote the leverage (asset-to-equity ratio). A banker will never default if her net worth is still positive even in a bad state ($Z_1 = -1$), i.e.,

$$\underbrace{x_0 e_0}_{\text{total assets}} \underbrace{[1 + R_0 - \pi_D(-1)]}_{\text{loan return}} \geq \underbrace{(x_0 - 1) e_0}_{\text{total debt}} \underbrace{(1 + r_0)}_{\text{deposit repayment}} .$$

This incentive or solvency constraint can be rewritten as a limit on leverage:

$$x_0 \leq \frac{r_0 + 1}{r_0 + \delta + \sigma - R_0} := \bar{x}_0 \quad (5)$$

Finally, I introduce the third and last friction – banks' equity issuance cost. At $t = 0$, bankers may raise equity subject to a proportional dilution cost χ . To raise one dollar, a bank needs to give $1 + \chi$ worth of equity to investors.²⁴ I will consider $\chi < \infty$ in the continuous-time analysis. For now, $\chi = \infty$ and banks do not issue equity. As a result, inside money creation is limited by bankers' equity or balance-sheet capacity: total deposits cannot exceed $(\bar{x}_0 - 1)E_0$.

The frictions form three pillars of the model: firms' money demand, bank debt as money; banks' equity constraint. Insufficient inside money supply leads to underinvestment by compromising entrepreneurs' liquidity management. The next section recasts the model in a continuous-time framework and delivers the main results. Before that, I will close this section by showing several features of the static model that are shared with the continuous-time Markov equilibrium.

²⁴ χ is a reduced form representation of informational frictions in settings such as Myers and Majluf (1984) or Dittmar and Thakor (2007). The illiquidity of bank equity may also result from banks' intentional choice of balance-sheet opaqueness that protects the information insensitivity, and thus, the moneyness of deposits (Dang et al. (2014)).

2.2 Equilibrium

Lemma 1, 2, and 3 below summarize the optimal choices for firms and banks at $t = 0$. We will focus on an equilibrium where firms' liquidity constraint binds (i.e., $i_1 = m_0$). One more unit of deposits can be used to purchase one more unit of goods as inputs to create $F'(m_0)$ more units of capital (with productivity α) when λ shock hits. This has an expected net value of $\lambda [\alpha F'(m_0) - 1]$, making firms willing to accept a return lower than ρ , the discount rate.²⁵ The spread, $\rho - r_0$, is *money premium*, a carrying cost. Note that when $r_0 < \rho$, households do not hold deposits.

Lemma 1 (Money Demand) *Firms' equilibrium deposits, m_0 , satisfy the condition*

$$\lambda [\alpha F'(m_0) - 1] = \rho - r_0.$$

Firms also choose the amount they borrow from banks, which is subject to the collateral constraint that the expected repayment cannot exceed the total pledgeable value ($\alpha(1 - \delta)k_0$). Given the expected default probability $\mathbb{E}[\pi(Z_1)] = \delta$, the expected loan repayment is $(1 - \delta)(1 + R_0)$ per dollar borrowed, approximated by $1 + R_0 - \delta$ (the product of two percentages ignored). When $R_0 - \delta = \rho$, firms are indifferent; when $R_0 - \delta < \rho$, firms borrow to the maximum. The spread, $\rho - (R_0 - \delta)$, is collateral shadow value κ_0 (the Lagrange multiplier of collateral constraint).

Lemma 2 (Credit Demand) *The equilibrium loan rate is given by: $R_0 = \delta + \rho - \kappa_0$.*

Competitive bankers take as given the market loan rate R_0 and deposit rate r_0 . At $t = 0$, a representative banker chooses consumption-to-equity ratio y_0 (and retained equity $e_0 - y_0 e_0$), and the asset-to-equity ratio x_0 (leverage). Each dollar of retained equity is worth $x_0 [1 + R_0 - \pi(Z_1)] - (x_0 - 1)(1 + r_0)$ at $t = 1$, which is the difference between asset and liability value. Because $\mathbb{E}[\pi_D(Z_1)] = \delta$, the expected return on retained equity is $1 + r_0 + x_0(R_0 - \delta - r_0)$. The return in

²⁵Because bankers' only endowments are goods that cannot be stored, to carry net worth to date 1, bankers must lend some goods to firms in exchange for loans, i.e., the instruments that bankers use to transfer wealth intertemporarily. Since goods cannot be stored, entrepreneurs must consume at $t = 0$ in equilibrium. To make risk-neutral entrepreneurs indifferent between consumption and savings, the price of capital q_0^K adjusts so that acquiring capital delivers an expected return equal to ρ , which is precisely the opportunity cost of holding deposits instead of capital.

a bad state is $1 + r_0 + x_0 (R_0 - \delta - \sigma - r_0)$. Let ξ_0 denote the Lagrange multiplier of the solvency constraint, i.e., the shadow value of bank equity.²⁶ The value function is

$$v(e_0; R_0, r_0) = \max_{y_0 \geq 0, x_0 \geq 0} y_0 e_0 + \frac{(e_0 - y_0 e_0)}{(1 + \rho)} \{1 + r_0 + x_0 (R_0 - \delta - r_0) + \xi_0 [1 + r_0 + x_0 (R_0 - \delta - \sigma - r_0)]\}.$$

Lemma 3 (Bank Optimization) *The first-order condition (F.O.C.) for bank leverage x_0 is*

$$R_0 - r_0 = \delta + \gamma_0^B \sigma, \quad (6)$$

where $\gamma_0^B = \left(\frac{\xi_0}{1 + \xi_0} \right) = \frac{R_0 - \delta - r_0}{\sigma} \in [0, 1)$ is the banker's effective risk aversion or price of risk. Substituting the F.O.C. into the value function, we have

$$v(e_0; q_0^B) = y_0 e_0 + q_0^B (e_0 - y_0 e_0), \text{ where, } q_0^B = \frac{(1 + r_0)(1 + \xi_0)}{(1 + \rho)}. \quad (7)$$

The banker consumes if $q_0^B \leq 1$; if $q_0^B > 1$, $y_0 = 0$ so that the entire endowments are lent out.

The equilibrium credit spread, $R_0 - r_0$, has two components: the expected default probability δ and the risk premium $\gamma_0^B \sigma$. Each dollar lent adds σ units of downside risk at date 1, which tightens the capital adequacy constraint. γ_0^B is the price of risk charged by bankers, the Sharpe ratio of risky lending financed by risk-free deposits. q_0^B is the marginal value of bank equity (Tobin's Q). Retained equity has a compounded payoff of $(1 + r_0)(1 + \xi_0)$ from reducing the external financing (debt) cost and relaxing the solvency constraint, so its present value is $\frac{(1 + r_0)(1 + \xi_0)}{(1 + \rho)}$. When $q_0^B > 1$, bankers lend out all endowments in order to carry their wealth to $t = 1$ in the form of loans.

Substituting the equilibrium loan rate into Equation (6), we can solve for the money premium $\rho - r_0$, as the sum of $\gamma_0^B \sigma$, bankers' risk compensation, and κ_0 , the collateral shadow value.

Proposition 1 (Money Premium Decomposition) *The equilibrium money premium is given by*

$$\rho - r_0 = \gamma_0^B \sigma + \kappa_0. \quad (8)$$

Equation (8) decomposes the money premium into an *intermediary wedge*, $\gamma_0^B \sigma$, that measures the scarcity of bank equity, and a collateral wedge κ_0 . Since the money premium equals the

²⁶Note that ξ_0 is known at $t = 0$, so its subscript is 0 instead of 1.

expected value of foregone marginal investment (Lemma 1), Equation (8) offers an anatomy of investment inefficiency. To support the first-best investment, i_{FB} , each firm must carry at least $i_{FB}K_0$ deposits in aggregate, which requires a minimum level of bank equity:

Condition 1 $E_0 \geq E_{FB}$, where $E_{FB} := \frac{i_{FB}K_0}{\bar{x}_{FB}-1} = \frac{i_{FB}}{\frac{1+\rho}{\sigma}-1}K_0$, and i_{FB} is defined in Equation (1).

\bar{x}_{FB} is solved as follows: under the first-best investment, the money premium is zero, so $\kappa_0 = 0$. Substituting $r_0 = \rho$ and $R_0 = \delta + \rho$ into the solvency constraint yields $\bar{x}_{FB} = \frac{1+\rho}{\sigma}$. When the size of aggregate shock is larger (higher σ), the required bank equity as risk buffer (E_{FB}) is larger.

First-best deposit creation also requires a minimum stock of collateral to back bank loans. The minimum bank lending that supports the first-best investment is $\bar{x}_{FB}E_{FB}$, so that collateral must be sufficient to cover firms' expected debt repayment: $\alpha(1-\delta)K_0 \geq \bar{x}_{FB}E_{FB}(1+\rho)$, or,

$$K_0 \geq K_{FB} := \frac{\bar{x}_{FB}E_{FB}(1+\rho)}{\alpha(1-\delta)} = \left(\frac{\frac{1+\rho}{\sigma}}{\frac{1+\rho}{\sigma}-1} \right) \left(\frac{i_{FB}(1+\rho)}{\alpha(1-\delta)} \right) K_0,$$

This condition can be simplified into the following parameter restrictions:

Condition 2 $\frac{\alpha(1-\delta)}{1+\rho} \geq \left(\frac{1}{1-\frac{\sigma}{1+\rho}} \right) i_{FB}$, where i_{FB} is defined in Equation (1).

Condition 2 is more likely to be violated when the expected collateral destruction rate δ is higher. Thus, δ measures the severity of collateral shortage that is studied by Holmström and Tirole (1998).²⁷ This paper focuses on the scarcity of intermediation capacity. As shown in Condition 1, such scarcity is more severe if σ is larger. Therefore, two parameters, δ and σ , correspond to the strengths of two limits on inside money creation. Corollary 1 summarizes the analysis.

Corollary 1 (Sufficient Conditions for a Money Shortage) *The equilibrium money premium is positive, and investment is below the first-best level, if either Condition 1 or 2 is violated.*

²⁷See also the literature of asset shortage, such as Woodford (1990b), Kocherlakota (2009), Kiyotaki and Moore (2005), Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Giglio and Severo (2012) among others.

3 Dynamic Model: Procyclical Money Creation

To study the cyclicity of bank leverage and the frequency and duration of crisis, I recast the model in continuous time. New mechanisms arise from agents' intertemporal decision making. The analysis focuses on the intermediary wedge, assuming a corresponding version of Condition 2 holds, so the economy has enough capital as collateral, but not enough bank equity as risk buffer.

3.1 Setup

Continuous-time setup. All agents maximize risk-neutral life-time utility with discount rate ρ . Households consumes the generic goods and can invest in securities issued by firms and banks.²⁸ Firms trade capital at price q_t^K . One unit of capital produces α units of goods per unit of time. They can issue equity to households, promising an expected return of ρ per unit of time (i.e., their cost of equity). Given the deposit rate r_t , the deposit carry cost or the money premium, is defined by the spread between ρ and deposit rate r_t as in the static model.²⁹

At idiosyncratic Poisson times (intensity λ), firms are hit by a liquidity shock and cut off from external financing. These firms either quit or invest. Let k_t denote a firm's capital holdings. Investing $i_t k_t$ units of goods can preserve the existing capital and create $F(i_t)k_t$ units of new capital. Investment is constrained by the firm's deposit holdings, $i_t \leq m_t$, where m_t is the deposits per unit of capital on its balance sheet.³⁰ Holding deposits allows firms' wealth to jump up at these Poisson times through the creation of new capital. I assume the technology $F(\cdot)$ is sufficiently productive, so we focus on an equilibrium where the liquidity constraint always binds.

The aggregate shock Z_t is a standard Brownian motion. Every instant, $\delta dt - \sigma dZ_t$ fraction

²⁸Risk-neutral households' required return is fixed at ρ because negative consumption is allowed, which is interpreted as dis-utility from additional labor to produce extra goods as in Brunnermeier and Sannikov (2014). Allowing negative consumption serves the same purpose as assuming large endowments of goods per period in the static model.

²⁹Nagel (2016) emphasizes the variation in illiquid return (i.e. ρ in the model) as a driver of the money premium dynamics in data. This paper provides an alternative model that focuses on the yield on money-like securities, r_t .

³⁰The idiosyncratic nature of liquidity shock and the assumption that firms can access external funds in normal times imply that firms' money demand does not contain hedging motive that complicates model mechanism. Bolton, Chen, and Wang (2013) model the market timing motive of corporate liquidity holdings in the presence of technological illiquidity and state-dependent external financing costs. He and Kondor (2016) examine how the hedging motive of liquidity holdings amplifies investment cycle through pecuniary externality in the market of productive capital.

of capital is destroyed. Firms default on loans backed by the destroyed capital. Let R_t denote the loan rate. For one dollar borrowed from banks at t , firms expect to pay back

$$\underbrace{(1 + R_t dt)}_{\text{principal + interest payments}} \underbrace{[1 - (\delta dt - \sigma dZ_t)]}_{\text{default probability}} = 1 + R_t dt - (\delta dt - \sigma dZ_t),$$

where high-order infinitesimal terms are ignored. The default probability is a random variable that loads on dZ_t .³¹ Both loans and deposits are short-term contracts, initiated at t and settled at $t + dt$.³²

Let r_t denote the deposit rate, and x_t banks' asset-to-equity ratio. Let c_t^B denote a bank's cumulative dividend. $dc_t^B > 0$ means consumption and paying dividends to outside shareholders (households); $dc_t^B < 0$ means raising equity. As in the static model, we can define $dy_t = \frac{dc_t^B}{e_t}$ as the payout or issuance ratio, which is an impulse variable, so bank equity e_t follows a regulated diffusion process, reflected at payout and issuance (i.e., when $dy_t \neq 0$):

$$de_t = \underbrace{e_t x_t}_{\text{Loan value}} \underbrace{[R_t dt - (\delta dt - \sigma dZ_t)]}_{\text{Loan return}} - \underbrace{e_t (x_t - 1)}_{\text{Debt value}} \underbrace{r_t dt}_{\text{Interest payment}} - \underbrace{e_t dy_t}_{\text{Payout or issue}} - \underbrace{e_t \iota dt}_{\text{Cost of operations}}. \quad (9)$$

Because in equilibrium, banks earn a positive expected return on equity, the operation cost ι is introduced to motivate payout so that the banking sector does not outgrow the economy.³³

Bankers maximize life-time utility, subject to a proportional equity issuance cost:

$$\mathbb{E} \left\{ \int_{t=0}^{\tau} e^{-\rho t} [\mathbb{I}_{\{dy_t \geq 0\}} - (1 + \chi) \mathbb{I}_{\{dy_t < 0\}}] e_t dy_t \right\}. \quad (10)$$

\mathbb{I}_A is the indicator function of event A .³⁴ The solvency constraint in the static setting boils down to the requirement of non-negative equity. Unlike the static setting, in equilibrium, bankers always preserve a slackness, so $\tau := \inf\{t : e_t \leq 0\} = \infty$. As will be shown later, even in the absence of a binding solvency constraint, bankers are still risk-averse due to the recapitalization friction χ .

³¹Probit transformation can guarantee $\pi(dZ_t) \in (0, 1)$ but complicates expressions. See also Klimenko et al. (2016).

³²I assume banks repay deposits after investment takes place, so that investing firms cannot be buy-and-hold investors, and thus, have to actually sell deposits in exchange for goods. Long-term deposits avoid this assumption, but would introduce other mechanisms, such as the Fisherian deflationary spiral in Brunnermeier and Sannikov (2016). Similarly, long-term loan contracts introduce the fire sale mechanism in Brunnermeier and Sannikov (2014).

³³The cost of operations is mathematically equivalent to a higher time-discount rate for bankers, common in the literature of heterogeneous-agent models (e.g., Kiyotaki and Moore (1997)). It can also be interpreted as agency cost.

³⁴In different settings, Van den Heuvel (2002), Phelan (2016), and Klimenko et al. (2016) also introduce issuance frictions in dynamic banking models. Dilution cost is just one form of frictions that lead to the endogenous variation of intermediaries' risk-taking capacity. He and Krishnamurthy (2012) use a minimum requirement of insiders' stake.

State variable. At time t , the economy has K_t units of capital and aggregate bank equity E_t . In principle, a time-homogeneous Markov equilibrium would have both as state variables. Because production has constant return-to-scale and the investment technology is homogeneous of degree one in capital, the Markov equilibrium has only one state variable:

$$\eta_t = \frac{E_t}{K_t}.$$

Since the model highlights the interaction between money supply and demand, intuitively, η_t measures the size of liquidity suppliers (banks) relative to that of liquidity demanders (firms).

Because there is a unit mass of homogeneous bankers, E_t follows the same dynamics as e_t , so the instantaneous expectation and standard deviation of $\frac{dE_t}{E_t}$ (μ_t^e and σ_t^e) are $r_t + x_t(R_t - \delta - r_t) - \iota$ and $x_t\sigma$ respectively (from Equation (9)). By Itô's lemma, η_t follows a regulated diffusion process

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ_t - dy_t, \quad (11)$$

where μ_t^η is $\mu_t^e - [\lambda F(m_t) - \delta] - \sigma_t^e \sigma + \sigma^2$, with bracket term being the expected growth rate of K_t , and the shock elasticity σ_t^η is $(x_t - 1)\sigma$, which is positive because banks lever up ($x_t > 1$). Positive shocks increase η_t , so banks become relatively richer; negative shocks make banks relatively undercapitalized. As η_t evolves over time, the economy repeats the timeline in Figure 1 with date 0 replaced by t and date 1 replaced by $t + dt$. Let intervals $\mathbb{B} = [0, 1]$ and $\mathbb{K} = [0, 1]$ denote the sets of banks and firms respectively. The Markov equilibrium is formally defined below.

Definition 1 (Markov Equilibrium) *For any initial endowments of firms' capital $\{k_0(s), s \in \mathbb{K}\}$ and banks' goods (i.e., initial bank equity) $\{e_0(s), s \in \mathbb{B}\}$ such that*

$$\int_{s \in \mathbb{K}} k_0(s) ds = K_0, \text{ and } \int_{s \in \mathbb{B}} e_0(s) ds = E_0,$$

a Markov equilibrium is described by the stochastic processes of agents' choices and price variables on the filtered probability space generated by the Brownian motion $\{Z_t, t \geq 0\}$, such that:

- (i) Agents know and take as given the processes of price variables, such as the price of capital, the loan rate, and the deposit rate (i.e., agents are competitive with rational expectation);*
- (ii) Households optimally choose consumption and savings that are invested in securities iss-*

ued by firms and banks;

(iii) Firms optimally choose capital holdings, deposit holdings, investment, and loans;

(iv) Bankers optimally choose leverage, and consumption/payout and issuance policies;

(v) Price variables adjust to clear all markets with goods being the numeraire;

(vi) All the choice variables and price variables are functions of η_t , so Equation (11) is an autonomous law of motion that maps any path of shocks $\{Z_s, s \leq t\}$ to the current state η_t .

3.2 Markov Equilibrium

The risk cost of money creation. In analogy to Proposition 1, I will show a risk cost of money creation that ties inside money supply to bank equity. I start with firms' demand for bank deposits.

Lemma 1' gives firms' optimal deposit demand in analogy to Lemma 1, with one modification that capital is valued at the market price q_t^K instead of the terminal value α in the static setting.³⁵ As will be shown, this difference is critical, as it leads to a unique intertemporal feedback mechanism that amplifies the procyclicality of money creation. As in the static model, households do not hold deposits when $r_t < \rho$, so firms' deposit demand is the aggregate demand for bank debt.

Lemma 1' (Money Demand) *Firms' equilibrium deposits, m_t , satisfy the condition*

$$\lambda [q_t^K F'(m_t) - 1] = \rho - r_t. \quad (12)$$

The static model highlights two limits on inside money creation: the scarcities of collateral and bank equity. Here I will focus on the latter, and later confirm that firms' collateral constraint never binds in the calibrated equilibrium. Thus, the expected loan repayment $R_t - \delta$, is equal to ρ .

Lemma 2' (Credit Demand) *The equilibrium loan rate is given by: $R_t = \delta + \rho$.*

Bankers solve a dynamic problem. Following Lemma 3, I conjecture bankers' value function is linear in equity, $v(e_t; q_t^B) = q_t^B e_t$, where q_t^B summarizes the investment opportunity set. Define

³⁵To be precise, the liquidity shock hits at $t + dt$, and by then the capital created will be worth $q_{t+dt}^K = q_t^K + dq_t^K$. In equilibrium, q_t^K is a diffusion process with continuous sample paths, so dq_t^K is infinitesimal, and thus, ignored.

ϵ_t^B as the elasticity of q_t^B : $\epsilon_t^B := \frac{dq_t^B/q_t^B}{d\eta_t/\eta_t}$. Intuitively, q_t^B signals the scarcity of bank equity, so I look for an equilibrium in which $\epsilon_t^B \leq 0$. Individual bankers take as given the equilibrium dynamics of q_t^B . Let μ_t^B and σ_t^B denote the instantaneous expectation and standard deviation of $\frac{dq_t^B}{q_t^B}$ respectively. The Hamilton-Jacobi-Bellman (HJB) equation can be written as

$$\rho = \max_{dy_t^B \in \mathbb{R}} \left\{ \frac{(1 - q_t^B)}{q_t^B} \mathbb{I}_{\{dy_t > 0\}} dy_t + \frac{(q_t^B - 1 - \chi)}{q_t^B} \mathbb{I}_{\{dy_t < 0\}} (-dy_t) \right\} + \mu_t^B + \max_{x_t \geq 0} \{ r_t + x_t (R_t - \delta - r_t) - x_t \gamma_t^B \sigma \} - \iota, \quad (13)$$

where the effective risk aversion is defined by $\gamma_t^B := -\sigma_t^B$. By Itô's lemma, $\gamma_t^B = -\epsilon_t^B \sigma_t^\eta \geq 0$.

Lemma 3' (Bank Optimization) *The first-order condition for bank leverage x_t is*

$$R_t - \delta - r_t = \gamma_t^B \sigma, \quad (14)$$

The banker pays dividends ($dy_t > 0$) if $q_t^B \leq 1$, and raises equity ($dy_t < 0$) if $q_t^B \geq 1 + \chi$.

Bankers' issuance and payout policies imply that η_t is bounded by two reflecting boundaries: the issuance boundary $\underline{\eta}$, given by $q^B(\underline{\eta}) = 1 + \chi$, and the payout boundary $\bar{\eta}$ given by $q^B(\bar{\eta}) = 1$. When η_t falls to $\underline{\eta}$, banks raise equity and η_t never decreases further; When η_t rises to $\bar{\eta}$, banks pay out dividends and η_t never increases further. When $\eta_t \in (\underline{\eta}, \bar{\eta})$, bankers neither issue equity nor pay out dividends, because $q_t^B \in (1, 1 + \chi)$ by the monotonicity of $q^B(\eta_t)$. As η_t rises following good shocks and falls following bad shocks, banks follow a countercyclical equity management strategy, paying out dividends in good times and issuing shares in bad times, which is consistent with the evidence (Baron (2014); Adrian, Boyarchenko, and Shin (2015)).

Lemma 4 (Reflecting Boundaries) *The economy moves within bank issuance boundary $\underline{\eta}$ and payout boundary $\bar{\eta}$. In $[\underline{\eta}, \bar{\eta}]$, the law of motion of the state variable η_t is given by Equation (11).*

Bankers are risk-averse because of the recapitalization friction. From an individual banker's perspective, the issuance cost causes her marginal value of equity q_t^B to be negatively correlated with shocks. Following a negative shock, bankers will not raise equity unless q_t^B reaches $1 + \chi$, so the whole industry shrinks (i.e. the aggregate bank equity decreases), and intuitively, Tobin's

Q , q_t^B , increases. Following a positive shock, bankers will not immediately pay out dividends unless q_t^B drops to 1, so the whole industry expands and q_t^B decreases. Thus, bankers require a risk premium for holding any asset whose return is positively correlated with dZ_t (i.e., negatively correlated with q_t^B). In particular, bankers require a risk compensation for extending loans.

On the left-hand side of Equation (14) is the net interest margin, $R_t - \delta - r_t$, the marginal benefit of issuing deposits backed by risky loans. The right-hand side is the marginal cost, that is the σ units of risk exposure, priced at γ_t^B per unit.³⁶ We can interpret the equilibrium γ_t^B as the expected profit per unit of risk (i.e. the Sharpe ratio), from creating deposits backed by risky loans:

$$\gamma_t^B = \frac{R_t - \delta - r_t}{\sigma}.$$

Banks face two markets, the loan market and the money market. With the loan rate R_t equal to $\rho + \delta$ (Lemma 2'), there is a one-to-one mapping between the deposit rate r_t and γ_t^B .

Interpreting γ_t^B as the Sharpe ratio or profitability of money creation helps us build an intuitive connection between q_t^B and γ_t^B . As a summary statistic for banks' investment opportunity set, q_t^B reflects the expectation of future profits from money creation (i.e. the future paths of γ_t^B). Intuitively, when the banking sector is relatively large, i.e., η_t is high, banks' profit per unit of risk, γ_t^B , declines. This is a key equilibrium property, later confirmed by the full solution. Substituting the equilibrium loan rate into Equation (14), we have the dynamic counterpart of Proposition 1.³⁷

Proposition 1' (Money Premium) *The equilibrium money premium is given by*

$$\rho - r_t = \gamma_t^B \sigma. \quad (15)$$

Figure 2 takes a snapshot of the deposit market, given capital value q_t^K and γ_t^B . In the Markov equilibrium, these variables vary continuously with η_t . The horizontal axis is m_t , the representative firm's deposits per unit of capital. The vertical axis is the money premium. The investment technology $F(\cdot)$ is concave, so firms' indifference curve from Lemma 1' gives a downward-sloping

³⁶ $\gamma_t^B \sigma$ opens up a wedge between the credit spread, $R_t - r_t$, and δ the expected default rate. This intermediary premium shares the insight of He and Krishnamurthy (2013), but here, the purpose of intermediation is to create inside money. Bankers need loans to back deposits, and all that households need is to break even as shown in Lemma 2'.

³⁷Recall that for the transparency of the dynamic mechanisms, I assume firms' collateral constraint never binds, so the collateral shadow value disappears. This assumption is confirmed later by the solution of calibrated model.

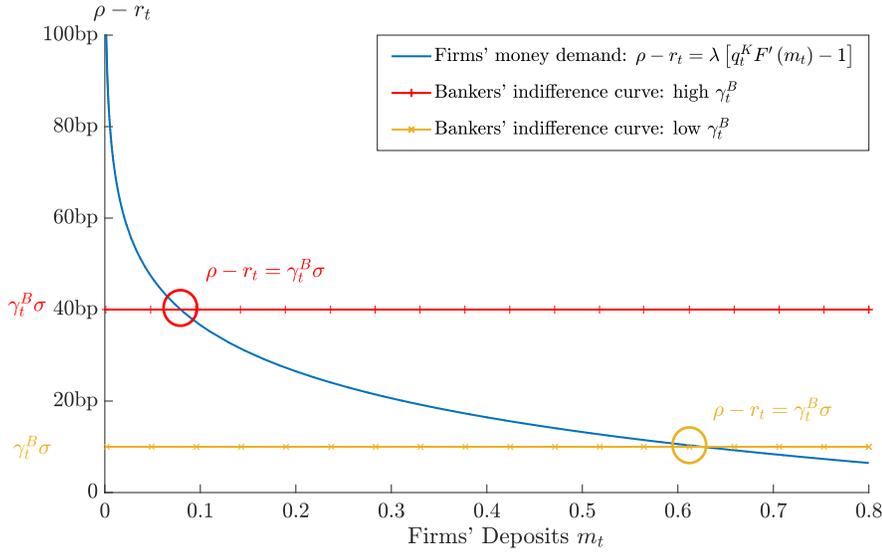


Figure 2: Money Market. This figure plots the money demand curve of firms and two indifference curve of banks corresponding to high and low γ_t^B respectively. The intersection points are the money market equilibrium.

demand curve. The supply curve is represented by bankers' indifference curve $\rho - r_t = \gamma_t^B \sigma$. Two equilibrium points are circled. When banks undercapitalized (low η_t), γ_t^B is high. The equilibrium money premium must compensate bankers' risk exposure from issuing money backed by risky loans. When banks are well capitalized (high η_t), γ_t^B is low, and the money premium is low.³⁸

With money being risk-free and loan being risky, money creation induces risk mismatch on bank balance sheets, with precisely σ units of risk per dollar of money created. As γ_t^B varies with η_t , this *risk cost* of money production links banks' balance-sheet capacity to the real economy through the liquidity constraint on firms' investments. This risk cost of money creation adds to the literature of balance-sheet channel (Bernanke and Gertler (1989); Kiyotaki and Moore (1997)). By modeling banks as money creators, this paper offers a bank balance-sheet perspective on liquidity shortage that was previously studied by Woodford (1990b) and Holmström and Tirole (1998).

Corollary 1' (Investment Inefficiency) *From Lemma 1' and Proposition 1', we have*

$$\lambda [q_t^K F'(m_t) - 1] = \gamma_t^B \sigma. \quad (16)$$

³⁸Drechsler, Savov, and Schnabl (2016) provide evidence on banks' unique position in creating deposits. In contrast with this paper, they emphasize banks' market power as the driver of deposit rate instead of balance-sheet capacity.

The risk compensation charged by bankers is exactly the net present value of foregone marginal investment. When banks are undercapitalized and γ_t^B is high, firms hold less liquidity and invest less. This result echoes Corollary 1 of the static model, except that now bank equity evolves endogenously. Bad shocks destroy bank equity, so inside money supply contracts, slowing down resources reallocation towards investing firms. Following good shocks, more inside money is created, facilitating reallocation. Eisfeldt and Rampini (2006) find procyclical reallocation among firms. Bachmann and Bayer (2014) find procyclical dispersion of firms' investment rates. Here, procyclical reallocation is driven by procyclical money creation and transaction volumes.

So far, we have revisited the main results of the static model in a dynamic setting. Next, I will discuss intertemporal feedback mechanisms that amplify the procyclicality of money creation. **Intertemporal feedback.** The endogenous capital price plays a critical role in generating a feedback mechanism. Proposition 2 shows firms' indifference condition as a capital pricing formula.

Proposition 2 (Capital Valuation) *The equilibrium price of capital satisfies*

$$q_t^K = \frac{\underbrace{\alpha}_{\text{Production}} + \lambda \underbrace{[q_t^K F(m_t) - m_t]}_{\text{Expected net investment gain}} - \underbrace{(\rho - r_t) m_t}_{\text{Deposit carry cost}}}{\underbrace{\rho}_{\text{Discount rate}} - \left(\underbrace{\mu_t^K}_{\text{Expected price appreciation}} - \underbrace{\delta}_{\text{Expected capital destruction}} + \underbrace{\sigma_t^K \sigma}_{\text{Quadratic covariation}} \right)}, \quad (17)$$

where μ_t^K and σ_t^K are defined in the equilibrium price dynamics: $dq_t^K = \mu_t^K q_t^K dt + \sigma_t^K q_t^K dZ_t$.

Capital value q_t^K is procyclical. Consider a positive shock, $dZ_t > 0$, at an interior state, $\eta_t \in (\underline{\eta}, \bar{\eta})$. Since fewer loans default than expected, banks receive a windfall. Given the wedge between q_t^B and 1 that is created by the issuance cost, q_t^B does not immediately jump down to one and trigger payout. So, banks' equity increases, and *in expectation*, the shock's impact on the bank equity will only dissipate gradually into the future. Thus, a positive shock increases current bank equity, and due to the persistence of its impact, it lifts up the expectation of future bank equity.

Thus, a positive shock increases capital value through two channels. As banks' equity increases, they charge a lower price of risk for deposit creation, so firms pay a lower deposit carry cost, $\rho - r_t$, and hold more deposits from t to $t + dt$. Capital is expected to grow faster in dt thanks

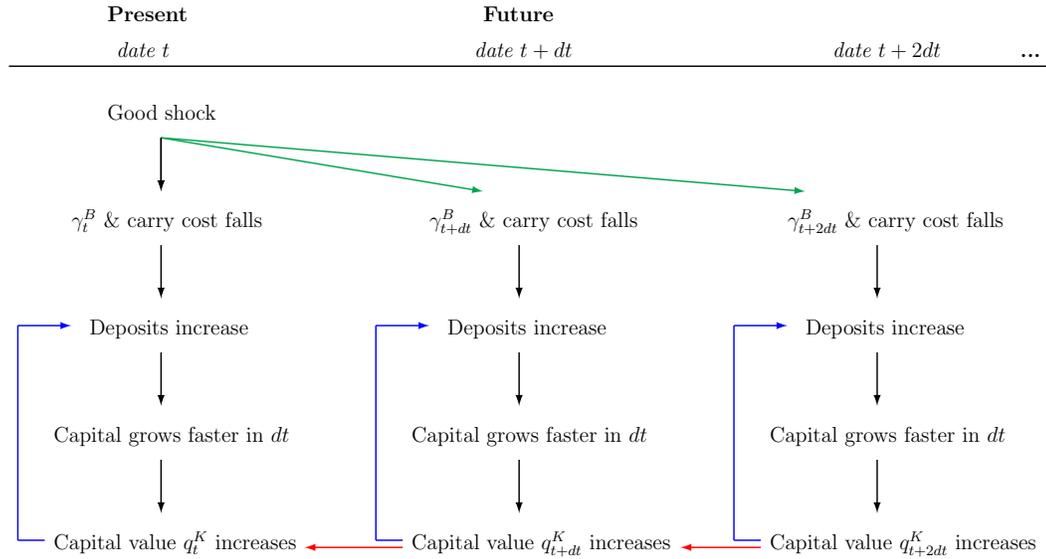


Figure 3: Intertemporal Feedback and Procyclicity. This figure illustrates the mechanism behind the procyclicity of capital value q_t^K . Following good shocks, bank risk price γ_t^B declines, and due to the persistence of shock impact, the path of expected γ_t^B shifts down. Firms face a lower money premium now and hold more money, and they expect so in the future, which translates into a higher growth path of capital in expectation and higher value of capital.

to more investments financed by these deposits, which directly leads to a higher market price of capital. This is the *contemporaneous* channel of procyclicity of q_t^K .

An *intertemporal* channel further increases capital value. Due to the persistent impact of the shock, firms expect the banking sector to be better capitalized for an extended period of time, and thereby, they expect to hold more deposits and capital to grow faster going forward. This lifts up the expectation of future capital prices, which feeds back into an even higher current price through the expected price appreciation μ_t^K . Figure 3 illustrates the two channels of procyclical q_t^K .³⁹

Procyclical bank leverage. Following good shocks, banks' equity increases and they charge a lower price of risk γ_t^B . As illustrated by Figure 4, the money market moves from "1" to "2". The equilibrium quantity of deposits increases. Whether it increases faster or slower than banks' equity determines whether bank leverage is procyclical or countercyclical.

Because capital value q_t^K is procyclical, firms' money demand will also shift outward, so

³⁹Note that an additional channel of procyclical q_t^K has been shut down by the assumption that the economy has enough collateral to back loans. Following good shocks, banks expand and become willing to lend more. When firms' borrowing constraint binds, a collateral shadow value (κ in the static model) arises, which increases capital price.

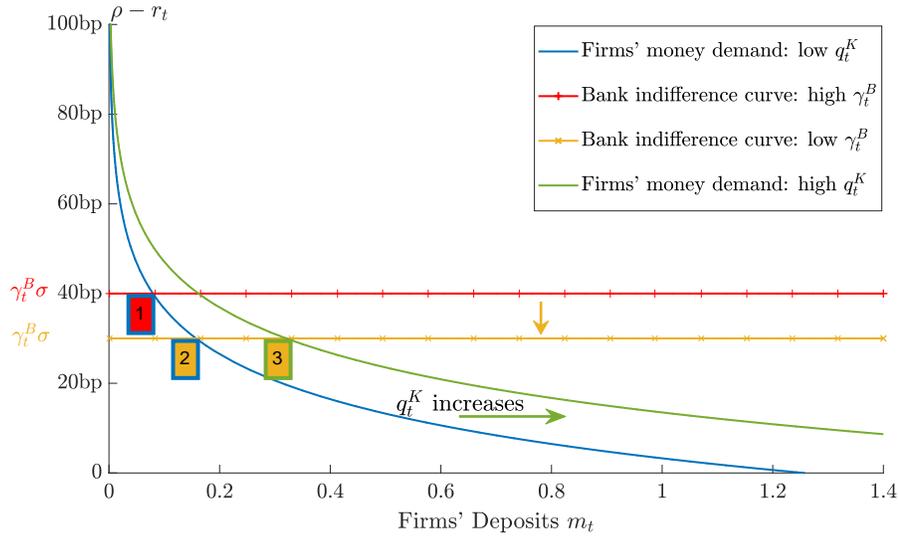


Figure 4: **Money Market Response to A Positive Shock.** First, the bank indifference curve shifts downward because bank risk price γ_t^B declines (i.e., from (1) to (2)). Then firms’ money demand curve shifts outward because capital price q_t^K rises (i.e., from (2) to (3)), which is in turn due to a higher growth path in expectation as in Figure 3.

the equilibrium point moves further from “2” to “3,” which increases the equilibrium quantity of deposits even further. This endogenous expansion of firms’ money demand allows banks’ debt to grow faster than their equity, contributing to the procyclicality of bank leverage.

One aspect of the model that distinguishes itself from other macro-finance models is this endogenous expansion of the demand for intermediaries’ debt. A static demand may lead to countercyclical leverage (e.g. He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)).⁴⁰

This paper shares with Kiyotaki and Moore (1997) the idea that intertemporal complementarity amplifies fluctuations. Capital becomes more valuable (higher q_t^K) because it grows faster, which is in turn due to more money held by firms in the *future*. Through q_t^K , firms’ *current* money demand rises in the expectation of future market conditions. The procyclicality of money demand contributes to the procyclicality of bank leverage and the resulting risk accumulation. As leverage rises, bank equity becomes more sensitive to shocks, so does the whole economy through η_t . Asset price here plays a key role in intertemporal feedback, but it differs from a typical balance-

⁴⁰In this type of models, intermediaries bet on asset prices. Asset price volatility is countercyclical, so a value-at-risk constraint can make leverage procyclical (Adrian and Boyarchenko (2012); Danielsson, Shin, and Zigrand (2012)).

sheet effect in Kiyotaki and Moore (1997), and more recently, Brunnermeier and Sannikov (2014).

Even though the model features a specific type of procyclical money demand motivated by corporate cash holdings, the insight that leverage procyclicality results from money demand procyclicality is general. We would expect a booming economy to have strong transaction demand for money-like assets issued by intermediaries. The model assumes that when firms are not experiencing the Poisson liquidity shock, they can immediately respond to changes in capital value by raising funds from banks or households to build up savings. In reality, firms may not act so swiftly. To what extent it affects the procyclicality of bank leverage is an interesting empirical question.⁴¹

Dynamic investment inefficiency. The endogenous variation in capital price leads to a new form of investment inefficiency. Given q_t^K , Corollary 1' reveals a form of *static inefficiency*, measured by the wedge between firms' deposits m_t and the contemporaneous investment target i_t^* , defined by $q_t^K F'(i_t^*) = 1$. Static inefficiency is due to bankers' *current* capacity to create money is limited. This echoes Corollary 1 of the static model. In a dynamic setting, the target i_t^* also varies with capital value. Due to the necessity and cost of carrying deposits, q_t^K is smaller than q_{FB}^K , the capital value in an unconstrained economy in which firms finance investment freely (defined below).

$$q_{FB}^K = \frac{\alpha + \lambda [q_{FB}^K F(i_{FB}) - i_{FB}]}{\rho + \delta}, \quad (18)$$

where the first-best investment is given by $q_{FB}^K F'(i_{FB}) = 1$. Because $q_t^K < q_{FB}^K$, the target i_t^* is below the first-best investment rate i_{FB} . Since q_t^K incorporates the expectation of future paths of deposit carry cost (the money premium), $i_{FB} - i_t^*$, measures a form of *dynamic inefficiency*.⁴²

Stagnation and instability. Recession states are states with negative expected growth rate of K_t . Growth is driven by investment, which is tied to firms' money holdings. Negative shocks deplete

⁴¹Related, since deposits can be regarded as insurance against the Poisson shock and the demand for such insurance is procyclical, this paper shares the insight of Rampini and Viswanathan (2010) that when firms are richer, they hedge more. In Rampini and Viswanathan (2010), hedging competes with investing for limited resources, and thus, richer firms, with more resources at hand, hedge more. In contrast, firms in this paper are always unconstrained outside of the Poisson times, so the procyclical demand for insurance is not driven by more resources available, but rather, by the procyclical benefit of hedging (i.e., more valuable investment), which is in turn due to the procyclicality of q_t^K .

⁴²Note that this dynamic inefficiency is about the lack of investment (productive reallocation across firms), *not* the dynamic inefficiency in overlapping-generation models that is due to the lack of intergenerational trade.

banks' equity and elevate the required risk compensation $\gamma_t^B \sigma$. At the same time, capital value decreases. As illustrated by Figure 4, bankers' indifference curve shifts upward and firms' money demand curve shifts inward, so firms hold less deposits and invest less, and the economy grows slower. Banking crises affect the real economy through the contraction of inside money supply, which echoes the classic account of the Great Depression by Friedman and Schwartz (1963).

Procyclical bank leverage implies stagnant recessions. Recessions happen near the issuance boundary $\underline{\eta}$ where banks are undercapitalized (η_t is low) and leverage is low. Low leverage limits the impact of good shocks on bank equity, so banks have to rebuild equity slowly. Low leverage also limits the impact of bad shocks, but this benefit is small. Near $\underline{\eta}$, the impact of bad shocks is already bounded: bank equity never decreases below $\underline{\eta}$. Low leverage and such *asymmetric impact* of shocks near boundary implies the economy is stuck with undercapitalized banks for a long time.

Procyclical leverage leads to downside risk accumulation in booms. Following good shocks, banks build up equity and leverage. As the economy approaches the payout boundary $\bar{\eta}$, shock impact becomes increasingly asymmetric. Since bank equity never rises above $\bar{\eta}$, the impact of good shocks is bounded, so high leverage only serves to amplify the impact of bad shocks on bank equity. Therefore, as a boom prolongs, leverage builds up, and the economy becomes increasingly fragile. Even small bad shocks can significantly deplete bank equity and trigger a recession.

Proposition 3 solves the stationary probability density of η_t (i.e., the likelihood of different states in the long run) and the expected time to reach $\eta \in [\underline{\eta}, \bar{\eta}]$ from $\underline{\eta}$ ("recovery time").

Proposition 3 *The stationary probability density of state variable η_t , $p(\eta)$ can be solved by:*

$$\mu^\eta(\eta) p(\eta) - \frac{1}{2} \frac{d}{d\eta} (\sigma^\eta(\eta)^2 p(\eta)) = 0,$$

where $\mu^\eta(\eta)$ and $\sigma^\eta(\eta)$ are defined in Equation (11). The expected time to reach η from $\underline{\eta}$, $g(\eta)$ can be solved by:

$$1 - g'(\eta) \mu^\eta(\eta) - \frac{\sigma^\eta(\eta)^2}{2} g''(\eta) = 0,$$

with the boundary conditions $g(\underline{\eta}) = 0$ and $g'(\underline{\eta}) = 0$.

Solving the equilibrium. The solution of the model is a set of functions defined on $[\underline{\eta}, \bar{\eta}]$. Each function maps the value of state variable η_t to the value of an endogenous variable. These functions

are separated into two sets. The first set includes the forward-looking variables $(q^B(\eta_t), q^K(\eta_t))$. The second includes variables, such as banks' leverage x_t , firms' deposits-to-capital ratio m_t , and deposit rate r_t that can be solved once we know the first set of functions. We solve the second set of variables as functions of $(q^B(\eta_t), q^K(\eta_t))$ and their derivatives to transform Equation (13) and (17) into a system differential equations of $(q^B(\eta_t), q^K(\eta_t))$ using Itô's lemma.

A key step is to solve bank leverage from the intersection of money demand and supply curves. We can use the deposit market clearing condition

$$m_t K_t = (x_t - 1) E_t, \text{ i.e., } m_t = (x_t - 1) \eta_t,$$

to substitute out m_t with $(x_t - 1) \eta_t$ on the left hand side of Equation (16). On the right hand side is the intermediary wedge, $\gamma_t^B \sigma$. By knowing the function $q^B(\eta_t)$, we know the elasticity ϵ_t^B , so banks' risk price, γ_t^B , is directly linked to the equilibrium leverage as follows.

$$\gamma_t^B = -\epsilon_t^B \sigma_t^\eta, \text{ where the } \eta_t \text{'s instantaneous shock elasticity is } \sigma_t^\eta = (x_t - 1) \sigma. \quad (19)$$

Equation (16) has a unique solution ($F''(\cdot) < 0$) of leverage x_t as a function of η_t , q_t^K , and ϵ_t^B . m_t is given by deposit market clearing condition, and r_t from Equation (12). Details are in Appendix I, which also shows the existence and uniqueness of Markov equilibrium in a constructive manner.

Proposition 4 (Markov Equilibrium) *There exists a unique Markov equilibrium with state variable η_t that follows an autonomous law of motion in $[\underline{\eta}, \bar{\eta}]$. Given functions $q^B(\eta_t)$ and $q^K(\eta_t)$, agents' optimality conditions and market clearing conditions solve bank leverage, firms' deposits, and deposit rate as functions of η_t . Substituting these variables into bankers' HJB equation and the capital pricing formula (Equations (13) and (17)), we have a system of two second-order ordinary differential equations that solves $q^B(\eta_t)$ and $q^K(\eta_t)$ under the following boundary conditions:*

$$\begin{aligned} \text{At } \underline{\eta}: \quad (1) \frac{dq^K(\eta_t)}{d\eta_t} = 0; \quad (2) q^B(\bar{\eta}) = 1 + \chi; \quad (3) \frac{d(q_t^B \eta_t)}{d\eta_t} = 0; \\ \text{At } \bar{\eta}: \quad (4) \frac{dq^K(\eta_t)}{d\eta_t} = 0; \quad (5) q^B(\underline{\eta}) = 1; \quad (6) \frac{d(q_t^B \eta_t)}{d\eta_t} = 1. \end{aligned}$$

We need exactly six boundary conditions for two second-order ordinary differential equations and two endogenous boundaries to pin down the solution. (1) and (4) prevent capital price from jumping upon reflection, ruling out arbitrage in the market of capital. (2) and (5) are the

value-matching conditions for banks' issuance and payout respectively. (3) and (6) are the smooth-pasting conditions that guarantee that the bank shareholders' value does not jump at the reflecting boundaries (similar to those in Brunnermeier and Sannikov (2014) and Phelan (2016)).⁴³

3.3 Solution

Calibration. To numerically solve the differential equations, I calibrate the model as follows. One unit of time is set to one year. δ and σ are the mean and standard deviation of loan delinquency rates (source: FRED). The other parameters are set to match model moments to data, such as interest rate, corporate cash holdings, and economic growth. All model moments are based on the stationary distribution. In particular, I use the mean and standard deviation of money premium to calibrate λ , the arrival rate of liquidity shocks, and χ , the issuance cost that governs the tail behavior of the model. Money premium data is from Nagel (2016) (GC repo/T-bill spread). Since in reality, money premium varies due to forces beyond the model mechanism, χ is set to a conservative value so that model-generated standard deviation is two thirds of data standard deviation. Bank leverage is intentionally left out of the calibration, so the leverage dynamics, and the associated boom-bust cycle, may serve for external validation. Appendix II summarizes the calibration.

Note that as in the static model, firms' external financing capacity cannot exceed their collateral value, i.e., $q_t^K K_t$ in aggregate. The analysis so far has focused on the case that this constraint never binds. This assumption is satisfied by the calibrated solution: the ratio of bank loans to collateral value varies from 4.6% to 83.3% depending on the state of the world (i.e., η_t).⁴⁴

Bank balance-sheet cycle. The state variable η_t measures the aggregate bank equity, i.e., the size of money suppliers, relative to the size of firms (money demanders). Bank equity affects the real economy through deposit creation. When η_t increases, banks expand balance sheets and issue more

⁴³The market value of bank equity is $q_t^B \eta_t K_t$. (3) guarantees the value of existing shares does not jump when banks issue new shares. (6) guarantees the value of bank equity declines exactly by the amount of dividends paid out. If (3) or (6) is violated, taking as given aggregate issuance and payout, individual banks have incentive to deviate.

⁴⁴The model will have two state variables, η_t and firm equity scaled by capital stock, if in some states of the world, firms' collateral constraint binds. This certainly enriches the model and improves its quantitative performance, but sacrifices the transparency of mechanisms. Rampini and Viswanathan (2017) study the joint dynamics of firm and intermediary equities (see also Elenev, Landvoigt, and Nieuwerburgh (2017)).

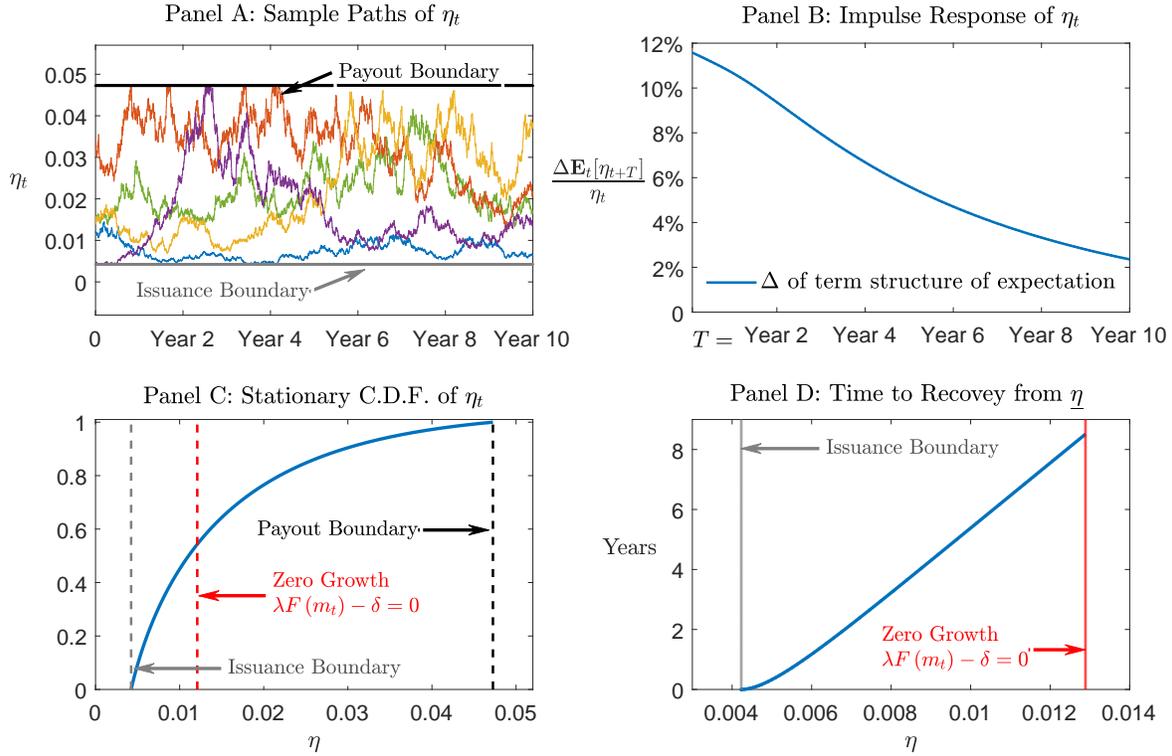


Figure 5: State Variable Dynamics. This figure shows the statistical properties of state variable η_t , the ratio of banking capital to illiquid, productive capital. Panel A plots the simulated paths of η_t . Panel B plots the percentage change of the expected value of η at different horizons in response to a positive shock. It shows the persistence of shock impact. Panel C plots the stationary cumulative distribution function (C.D.F.) of η_t that starts from the issuance boundary, passes the zero growth point, and ends at the payout boundary. Panel D plots expected years to reach a value of η (horizontal axis) when the current state is at the issuance boundary. It ends at the zero growth point.

deposits, so firms can hold more liquidity for investment, and the economy grows faster. Figure 5 shows the statistical properties of η_t . Panel A plots several sample paths of η_t by simulating the law of motion (Equation (11)).⁴⁵ The paths are bounded by the issuance and payout boundaries.

Panel B of Figure 5 plots the impulse response function of η_t , showing that shocks have persistent effects. Because the law of motion of η_t is non-linear and state-dependent, we cannot define impulse response functions as in linear time series analysis. Thus, to illustrate the persistent impact of shocks, I fix the value of η_t to the median value under the stationary distribution, and consider an increase. The figure plots the percentage change of the term structure of expectation,

⁴⁵One step is one day so the shock ΔZ_t is drawn from normal distribution $N(0, 1/365)$. Asmussen, Glynn, and Pitman (1995) show a weak order of convergence of 1/2 for discretized regulated diffusion.

i.e., the expected value of η_{t+T} with T ranging from one month to ten years.⁴⁶ Shock impact dissipates gradually. The initial increase of 11.6% raises the expected value in ten years by 2.4%.

The persistence is caused by banks' precautionary behavior, which is in turn due to the equity issuance cost. If bankers could issue equity freely (i.e. $\chi = 0$), they no longer need to retain equity. Whenever q_t^B is above one, signaling an improvement of the investment opportunity set, bankers raise equity from households; whenever q_t^B is below one, bankers distribute dividends. The dilution cost implies that equity is only raised infrequently when q_t^B reaches $1 + \chi$, signaling severe capital shortage. χ opens up a wedge between q_t^B , the value of one dollar as banks' retained equity, and 1, the value of one dollar paid out as dividends. As a result, banks preserve a financial slackness. Unless the economy hits the payout boundary, banks accumulate equity.

Panel C shows the stationary cumulative probability function (c.d.f.) from Proposition 3. The curve starts from zero at the issuance boundary, and ends at one at the payout boundary. Around 50% of the time, the economy is in a region with negative growth. I calibrate the mean growth rate to a relatively low number, 0.74% per year, which is the growth of output attributed to intangible investment from 1995 to 2007 (Corrado and Hulten (2010)). Intangible investment, such as R&D, relies heavily on internal liquidity, and thus, corresponds well to investment in the model.⁴⁷

Panel D of Figure 5 plots the expected time to reach different values of η_t from the issuance boundary $\underline{\eta}$ (Proposition 3). The right bound marks the lowest value of η_t that delivers a non-negative economic growth rate. In expectation, it takes more than eight years to recover from the bottom of a recession. As previously discussed, stagnation results from bank's deleveraging in the bad states and the asymmetric impact of shocks on bank equity near the reflecting boundaries.

Procyclicality. As illustrated by Figure 4, the money market moves with (γ_t^B, q_t^K) , which in turn varies with the state variable, η_t . γ_t^B , the risk price that bankers charge for issuing safe deposits

⁴⁶The expectation is calculated by the Kolmogorov backward equation (i.e. the Feynman–Kac formula), for reflected diffusion processes. The partial differential equations are solved by the Method of Lines (Schiesser and Griffiths (2009)). Borovička, Hansen, and Scheinkman (2014) provide an alternative, systematic framework to define and calculate impulse responses and the term structure of shock elasticity for non-linear diffusion processes.

⁴⁷Since Hall (1992) and Himmelberg and Petersen (1994), it has been well documented that R&D heavily relies on internal financing (see Hall and Lerner (2009) for a survey on innovation financing). A difficulty of external financing is that the knowledge asset created by R&D is intangible, partly embedded in human capital, and is often specialized to the particular firm in which it resides. It is difficult for investors to repossess such intangible assets in case of default.

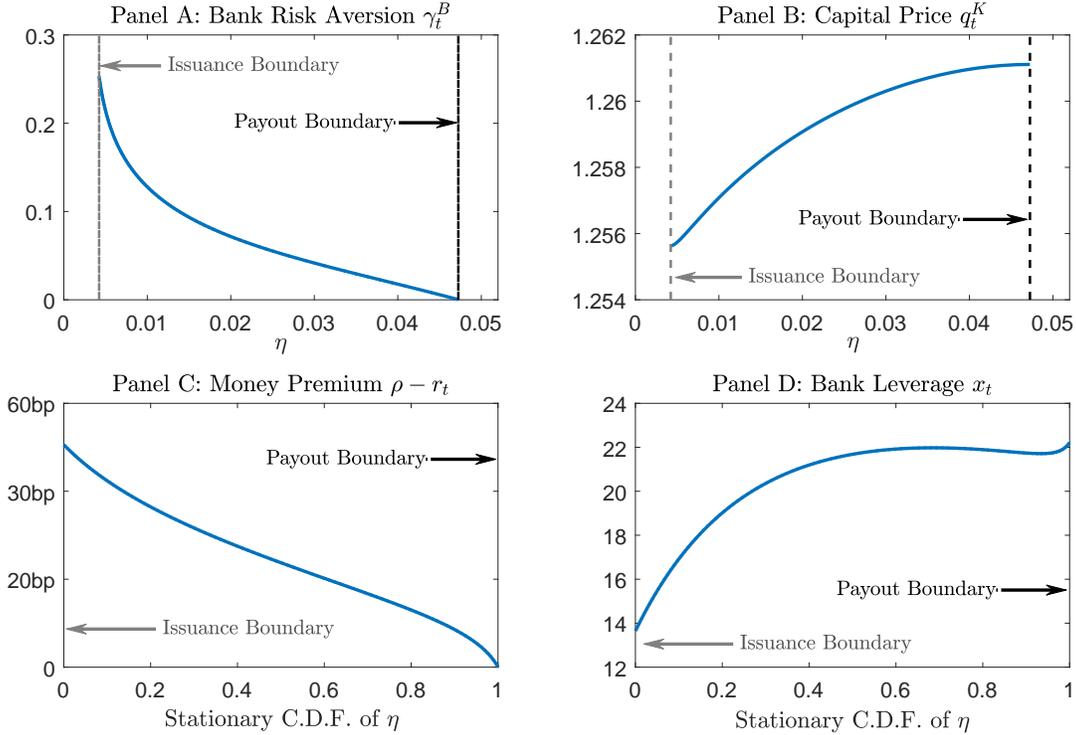


Figure 6: Proccyclical Leverage. This figure plots bank risk price γ_t^B (Panel A) and capital price q_t^K (Panel B) against the state variable η_t . These two variables determine the locale of money market equilibrium (Figure 2). Panel C and D plot money premium and bank book leverage respectively against the stationary cumulative distribution function (C.D.F.). They show how often (horizontal axis) the variable of interest stays in certain regions (vertical axis).

backed by risky loans, drives the money supply, while q_t^K , the capital value, shifts firms' money demand curve. Panel A of Figure 6 shows γ_t^B as a function of η_t . Because one unit of time is set to one year, γ_t^B is the annual Sharpe ratio of risky lending financed by risk-free deposits. γ_t^B decreases in η_t . When the economy is close to the bank recapitalization boundary $\underline{\eta}$, banks charge a price of risk close to 0.25; at the payout boundary, γ_t^B is zero. Good shocks increase η_t and decrease γ_t^B , shifting downward the bankers' indifference curve in Figure 4. Panel B shows that q_t^K increases in η_t . Even if the variation of q_t^K is not quantitatively large, it is sufficient to generate strong procyclicality of firms' money demand that leads to procyclical bank leverage.⁴⁸

⁴⁸As well documented in the empirical literature, asset price variation is dominated by the variation in discount rate (e.g. Cochrane (2011)). In the model, discount rate is fixed at ρ , so the variation in q_t^K is purely driven by firms' cost of liquidity management (i.e. the money premium), and their choice of liquidity holdings that determines the capital growth. Thus, quantitatively, we would not look for large variation in capital price over the cycle.

Panel C plots the equilibrium money premium, $\rho - r_t$, against the stationary cumulative probability of η_t . For instance, 0.2 on the horizontal axis is mapped to a money premium equal to 36bps, meaning that 20% of the time, money premium is larger than or equal to 36 basis points. The width of an interval on horizontal axis shows how much time the economy spends in the region. The interval $[0.2, 0.3]$ is mapped to $[32\text{bps}, 36\text{bps}]$, so $30\% - 20\% = 10\%$ of the time, the money premium is between 32bps and 36bps. Reading the graph from left to right, we follow a path of positive shocks, and see as the banking sector builds up equity, their risk-taking capacity expands, and thus, the equilibrium money premium, i.e., firms' cost of liquidity management, declines.

Panel D plots bank leverage, x_t , against the stationary cumulative probability of η_t . Leverage is mostly procyclical. Reading the graph from left to right, we see that when bank equity increases, banks issue even more deposits, so their leverage increases. The reason is that firms foresee a lower cost of liquidity management going forward, and thus, assign a higher valuation of capital, making the incentive stronger to hoard liquidity in case the investment opportunity arrives the very next instant (Figure 3 and 4). From right to left, we see how a crisis unfolds and banks deleverage.

There is a small region near the 80th percentile, where bank leverage rises following bad shocks (moving left). In reality, balance-sheet cyclicalities may differ by the types of financial intermediaries. Adrian and Shin (2010) find the book leverage of broker-dealers is procyclical. He, Khang, and Krishnamurthy (2010) show that commercial banks' leverage increased in the 2007-09 crisis. Commercial banks' ability to issue deposits depends not only on equity as risk buffer but also regulatory constraints and government guarantees. Banks in the model are actually closer to shadow banks. Many have argued that the demand for money-like securities is one of the major drivers behind shadow banking developments (e.g., Gorton (2010); Gennaioli, Shleifer, and Vishny (2013); Pozsar (2014)). The model does not have heterogeneous intermediaries, but the countercyclicality of leverage at high η_t reflects some complexity of the leverage cycle.

The procyclicality of bank leverage helps explain the statistical properties of the model in Figure 5. In Panel C, there is a relatively small probability of states with high banks' equity. In good times, banks' leverage is high, so the economy is sensitive to shocks. When the economy is close to the payout boundary, the impact of good shocks is limited, because large good shocks

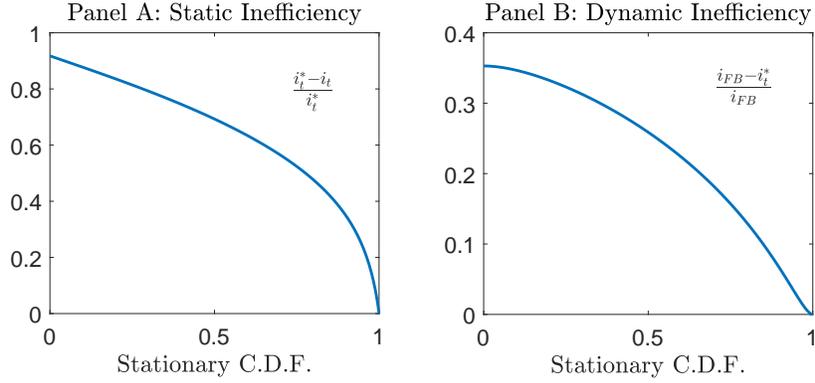


Figure 7: Static and Dynamic Investment Inefficiencies. This figure plots static investment inefficiency (Panel A), measured by the percentage deviation of equilibrium investment rate (i_t) from the target rate implied by the equilibrium capital price (i_t^*), and dynamic investment inefficiency (Panel B), measured by the percentage deviation of target rate (i_t^*) from the first-best investment rate (i_{FB}), against the stationary cumulative distribution function (C.D.F.). The plots show how often (horizontal axis) the variable of interest stays in certain regions (vertical axis).

trigger dividend distribution, meaning that the banking sector cannot grow beyond $\bar{\eta}$. High leverage only serves to amplify the impact of negative shocks. Therefore, the downside risk accumulates as leverage rises. Because of this fragility, the economy spends less time in boom. Panel D of Figure 5 also shows the slow recovery. When the economy is close to the issuance boundary, the impact of negative shocks is bounded. Low bank leverage only serves to reduce the impact of good shocks on bank equity, so banks accumulate equity slowly and the economy tends to get stuck in recessions. Countercyclical leverage would have led to the exactly opposite pattern.⁴⁹

Static and dynamic inefficiencies. In Corollary 1', the money premium is equal to bankers' required risk compensation $\gamma_t^B \sigma$. A higher γ_t^B , and thus, a higher money premium, directly translates into a larger gap between the cash-constrained investment rate and the current target rate i_t^* , defined by $q_t^K F'(i_t^*) = 1$. Panel A of Figure 7 shows the static inefficiency, measured by the percentage deviation of the actually investment rate from the target, i_t^* . As bankers' risk capacity increases, the money premium declines, and firms hold more deposits for investment. Moving from the left to the right, the investment wedge declines from 95% at the depth of recession to 0% at $\bar{\eta}$.

⁴⁹Models with countercyclical leverage generate instability through other mechanisms, such as fire sale in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014), which need intermediaries to hold long-term assets, and thereby, are exposed to endogenous volatility of asset prices. To highlight the novel link between leverage and money demand, I shut down this channel by restricting banks' investment to short-term loans.

Panel B of Figure 7 shows the dynamic inefficiency, measured by the percentage deviation of the current target from the first-best investment rate defined by Equation (18). The wedge varies with capital price, and declines as η_t increases, because q_t^K increases in η_t (as shown in Panel B of Figure 6). Around 50% of the time, the current target is 25% or more below the first-best level.

Investment inefficiencies decrease welfare. Given the constant productivity α , the aggregate consumption is determined by the capital stock K_t . Under risk-neutral preference, what matters, from a welfare perspective, is the expected growth rate of K_t , i.e., $\lambda F'(m_t) - \delta$, newly created capital net of destroyed existing capital. Therefore, through firms' liquidity constraint on investment, welfare is tied to money creation. Insufficient inside money suggests the government has a role in supplying *outside money*. Does outside money really improve welfare? How will banks respond? Will outside money stabilize the economy? The next section aims to answer these questions.

4 Government Debt: Instability, Stagnation, and Welfare

It has long been recognized that government debt offers monetary services (Patinkin (1965); Friedman (1969)). Repo market developments since 1980s enhanced the liquidity of Treasury securities (Fleming and Garbade (2003)). In this section, I introduce government debt as “outside money”, an alternative to bank debt. Whether outside money alleviates the money shortage faced by firms depends on how banks react. The competition between inside and outside money can *destabilize* the economy by amplifying bank leverage cycle, and thereby, exacerbate investment inefficiencies. These findings complement the literature on government debt as a means to financial *stability* (Greenwood et al. (2015); Krishnamurthy and Vissing-Jørgensen (2015); Woodford (2016)).

Setup. Firms can hold bank debt or government debt to relax the liquidity constraint on investments. Thus, issued at t , government debt pays the same risk-free rate as deposits, $r_t dt$, at $t + dt$. To focus on the liquidity provision role of government debt, I abstract away other fiscal distortions: issuance proceeds are distributed as lump-sum payments and debt is repaid with lump-sum tax on households. Moreover, I assume the government faces a debt limit that is proportional to the scale of the economy, i.e., $M^G K_t$, and consider a debt management strategy in line with Friedman's rule

– the government always issues the maximum amount when the money premium is positive.⁵⁰

Firms' money holdings per unit of capital are now $m_t + M^G$. Substituting it into the optimality condition of deposit holdings in Lemma 1', we have a new deposit demand curve:

$$\rho - r_t = \lambda [q_t^K F'(m_t + M^G) - 1].$$

Because $F(\cdot)$ is a concave function, the demand curve is shifted inward. Introducing government debt reduces the marginal benefit of deposit holdings (m_t), and the equilibrium money premium. By helping firms manage liquidity, government debt is likely to have a positive effect on investment and growth, which is similar to the investment crowding-in effect in Woodford (1990b) and Holmström and Tirole (1998).⁵¹ However, the actual effect depends on how banks react.

This setup does not distinguish Treasury securities from central bank liabilities that pay interests. It intends to capture outside money supply from the traditional expansion of monetary base and from government issuing liquid securities. Accordingly, the setup has an alternative interpretation. Government debt is held by a central bank, who issues an equal amount of reserves that pay the same interest rate. Reserves are held by banks who in turn issue an equal amount of deposits to firms. On this chain, both the central bank and banks are just pass-throughs. Intermediating between risk-free assets and liabilities does not require additional bank equity. In reality, the line between short-term Treasury securities and reserves is becoming increasingly blurry.⁵² Interest on reserves has been introduced in countries such as the U.K. and the U.S. Federal Reserve now allows the public to hold reserves through money market funds that are reverse repo counterparties.

Leverage cycle and instability. Figure 8 compares the model's performances when the govern-

⁵⁰Friedman's rule says individuals' opportunity cost to hold money should be equal to the social cost of creating money (Friedman (1969); Woodford (1990a)). Firms' opportunity cost to hold money is $\rho - r_t$, the money premium. The private sector's marginal cost of money creation is $\gamma_t^B \sigma$, the risk compensation charged by bankers. In contrast, the government's cost of money creation is zero as I already assume away any fiscal distortions. Therefore, Friedman's rule suggests that the government should maximize its debt issuance (i.e., outside money supply). However, as will be discussed later, because outside money crowds out inside money, Friedman's rule is not necessarily optimal.

⁵¹The traditional crowding-out effect describes how the government debt supply raises interest rate in general, and thereby crowds out private investment through a higher financing cost. My model does not entertain this effect, because by shutting down external financing completely, the model has cash being the only determinant of investment.

⁵²The liquidity coverage ratio (Basel Committee on Bank Supervision (2013)), which counts government securities as banks' liquidity holdings, is introduced as a modern version of reserves requirement.

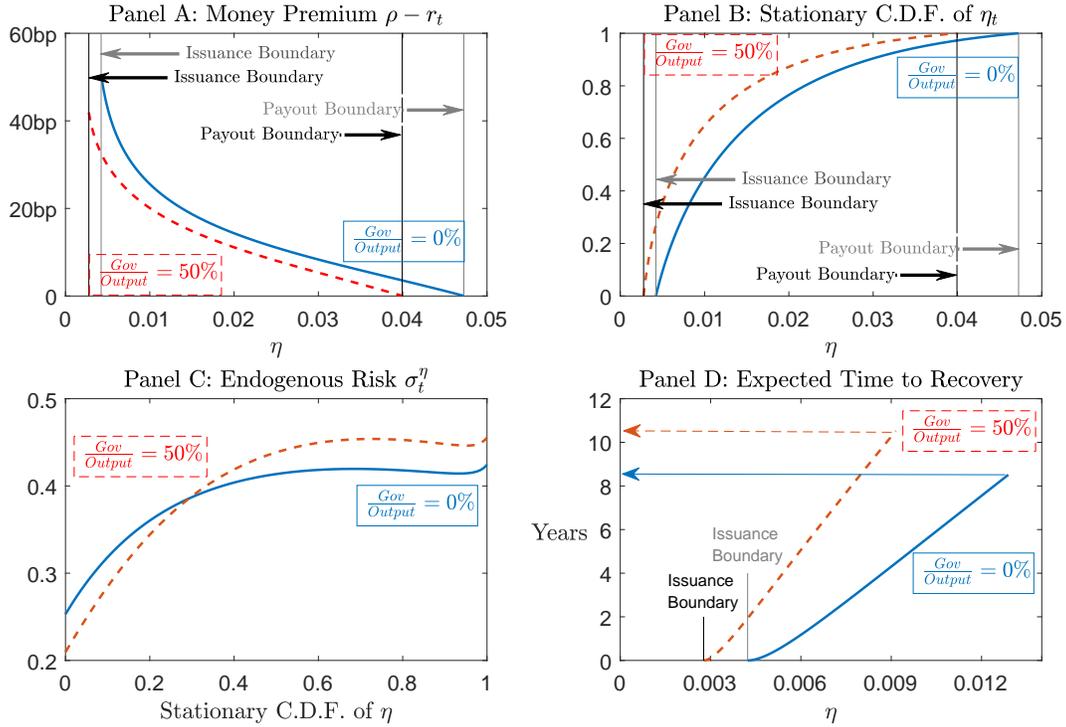


Figure 8: Outside money: Instability and Stagnation. This figure plots variables of the benchmark model without government debt and model with 50% government debt-to-output ratio (dotted line). Panel A plots money premium $\rho - r_t$ against state variable η_t . Panel B plots the stationary C.D.F. of η_t . Panel C plots endogenous risk, σ_t^η , i.e., the instantaneous standard deviation of η_t against the stationary C.D.F. Panel D shows expected years to reach a value of η_t (horizontal axis) when the current state is at the issuance boundary (ending at the zero growth point).

ment debt-to-output ratio equals to 0%, the benchmark case, and 50%.⁵³ Panel A plots the money premium, $\rho - r_t$, against the state variable, η_t . Government debt supply reduces the money premium, which is in line with the evidence (Krishnamurthy and Vissing-Jørgensen (2012); Greenwood and Vayanos (2014); Greenwood, Hanson, and Stein (2015); Sunderam (2015)). However, by raising r_t , outside money increases banks' debt cost, and thus, reduces their return on equity. In response, banks reset payout and issuance policies, shifting both boundaries to the left. This *bank equity crowding-out* effect is also shown in Panel B, the stationary cumulative distribution of η_t .⁵⁴

⁵³Comparative statics show the model's performances in response to *unexpected* and *permanent* changes in the government debt supply. The current practice of U.S. Treasury debt management emphasizes predictability (Garbade (2007)), but since the financial crisis, there has been a considerable amount of uncertainty on fiscal policies, especially on the debt level (e.g. Baker, Bloom, and Davis (2015); Kelly, Pástor, and Veronesi (2016))

⁵⁴The stationary distribution is calculated using Proposition 4 using the model solutions under different M^G .

Panel C of Figure 8 shows that government debt makes endogenous risk accumulate faster in booms (i.e., as η_t increases). Endogenous risk is measured by σ_t^η , the instantaneous shock elasticity of η_t . σ_t^η is plotted against the stationary cumulative probability of η_t , so we can compare the two models in the corresponding phases of their cycles. Reading the graph from left to right, we see σ_t^η increases faster when government debt-to-output ratio is 50%.

Endogenous risk is directly linked to leverage, $\sigma_t^\eta = (x_t - 1) \sigma$ (Equation (11)), so Panel C also shows that government debt amplifies the bank leverage cycle. Stronger leverage procyclical-ity makes the economy more sensitive to shocks in good times (high η_t) and less sensitive to shocks in bad times (low η_t). As previously discussed, shock impact is asymmetric near the boundaries, so the economy gets stuck near $\underline{\eta}$ and very responsive to bad shocks near $\bar{\eta}$. As a result, probability mass is shifted towards low η_t states, as shown in Panel B (i.e., a more concave c.d.f.).

To understand how outside money amplifies the leverage cycle, consider the economy at the issuance boundary $\underline{\eta}$. As shown in Panel A of Figure 8, outside money reduces the money premium at $\underline{\eta}$, so banks' risk price γ_t^B must decline so that their indifference condition, $\rho - r_t = \gamma_t^B \sigma$, holds. By Itô's lemma, we can decompose risk price (see also Equation (19)): $\gamma_t^B = -\sigma_t^B = -\epsilon_t^B \sigma_t^\eta$, where $\sigma_t^\eta = (x_t - 1) \sigma$ is the instantaneous shock elasticity of η_t . The boundary condition (3) in Proposition 4 implies that $\epsilon_t^B = -1$ at $\underline{\eta}$, so to reduce banks' risk price, the state variable η_t needs to be less volatile (i.e., a lower σ_t^η), meaning that the equilibrium bank leverage x_t has to be lower. Therefore, outside money reduces bank leverage at the issuance boundary $\underline{\eta}$.

However, due to equity issuance cost, outside money increases banks' leverage *away* from $\underline{\eta}$, and thereby, amplifies leverage procyclicality. The wedge between q_t^B and 1 measures future profits (return on equity) that come from leverage, i.e., financing loans with inside money, and the money premium. At $\underline{\eta}$, this wedge is χ , as it must compensate the issuance cost. When the money premium is squeezed by outside money in every state (Panel A of Figure 8), banks' leverage has to increase on future equilibrium paths to sustain this level of profits. Thus, while reducing banks' leverage at $\underline{\eta}$, outside money increases leverage where $\eta_t > \underline{\eta}$, making leverage more procyclical.

Greenwood, Hanson, and Stein (2015), Krishnamurthy and Vissing-Jørgensen (2015), and Woodford (2016) also explore the financial stability implications of government debt when it serves

as a substitute for bank money. A common prediction is that by squeezing the money premium, government debt crowds out bank debt, and thereby, decreases banks' leverage and stabilizes the economy.⁵⁵ What is missing in these models is banks' dynamic equity management under frictions.

My model also highlights the competition between bank and government debt in the money market, but it makes two unique predictions regarding the destabilizing effect of government debt. By crowding out banks' profit, government debt crowds out bank equity. It also amplifies the bank leverage cycle, as banks try to sustain a level of return on equity that compensates issuance costs.

Stagnation. Panel D of Figure 8 plots the recovery paths from the bank issuance boundary. The Y-axis shows the expected number of years it takes to travel from $\underline{\eta}$ to different values of η on the X-axis. For instance, when the government debt-to-output ratio is 0%, it takes more than one year to reach $\eta = 0.006$. Both curves end at the recovery point, which is the lowest value of η that has a non-negative economic growth rate, i.e., $\lambda F(m_t + M^G) - \delta \geq 0$.

Government debt prolongs recessions. Raising government debt-to-output ratio from 0% to 50% delays the recovery by two years. The reason is that because firms still rely on banks as the *marginal* supplier of money, the profit crowding-out effect delays the recovery of banks. Lower return on equity slows down the accumulation of bank equity. Admittedly, more government debt also makes banks less relevant, because firms already hold outside money, and thus, the marginal value of inside money declines. In the extreme case where the government has a large debt capacity to satiate firms' money demand, the economy grows with the first-best investments.

Growth and welfare. Figure 9 shows the mean growth rate under stationary distribution for different levels of government debt. Since the capital productivity is constant and agents are indifferent about consumption timing, the mean growth rate of K_t proxies welfare. Before the government debt-to-output ratio reaches around 130%, more government debt decreases welfare, because *on average*, one dollar more outside money crowds out more than one dollar of inside money. It might seem difficult to reconcile this with Panel A of Figure 8, which shows more government debt decreases the money premium in *every* state of the world, and thus, must raise firms' investment rate

⁵⁵Bank debt crowding-out effect of government debt has been documented by Bansal, Coleman, and Lundblad (2011), Greenwood, Hanson, and Stein (2015), Krishnamurthy and Vissing-Jørgensen (2015), and Sunderam (2015).

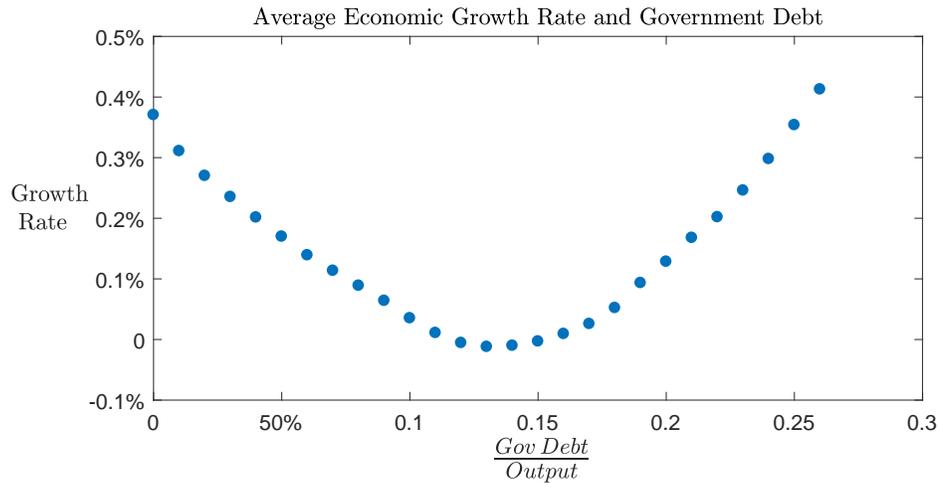


Figure 9: **Non-monotonic Effects of Outside Money on Investment and Growth.** The figure plots the mean growth rate (based on stationary distribution) for models with different levels of government debt-to-output ratio.

in *every* state of the world. The key lies in the shift of probability distribution.

Outside money amplifies the bank leverage cycle and shifts the probability mass towards states where banks are relatively undercapitalized and inside money supply is weak (Panel B of Figure 8). Therefore, even if every state of the world has a higher growth rate, the average growth rate can decrease when more probability is assigned to bad states. However, when outside money almost satiates firms' money demand, bank equity and inside money become less relevant. Once passing the point of 130% government debt-to-output ratio, government debt improves welfare.

The decreasing leg in Figure 9 is particularly relevant for understanding the U.S. economy around the Great Recession. From 2001 to 2008, the public debt-to-GDP ratio rose from c.55% to c.70%. This coincided with a period of strong leverage procyclicality in the financial sector. Many argue that downside risk accumulated as a result (FSB (2009)). The post-crisis period saw an even more dramatic increase in government debt, partly due to quantitative easing. By 2012, the public debt-to-GDP ratio had reached its current level, c.100%. Meanwhile, economic recovery was slow.

5 Conclusion

This paper revisits the money view of banking. To buffer liquidity shocks, firms hold inside money issued by banks. Banking crises cause the contraction of inside money supply, which compromises firms' liquidity management. The frequency and duration of crisis depend on the bank leverage cycle and banks' payout and equity issuance decisions. Another theme is how government debt (outside money) may contribute to financial instability. By squeezing banks' profits from money creation, government debt crowds out bank equity and amplifies the leverage cycle, and thus, may reduce welfare. The key to this destabilizing effect is banks' dynamic balance-sheet management under equity issuance frictions, which has been ignored by the pioneer works in the literature.

In the model, the government walks on a tightrope. Increasing government debt temporarily alleviates the liquidity shortage that firms face, but in the long run, by crowding out bank equity and amplifying bank leverage cycle, it can crowd out more inside money, causing more severe liquidity shortage. This trade-off suggests that optimal strategy of outside money supply is macroprudential, i.e., contingent on the relative tightness of banks' and firms' financial constraints.

In this paper, outside money crowds out inside money. In a richer environment, outside money can crowd in inside money. Banks hold outside money to buffer their own liquidity shocks, so by relaxing banks' liquidity constraint, outside money allows banks to expand balance sheet and issue more inside money. The proceeds from outside money issuance can be used by governments to recapitalize banks, preventing a sudden evaporation of inside money due to banks' default.

The demand for money-like securities, or safe assets in general, has attracted enormous attention. Theoretical studies in the literature devote tremendous effort in characterizing the issuers, intermediaries for instance, while relegate the modeling of investors who demand money or safe assets, often by assuming money in utility. This paper takes a step forward, relating money demand to corporate liquidity management. Other sources of demand, such as foreign sovereigns and institutional investors, should also be modeled carefully. The endogenous and distinct dynamics of different money market investors is as important as the behavior of issuers (financial intermediaries), when it comes to understand the causes and consequences of financial crisis.

Appendix I Proofs

I.1 Static Model

Firms' problem. Let $\underline{k}_0(s)$ denote capital endowments of a firm s at $t = 0$, and $k_0(s)$ its capital demand. Aggregate stock K_0 is $\int_{s \in [0,1]} \underline{k}_0(s) ds$. Capital market clears, so $K_0 = \int_{s \in [0,1]} k_0(s) ds$. To save notations, I suppress firm index s . Given q_0^K , a firm's wealth is $w_0 = q_0^K \underline{k}_0$. At $t = 0$, a firm chooses capital k_0 , deposits per unit of capital m_0 , consumption c_0 , the value of bank loan l_0 , and funds raised by issuing securities to households h_0 , and at $t = 1$, chooses investment rate i_1 .

By promising to *expected* payments of v_1^H at $t = 1$, a firm raises h_0 from households at $t = 0$. Given household discount rate ρ , a competitive security market implies $v_1^H = h_0(1 + \rho)$.

Let v_0 denote the firm's value function, which is equal to $\max_{c_0 \geq 0, h_0 \geq 0} c_0 + \frac{1}{1+\rho} (v_1^* - v_1^H)$, where v_1^* is the *maximized expected* value before repaying households, a function of other optimal choices (k_0, m_0, l_0, i_1) . Substituting $v_1^H = h_0(1 + \rho)$ into the expression, we have

$$v_0 = \max_{c_0 \geq 0, h_0 \geq 0} c_0 - h_0 + \frac{1}{1 + \rho} v_1^*.$$

Therefore, what matters is the net consumption $(c_0 - h_0)$ or net financing $(h_0 - c_0)$. Going forward, we allow c_0 to take positive or negative values. When $c_0 > 0$, the entrepreneur consumes, and when $c_0 < 0$, the entrepreneur raises funds from households. We can write the firms' value function as

$$v_0 = \max_{c_0^B \in \mathbb{R}} c_0 + \frac{1}{1 + \rho} v_1^*.$$

The firm can also issue securities to banks. Let v_1^B denote the firm's *expected* repayment to banks at $t = 1$. Note that in the analysis of the firm's problem, we do not need to specify the contractual form of securities issued to banks or households. All that matters is the expected repayment. Let v_1^{**} denote the *maximized expected* value of the firm before repaying both households and banks, so $v_1^{**} = v_1^* + v_1^B$ by definition. We can rewrite the value function as

$$v_0 = \max_{c_0^B \in \mathbb{R}} c_0 + \frac{1}{1 + \rho} (v_1^{**} - v_1^B).$$

Because bank debt earns a money premium, the required rate of return of banks can be different from that of households. Let ρ_0^B denote banks' required expected return in equilibrium. For the

firm to raise l_0 from banks at $t = 0$, it must deliver an expected repayment to competitive banks equal to $v_1^B = l_0 (1 + \rho_0^B)$. Thus, we can rewrite the value function as

$$v_0 = \max_{c_0^B \in \mathbb{R}, l_0 \geq 0} c_0 - \frac{1 + \rho_0^B}{1 + \rho} l_0 + \frac{1}{1 + \rho} v_1^{**}, \quad (20)$$

where v^{**} is a function of other optimal choices (k_0, m_0, i_1) . If $\rho_0^B < \rho$, the firm only issue securities to banks; likewise, if $\rho < \rho_0^B$, the firm only issue securities to households if at all. Since we study an equilibrium where firms do borrow from banks, it must be true that $\rho_0^B \leq \rho$.

The firm's financing capacity depends on its pledgeable value at $t = 1$. Newly created capital is not pledgeable, and a fraction $\delta - \sigma Z_1$ of existing capital will be gone by $t = 1$ (with $\mathbb{E}_0 [Z_1] = 0$), so the expected pledgeable value is $\alpha k_0 (1 - \delta)$. The firm faces the following financing constraint:

$$l_0 (1 + \rho_0^B) + \mathbb{I}_{\{c_0 < 0\}} (-c_0) (1 + \rho) \leq \alpha k_0 (1 - \delta), \quad (21)$$

where $\mathbb{I}_{\{\cdot\}}$ is an indicator function. Note that when $-c_0 < 0$, the firm raises $|c_0|$ from households.

The firm also faces the budget constraint and the liquidity constraint:

$$c_0 + q_0^K k_0 + m_0 k_0 \leq w_0 + l_0, \text{ where } c_0 \in \mathbb{R}, \quad (22)$$

$$i_1 \leq m_0. \quad (23)$$

Given capital price q_0^K , the deposit rate r_0 , and the required expected return of banks ρ_0^B , the firm maximizes the objective in Equation (20) subject to constraints (21), (22), and (23), with the expected total firm value (before repayment to investors) at $t = 1$ given by

$$v^{**} = \underbrace{\alpha k_0 (1 - \delta)}_{\text{expected surviving capital value}} + \underbrace{(1 + r_0) m_0 k_0}_{\text{deposits}} + \underbrace{\lambda (\alpha F (i_1) - i_1) k_0}_{\text{expected new capital net value}}. \quad (24)$$

Let κ_0 , ψ_0 , and θ_0 denote the Lagrange multipliers of the financing, budget, and liquidity constraints respectively. We can write the Lagrange (omitting the non-negativity constraints):

$$\begin{aligned} v_0 = \max_{c_0^B \in \mathbb{R}, l_0 \geq 0, k_0 \geq 0, m_0 \geq 0, i_1 \geq 0} & c_0 - \frac{1 + \rho_0^B}{1 + \rho} l_0 + \frac{1}{1 + \rho} [\alpha k_0 (1 - \delta) + (1 + r_0) m_0 k_0 \\ & + \lambda (\alpha F (i_1) - i_1) k_0] + \theta_0 (m_0 - i_1) \\ & + \kappa_0 [\alpha k_0 (1 - \delta) - l_0 (1 + \rho_0^B) - \mathbb{I}_{\{c_0 < 0\}} (-c_0) (1 + \rho)] \\ & + \psi_0 (w_0 + l_0 - c_0 - q_0^K k_0 - m_0 k_0). \end{aligned}$$

Proof of Lemma 2. First, we solve ψ_0 . We must have the coefficient of c_0 equal to zero

$$1 + \kappa_0 \mathbb{I}_{c_0 < 0} (1 + \rho) - \psi_0 = 0, \quad (25)$$

Because c_0 can take either positive or negative values. In equilibrium, banks lend out at least some of their goods endowments to firms to carry net worth to $t = 1$. Because goods cannot be stored, in aggregate, entrepreneurs must consume, so $c_0 > 0$, so from Equation (25), $\psi_0 = 1$.

Next, we solve κ_0 , the shadow value of financing. In equilibrium $l_0 > 0$ (i.e., not a corner solution), so locally the entrepreneur must be indifferent. Thus, the coefficient of l_0 is equal to zero

$$-\frac{1 + \rho_0^B}{1 + \rho} - \kappa_0 (1 + \rho_0^B) + \psi_0 = 0. \quad (26)$$

Substituting $\psi_0 = 1$ into Equation (26), we have

$$\kappa_0 = \frac{1}{1 + \rho_0^B} - \frac{1}{1 + \rho}. \quad (27)$$

If the firm promises one unit of goods in expectation at $t = 1$, it can obtain $\frac{1}{1 + \rho_0^B}$ bank financing and $\frac{1}{1 + \rho}$ household financing at $t = 0$. Intuitively, κ_0 is the difference between the price of securities issued to banks and the price of securities issued to households. $\kappa_0 > 0$ if and only if $\rho_0^B < \rho$.

Multiplying both sides of Equation (27) by $(1 + \rho_0^B)(1 + \rho)$, we have

$$\kappa_0 (1 + \rho_0^B) (1 + \rho) = \rho - \rho_0^B$$

Expanding the left hand side we have, $\kappa_0 + \kappa_0 \rho_0^B + \kappa_0 \rho + \kappa_0 \rho_0^B \rho$. The current time interval is 1, but if we let Δ denote the length of time between date 0 and 1, the product terms on the left-hand side are of the order of Δ^2 or higher. As Δ shrinks to zero, these terms approach to zero at a faster pace. To approximate the continuous-time expression, we ignore those product terms, so we have

$$\kappa_0 = \rho - \rho_0^B = \rho - (R_0 - \delta). \quad (28)$$

Proof of Lemma 1. The firm's first order condition with respect to m_0 is

$$\frac{1}{1 + \rho} (1 + r_0) k_0 - \psi_0 k_0 + \theta_0 = 0. \quad (29)$$

The first order condition with respect to i_1 is

$$\frac{1}{1 + \rho} \lambda (\alpha F' (i_1) - 1) k_0 - \theta_0 = 0. \quad (30)$$

Summing up Equation (29) and (30), we have

$$\frac{1}{1+\rho} (1+r_0) k_0 - \psi_0 k_0 + \frac{1}{1+\rho} \lambda (\alpha F'(i_1) - 1) k_0 = 0. \quad (31)$$

We focus on the situation where the investment technology $F(\cdot)$ is so productive that the liquidity constraint always binds, so $i_1 = m_0$. Substituting $\psi_0 = 1$ and rearranging the equation, we have

$$r_0 - \rho + \lambda (\alpha F'(m_0) - 1) = 0. \quad (32)$$

Proof of Lemma 3. The expected return on bank equity is:

$$x_0 (1 + R_0 - E[\pi(Z_1)]) - (x_0 - 1)(1 + r_0) = 1 + r_0 + x_0 (R_0 - E[\pi(Z_1)] - r_0).$$

Note that $E[\pi_D(Z_1)] = \delta$. When $Z_1 = -1$, the realized return on bank equity is:

$$1 + r_0 + x_0 (R_0 - \delta - \sigma - r_0).$$

The representative bank starts with equity e_0 and has the following value function:

$$\begin{aligned} v(e_0; R_0, r_0) = & \max_{y_0 \geq 0, x_0 \geq 0} y_0 e_0 + \frac{e_0 (1 - y_0)}{(1 + \rho)} \{1 + r_0 + x_0 (R_0 - \delta - r_0) \\ & + \xi_0 [1 + r_0 + x_0 (R_0 - \delta - \sigma - r_0)]\}, \end{aligned}$$

where y_0 is the banker's consumption-to-wealth ratio. The first order condition for x_0 is:

$$R_0 - r_0 = \delta + \gamma_0^B \sigma, \quad (33)$$

where $\gamma_0^B = \frac{\xi_0}{1+\xi_0} \in [0, 1)$ because $\xi_0 \geq 0$. Rearranging the equation, we have γ_0^B equal to the Sharpe ratio of loans: $\gamma_0^B = \frac{R_0 - \delta - r_0}{\sigma}$. When $\gamma_0^B > 0$, the capital adequacy constraint binds. Substituting the F.O.C. for x_0 into the value function, we have: $v(e_0; R_0, r_0) = y_0 e_0 + q_0^B (e_0 - y_0 e_0)$, where $q_0^B = \frac{(1+r_0)(1+\xi_0)}{(1+\rho)}$. The bank chooses $y_0 > 0$ only if $q_0^B \leq 1$.

Proofs of Proposition 1 and Corollary 1. Proofs are provided in the main text.

I.2 Continuous-time Model

Firms' problem: proof of Lemma 1', Lemma 2', and Proposition 2. Entrepreneurs ("firms") maximize life-time utility, $\mathbb{E} \left[\int_{t=0}^{+\infty} e^{-\rho t} dc_t \right]$, subject to the following wealth (equity) dynamics:

$$dw_t = -dc_t + \mu_t^w w_t dt + \sigma_t^w w_t dZ_t + (\widehat{w}_t - w_t) dN_t,$$

$\mu_t^w w_t$ and $\sigma_t^w w_t$ are the drift and diffusion terms that depend on choices of capital and deposit holdings and will be elaborated later. dN_t is the increment of the idiosyncratic counting (Poisson) process. $dN_t = 1$ if an investment opportunity arrives. At the Poisson time, firm equity jumps to

$$\widehat{w}_t = w_t + q_t^K F(m_t) k_t - m_t k_t.$$

We may conjecture that the value function is linear in equity w_t : $V_t = \zeta_t w_t$, where ζ_t is the marginal value of equity, and in equilibrium, follows a diffusion process:

$$d\zeta_t = \zeta_t \mu_t^\zeta dt + \zeta_t \sigma_t^\zeta dZ_t,$$

where $\zeta_t \mu_t^\zeta$ and $\zeta_t \sigma_t^\zeta$ are the drift and diffusion terms respectively. Note that firms' marginal value of wealth, ζ_t , is a summary statistic of firms' investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual firm is hit by investment opportunities.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho V_t dt = \max_{dc_t \in \mathbb{R}, k_t \geq 0, m_t \geq 0, l_t \geq 0} dc_t - \zeta_t dc_t + \{w_t \zeta_t \mu_t^\zeta + w_t \zeta_t \mu_t^w + w_t \zeta_t \sigma_t^\zeta \sigma_t^w + \lambda \zeta_t [\widehat{w}_t - w_t]\} dt.$$

By the same logic in the analysis of firms' problem in the static model, firms' negative consumption is equivalent to raising funds from households. Firms can choose any $dc_t \in \mathbb{R}$, so ζ_t must be equal to one, and thus, I have also confirmed the value function conjecture.

Since ζ_t is a constant equal to one, μ_t^ζ and σ_t^ζ are both zero. The HJB equation is simplified:

$$\rho V_t dt = \max_{k_t \geq 0, m_t \geq 0, l_t \geq 0} \mu_t^w w_t dt + \lambda [q_t^K F(m_t) - m_t] k_t dt. \quad (34)$$

Equity drift has production, value change of capital holdings, deposit return, and loan repayment:

$$\mu_t^w w_t dt = \alpha k_t dt + \mathbb{E}_t (q_{t+dt}^K k_{t+dt} - q_t^K k_t) + r_t m_t k_t dt - l_t (R_t - \delta) dt$$

Let $d\psi_t$ denote the Lagrange multiplier of the budget constraint, $q_t^K k_t + m_t k_t \leq w_t + l_t$. The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: $m_t \geq 0$, and

$$m_t \{ r_t dt + \lambda [q_t^K F'(m_t) - 1] dt - d\psi_t \} = 0.$$

A fraction $(\delta dt - \sigma dZ_t)$ of capital is to be destroyed, so the capital evolves as

$$k_{t+dt} = k_t - (\delta dt - \sigma dZ_t) k_t.$$

Given the equilibrium capital price dynamics, $dq_t^K = q_t^K \mu_t^K dt + q_t^K \sigma_t^K dZ_t$, we have

$$q_{t+dt}^K k_{t+dt} - q_t^K k_t = q_t^K k_t [-(\delta dt - \sigma dZ_t) + \mu_t^K dt + \sigma_t^K dZ_t + \sigma \sigma_t^K dt].$$

The F.O.C. for optimal capital holdings is : $k_t \geq 0$, and

$$k_t \{ \alpha dt + q_t^K (-\delta + \mu_t^K + \sigma \sigma_t^K) dt + r_t m_t dt + \lambda dt [q_t^K F'(m_t) - m_t] - (q_t^K + m_t) d\psi_t \} = 0.$$

The F.O.C. for optimal borrowing from banks is: $l_t \geq 0$, and

$$-(R_t - \delta) dt + d\psi_t = 0.$$

Finally, we have the complementary slackness condition: $d\psi_t \geq 0$, and

$$(w_t + l_t - q_t^K k_t - m_t k_t) d\psi_t = 0.$$

Substituting these optimality conditions into the HJB equation, we have

$$\rho V_t dt = w_t d\psi_t.$$

Because $\zeta_t = 1$, $V_t = w_t$, and $d\psi_t = \rho dt$. Substituting $d\psi_t = \rho dt$ into the F.O.C. for m_t , we have

$$r_t + \lambda [q_t^K F'(m_t) - 1] = \rho.$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for k_t and rearranging the equation, we have

$$q_t^K = \frac{\alpha - (\rho - r_t) m_t + \lambda [q_t^K F'(m_t) - m_t]}{\rho - (\mu_t^K - \delta + \sigma \sigma_t^K)}.$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for l_t , we have

$$R_t = \rho + \delta.$$

Banks' problem: proof of Lemma 3'. Conjecture that the bank's value function takes the linear

form: $v_t = q_t^B e_t$. In equilibrium, the marginal value of equity, q_t^B , evolves as follows

$$dq_t^B = q_t^B \mu_t^B dt + q_t^B \sigma_t^B dZ_t.$$

Under this conjecture, the HJB equation is

$$\begin{aligned} \rho v_t dt &= \max_{dy_t \in \mathbb{R}} \left\{ (1 - q_t^B) \mathbb{I}_{\{dy_t > 0\}} e_t dy_t + (q_t^B - 1 - \chi) \mathbb{I}_{\{dy_t < 0\}} e_t (-dy_t) \right\} \\ &\quad + \mu_t^B q_t^B e_t + \max_{x_t \geq 0} \left\{ r_t + x_t (R_t - \delta - r_t) - x_t \gamma_t^B \sigma \right\} q_t^B e_t - \iota q_t^B e_t, \end{aligned}$$

where $\gamma_t^B = -\sigma_t^B$. Dividing both sides by $q_t^B e_t$, we eliminate e_t in the HJB equation,

$$\begin{aligned} \rho &= \max_{dy_t \in \mathbb{R}} \left\{ \frac{(1 - q_t^B)}{q_t^B} \mathbb{I}_{\{dy_t > 0\}} dy_t + \frac{(q_t^B - 1 - \chi)}{q_t^B} \mathbb{I}_{\{dy_t < 0\}} (-dy_t) \right\} \\ &\quad + \mu_t^B + \max_{x_t \geq 0} \left\{ r_t + x_t (R_t - \delta - r_t) - x_t \gamma_t^B \sigma \right\} - \iota, \end{aligned}$$

and thus, confirm the conjecture of linear value function.

q_t^B is the marginal value of equity. Paying out one dollar of dividend, the bank's shareholders receive 1, but lose q_t^B . Only when $q_t^B \leq 1$, $dy_t > 0$. When the bank issues equity, it incurs a dilution cost. From the existing shareholders' perspective, one dollar equity is sold to outside investors at price $\frac{q_t^B}{1+\chi}$. To raise $(-dy_t) e_t$ that is worth $q_t^B (-dy_t) e_t$, the bank must issue $\frac{(1+\chi)(-dy_t)e_t}{q_t^B}$ shares, and thus, the existing shareholders give up total value of $q_t^B \frac{(1+\chi)(-dy_t)e_t}{q_t^B} = (1+\chi)(-dy_t) e_t$. Therefore, the bank raises equity only if $q_t^B \geq 1 + \chi$. Finally, the indifference condition for x_t is $R_t - \delta - r_t = \gamma_t \sigma$. If $R_t - \delta - r_t < \gamma_t \sigma$, x_t is set to zero; if $R_t - \delta - r_t > \gamma_t \sigma$, x_t is set to infinity.

Proof of Proposition 1'. Proof is provided in the main text.

Proof of Corollary 1'. Proof is provided in the main text.

Proof of Lemma 4. First, I derive equation (11). Because individual banks share the same μ_t^e , σ_t^e , and payout/issuance rate dy_t , aggregating over banks, the law of motion of E_t is

$$dE_t = \mu_t^e E_t dt + \sigma_t^e E_t dZ_t - dy_t E_t.$$

Given the expected growth rate, $\lambda F(m_t) - \delta$, which is the investment net of expected depreciation, the aggregate capital stock, K_t , evolves as: $dK_t = [\lambda F(m_t) - \delta] K_t dt + \sigma K_t dZ_t$.

By Itô's lemma, the ratio, $\eta_t = \frac{E_t}{K_t}$, has the following law of motion:

$$d\eta_t = \frac{1}{K_t}dE_t - \frac{E_t}{K_t^2}dK_t + \frac{1}{K_t^3} \langle dK_t, dK_t \rangle - \frac{1}{K_t^2} \langle dE_t, dK_t \rangle,$$

where $\langle dX_t, dY_t \rangle$ denotes the quadratic covariation of diffusion processes X_t and Y_t , so we have $\langle dK_t, dK_t \rangle = \sigma^2 K_t^2 dt$, and $\langle dE_t, dK_t \rangle = \sigma_t^e \sigma E_t K_t dt$. Dividing both sides by η_t , we have

$$\frac{d\eta_t}{\eta_t} = \frac{dE_t}{E_t} - \frac{dK_t}{K_t} + \sigma^2 dt - \sigma_t^e \sigma dt.$$

Substituting the law of motions of E_t and K_t into the equation above, we have Equation (11). The boundaries are given by banks' optimal payout and issuance policies in Proposition 3'.

Proof of Proposition 3. Following Brunnermeier and Sannikov (2014), I derive the stationary probability density. Probability density of η_t at time t , $p(\eta, t)$, has Kolmogorov forward equation

$$\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta} (\eta \mu^\eta(\eta) p(\eta, t)) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} (\eta^2 \sigma^\eta(\eta)^2 p(\eta, t)).$$

Note that in a Markov equilibrium, μ_t^η and σ_t^η are functions of η_t . A stationary density is a solution to the forward equation that does not vary with time (i.e. $\frac{\partial}{\partial t} p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over η , $p(\eta)$ solves the following first-order ordinary differential equation within the two reflecting boundaries:

$$0 = C - \eta \mu^\eta(\eta) p(\eta) + \frac{1}{2} \frac{d}{d\eta} (\eta^2 \sigma^\eta(\eta)^2 p(\eta)), \quad \eta \in [\underline{\eta}, \bar{\eta}].$$

The integration constant C is zero because of the reflecting boundaries. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_{\underline{\eta}}^{\bar{\eta}} p(\eta) d\eta = 1$).

Next, I solve the expected time to reach from $\underline{\eta}$. Define $f_{\eta_0}(\eta)$ the expected time it takes to reach η_0 starting from $\eta \leq \eta_0$. Define $g(\eta_0) = f_{\eta_0}(\underline{\eta})$ the expected time to reach η_0 from $\underline{\eta}$. One has to reach $\eta \in (\underline{\eta}, \eta_0)$ first and then reach η_0 from η . Therefore, $g(\eta) + f_{\eta_0}(\eta) = g(\eta_0)$. Since $g(\eta_0)$ is constant, we differentiate both sides to have $g'(\eta) = -f'_{\eta_0}(\eta)$ and $g''(\eta) = -f''_{\eta_0}(\eta)$.

From η_t , the expected time to reach η_0 , denoted by $f_{\eta_0}(\eta_t)$, is decomposed into $s - t$, and $E_t[f_{\eta_0}(\eta_s)]$, i.e., the expected time to reach η_0 from η_s ($s \geq t$) after $s - t$ has passed. We have $f_{\eta_0}(\eta_t)$ equal to $E_t[f_{\eta_0}(\eta_s)] + s - t$. Therefore, $t + f_{\eta_0}(\eta_t)$ is a martingale, so f_{η_0} satisfies the ordinary differential equation: $1 + f'_{\eta_0}(\eta) \mu^\eta(\eta) + \frac{\sigma^\eta(\eta)^2}{2} f''_{\eta_0}(\eta) = 0$. Therefore, $g(\eta)$ must satisfy

$$1 - g'(\eta) \mu^\eta(\eta) - \frac{\sigma^\eta(\eta)^2}{2} g''(\eta) = 0.$$

It takes no time to reach $\underline{\eta}$, so $g(\underline{\eta}) = 0$. Moreover, since $\underline{\eta}$ is a reflecting boundary, $g'(\underline{\eta}) = 0$.

Proof of Proposition 4. The Markov equilibrium is time-homogeneous, so I suppress time subscripts. In the main text, I show how to uniquely solve x , m , and r as functions of $(q^K(\eta), q^B(\eta))$ and their derivatives. Once we know these variables, we solve the dynamics of E_t :

$$\mu^e = r + x^B (R - \delta - r), \text{ and } \sigma^e = x^B \sigma,$$

and the economic growth rate, $\lambda F(m_t) - \delta$. So, then we have the drift and diffusion of η_t :

$$\mu^\eta = \mu^e - [\lambda F(m_t) - \delta] - \sigma^e \sigma + \sigma^2, \quad \text{and} \quad \sigma^\eta = (x - 1) \sigma.$$

Next, we can use bankers' HJB equation and the capital pricing formula (i.e. Equations (13) and (17)) to form a system of differential equations for $(q^K(\eta), q^B(\eta))$, i.e., a mapping from $(\eta, q^B, q^K, \frac{dq^B}{d\eta}, \frac{dq^K}{d\eta})$ to $(\frac{d^2 q^B}{d\eta^2}, \frac{d^2 q^K}{d\eta^2})$. Instead of the first derivatives, we can work with elasticities of (q^B, q^K) , $\epsilon^X = \frac{dq^X/q^X}{d\eta/\eta}$, $X \in \{B, K\}$ to simplify the expressions. Using Itô's lemma, we know

$$\mu^X = \epsilon^X \mu^\eta + \frac{1}{2q^X} (\sigma^\eta \eta)^2 \frac{d^2 q^X}{d\eta^2}, \text{ i.e., } \frac{d^2 q^X}{d\eta^2} = 2q^X \frac{(\mu^X - \epsilon^X \mu^\eta)}{(\sigma^\eta \eta)^2}, X \in \{B, K\}.$$

To calculate μ^K and μ^B , we use banks' HJB equation (Equation (13)):

$$\mu^B = \rho + \iota - r,$$

and the capital pricing formula (Equation (17)),

$$\mu^K = \rho + \delta - \sigma^K \sigma - \frac{\alpha}{q^K} + (\rho - r) m - \lambda \left[F(m) - \frac{m}{q^K} \right], \text{ where } \sigma^K = \epsilon^K \sigma^\eta.$$

Because $F(\cdot)$ is concave, Equation (16)) has a unique solution of x . Following from it, we solve m_t and r_t uniquely as shown in the main text. Thus, the mapping from $(\eta, q^B, q^K, \frac{dq^B}{d\eta}, \frac{dq^K}{d\eta})$ to $(\frac{d^2 q^B}{d\eta^2}, \frac{d^2 q^K}{d\eta^2})$ is unique. Given the boundary conditions, the system of differential equations uniquely pins down a solution $(q^B(\eta), q^K(\eta))$ under proper parameter range that guarantees existence, so we have a unique Markov equilibrium with state variable η_t . To solve the problem with government debt, we only need to change firms' deposit demand as shown in the main text.

Appendix II Calibration

Table A.1: Calibration.

This table summarizes the parameter values of the solution and corresponding model and data moments (including the sources and sample size) used in calibration. Model moments are calculated using the stationary distribution of the model solution.

	Parameters		Model Moments	Data	Source	
(1)	ρ	4.00%	Interest rate $\mathbb{E}[r_t]$	3.78%	3.77%	Average of MZM and T-bill rate FRED (1974Q1-2015Q4)
(2)	Inv. tech.	$F(i) = \omega_0 i^{\omega_1}$				
	ω_0	0.801	Expected capital growth	0.74%	0.74%*	Corrado and Hulten (2010)
	ω_1	0.99	Cash to net assets $\mathbb{E}[m_t]$	29.3%	29.2%	Compustat (1971-2015)**
(3)	λ	1/5	Expected liquidity premium $\mathbb{E}[\rho - r_t]$	24.59bps	23.65bps	GC Repo/T-bill spread Nagel (2016)
(5)	χ	1	Liquidity premium s.d. $\text{std}[\rho - r_t]$	12.35bps	18.19bps	GC Repo/T-bill spread Nagel (2016)
(6)	α	0.1	Firms' equity P/E ratio	25.6	24.9	S&P 500 (Jan1991-Dec2015)
(7)	ι	2.55%	Operation cost / bank total income $\mathbb{E}\left[\frac{\iota e_t}{(R_t - \delta - r_t)x_t e_t}\right]$	90.3%	91.4%	Opex / (Opex + Net Income) Call Reports (1976Q1-2015Q4)
(8)	δ	4.00%			3.67%	Average Loan delinquency rate FRED (1985Q1-2015Q4)
(9)	σ	2.00%			1.62%	Loan delinquency rate s.d. FRED (1985Q1-2015Q4)

* In reality, economic growth is driven by both cash-intensive investments and investments that can largely rely on external financing. I set ω_0 to 0.801, so the mean growth rate matches U.S. economic growth from intangible investment (Corrado and Hulten (2010)).

** Before 1971, money market funds hadn't developed, so under Regulation Q, firms' deposit holdings did not pay interests, which is different from model.

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Procyclical Finance: The Money View

Internet Appendix

Ye Li*

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1 Preliminary Evidence

1.1 The Structure of Inside Money

Figure 1 shows that the model setup and its mechanism are quantitatively important. The left panel shows that to nonfinancial firms, financial intermediaries are the most important suppliers of money-like securities. The right panel shows that nonfinancial firms are among the most important buyers of intermediaries' money-like liabilities. Money-like assets include various financial instruments. Accordingly, "deposits" in the model should be interpreted broadly, including short-term debt issued by financial intermediaries that either serves as a means of payment, such as deposits at commercial banks, or close substitutes (usually held through money market funds by corporate treasuries), such as repurchase agreements and high-quality asset-backed commercial papers.

The left panel of Figure 1 decomposes the money-like assets of U.S. non-financial corporations by the types of securities, and for each security, by the types of its issuers. I use the data in December 2015 from the March 10, 2016 release of the Financial Accounts of the United States

*The Ohio State University. E-mail: li.8935@osu.edu

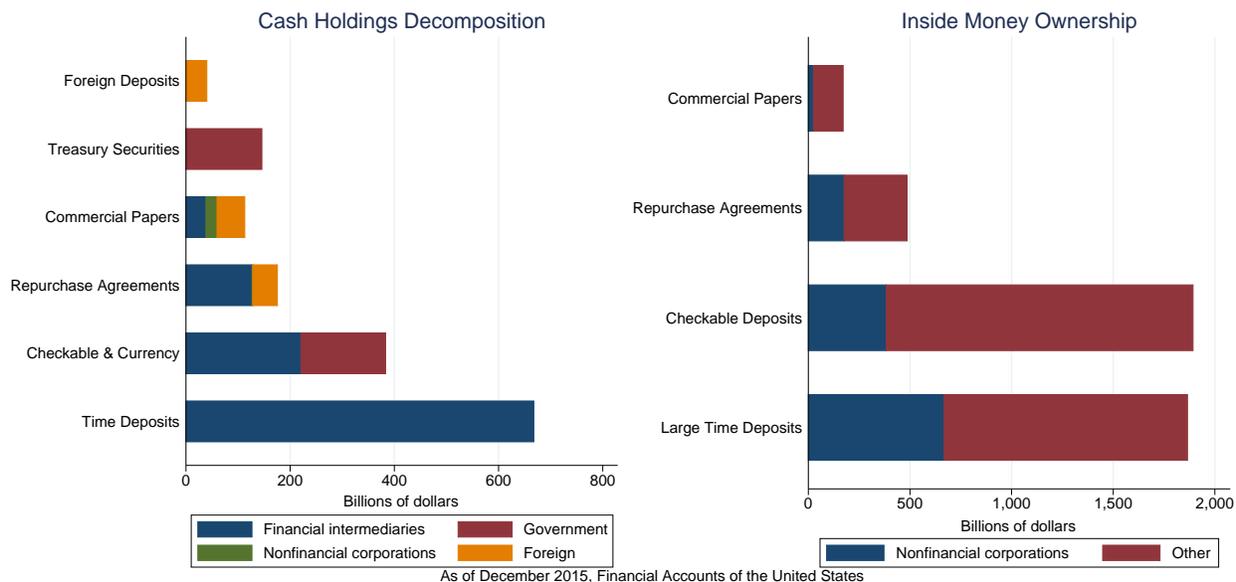


Figure 1: Inside Money Demand and Supply. This figure plots the issuers (by value) of different components of liquidity holdings of nonfinancial businesses (Panel A) and the holders (by value) of each type of money-like liabilities issued by the financial intermediation sector as of December 2015 (source: the Financial Accounts of the United States). The financial intermediation sector includes U.S.-chartered depository institutions, foreign banking offices in U.S., banks in U.S. affiliated areas, credit unions, issuers of asset-backed securities, finance companies, mortgage real estate investment trusts, security brokers and dealers, holding companies, and funding corporations.

(previously known as the “Flow of Funds”). From this graph, we can understand who are supplying money to the U.S. nonfinancial corporations and in what forms. Foreign deposits and time deposits are issued by depository institutions, and Treasury securities by the government. 19% of commercial papers are issued by the nonfinancial corporations themselves, 34% by domestic financial intermediaries, and 47% by foreign entities, of which 90% are issued by “foreign financial firms” (defined in the Financial Accounts). 72% of repurchase agreements are issued by the financial sector, and 27% by the foreign entities. Checkable deposits and currency are reported together in the Financial Accounts, of which 42% issued by the government are “currency outside banks”, and the remaining 58% are the liabilities of depository institutions. Given that firms usually do not hold currency directly, 58% underestimates the contribution of financial intermediaries.

The right panel of Figure 1 decomposes the outstanding money-like securities issued by financial intermediaries by the types of owners. Mainly through money market funds, nonfinancial

corporations hold a little less than a third of outstanding repurchase agreements and some commercial papers that account for a smaller fraction of the outstanding amount. They hold a significant share of checkable deposits and large time deposits.

Here are some details on how to construct this graph. Table L.103 records nonfinancial corporations' assets and liabilities. I break down the holdings of money market fund shares and mutual fund shares into financial instruments using Table L.121 and L.122 respectively, under the assumption that funds held by nonfinancial firms invest in the same portfolio as the aggregate sector does.¹ Corporate and foreign bonds, loans, and miscellaneous assets, all held indirectly through money market mutual funds and mutual funds, are excluded. Agency- and GSE-backed securities are excluded because of the potential spikes of repo haircuts and the secondary market illiquidity during crisis times. Municipal securities are excluded because of secondary market illiquidity.

Next, for each financial instrument, I calculate the net supply by each type of issuers using the instrument-level tables. Financial intermediaries are defined as in Krishnamurthy and Vissing-Jørgensen (2015), including the following financial institutions: U.S.-chartered depository institutions, foreign banking offices in U.S., banks in U.S. affiliated areas, credit unions, issuers of asset-backed securities, finance companies, mortgage real estate investment trusts, security brokers and dealers, holding companies, and funding corporations. Insurance companies are not included as financial intermediaries because their liabilities are long-term and usually held until maturity by insurance policy holders instead of resold in secondary market. Their liabilities are not money-like.

1.2 Cyclical money creation

The model predicts procyclical quantity of money and countercyclical price (i.e., the money premium). The banking sector expands its supply of inside money in goods times, allowing firms to better hedge against their liquidity shocks. In the model, all quantity variables are proportional to K_t , the capital stock, and thus, the output αK_t , so I scale all quantities variables in data by the nominal GDP. Figure 2 plots the quarterly data of money premium (the solid line) and the

¹It is implicitly assumed that funds are just pass-through and do not provide any liquidity services beyond the securities they hold. This ignores the potential sharing of idiosyncratic liquidity shocks through funds.

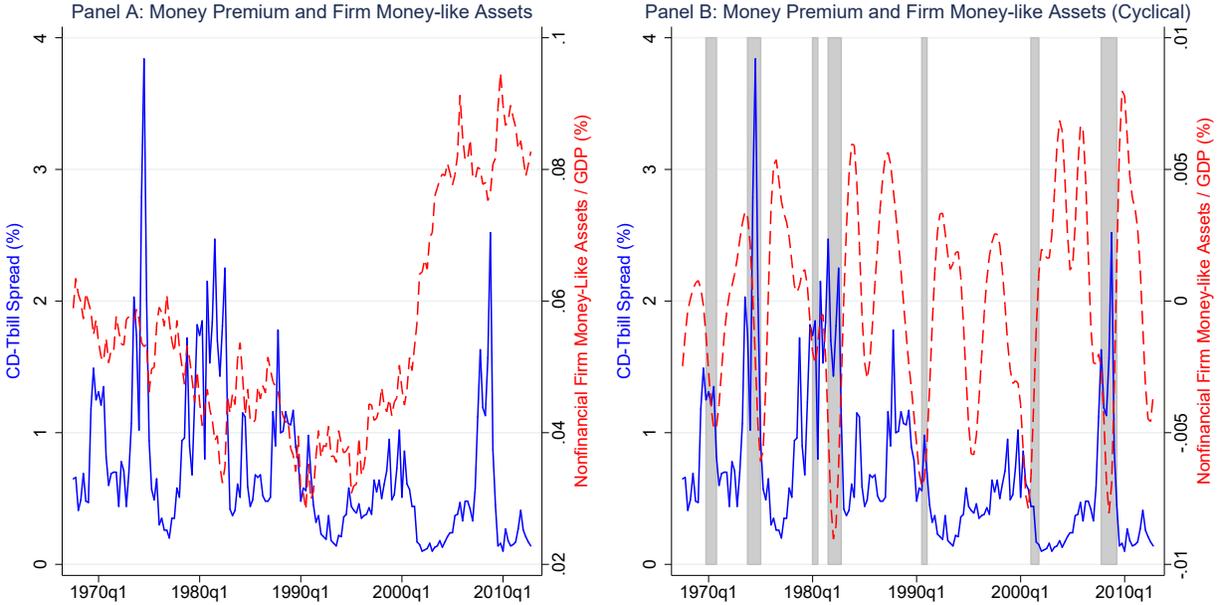


Figure 2: Money Premium and Nonfinancial Firms’ Holdings of Money-like Securities. Panel A plots money-like assets held by U.S. nonfinancial business (defined in Appendix III.1) and the money premium (solid line), measured by the spread between the three-month Certificate of Deposit rate and three-month Treasury Bill rate. Panel B plots the cyclical components of these two variables. I use the Baxter-King filter that removes frequencies longer than 69 months, the average length of US business cycle after World War II. Plots start from 1967Q3 and ends in 2012Q4.

U.S. nonfinancial firms’ money-like assets (defined in Appendix III.1). Following Nagel (2016), I define money premium as the spread between the three-month Certificate of Deposit rate and three-month Treasury Bill rate.² Panel B plots the cyclical component of firms’ money holdings applying the Baxter-King filter.³ NBER recession periods are marked by gray shades.

Figure 2 shows that firms’ money holdings tend to be procyclical, and the money premium tends to be countercyclical, consistent with the equilibrium behavior of the model. The correlation between the cyclical component of firms’ money holdings and the recession dummy is -47.3% , and the correlation between the money premium and the recession dummy is 53.4% . The correlation between the cyclical component of firms’ money holdings and the money premium is -35.0% .

²In Nagel (2016), the preferred measure of money premium is the spread between general collateral repo rate (GC) and Treasury bill rate. However, repo rates are available only since 1991. Nagel (2016) argues that outside the crisis periods (savings and loans crisis of 1980s; the financial crisis of 2007-09), the credit risk of CD component is small, and shows that when both CD rates and GC repo rates are available, there is only a small difference between the two.

³The filter remove frequencies longer than 69 months, the average length of NBER business cycle in 1945–2009.

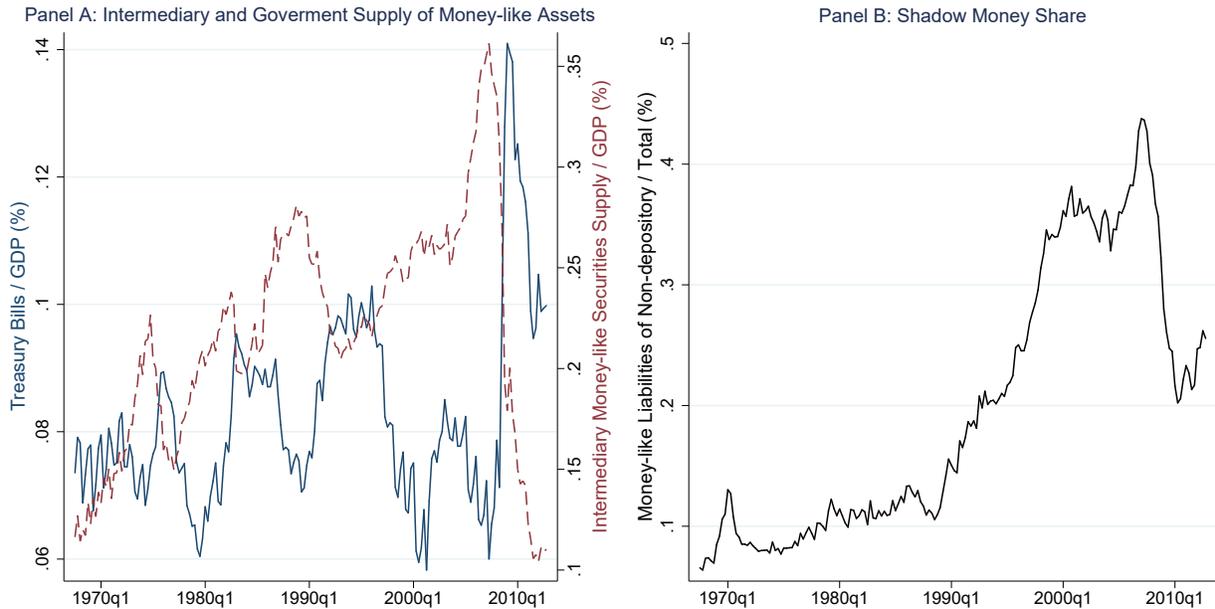


Figure 3: Money-like Securities Issued by Financial Intermediaries and Government. Panel A plots the outstanding Treasury bills (solid line) scaled by nominal GDP and the money-like liabilities of financial intermediaries scaled by GDP (defined in Figure 1 in Appendix III.1). Panel B plots the share of inside money, i.e., total money-like liabilities of financial intermediaries, that is from the non-depository intermediaries (“shadow banks”). The data source is the Financial Accounts of the United States from 1967Q3 to 2012Q4.

There is a large literature on the secular trends in corporate liquidity holdings (e.g., Bates, Kahle, and Stulz (2009)), but relatively few on its cycle. Eisfeldt and Rampini (2009) document a positive correlation between the money premium and the asynchronicity between the sources and uses of funds in the productive sector. The model by He and Kondor (2016) also predicts a negative correlation between the money premium and firms’ cash holdings, but they focus on the pecuniary externality in the market of productive capital and the resulting inefficient investment waves.

Using data from the Financial Accounts of the United States, Panel A of Figure 3 plots the outstanding Treasury bills (the solid line) and the *net* supply of money-like securities by intermediaries, i.e., the sum of intermediaries’ net liabilities in each instrument listed in Figure 1 minus their holdings of government securities.⁴ A key message is the collapse of inside money supply in the Great Recession, a more than 50% decline from the second quarter of 2007 (the peak) to end of

⁴Government securities include Treasury securities and vault cash held by depository institutions, and Treasury securities indirectly held through money market funds by other financial intermediaries.

2009 and a continuing contraction afterwards to a level lower than 1960s. This secular depression of inside money creation is consistent with the model's prediction on stagnant crises.

Treasury Bill supply increased by more than 100% during the Great Recession. The model predicts that such an increase prolongs the crisis by crowding out intermediaries' profit, and thus, delaying the repairment of intermediaries' balance sheet. To some extent, Treasury Bill supply mitigates the money shortage that the productive sector faces, so we do not see a slump in nonfinancial firms' money holdings (Figure 2) that is as large as the slump of inside money supply. However, the yield on money-like securities has been extremely low in the post-crisis period, which signals a persistent scarcity very likely due to the depressed inside money creation.

Panel B of Figure 3 plots the share of inside money supplied by non-depository financial intermediaries, i.e. the "shadow banks", such as broker-dealers, finance companies, and issuers of asset-backed securities. Many have argued that other sectors' demand for money-like securities is a major driver of the growth of shadow banking (Gorton (2010), Stein (2012), and Pozsar (2011) and (2014)). In comparison with traditional banks (depository institutions), shadow banks seemed to be more responsive to rise of money demand in the productive sector in the last two decades, and until the global financial crisis, took an increasingly large share of inside money supply.

1.3 Government debt and intermediary leverage cycle

This section provides evidence on the impact of outside money on the intermediary leverage cycle. I focus on one type of intermediaries, broker-dealers (or investment banks). Adrian and Shin (2010) document the procyclicality of broker-dealer leverage: when their assets grow, their debt grows faster, leading to higher leverage. Broker-dealers issue money-like securities (mainly repurchase agreements) that are held by firms and other entities through money market mutual funds. I focus on investment banks because relative to commercial banks, their choice of leverage is subject to less regulatory constraints, and thereby, they are closer to the laissez-faire banks in the model, and as shown in Figure 3, shadow banks play a more important role than traditional banks in the pre-crisis boom of inside money creation. Leverage is defined as the ratio of total book assets to book equity, using the data from the Financial Accounts, following Adrian and Shin (2010).

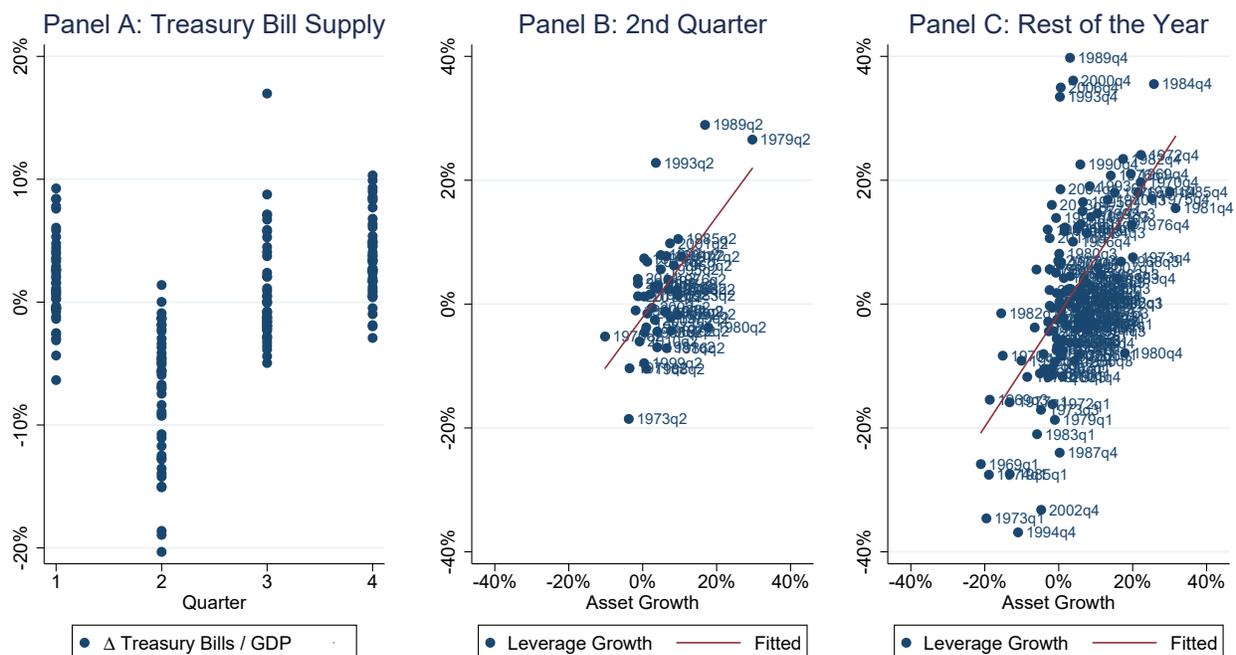


Figure 4: Intermediary Leverage Cycle and Government Debt. Panel A plots the change of Treasury bills outstanding scaled by nominal GDP by quarter. Panel B plots the quarterly growth rate (log difference) of broker-dealer book leverage (Adrian and Shin (2010)) against the growth rate of broker-dealer book assets in the second quarter, and Panel C plots the quarterly growth rate of leverage against the growth rate of assets in the first, third, and fourth quarters. The data source is the Financial Accounts of the United States from 1968Q3 to 2015Q3.

The sample is 1968Q3–2015Q3.⁵ He, Kelly, and Manela (2016) discuss issues related to internal capital markets that may induce measurement errors in leverage. For government debt, I use the ratio of Treasury Bills to nominal GDP. Short-term debt is more money-like in the sense that their value is more stable, their secondary market more liquid, and repo haircuts smaller.

In identifying the impact of government debt on intermediary leverage, one of the challenges is the endogeneity of government debt supply. Greenwood, Hanson, and Stein (2015) argue that Treasury Bill supply has seasonality. It expands ahead of statutory tax deadlines (e.g., April 15th) to meet its ongoing needs, and contracts following these deadlines. Greenwood, Hanson, and Stein (2015) and Nagel (2016) use week and month dummies respectively to instrument Treasury Bill supply. Panel A of Figure 4 shows that the strongest seasonality is in the second quarter of the year. Because my data is quarterly, I use the Q2 dummy as an instrument for T-bill supply.

⁵Adrian, Etula, and Muir (2014) discuss the data quality concerns in the pre-1968 sample.

Panel B and C of Figure 4 plot the log difference of book leverage (i.e., quarterly growth) against the book asset growth, extending the main result of Adrian and Shin (2010) to this longer sample. Panel B separates out the observations from the second quarters, and Panel C shows the rest of the year. The fitted line is steeper in Panel C, which seems to be consistent with the model’s prediction – increases in government debt amplify the procyclicality of intermediary leverage.

Another identification challenge is the endogeneity of asset growth. In the model, the cyclicity of leverage is defined with respect to *exogenous* shocks that increase or decrease banks’ asset value through their impact of collateral quality. Such structural shocks are rarely observed in reality. However, for the purpose of empirical analysis, any exogenous shocks that affect banks’ asset value can be used as instruments. I use monetary policy shocks as such instruments, and consider two measures: the unanticipated changes in the Fed Funds Rate around the FOMC (Federal Open Market Committee) announcements (“MP-FFR”), and a composite measure of unanticipated changes in several interest rates around the FOMC announcements proposed by Nakamura and Steinsson (2017) (“MP-Comp”).⁶ Shocks are aggregated to quarterly level. Calculated from high-frequency data, these policy shocks arguably reflect the changes in interest rates only from the unexpected content of FOMC announcements, and thus, tend to be orthogonal to contemporaneous variations of other economic variables. Through the impact on interest rates and various spreads, monetary policy shocks affect the value of banks’ assets.⁷

Table A.2 reports the results. The first column replicates the main regression in Adrian and Shin (2010). The left-hand side is leverage growth and the right-hand side is asset growth. A positive coefficient shows the procyclicality of leverage. The fourth column of Table A.2 shows the coefficients from regressing leverage growth on both asset growth and the interaction between asset and Treasury bill growth. A positive coefficient on the interaction term suggests when government debt increases, leverage becomes more procyclical, consistent with the model’s prediction.

Column 2 and 3 show leverage procyclicality using different measures of monetary policy

⁶A monetary policy shock is calculated using a 30-minute window from 10 minutes before the FOMC announcement to 20 minutes after it. The calculation follows Nakamura and Steinsson (2017). For earlier contributions, please refer to Cook and Hahn (1989), Kuttner (2001), and Cochrane and Piazzesi (2002) among others.

⁷Gertler and Karadi (2015) show that these high-frequency monetary policy shocks affect term premia and credit spreads, and Hanson and Stein (2015) show the strong effect on forward real rates even in the distant future.

Table A.2: Government Debt and Procyclical Leverage.

This table reports the evidence on the impact of government debt on the relation between broker-dealer asset growth and leverage growth. Column (1) reports the results of regressing leverage growth (quarterly log difference) on asset growth (as in Adrian and Shin (2010)). The sample is from 1968Q3 to 2015Q3. Column (2) and (3) repeat the regression in Column (1) with two types of monetary policy shocks, MP-FFR and MP-Comp, as instrument variables (IVs) for broker-dealer asset growth. MP-FFR and MP-Comp are the quarterly sums of unexpected Fed Funds rate change and composite rate changes respectively around FOMC announcements (available from Nakamura and Steins-son (2017) from 1995 to 2014). Column (4) reports the results of regressing broker-dealer leverage growth on asset growth and the interaction (product) between asset growth and the growth rate of Treasury bills scaled by nominal GDP. Column (5) and (6) repeat the regression in Column (4) with the second quarter dummy as IV for Treasury bill growth, and with MP-FFR (Column 5) or MP-Comp (Column 6) as IV for broker-dealer asset growth. All coefficients are estimated using GMM with Newey-West HAC standard errors reported in parentheses (optimal number of lags chosen following Newey and West (1994)). *, **, *** represent $p < 0.10$, $p < 0.05$, $p < 0.01$ respectively.

$\Delta \ln (Leverage)$	(1)	(2)	(3)	(4)	(5)	(6)
IV for $\Delta \ln (Assets)$		MP-FFR	MP-Comp		MP-FFR	MP-Comp
IV for $\Delta \ln (\frac{T-Bill}{GDP})$					Q2	Q2
$\Delta \ln (Assets)$	0.910*** (0.113)	3.873*** (1.451)	2.527*** (0.855)	0.203 (1.294)	0.991** (0.449)	0.867** (0.348)
$\Delta \ln (\frac{T-Bill}{GDP}) \cdot \Delta \ln (Assets)$				24.40 (38.57)	10.28*** (2.174)	8.999*** (2.220)
Observations	189	77	77	188	77	77

shocks as instruments for asset growth. Both estimates are positive and significant. Column 5 and 6 use the Q2 dummy as an instrument for Treasury Bill supply, and use two measures of monetary policy shocks respectively to instrument asset growth. The coefficient on the interaction term is positive and significant. These confirm the findings in baseline specifications.

Table A.2 provides evidence that supports the predictions of the model in main text. But it is far from conclusive. Future research based on longer samples, international data, and alternative identification strategies, shall provide a better evaluation of the model's predictions on intermediary leverage cycle, the impact of government debt on it, and other results such as how banks' payout and issuance policies respond to government debt supply.

2 Discussion: Government Debt as Outside Money

Optimal timing of government debt supply. This paper only considers the simplest case of fixed government debt supply, but the model does reveal an interesting trade-off that calls for an optimal strategy of government debt management. When the government issues more debt, it benefits the firms by reducing the money premium, but at the same time, it hurts the bankers who rely on the money premium as a source of profit. Its decision to increase or decrease its debt should balance the impact on both sectors, and in particular, depend on which sector is more constrained.

When η_t is high and banks are already supplying a large amount of money, the marginal benefit of increasing government debt is small. When banks are undercapitalized and not creating enough deposits, raising government debt can significantly alleviate the money shortage. This suggests a countercyclical government debt supply. However, the bank crowding-out effect favors procyclical government debt supply. In good times, banks' leverage rises, making the economy unstable. The government should increase its debt to crowd out bank leverage. And since banks are well capitalized, the government worries less about crowding out banks' profit.⁸ In low η_t states, the equity crowding-out effect becomes a major concern. The government may want to reduce its debt, allowing banks to rebuild equity by earning a high money premium.

Bank equity crowding in. So far, we have only considered the competition between government and banks as money suppliers. Under additional frictions, government debt may be held by banks for their own liquidity needs (Bianchi and Bigio (2014); Drechsler, Savov, and Schnabl (2017)) or as collateral (Saint-Paul (2005); Bolton and Jeanne (2011)). Banks may also hold government securities for regulatory purpose, for example to meet the liquidity coverage ratio (Basel Committee on Bank Supervision (2013)). Therefore, increases in government debt could relax banks' liquidity or regulatory constraints, and thereby, may increase their profit and crowd in equity.

Optimal bailout scale. In bad states, the government may intervene to recapitalize the banking sector, like the Troubled Asset Relief Program ("TARP") in the financial crisis of 2007-09. On the

⁸But by crowding out banks' profit, raising government debt gives banks more incentive to pay out dividends at a lower level of equity. By decreasing the payout boundary, it strengthens the asymmetric impact of shocks, making the impact of negative shocks stronger relative to good shocks. This negative effect needs to be weighed in against the positive effect on stability from the reduction of bank leverage.

one hand, the banking sector benefits from equity injection. On the other hand, banks' profit from money creation is squeezed by the government debt that finances the bailout program. Balancing the two effects, the government may find an optimal scale of bailout financed by government debt in the sense that the expected time to recover is minimized.

Instability from coexistence of regulated and unregulated banks. We can reinterpret M^G as money supplied by a separate banking sector that is fully regulated and backs deposits with 100% reserves in the form of government debt. Regulated banks' balance sheets are just a pass-through. Their money supply grows proportionally with the economy. Such a banking sector was proposed in the Chicago Plan.⁹ The competition between government and banks in supplying money can thus be reinterpreted as the competition between fully regulated banks and unregulated banks, or "shadow banks". Besides many issues surrounding the Chicago Plan, this paper points to a particular concern. If private money cannot be completely forbidden, the leverage cycle in unregulated shadow banking can be amplified by money that is fully backed government securities.

Instability from market liquidity. In reality, firms can and do hold some of other firms' or households' liabilities in their liquidity portfolios, provided that these securities have sufficiently liquid secondary markets. We can define $M^F K_t$ as the maximum amount of liquid securities that firms and households can issue in aggregate, so M^F is almost a measure of financial market development. Since firms and households are willing to issue any securities that promise an expected return less or equal to ρ , firms and households will maximize their issuance in the presence of money premium. To analyze the effects of market liquidity on growth and stability, we can simply follow the analysis of government debt, and all the conclusions carry through. Through the lens of the model, larger and deeper secondary security markets (i.e., higher M^F) can amplify the bank leverage cycle and prolong recessions. From 1975 to 2014, the stock market capitalization in the United States increased from c.40% of GDP to c.140%. The competition between market and intermediated liquidity is not the focus of this paper, but the model does point out a possibility that from a liquidity provision perspective, financial markets can destabilize financial intermediaries.

⁹It is a banking reform initially proposed by Frank Knight and Henry Simons of the University of Chicago and supported by Irving Fisher of Yale University (Phillips (1996)). Benes and Kumhof (2012) revisited the plan in the interest of financial stability.

Relaxing the financial constraint. It has been assumed that when the liquidity shock hits, a firm cannot mobilize any resources other than its money holdings because existing capital has been destroyed and the investment project is not pledgeable. One way to relax the financial constraint is to introduce a liquidation value of the firm, say $M^L k_t$, where M^L is a constant, so creditors can liquidate the firm and seize $M^L k_t$ when the firm defaults. Thereby, when the liquidity shock arrives, the firm has liquidity equal to $(m_t + M^L) k_t$, the sum of money holdings and the pledgeable value of the firm. Now, we have a new money demand curve:

$$\rho - r_t = \lambda [q_t^K F'(m_t + M^L) - 1].$$

The role that M^L plays is the same as M^G (and M^F in the analysis of market liquidity). Therefore, the previous analysis carries through.¹⁰ Relaxing the financial constraint indeed reduces firms' demand for bank debt, but it may amplify the bank leverage cycle, prolong recessions, and actually make the economy worse. To understand the financial stability implications of liquidity provided by the government (M^G), the secondary market of securities (M^F), and the productive capital (M^L), we must take into account the endogenous response of the banking system. More liquidity is not always beneficial.

¹⁰There is one caveat – more capital holdings (i.e., larger k_t) relaxes the liquidity constraint on future investments by raising the liquidation value, so capital price q_t^K is likely to increase in comparison with the benchmark case. However, this mechanism enhances the procyclicality of q_t^K , and thus, amplifies the procyclicality of bank leverage.

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