

# Dynamic Banking and the Value of Deposits\*

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## Abstract

We propose a dynamic theory of banking where the role of deposits is akin to that of productive capital in the classical Q-theory of investment for non-financial firms. As a key source of leverage, deposits create value for well-capitalized banks. However, unlike productive capital of nonfinancial firms that typically has a positive marginal  $q$ , the deposit  $q$  can turn negative for undercapitalized banks. Demand deposit accounts commit banks to allow holders to withdraw or deposit funds at will, so banks cannot perfectly control leverage. Therefore, for banks with insufficient equity-capital to buffer risk, deposit inflows and the associated uncertainty in future leverage can destroy value. Our model predictions on bank valuation and dynamic asset-liability management are broadly consistent with the evidence. Moreover, our model lends itself to a re-evaluation of the costs and benefits of leverage regulation and offers alternative perspectives on banking in a low interest rate environment.

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# 1 Introduction

During the unfolding COVID-19 pandemic, the U.S. banks have undergone unprecedented balance-sheet expansions as a result of massive inflows into deposit accounts. Most dramatically, deposits of US banks increased by \$865 billion just in April 2020 alone. From Q4 2019 to Q1 2020, JPMorgan Chase experienced an increase of 18% percent of its deposit base, and the deposit liabilities of Citigroup and Bank of America increased by 11% and 10%, respectively.<sup>1</sup> Contrary to the conventional wisdom, abundant funding liquidity did not benefit bank valuation. The banking sector is among the slowest sectors to recover from the pandemic lows of equity valuation.

Large deposit inflows are both an opportunity and a challenge for banks. As the literature has emphasized, the bank business model rests on deposit taking. Demand deposit account is a source of cheap funding that banks rely on to finance their lending and trading activities. Depositors accept relatively low rates for the convenience of using deposits as means of payment. But the consequence of allowing depositors to freely move funds in and out of their accounts is that banks cannot perfectly control the size of their deposit base and balance sheet. Therefore, deposit inflows present a challenge to banks' risk management because it is uncertain whether the new deposits will stay in the customers' accounts or be paid out in the near future.

To the extent that the banking literature is modeling the deposit-flow risk, it has done so only by assuming that loss of control of the balance sheet manifests itself by deposit outflows in a bank-run equilibrium (Diamond and Dybvig, 1983). Such models assume that, in the no-run equilibrium, banks face no risk with respect to the size of deposit base so that deposits can be treated as fixed-maturity debts. We depart from this framing and treat banks' deposit base more generally as a persistent and stochastic process subject to both inflow and outflow shocks in a fully dynamic setting. The stochastic law of motion of deposit stock captures the aggregate behavior of a large number (continuum) of atomic depositors. As in Drechsler, Savov, and Schnabl (2020), deposits are effectively term debts, but different from their paper, the maturities of debts are random in our model. The key ingredient is that the law of motion of deposit stock can be only partially

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<sup>1</sup>See "U.S. Banks are 'Swimming in Money' as deposits increase by 2 trillion dollars amid the coronavirus" by Hugh Son, CNBC June 21, 2020. Such deposit influx also happened in the financial crisis of 2007–2008.

controlled through deposit rate. Such uncertainty in the size and composition of balance sheet distinguishes a bank from a non-financial firm. We then add two important frictions to this model.

First, we assume in line with the evidence that it is costly for a bank to issue new equity. As in Brunnermeier and Sannikov (2014) and Klimenko, Pfeil, Rochet, and Nicolo (2016), the issuance costs cause the bank to be endogenously risk averse and make maintaining equity capital essential. The degree of this endogenous risk aversion varies with  $k$ , the ratio of equity capital to deposit stock, which is the key state variable. When the ratio  $k$  is high, the bank is well-capitalized and deposit inflow creates value by allowing the bank to cheaply finance risky lending. The bank's risk-taking behavior is summarized by a formula akin to that of Merton (1969) but with endogenous risk aversion. When the ratio  $k$  is low, the bank has relatively less equity capital to buffer the shocks to its deposit base and the risk from lending activities. As a result, deposit inflow becomes burdensome as it further reduces the equity capital-to-deposit ratio, making risk management more challenging and costly equity issuance more likely.

The bank adjusts deposit flows through deposit rate. When it is well-capitalized, the bank raises deposit rate to attract deposits. In our model and as documented by Drechsler, Savov, and Schnabl (2017), deposit base is sticky with random yet persistent flows. Setting a higher rate to attract deposits is just like investing in a customer base that brings cheap financing for future risk-taking. Indeed, the optimal deposit-rate policy resembles a nonfinancial firm's investment policy in Hayashi (1982). When the bank is undercapitalized (i.e., the equity capital-to-deposit ratio  $k$  is low), risk-management considerations dominate. The bank lowers its deposit rate to discourage deposit inflows mitigating unintended expansion of its leverage (i.e., a further decline of  $k$ ).

The second friction we add is a lower bound for the deposit rate. A natural bound is zero, which can be motivated by the threat of withdrawal en masse and liquidation. While this friction is not required to generate endogenous risk aversion, it further limits the bank's ability to adjust the size of its deposit liabilities and balance sheet, strengthening the mechanism. Empirically, this deposit rate lower bound has become increasingly binding in the current low-rate environment. Indeed, banks are loath to impose negative rates on deposits in practice even if this could help stem

an inflow of new deposits and the associated involuntary expansion of leverage.<sup>2</sup>

Finally, another distinguishing feature of banks is that to be able to continue operating, a bank must sustain a level of equity capital that is above a regulatory minimum. Due to the deposit-flow risk, a bank does not have full control over its balance-sheet size and composition. Unexpected deposit inflows increase leverage, so when the bank is undercapitalized, it has to raise costly equity to avoid violating the regulatory restriction. Regulation does not cause endogenous risk aversion (equity issuance costs do) but regulation amplifies it. Therefore, leverage regulation can make deposit-taking more costly and make banks more reluctant to lend. During the Covid-19 pandemic, the U.S. banking regulators relaxed the supplementary leverage ratio requirements. Our model provides a rationale for such a response.

Next, we provide a more detailed summary of the bank's dynamic asset-liability management decisions including optimal deposit rate, short-term borrowing, lending, equity issuance, and dividend payout.<sup>3</sup> We solve for the franchise value of the bank and show how it varies with the equity capital-to-deposit ratio  $k$ , generating endogenous risk aversion. We also solve for the marginal value of deposits ("marginal  $q$  of deposits") and show that it can be negative when the bank is undercapitalized. This key result explains why deposit influx during the Covid-19 pandemic can hurt bank valuation when bank capital was weakened by enormous loan loss provisions. After presenting the model solution, we summarize its implications on capital regulation, banking in a low interest-rate environment, and the impact of deposit market power on bank valuation.

Because it is costly to issue equity, the marginal value of equity capital can be greater than one, and depends on the value of equity capital-to-deposit ratio,  $k$ , which is bounded by two endogenous reflecting boundaries. The marginal value of equity capital varies over a wide range: it is equal to one at the dividend payout boundary, when  $k$  is high and the bank is indifferent between retaining an extra unit of earnings or paying it out (that is how the endogenous upper bound is

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<sup>2</sup>Heider, Saidi, and Schepens (2019) document the bunching of deposits rates at zero in the Euro area.

<sup>3</sup>We do not add any other frictions that may be relevant in practice. One friction that is absent is fire-sale pricing (or other forms of asset adjustment costs), which happens when a bank seeks to quickly shrink its asset base. In our model, the bank can costlessly change the size of its asset portfolio, so our focus is entirely on the lack of control of deposit liabilities rather than assets. This is a reasonable assumption for financial securities. Admittedly, this is a stronger assumption as far as the bank's loan book is concerned. Adding an adjustment cost to the loan book, however, is a major complication as this would introduce a second state variable, rendering the mechanism much less transparent.

characterized), and when  $k$  is low, it can rise to above nine at the equity issuance boundary (the lower endogenous boundary of  $k$ ), even under conservative values for equity issuance costs from the empirical literature. When the bank is close to hitting its regulatory leverage restriction, equity capital is scarce and highly valuable as it reduces the likelihood of costly equity issuance.

The marginal value of equity capital is highly nonlinear in  $k$ . The value function is strictly concave in  $k$ , meaning that the bank is endogenously risk-averse even though shareholders are assumed to be risk-neutral. We show that it is optimal for the bank to substantially reduce lending as  $k$  declines and the bank approaches the equity issuance boundary. This is consistent with empirical findings linking changes in bank equity capital to bank lending. When  $k$  increases and the bank approaches the payout boundary, the marginal value of equity quickly converges to one. Indeed, at the peak of the stationary density of  $k$  the marginal value of equity is only slightly above one, so that, for the majority of time, the bank does not seem to be financially constrained. This nonlinearity captures a sharp contrast between the normal times and the crisis times when  $k$  is low, close to the equity issuance boundary, and the marginal value of equity capital shoots up dramatically.

In our model, deposits are valuable because depositors are willing to accept a deposit rate that is below the risk-free rate. This can be motivated by the convenience of payment services offered by deposit accounts, which also exposes the bank to deposit-flow uncertainty. When the bank has sufficient equity capital, deposits create value by allowing the bank to finance risky lending with cheap sources of funds. As a pool of cheap funding, the deposit stock in effect serves as a form of productive capital for the bank. However, when the bank's equity capital is depleted, the marginal value of deposits can be negative. The reason is that, near the (lower) boundary of costly equity issuance, uncontrollable deposit inflows destroy value for the bank's shareholders by increasing the bank's leverage and amplifying the likelihood of costly equity issuance. The bank then wants to deleverage and turn away deposits. However, the bank can only go as far as setting the deposit rate at the lower bound; it cannot turn down deposits by further lowering the rate.

Losing control of its deposit base is a problem when the bank faces equity issuance costs, for then, in effect, it also loses control of its leverage and runs the risk of paying the issuance costs. In our model, we draw a sharp distinction between deposits and short-term debt. With short-

term debt, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debt. Deposits are long-term contracts and do not have a well-defined maturity. Deposits leave the bank only when depositors choose to withdraw funds. When the equity capital-to-deposit ratio,  $k$ , is high, the bank issues short-term debt to obtain additional leverage for lending. If  $k$  declines, the bank deleverages by reducing short-term debt. If  $k$  declines further and approaches the lower boundary of costly equity issuance, the bank can even switch from short-term borrowing to holding risk-free assets thereby reducing its leverage and de-risking its asset portfolio. In contrast, the deposit stock is subject to shocks and cannot be perfectly controlled. Moreover, once the deposit rate hits the lower bound, the bank completely loses control of deposit stock. Managing the deposit risk is a key task that distinguishes banks from other financial intermediaries.

Capital regulation that imposes a cap on leverage amplifies the cost of deposit-taking because unexpected deposit inflow drives up leverage, pushing the bank closer to the violation of regulation and costly recapitalization. During the Covid-19 pandemic, the U.S. banking regulators relaxed leverage regulation. Our model shows that such responses stimulate lending and deposit-taking. Such measures can be particularly effective in a low interest rate environment where the deposit rate lower bound is likely to bind and the bank is very concerned about losing control of leverage. In contrast, tightening leverage regulation reduces deposit  $q$  and discourages deposit-taking.

Our model predicts a permanent negative impact of heightened leverage regulation on bank value. Kashyap, Stein, and Hanson (2010) argue that the impact should be temporary instead, because in a deterministic environment, the bank pays the equity issuance costs once and then settles on a lower leverage. With deposit and loan-return shocks, costly equity issuance is recurrent in our model. The issuance costs are therefore reflected in bank value even away from the equity issuance boundary. Tightening leverage regulation permanently reduces bank value by making costly equity issuance more frequent. An unintended consequence is that to offset the increase in issuance costs, the bank has to generate more earnings for shareholders to break even in expectation, and to achieve that, the bank increases risk exposure per unit of equity capital. Leverage regulation, while successfully builds up bank capital, fails its original purpose of taming risk-taking.

Our model also sheds light on the critical role of interest rate level in bank valuation and

balance-sheet management. As in Drechsler, Savov, and Schnabl (2017), the bank earns a deposit spread (the wedge between the risk-free rate and the lower deposit rate). The bank raises deposit rate when  $k$  is higher so that when  $k$  declines in the future (for example, following unexpected deposit inflows), the bank will have more room to reduce deposit rate before hitting the deposit rate lower bound. Therefore, when the risk-free rate is high, the bank has more flexibility in raising deposit rate in the high- $k$  region without squeezing the deposit spread too much. The distance between the risk-free rate and deposit rate lower bound essentially determines the degree of flexibility to regulate deposit flows through deposit rate. In a low interest rate environment, the bank has less flexibility, so deposit  $q$  declines. Moreover, bank franchise value suffers in a low interest rate environment, which causes the bank to become more aggressive in shareholder payout. This helps explain the massive bank stock buybacks in the last decade of low interest rates.

**Literature.** In the banking literature, the focus is on bank runs when it comes to banks' commitment to allow depositors to withdraw funds without prior notice (Diamond and Dybvig, 1983; Allen and Gale, 2004; Goldstein and Pauzner, 2005). However, the deposit flow risk is more ubiquitous than the dramatic bank runs and influences banks' daily operation. Moreover, our paper is the first to show that the uncertainty in both deposit outflows and inflows poses a challenge to banks under equity issuance costs. Equity issuance costs have been incorporated in the models of bank capital structure (Bolton and Freixas, 2000; Allen, Carletti, and Marquez, 2015).

Depositors are willing to accept relatively low rates because of the convenience of using deposits as means of payment. What enables deposits as money is precisely banks' commitment to allow depositors to move funds in and out of their accounts. Banks are thus exposed to large payment flow shocks (Furfine, 2000; Bech and Garratt, 2003; Denbee, Julliard, Li, and Yuan, 2018; Choudhary and Limodio, 2017). Deposit maturities are not chosen by the bank and depend on depositors' payment needs that are uncertain (Freixas, Parigi, and Rochet, 2000; Donaldson, Piacentino, and Thakor, 2018; Parlour, Rajan, and Walden, 2020). With a diversified depositor base, a bank views deposits as debts that retire at a stochastic rate. Bianchi and Bigio (2014), De Nicolò, Gamba, and Lucchetta (2014), and Vandeweyer (2019) also recognize such shocks. They model deposits as one-period debts with intra-period shocks, so banks can freely adjust the

deposit base every period. In contrast, deposit contract in reality (and in our model) is long-term – depositors can hold deposits as long as they want (Drechsler, Savov, and Schnabl, 2020). To adjust deposit stock, banks can change deposit rate but completely loses control of deposit stock when its deposit rate hits the lower bound. The key to our main results is precisely banks’ lack of control of the deposit base, which exposes banks to the risk of incurring equity issuance costs.

Deposits are inside money of the private sector – stores of value and means of payment issued by banks. This feature has been well recognized in the recent macro-finance literature. Deposits are modelled as short-term debts (Piazzesi and Schneider, 2016; Drechsler, Savov, and Schnabl, 2018) with interest rates below the risk-free rate by a money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2019; Begenau, 2019). Brunnermeier and Sannikov (2016) and Drechsler, Savov, and Schnabl (2020) are notable exceptions. Brunnermeier and Sannikov (2016) model deposits as infinite-maturity nominal liabilities and study the general equilibrium implications. Our model is more related to Drechsler, Savov, and Schnabl (2020) who find that sticky deposit liabilities have long duration. Our contribution is to incorporate the random evolution of deposit liabilities and analyze the impact of deposit risk on bank valuation and asset-liability management. Dynamic banking models typically differentiate short-term debts and deposits in both interest rate and operation costs (Hugonnier and Morellec, 2017; Van den Heuvel, 2018; Begenau, 2019). In these models, banks do not face uncertainty in the size of deposit liabilities.

## 2 Model

We model the decisions of a single bank that maximizes risk-neutral shareholders’ value.<sup>4</sup> Let  $r$  denote the risk-free rate, which is also the discount rate of bank shareholders.

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<sup>4</sup>Risk-neutrality can be reinterpreted as modelling under the risk-neutral measure by taking as exogenous a pricing kernel (stochastic discount factor) that depends on the aggregate dynamics of the broader economy. Then the risk-free rate,  $r$ , is the expected return under the risk-neutral measure of all financial assets that are traded by bank shareholders.

**Risky Assets.** We use  $A_t$  to denote the value of the bank's holdings of loans and other revenue-generating assets at time  $t$ . The value of the asset evolves as follows:

$$dA_t = A_t (r + \alpha_A) dt + A_t \sigma_A d\mathcal{W}_t^A. \quad (1)$$

The parameter  $\alpha_A$  reflects the return from the bank's expertise.<sup>5</sup> The second term in (1) describes the shock to the asset value, where  $\sigma_A$  is the diffusion-volatility parameter and  $\mathcal{W}^A$  is a standard Brownian motion. Examples of these shocks include unexpected charge-offs of delinquent loans. At any time  $t$ , the bank may adjust its risky assets and the liability structure.

**Deposits.** Deposits are at the core of our model. The stock of deposits at time  $t$ , which we denote by  $X_t$ , evolves as follows:

$$dX_t = -X_t (\delta_X dt - \sigma_X d\mathcal{W}_t^X) + X_t n(i_t) dt, \quad (2)$$

where  $\mathcal{W}_t^X$  is a standard Brownian motion driving the shock to the deposit base. Let  $\phi dt$  denote the instantaneous covariance between  $d\mathcal{W}_t^X$  and  $d\mathcal{W}_t^A$ . The deposit flow is given by  $(\delta_X dt - \sigma_X d\mathcal{W}_t^X)$  in  $dt$ . When  $(\delta_X dt - \sigma_X d\mathcal{W}_t^X) < 0$ , the depositors in aggregate withdraw deposits. The randomness in withdrawal, measured by  $\sigma_X$ , is in line with the traditional three-dates models (Diamond and Dybvig, 1983; Allen and Gale, 2004).<sup>6</sup> In reality, deposit flows are driven by payment activities. When the bank's depositors pay agents who hold accounts at the other banks, the bank incurs deposit outflow. When its depositors receive cash payments or electronic payments from other banks' depositors, the bank incurs deposit inflow, i.e.,  $(\delta_X dt - \sigma_X d\mathcal{W}_t^X) < 0$ .<sup>7</sup>

The bank optimally chooses the deposit rate,  $i_t$ , and adjusts the growth rate of deposits via  $n(i_t) dt$ . Raising the deposit rate induces deposits inflow, i.e.,  $n'(i_t) > 0$ . The deposit demand

<sup>5</sup>The bank may have expertise in monitoring (Diamond, 1984), loan screening (Ramakrishnan and Thakor, 1984), relationship lending (Boot and Thakor, 2000), restructuring (Bolton and Freixas, 2000), asset management (He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014, 2016), collateralization (Rampini and Viswanathan, 2018), and serving local credit markets (Gertler and Kiyotaki, 2010).

<sup>6</sup>In these models, agents' stochastic preferences over consumption timing causes uncertainty in deposit outflows.

<sup>7</sup>The value of  $\delta_X$  and  $\sigma_X$  largely depend on where the bank sits in the payment network, and the payment flow volatility  $\sigma_X$  can be significant in data (Denbee, Julliard, Li, and Yuan, 2018).

elasticity depends on the bank’s deposit market power (Drechsler, Savov, and Schnabl, 2017) and the convenience yield that agents derive from holding deposits as means of payment (DeAngelo and Stulz, 2015; Nagel, 2016; Piazzesi and Schneider, 2016; Li, 2018, 2019). For deposits to function as means of payment, depositors must be able to move funds in and out of their accounts freely, and this exposes the bank to unhedgeable deposit-base shock which we capture in (2) (Freixas, Parigi, and Rochet, 2000; Parlour, Rajan, and Walden, 2020). Moreover, following Hugonnier and Morellec (2017) and Drechsler, Savov, and Schnabl (2020), we assume that to maintain the existing deposits and attract new deposits, the bank pays a flow cost  $C(n(i_t), X_t) dt$ , which captures the expenses of maintaining branches, servicing customers, and marketing.

Deposits are essentially long-term debts with stochastic maturity and controllable drift. Not all depositors withdraw or deposit funds at the same time. Therefore, a diversified depositor base implies an effective duration of deposits that depends on the stochastic rate of net withdrawal. Our treatment of deposits stands in contrast with the macro-finance literature and dynamic banking literature that generay treats deposits simply as short-term debts. We share with Drechsler, Savov, and Schnabl (2020) the view that the right to withdrawal does not necessarily translate into a low duration of deposits. We emphasize that the right to withdrawal imposes a lower bound on the feasible deposit rate – the bank cannot set a negative deposit rate because depositors will withdraw en masse and earn the zero return on dollar bills that also work as means of payment. In reality, depositors may tolerate an effective negative deposit rate (often in the form of fees) due to the inconvenience of carrying dollar bills, but as long as there exists a lower bound, our results hold.

**Bonds.** The bank can trade standard risk-free bonds and it is costless to do so. Let  $B_t$  denote the value of bonds that the bank issues at  $t$  and will mature at  $t + dt$ . When  $B_t > 0$ , the bank issues bonds (e.g., commercial papers) and incurs interest expenses of  $B_t r dt$  over time interval  $dt$ . When  $B_t < 0$ , the bank holds bonds issued by other entities (e.g., the government).

**Equity, Dividends, and Costly Issuance.** Let  $K_t$  denote the bank's equity capital. The following identity describes the balance-sheet:

$$K_t = A_t - (B_t + X_t) . \quad (3)$$

The bank can pay out dividends that reduce  $K_t$ . We use  $U_t$  to denote the (undiscounted) cumulative dividends, so the amount of (non-negative) incremental payout is  $dU_t$ .

The bank may find it optimal to issue external equity. In reality, banks face significant external financing costs due to asymmetric information, incentive issues, and transaction costs. A large empirical literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction in response to the announcement of a new equity issue.<sup>8</sup> Let  $F_t$  denote the bank's (undiscounted) cumulative net external equity financing up to time  $t$  and  $H_t$  to denote the corresponding (undiscounted) cumulative costs of equity financing up to time  $t$ .

The bank maximizes the equityholders' value.<sup>9</sup> The objective function is given by

$$V_0 = \mathbb{E} \left[ \int_{t=0}^{\tau} e^{-\rho t} (dU_t - dF_t - dH_t) \right] , \quad (4)$$

where  $\tau$  is the stochastic stopping time of bank closure. Regulators will shut down the bank when its equity or net worth,  $K$ , turns negative. Moreover, we assume that shareholders are more impatient than creditors in that shareholders' required rate of return  $\rho$  is greater than  $r$ , a common assumption in dynamic corporate finance and macro-finance models, e.g., DeMarzo and Fishman (2006) and Brunnermeier and Sannikov (2014) among others.<sup>10</sup>

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<sup>8</sup>Explicitly modeling informational asymmetry would result in a substantially more involved analysis. Lucas and McDonald (1990) provides a tractable analysis by making the simplifying assumption that the informational asymmetry is short lived, i.e. it lasts one period. Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs) of equity, the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), the direct costs average 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was  $-3\%$  and the loss in equity value as a percentage of the size of the new equity issue was as high as  $-31\%$  (Eckbo et al., 2007).

<sup>9</sup>In our model, the bank manager's incentive is aligned with equityholders. The issuance cost partially captures insiders' distinct incentives, as a high insider ownership is empirically associated with reluctance to issue equity (Goetz, Laeven, and Levine, 2020). Becht, Bolton, and Röell (2011) review the issues of bank corporate governance

<sup>10</sup>This impatience can be microfounded by a Poisson exit rate that is equal to  $\rho - r$ .

**Capital Requirement.** The bank must meet a capital requirement. For example, the Basel III accords stipulate that banks must back a specific percentage of risk-weighted assets with equity.<sup>11</sup> As in Nguyen (2015), Davydiuk (2017), Van den Heuvel (2018), and Begenau (2019), we introduce the capital requirement as follows:

$$\frac{A_t}{K_t} \leq \xi_K. \quad (5)$$

In accordance with Basel III capital standards, banks maintains a minimal ratio of capital to risk-weighted assets of 7%. We set  $\xi_K$  equal to  $1/0.07 = 14.3$ .<sup>12</sup>

**Supplementary Leverage Ratio (SLR).** Since January 1, 2018, banks in the U.S. face an SLR restriction. It supplements the capital requirement that can be vulnerable to manipulation (Plosser and Santos, 2014). The U.S. bank holding companies that have been identified as global systemically important banks (“U.S. G-SIBs”) must maintain an SLR of greater than 5%, and if they fail to do so, they will be subject to increasingly stringent restrictions on its ability to make capital distributions to shareholders and discretionary bonus payments to the management.

The SLR requirement targets the ratio of the bank’s total assets (or total liabilities) to equity capital. When the bank issues bonds, i.e.,  $B > 0$ , the leverage ratio requirement restricts  $A/K$ ,

$$\frac{A}{K} = \frac{K + X + B}{K} \leq \xi_L, \quad (6)$$

just as the capital requirement does, and when  $B < 0$ , the leverage ratio requirement is given by

$$\frac{A - B}{K} = \frac{K + X}{K} \leq \xi_L. \quad (7)$$

An SLR capital requirement of 5% translates to  $\xi_L = 20$ .

<sup>11</sup>See Thakor (2014) for a review of the debate on bank capital and its regulations.

<sup>12</sup>Davydiuk (2017) and Begenau (2019) set  $\xi_K$  to be the sample average of the ratio of Tier 1 equity to risky assets for the reason that banks typically maintain a buffer to prevent regulatory corrective action. In our model, the buffer arises endogenously, so we set  $\xi_K$  to the regulatory threshold. In theoretical studies on banking regulations, De Nicolò, Gamba, and Lucchetta (2014) calibrate the capital requirements to 4% and 12%, Hugonnier and Morellec (2017) calibrate the thresholds to 4%, 7%, 9%, and 20% to investigate the effects of the proposal by Admati and Hellwig (2013), and Phelan (2016) calibrates the threshold to 7.7% and 10.6% in a macroeconomic model.

## 3 Dynamic Banking

### 3.1 Bank Optimization

We derive the optimality conditions for the bank's control variables and the Hamilton-Jacobi-Bellman (HJB) equation for the bank's value function. In the next subsections, we parameterize the deposit maintenance cost and provide intuitive characterizations of the bank's optimal policies.

**State and Control Variables.** The bank solves a dynamic optimization problem with two state variables, deposit stock  $X_t$  and equity capital  $K_t$ . Let  $V_t$  denote the shareholders' value at time  $t$ . The bank chooses its loan portfolio size  $A_t$ , its position in bonds  $B_t$ , the deposit rate  $i_t$ , the payout of dividends  $dU_t$ , and the value of newly issued equity shares  $dF_t$  to solve the following problem:

$$V_t = V(X_t, K_t) = \max_{\{A, B, i, U, F\}} \mathbb{E} \left[ \int_{t=0}^{\tau} e^{-\rho t} (dU_t - dF_t - dH_t) \right]. \quad (8)$$

The value function is a function of the state variables, i.e.,  $V_t = V(X_t, K_t)$ . To solve the bank's optimal decisions and value function, we need the laws of motion of state variables that show how the control variables affect their evolution. The law of motion for  $X_t$  is given by (2). For equity capital  $K_t$ , we have

$$dK_t = A_t [(r + \alpha_A) dt + \sigma_A d\mathcal{W}_t^A] - B_t r dt - X_t i_t dt - C(n(i_t), X_t) dt - dU_t + dF_t. \quad (9)$$

The first three terms on the right side record the return on risky assets, bond interest expenses if  $B_t > 0$  or interest payments if  $B_t < 0$ , and deposit interest expenses. The fourth term is the operation cost associated with adjusting and maintaining the deposit stock. The last two terms are the dividend payout and capital raised via equity issuance.

The deposit stock and equity capital are slow-moving state variables that constitute the long-term funds of the bank. Given  $X_t$  and  $K_t$ , the bank's choices of  $A_t$  and  $B_t$  resemble a portfolio problem (Merton, 1969).<sup>13</sup> Let  $\pi_t^A$  denote the portfolio weight on loans, i.e.,  $\pi_t^A (X_t + K_t) = A_t$ ,

<sup>13</sup>The bank may adjust the loan amount  $A_t$  by selling loans. Technological progress on the reduction of information

so the weight on bonds is  $(\pi_t^A - 1)$  as implied by the balance-sheet identity (3). We now rewrite the law of motion for  $K_t$  as

$$dK_t = (X_t + K_t) \left[ (r + \pi_t^A \alpha_A) dt + \pi_t^A \sigma_A d\mathcal{W}_t^A \right] - X_t i_t dt - C(n(i_t), X_t) dt - dU_t + dF_t. \quad (10)$$

Given the Markov nature of the bank's problem, we suppress the time subscripts for  $X$ ,  $K$ , and control variables going forward to simplify the notations wherever it does not cause confusion.

The regulatory requirements translate into constraints on the bank's control variables and state variables. If the bank issues bonds (i.e.,  $B > 0$  or  $\pi^A > 1$ ), the capital requirement (5) and SLR requirement (6) are both restrictions on  $A/K$  so the bank faces  $A/K \leq \min \{\xi_K, \xi_L\}$  or

$$\pi^A \leq \min \{\xi_K, \xi_L\} \left( \frac{K}{X + K} \right). \quad (11)$$

If the bank holds bonds (i.e.,  $B \leq 0$  or  $\pi^A \leq 1$ ), the capital requirement (5) is still a restriction on the control variable  $\pi^A$ ,

$$\pi^A \leq \xi_K \left( \frac{K}{X + K} \right), \quad (12)$$

while the SLR requirement, now given by (7) instead of (6), stipulates a boundary in the space of state variables  $X$  and  $K$ ,

$$\frac{X + K}{K} \leq \xi_L. \quad (13)$$

In the next section, our numerical solution will show that the bank holds bonds for risk management when its equity capital  $K$  is low relative to its deposit liabilities  $X$ . Therefore, given  $X$ , the bank has to pay the issuance costs and raise equity when  $K$  declines significantly following negative shocks and SLR requirement (13) binds. The newly introduced SLR requirement is a boundary condition on the state variables and is more effective a tool to motivate bank recapitalization, while the traditional capital requirement restricts the control variable  $\pi^A$  (risk-taking).

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asymmetries facilitates loan trading. The design of contract between loan buyers and originators alleviates the moral hazard (reduced monitoring incentive) on the part of loan originators (Pennacchi, 1988; Gorton and Pennacchi, 1995).

**The HJB Equation and Boundaries.** When the bank does not pay out dividends ( $dU = 0$ ) or issue equity ( $dF = 0$  and  $dH = 0$ ), the HJB equation for the value function is

$$\begin{aligned} \rho V(X, K) = \max_{\{\pi^A, i\}} & V_X(X, K) X [-\delta_X + n(i)] + \frac{1}{2} V_{XX}(X, K) X^2 \sigma_X^2 \\ & + V_K(X, K) (X + K) (r + \pi^A \alpha_A) + \frac{1}{2} V_{KK}(X, K) (X + K)^2 (\pi^A \sigma_A)^2 \\ & - V_K(X, K) [Xi + C(n(i), X)] + V_{XK}(X, K) X (X + K) \pi^A \sigma_A \sigma_X \phi \end{aligned} \quad (14)$$

The optimality conditions on dividend payout and equity issuance specify the boundaries of  $(X, K)$ , denoted by  $(\bar{X}, \bar{K})$ , the payout boundary and  $(\underline{X}, \underline{K})$ , the equity issuance boundary.

The bank pays out dividends only if the payout value overcomes the decrease of continuation value, i.e.,  $dU \geq V(\bar{X}, \bar{K}) - V(\bar{X}, \bar{K} - dU)$  or in the differential form,

$$V_K(\bar{X}, \bar{K}) \leq 1. \quad (15)$$

The optimality of payout also requires the following super-contact condition (Dumas, 1991):

$$V_{KK}(\bar{X}, \bar{K}) = 0. \quad (16)$$

The bank raises equity and pays the issuance costs only when the increase of existing shareholders' value after issuance overweighs the new equity investment,  $dF$ , and issuance costs,  $dH$

$$V(\underline{X}, \underline{K} + dF) - V(\underline{X}, \underline{K}) \geq dF + dH, \quad (17)$$

We assume that the issuance costs depend on both the issuance amount and the size of the bank, i.e.,  $dH = \phi_1 dF + \phi_0 \underline{X}$ . We use the deposit base to measure the size of the bank, because, as we will show shortly, the bank's problem has a homogeneity property that significantly simplifies the analysis and allows for an intuitive presentation of our results. Finally, the optimality of  $dF$  also

requires the following smooth-pasting condition

$$V_K(X, K) = 1 + \psi_1. \quad (18)$$

Equation (18) states that the marginal value of bank equity is equal to the marginal cost of issuance.

Equations (13) and (15)–(18) define the boundaries of  $(X, K)$  given the value function. The HJB equation (14) solves the value function given the boundary conditions. The solution structure is akin to the dynamic models of corporate liquidity and risk management under equity issuance costs (e.g., Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Décamps, Gryglewicz, Morellec, and Villeneuve, 2017). Note that when characterizing the boundaries of  $(X, K)$ , we do not consider bank closure. In our model, the bank does not default on its debts because under (continuous) diffusive shocks, the bank can and will preserve the positive continuation value for shareholders by immediately adjusting its balance sheet in response to shocks so that  $\tau = +\infty$  in (4). One implication is that deposits and bonds are risk-free, which is in line with the fact that banking regulators often intervene before banks default on their debts.<sup>14</sup>

**Homogeneity.** We specify the cost of maintaining deposits and managing deposit flows as

$$C(n(i), X) = c(n(i)) X, \quad (19)$$

where  $c(\cdot)$  is an increasing and strictly convex function. Under this functional form and the previous specification of  $dH$ , the bank's optimal choices of  $\pi^A$  and  $i$  become univariate functions of the equity capital-to-deposit ratio,

$$k \equiv \frac{K}{X}, \quad (20)$$

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<sup>14</sup>For example, on November 21, 2008, the FDIC implemented the Temporary Liquidity Guarantee Program that guarantee all newly issued senior unsecured debt and non-interest-bearing transaction accounts at FDIC-insured banks.

and the bank's value function becomes  $V(X, K) = v(k)X$ . We demonstrate these results as follows. First, given  $V(X, K) = v(k)X$ , we obtain the following derivatives

$$\begin{aligned} V_K(X, K) &= v'(k), \quad V_X(X, K) = v(k) - v'(k)k \\ V_{KK}(X, K) &= v''(k) \frac{1}{X}, \quad V_{XX}(X, K) = v''(k) \frac{k^2}{X}, \quad V_{XK}(X, K) = -v''(k) \frac{k}{X}. \end{aligned} \quad (21)$$

Substituting these expressions into the HJB equation (14) and dividing both sides by  $X$ , we obtain

$$\begin{aligned} \rho v(k) &= \max_{\pi^A, i} [v(k) - v'(k)k] [-\delta_X + n(i)] + \frac{1}{2} v''(k) k^2 \sigma_X^2 \\ &\quad + v'(k) (1+k) (r + \pi^A \alpha_A) + \frac{1}{2} v''(k) (1+k)^2 (\pi^A \sigma_A)^2 \\ &\quad - v'(k) [i + c(n(i))] - v''(k) k (1+k) \pi^A \sigma_A \sigma_X \phi. \end{aligned} \quad (22)$$

Therefore, the  $X$ -scaled HJB equation (22) is an ordinary differential equation (ODE) for the  $X$ -scaled value function,  $v(k)$ . From this equation, we can solve  $\pi^A$  and  $i$  as univariate functions of  $k$ . In the next subsections, we will discuss the implications of these optimal choices in details. The constraints (11) and (12) on  $\pi^A$  translate to

$$\pi^A \leq \min \{ \xi_K, \xi_L \} \left( \frac{k}{1+k} \right) \text{ if } \pi^A > 1, \quad (23)$$

and

$$\pi^A \leq \xi_K \left( \frac{k}{1+k} \right) \text{ if } \pi^A \leq 1, \quad (24)$$

respectively. And the SLR requirement (13) implies a lower bound on  $k$ :

$$k \geq \underline{k} \equiv \frac{1}{1 - \xi_L^{-1}} - 1 \text{ if } \pi^A \leq 1. \quad (25)$$

The boundary conditions (15) to (18) can also be transformed. Let  $\bar{k}$  and  $\underline{k}$  denote respectively the dividend payout and issuance boundaries, and let  $m \equiv dF/X$  denote the (scaled) equity

issuance. The payout boundary conditions (15) and (16) can be simplified as follows:

$$v'(\bar{k}) = 1, \quad (26)$$

and

$$v''(\bar{k}) = 0. \quad (27)$$

The equity issuance boundary conditions (17) and (18) are simplified as follows:

$$v(\underline{k} + m) - v(\underline{k}) = \psi_0 + (1 + \psi_1)m, \quad (28)$$

and

$$v'(\underline{k} + m) = 1 + \psi_1. \quad (29)$$

In our numerical solution,  $v(k)$  is globally concave, so the conditions (26) and (29) imply that the bank pays out dividend when  $k$  is high and raise equity when  $k$  is low, i.e.,  $\bar{k} > \underline{k}$ . Moreover, when  $k$  is low, our numerical solution features  $B < 0$  (or equivalently,  $\pi^A < 1$ ), so the equity issuance boundary  $\underline{k}$  is given by (25). Given  $\underline{k}$ , the boundary conditions (26)–(29) and second-order ODE (22) (the HJB equation) solve  $v(k)$ , the upper boundary  $\bar{k}$ , and the issuance amount  $m$ .

In Appendix A, we provide a setup where, as in Drechsler, Savov, and Schnabl (2018), the bank has to hold assets that are more liquid than loans and is subject to a regulatory liquidity requirement, such as reserve and liquidity coverage ratio requirements. In this richer setup, our results on the value of deposits and the optimal strategies of payout, equity issuance, risk-taking, and deposit rate still hold.<sup>15</sup> The only difference is that the liquidity requirement generates another lower bound for  $k$ . Therefore, the bank raises equity to meet either the SLR requirement binds (i.e., at  $\underline{k}$  given by (25)) or the liquidity requirement binds. Empirically, equity issuance can happen before a constraint binds because, once a bank is close to violating a regulatory constraint, regulators intervene and often restrict managerial compensation or payout to shareholders.

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<sup>15</sup>Our solution of optimal liquidity holdings resembles the classic money demand (Baumol, 1952; Tobin, 1956).

### 3.2 Optimal Risk-Taking

From the  $X$ -scaled HJB equation (22), we can solve  $\pi^A$ . Using  $\frac{A}{K} = \frac{\pi^A(X+K)}{K} = \pi^A \left(\frac{1+k}{k}\right)$ , we obtain the following formula for the risky loan-to-capital ratio:

$$\frac{A}{K} = \frac{\alpha_A}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \quad (30)$$

In (30),  $\gamma(k)$  is a measure of endogenous risk aversion based on the value function:

$$\gamma(k) \equiv -\frac{v''(k)k}{v'(k)}, \quad (31)$$

This solution resembles Merton's portfolio choice including both the mean-variance term and the hedging-demand term. In the numerator, a higher excess return,  $\alpha_A$ , increases lending. The bank's incentive to lend is also strengthened when deposits are natural hedge when the asset-side shock,  $d\mathcal{W}^A$ , and the liability-side (deposit) shock,  $d\mathcal{W}^L$  are positively correlated ( $\phi > 0$ ).

The bank's risk-taking is state-dependent and only depends on  $k$  through  $\gamma(k)$ . Even though the bank evaluates the equityholders' payoffs with a risk-neutral objective in (4), it is endogenously risk-averse, i.e.,  $\gamma(X, K) > 0$ , due to the equity issuance cost. When the effective risk aversion is low, the bank chooses a high loan-to-capital ratio; when the effective risk aversion is high, the bank reduces its risk exposure. In our numeric solution, we will show that  $\gamma(k)$  decreases in  $k$ , so the equity buffer is high relative to the deposit liabilities.

The correlation between the loan return shock and the deposit flow shock,  $\phi$ , induces a hedging demand. The risk of deposit flow is essentially the bank's background risk from the perspective of portfolio management. When  $\phi > 0$ , it captures the synergy between lending and deposit-taking that has been studied extensively in the literature (e.g., Calomiris and Kahn, 1991; Berlin and Mester, 1999; Kashyap, Rajan, and Stein, 2002; Gatev and Strahan, 2006; Hanson, Shleifer, Stein, and Vishny, 2015). This hedging mechanism also echoes the finding of Drechsler, Savov, and Schnabl (2020) that financing lending with deposits helps banks to hedge risk.<sup>16</sup>

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<sup>16</sup>The finding of Drechsler, Savov, and Schnabl (2020) focuses on interest-risk risk.

### 3.3 Optimal Deposit Rate

When the bank increases the deposit rate by 1, it obtains new deposits with the marginal value equal to  $V_X(X, K) X n'(i)$ , but it also reduces the return on equity capital through higher interest payments on existing deposits, which is valued at  $V_K(X, K) X$ , and through the marginal cost of maintaining a larger deposit franchise,  $V_K(X, K) X c'(n(i)) n'(i)$ . The optimal deposit rate is implicitly defined by the condition that the marginal benefit is equal to the marginal cost:

$$V_X(X, K) n'(i) X = V_K(X, K) [X + X c'(n(i)) n'(i)] . \quad (32)$$

Rearranging the equation, we obtain:

$$c'(n(i)) = \frac{V_X(X, K)}{V_K(X, K)} - \frac{1}{n'(i)} = \frac{v(k) - v'(k)k}{v'(k)} - \frac{1}{n'(i)} . \quad (33)$$

Because  $c(\cdot)$  is a strictly convex function and  $n(i)$  is an increasing function, the optimality condition (33) implies that the optimal deposit rate increases in the ratio of marginal value of deposits to marginal value of equity capital  $\frac{V_X(X, K)}{V_K(X, K)}$ . Intuitively, when deposits are more valuable relative to equity capital, the bank is willing to sacrifice return on equity for deposit-taking (via a higher deposit rate). Moreover, when deposit flow is more responsive to the adjustment of deposit rate, i.e.,  $n'(i)$  is high, the bank is willing to set a high deposit rate.

Since deposits are at the core of our model, we sharpen the intuitions about the optimal deposit rate by adopting the following functional forms. First, we specify  $n(i)$  as a linear function:

$$n(i) = \omega i , \quad (34)$$

where, as shown in (2),  $\omega$  is the semi-elasticity of deposits stock  $X$  with respect to  $i$ . Next, we specify the cost of attracting new deposits in a simple quadratic form

$$c(n(i)) = \frac{\theta}{2} n(i)^2 . \quad (35)$$

These functional forms lead to a Hayashi style optimal policy for the deposit rate. In Hayashi (1982), firms make investments in productive capital, while, in our model, the bank attracts depositors by raising the deposit rate, building up its customer capital. Using (33), we obtain

$$i = \frac{\frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega}}{\theta\omega} = \frac{\left(\frac{v(k)-v'(k)k}{v'(k)}\right) - \frac{1}{\omega}}{\theta\omega}. \quad (36)$$

The difference between our optimal deposit-rate policy and Hayashi's investment policy is two-fold. First, it is not a single Tobin's  $q$  that dictates the optimal decision but rather the ratio of marginal deposit  $q$ ,  $V_X(X, K)$ , to marginal equity  $q$ ,  $V_K(X, K)$  drives the optimal deposit rate. Second, through the ratio  $\frac{V_X(X,K)}{V_K(X,K)}$ , our optimal deposit rate is state-dependent.

An interesting feature of the optimal deposit rate is that it hits the zero lower bound when

$$\frac{V_X(X, K)}{V_K(X, K)} \leq \frac{1}{\omega}. \quad (37)$$

Once the deposit rate reaches zero, the bank cannot further decrease the deposit rate to reduce deposits. Later we show that this restriction makes deposits undesirable, especially when the bank is undercapitalized, and thus, is concerned of a high leverage from large deposits that amplifies the impact of negative shocks on equity, increasing the likelihood of costly equity issuance.

When the deposit demand is more elastic, i.e.,  $\omega$  is high, the bank has to pay a higher deposit rate, as shown in (36). However, given the value function, it is less likely for the condition (37) to hold, because a high demand elasticity allows the bank to control the deposit flow more effectively and thereby to avoid hitting the zero lower bound. This result suggests that the deposit-rate lower bound is more acute a problem for larger banks with greater deposit market power or stickier deposit base (i.e., smaller  $\omega$ ). Smaller banks with less deposit market power are less concerned of the deposit-rate lower bound, but they have to pay higher interest rates to attract depositors.

Table 1: PARAMETER VALUES

This table summarizes the parameter values for our baseline analysis. The unit of time is year.

Parameters	Symbol	Value
risk-free rate	$r$	1%
discount rate	$\rho$	4.5%
bank excess return	$\alpha_A$	0.2%
asset return volatility	$\sigma_A$	10%
deposit flow (mean)	$\delta_X$	0
deposit flow (volatility)	$\sigma_X$	5%
deposit maintenance cost	$\theta$	0.5
deposit demand semi-elasticity	$\omega$	5.3
corr. between deposit and asset shocks	$\phi$	0.8
equity issuance fixed cost	$\psi_0$	0.1%
equity issuance propositional cost	$\psi_1$	5.0%
SLR requirement parameter	$\xi_L$	20
capital requirement parameter	$\xi_K$	14.3

## 4 Quantitative Analysis

### 4.1 Functional Form and Parameter Choices

For the functional forms of  $n(\cdot)$  and  $c(\cdot)$ , we use (34) and (35) respectively.<sup>17</sup> In Table 4 we report our calibration and parameter choices. We set the unit of time to year and  $r$  to 1% in line with the average Fed funds rate in the last decade. Shareholders' discount rate  $\rho$  is set to 4.5% in line with the commonly used value in dynamic corporate finance models.<sup>18</sup> We set  $\alpha_A$  to 0.2% so that the model generates an average return on assets (ROA) of 1.05%, close to the average ROA of US banks in the last decade (source: FRED). Note that when  $k$  is large, the bank only holds risky assets (and the asset value is  $A_t$ ), but when  $k$  is small, the bank also holds risk-free assets ( $B < 0$ ) and the asset value is  $A_t - B_t$ . Therefore, the ROA is state-dependent. To calculate the average ROA

<sup>17</sup>We also experiment with an alternative specification of quadratic  $n(i)$  that allows the deposit flow to be increasingly sensitive to deposit rate as  $i$  approaches zero. The results are very similar and are available upon request.

<sup>18</sup>One example is Bolton, Chen, and Wang (2011). This is also consistent with the dynamic contracting literature (DeMarzo and Fishman, 2007; Biais, Mariotti, Plantin, and Rochet, 2007).

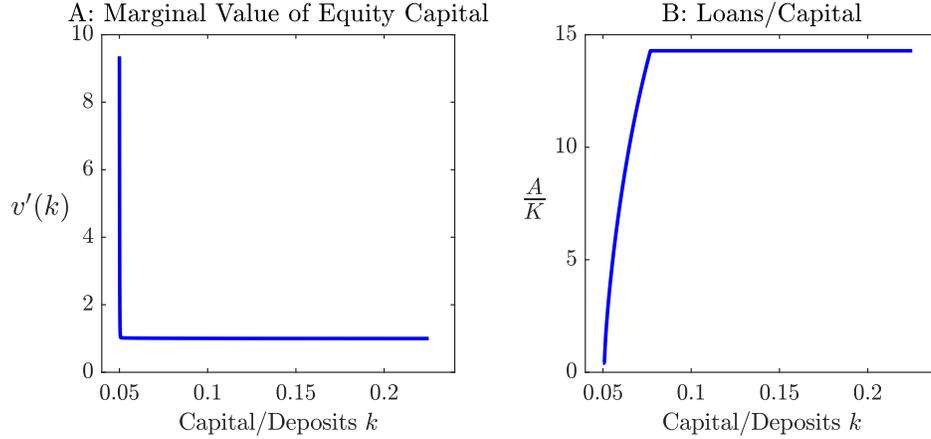


Figure 1: Marginal Value of Equity Capital and Bank Risk-Taking.

and other averages later, we use the stationary distribution of  $k$ . We set the asset return volatility,  $\sigma_A$ , to 10% as in Sundaresan and Wang (2014) and Hugonnier and Morellec (2017).<sup>19</sup>

For the deposit dynamics, we set  $\delta_X$  to 0% and  $\sigma_X$  to 5% following Bianchi and Bigio (2014). We further set  $\omega$ , the semi-elasticity of deposits to the deposit rate, to 5.3, an estimate from Drechsler, Savov, and Schnabl (2017). The correlation between asset-side and liability-side (deposit) shocks,  $\phi$ , directly affects  $A/K$  in (30) and is set to 0.8 so that the (stationary) probability of a binding capital requirement is consistent with the evidence (Begenau, Bigio, Majerovitz, and Vieyra, 2019). As for the cost of maintaining deposit franchise, we set the maintenance cost parameter,  $\theta$ , to 0.5. With this value, the model generates an average deposit-to-total liabilities ratio equal to 96% in line with the evidence (Drechsler, Savov, and Schnabl, 2017). We set the proportional issuance cost parameter,  $\psi_1$ , to 5% (Boyson, Fahlenbrach, and Stulz, 2016). The fixed cost parameter,  $\psi_0$ , is set to 0.1%, so the model generates an issuance-to-equity ratio of 1% in line with the evidence (Baron, 2020). The regulatory parameters were discussed in Section 2.

## 4.2 Marginal Value of Equity Capital and Risk-Taking

The marginal value of equity capital,  $V_K(K, M) = v'(k)$ , should be equal to one without financial frictions because the bank is indifferent between paying out one dollar and retaining one dollar of earnings. In other words, precautionary savings do not add value without financial frictions. Under the equity issuance costs, the marginal value of equity capital can be above one, and the wedge between  $v'(k)$  and one widens as the bank approaches the boundary of equity issuance. Panel A of Figure 1 plots the marginal value of equity capital,  $v'(k)$ , against the equity capital-to-deposit ratio,  $k$ . At the equity issuance boundary of  $k$ ,  $\underline{k}$ , a value of  $v'(k)$  close to nine means that one dollar of equity is worth nine dollars because of the imminence of costly equity issuance.<sup>20</sup>

The interior region ends at the endogenous payout boundary  $\bar{k}$ . At that point the the marginal value of equity capital is equal to one and bank has retained a sufficient amount of earnings, so that it is optimal to pay out dividends to shareholders as they discount cash flows at a higher rate  $\rho$  than  $r$ . Note that near the payout boundary,  $\bar{k}$ , the marginal value of equity capital is close to one and relatively insensitive to variations in  $k$  because, at that point, the likelihood of a large loss of equity or a large deposit inflow that dramatically decrease  $k$  to the equity issuance boundary  $\underline{k}$  is low. In other words, distress in the form of costly equity issuing is a distant scenario near  $\bar{k}$ .

Throughout the whole region of  $k$ , the marginal value of equity capital stays positive, which implies that when the bank accumulates more equity capital, *ceteris paribus*, the bank shareholders' value increases. This is in line with the empirical findings of Mehran and Thakor (2011) and Minton, Stulz, and Taboada (2019) that bank value is positively associated with bank capital. In the next subsection, we examine the marginal contribution of deposits to bank shareholders' value and discuss further the implications of our model on empirical analysis of bank valuation (Atkeson, d'Avernas, Eisfeldt, and Weill, 2019). Moreover, our model predicts that the bank pays dividend when equity capital is high relative to its deposit liabilities and raises equity when equity capital is low. The procyclical payout and countercyclical equity issuance are consistent with the evidence on bank equity management (Adrian, Boyarchenko, and Shin, 2015; Baron, 2020).

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<sup>19</sup>Sundaresan and Wang (2014) in turn refer to the calculation of Moody's KMV Investor Service.

<sup>20</sup>The proportional cost is only 5%, but due to the fixed cost, the marginal value of equity is much higher than 1.05.

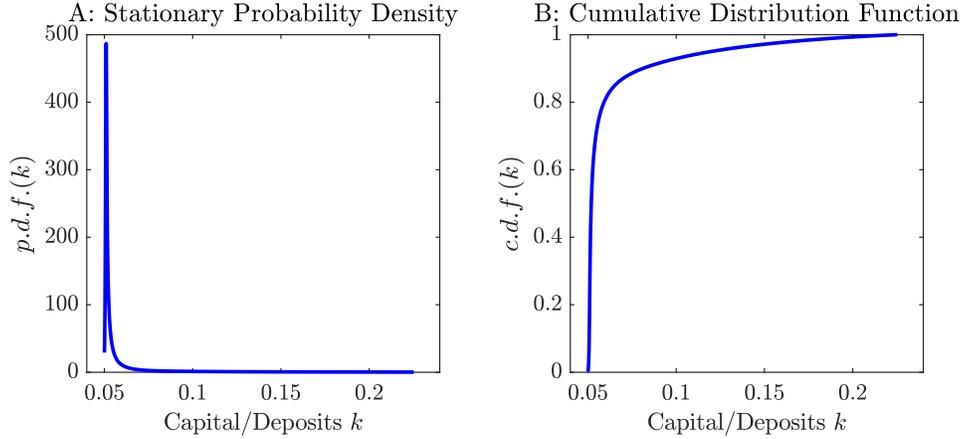


Figure 2: Stationary Probability Density and Cumulative Distribution Function of  $k$ .

As shown in the solution of optimal loan-to-capital ratio,  $A_t/K_t$ , given by (30), the marginal value of equity capital directly drives the bank's risk-taking behavior through  $\gamma(k)$ , the bank's endogenous relative risk aversion defined in (31). The decreasing marginal value of equity capital in Panel A of Figure 1 suggests that  $\gamma(k)$  decreases in  $k$ , because as  $k$  increases, the concavity of bank value in equity capital subdues quickly and, as  $k$  approaches  $\bar{k}$  (the payout boundary), bank value is almost linear in  $k$  with  $v'(k)$  close to one as previously discussed. Indeed, in Panel B of Figure 1, we show that the loan-to-capital ratio increases in  $k$ . The bank obviously cannot exceed the regulatory capital requirement (i.e.,  $A/K \leq \xi_K = 14.3$ ), but it can expand its balance sheet up to that limit. Our model predicts that risk-taking is procyclical. As equity capital increases relative to deposits (as  $k$  increases), the bank expands its balance sheet, financing the expansion through deposits and wholesale (short-term bond) funding. But when capital is depleted relative to deposits, the bank de-risks. This is consistent with the findings of Ben-David, Palvia, and Stulz (2020) that distressed banks decrease observable measures of riskiness.

Figure 1 reports the marginal value of equity capital and optimal loan-to-capital ratio given any value of  $k$ . To understand the long-run behavior of this model, i.e., how much time the bank spends in different regions of  $k$ , we examine the stationary density of  $k$ . Panel A of Figure 2 plots the stationary probability density of  $k$  and Panel B plots the corresponding cumulative distribution

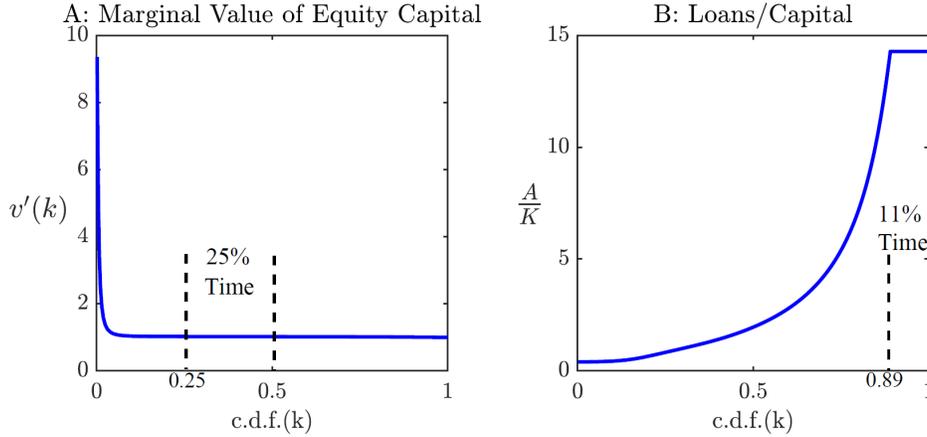


Figure 3: Long Run Distribution of Marginal Value of Equity Capital and Loan-Capital Ratio.

function (c.d.f.). While the probability mass is concentrated in the area where  $k$  is near the lower boundary  $\underline{k}$ , the marginal value of equity capital is only slightly above one (1.02) where the density function peaks. However, even if for the majority of time the bank does not seem to be financially constrained, the shadow value of equity rises dramatically when equity is depleted relative to the bank's deposit liabilities and  $k$  approaches  $\underline{k}$ , the boundary of costly equity issuance, as shown in Panel A of Figure 1. These results illustrate the sharp contrast between normal times, when the bank is comfortably meeting its leverage requirements, and crisis times, when it is in danger of violating its leverage requirement and triggering equity issuance.

With the stationary distribution of the key state variable  $k$ , we now report the model predictions on the distribution of marginal value of equity capital and loan-to-capital ratio. In Panel A of Figure 3, we plot the marginal value of equity capital against the stationary c.d.f. of  $k$  (note  $c.d.f.(\underline{k}) = 0$  and  $c.d.f.(\bar{k}) = 1$ ). The interval on the horizontal axis represents the fraction of time that the bank spends in the corresponding region of  $v'(k)$  on the vertical axis. For example, the bank spends 25% of the time with its marginal value of equity between 1.019 and 1.022. The bank spends less than 5% of the time in the region where it is in danger of violating the leverage requirement with  $v'(k)$  above 1.08. In other words, crisis states are rare but they cast a long shadow over the bank's management of its balance sheet. As the bank becomes better capitalized relative

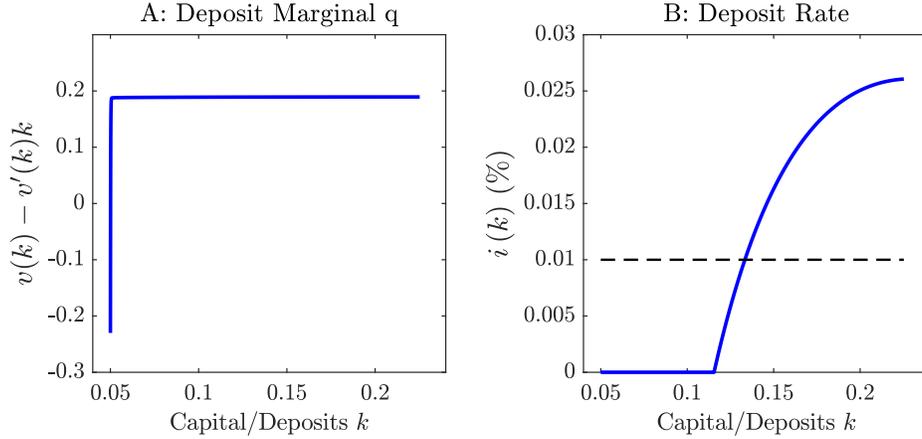


Figure 4: Deposit Marginal  $q$  and Deposit Rate.

to its deposit liabilities (as  $k$  increases), the marginal value of equity declines dramatically, so that the bank value is concave in equity and the bank is endogenously risk averse.

In Panel B of Figure 3, we plot the optimal loan-to-capital ratio,  $A_t/K_t$ , against the stationary c.d.f. of  $k$ . We show that capital requirement binds about 11% of the time (the horizontal part of the curve on the right end). Capital requirement becomes relevant when the bank is well-capitalized and the risk-taking incentive is strong. Such procyclicality suggests that capital requirement can act as a macroprudential tool as suggested by Gersbach and Rochet (2017). In contrast, the SLR requirement motivates the bank to replenish equity capital in bad times when its equity capital is low relative to its deposit liabilities (see (25)). While capital requirement and SLR requirement play distinct roles in our model, they both contribute to a form of parity between risk and capital with the former restricting risk-taking given equity capital and the latter triggering capital raising.

### 4.3 Deposit Marginal $q$

Bank value depends on equity capital,  $K$ , and deposit stock,  $X$ . Panel A of Figure 4 plots the marginal value of deposits (“deposit marginal  $q$ ”),  $V_X(X, K) = v(k) - v'(k)k$ . When the bank has ample capital relative to deposits, i.e., when  $k$  is large, deposit marginal  $q$  is positive. However, it turns sharply negative when  $k$  nears the lower boundary of costly equity issuance.

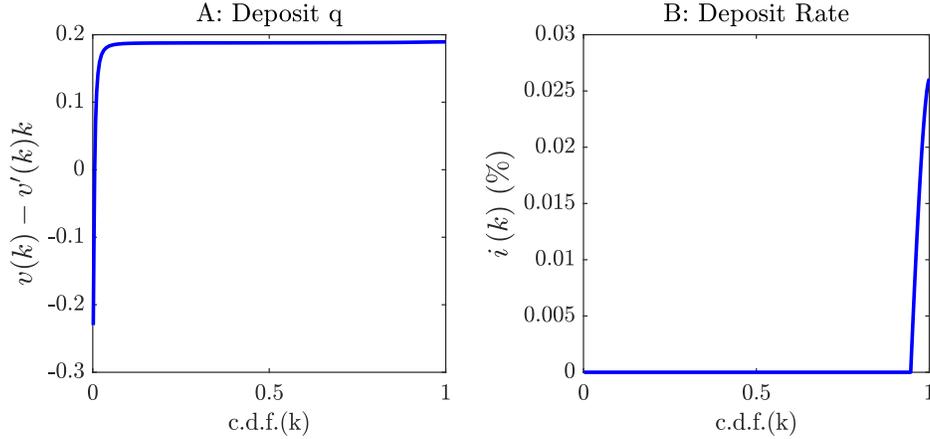


Figure 5: The Long Run Distribution of Deposit Marginal  $q$  and Deposit Rate.

Deposits create value by allowing the bank to finance risky lending with relatively cheap sources of funds. Therefore, deposit stock serves as a form of productive capital for the bank. Intuitively, when the bank becomes better capitalized, it raises deposit rate to attract more deposits for more risky lending. Panel B of Figure 4 shows that the deposit rate increases in  $k$  as the loan-to-equity ratio does in Panel B of Figure 1. The positive comovement of loan growth and deposit rate increase is consistent with the finding of Ben-David, Palvia, and Spatt (2017).

A key finding is that deposit marginal  $q$  declines sharply and can turn negative when the bank's equity capital is low relative to its deposit liabilities. The reason is that when  $k$  is near the equity issuance boundary,  $\underline{k}$ , deposits destroy value for the bank's shareholders by forcing the bank to sustain a high level of leverage that amplifies the impact of shocks on equity capital and makes the costly equity issuance more likely. The bank may want to delever, turning away deposits by lowering the deposit rate. However, as shown by Panel B of Figure 4, doing so has a limit, that is the zero lower bound of deposit rate. In practice, banks are reluctant to impose negative deposit rate on depositors. Consistent with our zero lower bound on the deposit rate, Heider, Saidi, and Schepens (2019) find that the distribution of deposit rates of euro-area banks is truncated at zero.<sup>21</sup>

In Figure 5, we plot deposit marginal  $q$  and optimal deposit rate against the stationary c.d.f.

<sup>21</sup>Moreover, when the ECB lowers the policy rate, more deposit rates bunch at zero.

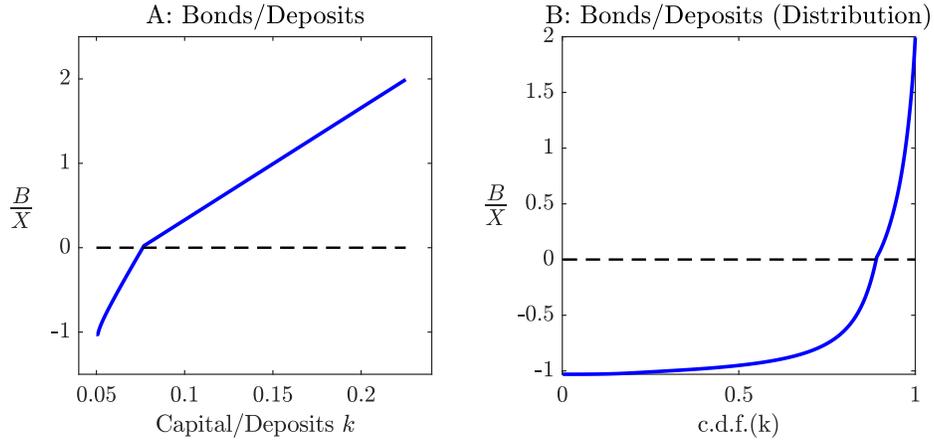


Figure 6: Short-Term Debt and Total Leverage

of  $k$ . Deposit marginal  $q$  is positive and larger than 0.185 in 81% of the time, but near the lower boundary of costly equity issuance (i.e.,  $c.d.f.(k) = 0$ ), it can drop to  $-0.23$ . The deposit rate hovers around the lower bound at zero, showing that the bank is very conservative in deposit-taking. The deposits attracted by high rate today is helpful in financing lending (i.e., earning  $\alpha_A$ ) but can become burdensome when negative shocks deplete bank equity capital and  $k$  declines. However, for a bank with sufficiently strong balance sheet, i.e., a higher value of the capital-to-deposit ratio  $k$ , the bank is willing to offer more attractive deposit rate to attract depositors.

Deposits are very different from short-term debt. For short-term debt, the bank can continuously and freely adjust its debt level, and therefore, does not face the problem of unwanted debts. However, deposit contracts do not have maturity. Deposits leave the bank only when depositors withdraw dollar bills or make payments to those who hold accounts at other banks. As long as depositors are willing to hold deposits, the bank cannot turn away the existing depositors. Moreover, the bank must accept any deposit inflow unconditionally, for example, when a depositor receives a payment or deposits cash. Therefore, after hitting the zero lower bound, the bank can no longer decrease its deposit rate further to reduce deposit inflow and thus loses control of its leverage. When the bank is sufficiently close to incur costly equity issuance (i.e.,  $k$  is close to  $\underline{k}$ ), the marginal value of deposits is negative for the bank's shareholders as the bank loses control of its leverage.

Figure 6 analyzes the the bank’s debt structure. Panel A plots the ratio of short-term debts to deposits,  $B/X$ , against  $k$  and Panel B plots this ratio against the stationary c.d.f. of  $k$  to how much time the bank spends in different regions of  $B/X$ . When capital is abundant relative to deposits, the bank raises funds from short-term debts for risky lending, i.e.,  $B_t > 0$  when  $k$  is high. As  $k$  increases, the bank becomes increasingly reliant on short-term debt as the source of financing instead of deposits. The substitution from deposits to short-term debts reflects the bank’s concern over the lack of control over deposit liabilities and the bank’s preference for more controllable short-term debts in spite of higher debt costs. In our solution, the deposit rate is below the risk-free rate  $r$  (1%) (see Panel B of Figure 4). This result captures the bank’s incentive to avoid deposit risk. Deposit risk management is a unique feature of our model and is distinct from the standard loan risk management (dictated by the Merton-style formula (30)).

Panel A of Figure 6 also shows that when the bank’s equity capital is scarce relative to its deposit liabilities, the bank switches its position in short-term debt, holding risk-free debts to reduce the overall riskiness of its asset portfolio, i.e.,  $B_t < 0$  when  $k$  is low. The bank pays the issuance costs to raise equity when  $k$  falls to  $\underline{k}$ , which happens because of either negative shocks to loan return that reduces bank equity  $K$  (the numerator) or positive shocks to deposit stock  $X$  (the denominator). To avoid paying the equity issuance costs, the bank has to manage its exposure to both loan return risk and deposit risk. When  $k$  declines, the optimal deposit rate approaches the lower bound. Once the deposit rate hits the lower bound, the bank loses control of its deposit liabilities and thus can no longer manage deposit risk. Therefore, it focuses on reducing the exposure to loan return risk on the asset-side of its balance sheet, and doing so requires holding risk-free assets, i.e.,  $B_t < 0$ . Our model reveals a new source of demand for safe assets – when the deposit rate is near zero, banks cannot actively offset positive deposit-flow shocks by lowering deposit rates, so they de-risk the asset-side of their balance sheets instead. This result sheds light on the enormous holdings of safe assets by U.S. banks as their deposit liabilities grow fast during the Covid-19 pandemic and other episodes of persistent deposit inflows.

## 5 Leverage Regulation

The supplementary leverage ratio (SLR) is the U.S. implementation of the Basel III Tier 1 leverage ratio. The SLR, which does not distinguish between assets based on risk, is conceived as a backstop to risk-weighted capital requirements. In our model, the SLR plays the critical role of pinning down the (lower) boundary of equity issuance for the state variable,  $k$ . In contrast, capital requirement imposes a restriction on the control variable,  $\pi^A$ , the loan-risk exposure, as previously discussed.

In response to the crisis provoked by the Covid-19 pandemic, U.S. banking regulators relaxed the supplementary leverage ratio (SLR) requirements. Jerome Powell, the Federal Reserve Chairman, emphasized that the SLR provision is straining banks' ability to handle large deposit inflows. *"Many, many bank regulators around the world have given leverage ratio relief,"* Powell said at a news conference following an FOMC meeting. *"What it's doing is allowing [banks] to grow their balance sheet in a way that serves their customers."*<sup>22</sup>

To shed light on this decision, we examine the effects of relaxing the SLR requirement on bank balance-sheet management and valuation. Relaxing the SLR stimulates lending immediately, but contrary to the conventional wisdom, it leads to less risk-taking over the long run. Moreover, our model shows that equity issuance costs generate a reach-for-yield incentive, so tightening the SLR can actually cause the bank to be more aggressive in risk-taking by increasing the frequency of costly equity issuance over the long run. Relaxing the SLR also increases the deposit marginal  $q$ , helping the bank to absorb deposit influx like the one we saw during the Covid-19 pandemic. However, if the deposit influx lasts for a long period time, the deposit marginal  $q$  can fall below the level before the SLR is relaxed. Finally, our model predicts a permanent decrease of bank shareholders' value when the SLR is tightened.

### 5.1 Lending and Risk-Taking

The bank must raise equity and incur issuance costs in order to stay in compliance with the leverage requirement, as shown in (25). In our model, the cost of financial distress or undercapitalization

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<sup>22</sup>See "Fed's Powell makes case why Congress should relax bank capital rule" by Hannah Lang, American Banker July 29, 2020.

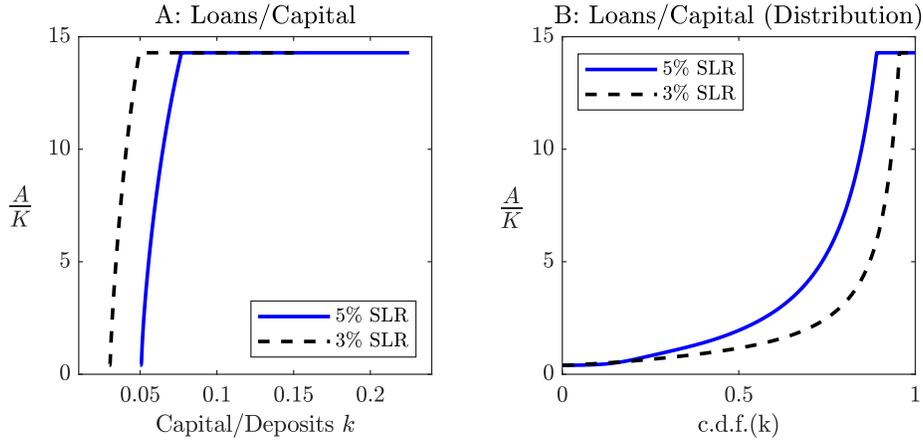


Figure 7: The Impact of Relaxing the SLR Requirement on Bank Lending

is in the form of equity issuance costs instead of bankruptcy costs. Given that the bank only faces small diffusive shocks, it can avoid insolvency by adjusting its balance sheet continuously, but when  $k = K/X$  hits the lower boundary  $\underline{k}$  – for example after an unexpected deposit inflow that increases  $X$  – the bank must raise equity. This is a realistic approach as in practice, bank insolvency is relatively rare, and recapitalization is often triggered by regulatory intervention.

Relaxing the SLR lowers  $\underline{k}$  so that given the value of  $k$ , i.e., the current balance-sheet status, costly equity issuance becomes a more distant event. A reduced likelihood of paying the equity issuance costs makes the bank less risk-averse and thereby stimulates lending as shown in Panel A of Figure 7 where we compare the loan-to-equity capital ratio,  $A/K$ , under the SLR requirement equal to 5% (the baseline value) and 3% (the dashed line). Given  $k$ ,  $A/K$  is higher when the SLR requirement is lower. In both cases,  $A/K$  peaks at the level given by the risk-based capital requirement (5). Note that it is not the SLR that causes risk aversion. Even without it, the bank still has to raise equity when  $k$  falls to zero. It is costly but optimal to do so since the continuation value is positive. The SLR simply pushes the equity issuance boundary  $\underline{k}$  above zero.

Many are concerned that relaxing leverage regulations will cause the bank to take on more risks over the long run.<sup>23</sup> Consistent with this intuition, Panel A of Figure 7 shows that the payout

<sup>23</sup>When discussing the relaxation of SLR requirement, Fed chairman Powell emphasized that “This will not be a permanent change in capital standards.” (see “Fed’s Powell makes case why Congress should relax bank capital rule”

and equity issuance boundaries both shift leftward after the regulatory change. Relaxing the SLR requirement makes the bank less risk-averse and maintain less equity (relative to deposit liabilities). However, this does not necessarily imply a higher risk exposure per unit of equity capital over the long run as shown in Panel B. Drawing the distinction between Panels A and B is important for understanding the result. Panel A shows the impact of relaxing the SLR requirement given  $k$ , which summarizes the current state of balance-sheet conditions of the bank. The move from the solid line to the dashed line mimics the *immediate effect* of regulatory change. In contrast, Panel B shows the *long-run effect*. The plot of  $A/K$  against the stationary c.d.f. of the state variable  $k$  shows how much time the bank spends (horizontal axis) at different values of  $A/K$  (vertical axis). Quite contrary to conventional wisdom, relaxing the SLR actually leads to a smaller risk exposure per unit of equity capital over the long run as the dashed line is below the solid line in Panel B.

Every time the bank raises equity it pays the issuance costs. Therefore, over the long run the bank must generate sufficient earnings to offset these costs. Relaxing the SLR requirement reduces the frequency of costly equity issuance, so the amount of earnings that the bank needs to generate declines. Therefore, the bank becomes less aggressive in earning the loan spread,  $\alpha_A$ , through risk-taking. By the same logic, tightening leverage regulations can actually lead to more aggressive risk-taking over the long run, as it means more frequent equity issuance. The bank has to engage in more risk-taking per unit of equity to generate earnings (return on equity) that offset issuance costs. Equity issuance costs generate a reach-for-yield incentive. Thus, tightening the SLR achieves the purpose of incentivizing the bank to maintain more equity over the long run but fails to tame risk-taking per unit of equity. The mechanism captures the real-world bankers' focus on return on equity and is similar to the channel of financial instability in Li (2019).<sup>24</sup>

**Discussion: SLR and Capital Requirement.** When  $k$  is low and  $B < 0$ , the SLR requirement implies a lower (equity issuance) bound on the state variable  $k$  (see (7)). When  $k$  is high and  $B > 0$ , the SLR requirement becomes a restriction on the control variable, loan-to-equity capital ratio (see

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by Hannah Lang, American Banker July 29, 2020).

<sup>24</sup>In Li (2019) presents a model of financial instability induced by government debt where the supply of government-issued money-like securities (e.g., Treasury bills) squeezes banks' profits from issuing money-like securities, so banks become more aggressive in risk-taking to sustain earnings that can offset the costs of issuing equity over the long run.

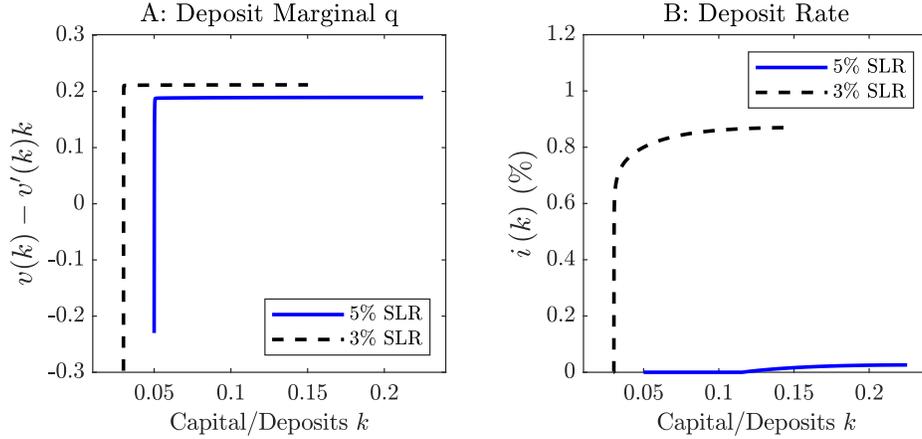


Figure 8: The Impact of Relaxing the SLR Requirement on Deposit Taking

(6)), just as the risk-based capital requirement does (see (5)). Under the current parameter values, the capital requirement binds before the SLR when  $k$  is high, so in our model, the two regulations play distinct roles: The SLR pins down the lowest amount of equity capital relative to deposits, i.e., the lower bound of  $k$ , and the capital requirement restricts the risk exposure per unit of equity capital. This seems to suggest that risk-based capital requirements are more direct in taming risk-taking than the SLR. However, this conclusion relies on an important assumption that the riskiness of loans, given by the parameter  $\sigma_A$ , is time-invariant. When loan risk is countercyclical, risk-based capital requirements amplify the procyclicality of bank risk-taking (Repullo and Suarez, 2012). Moreover, risk weights are vulnerable to manipulation (Plosser and Santos, 2014). Because our model is designed to focus on deposit risk and the bank's imperfect control of balance-sheet size and composition, we do not include the possibility of equilibrium bank failures in our model and the associated externalities that motivate both the leverage and risk-based capital requirements. Therefore, our analysis does not aim to provide a comprehensive evaluation of banking regulations.

## 5.2 Deposit Marginal $q$ and Deposit Rate

One key motivation for relaxing the SLR during the Covid-19 pandemic is allowing banks to accommodate the unprecedented deposit inflows without concerns over violating regulatory con-

straints. In Panel A of Figure 8, we plot the marginal value of deposits,  $V_X(X, K) = v(k) - v'(k)k$ , before (solid line) and after (dashed line) the SLR requirement is reduced. To see the model predictions, pick any value of  $k$  on the solid line and consider the vertical movement to the dashed line. This mimics the immediate response of a bank to the regulatory change given its balance-sheet condition (i.e., the value of  $k$ ). The regulatory change achieves its intended purpose of stimulating deposit-taking as the marginal value of deposits jumps up. The jump in deposit  $q$  is most significant at the low values of  $k$  where the deposit  $q$  turns sharply negative before the regulatory change.

If the deposit influx continues after the regulatory change (for example, due to new rounds of stimulus payments to households) and raises the bank's deposit liabilities,  $X$ , faster than the growth of its equity capital,  $K$ , via retained earnings, the bank moves along the dashed line to the left in Panel A of Figure 9 and its deposit marginal  $q$  declines. Note that after the SLR is relaxed, deposit marginal  $q$  is even more negative near the new and lower equity issuance boundary, because the equity capital is now lower relative to deposits at the new issuance boundary so that the effects of deposit inflows on  $k$  ( $= K/X$ ) are greater. Once the deposit influx pushes deposit marginal  $q$  into the negative territory, further relaxing the SLR becomes necessary to avoid the decline of bank shareholders' value as a result of deposit inflows.

We plot the deposit rate in Panel B of Figure 8. After the SLR requirement is reduced, the bank sets a higher rate to attract deposits because the deposit  $q$  is higher. As a result, the region of  $k$  where the deposit-rate lower bound binds shrinks significantly. By the same logic, tightening leverage regulation has the unintended consequence of making the deposit-rate lower bound a more binding constraint for the bank. The bank controls the size of its deposit liabilities through the deposit rate. When the deposit-rate lower bound is more binding, the bank has less control over the size and composition of its balance sheet. This unintended consequence of leverage regulation is a unique prediction of our model.

### 5.3 Bank Franchise Value

Finally, we examine the impact of the SLR requirement on bank shareholders' value. Panel A of Figure 9 shows a clear increase of bank franchise value (scaled by deposit stock),  $(V(X, K) -$

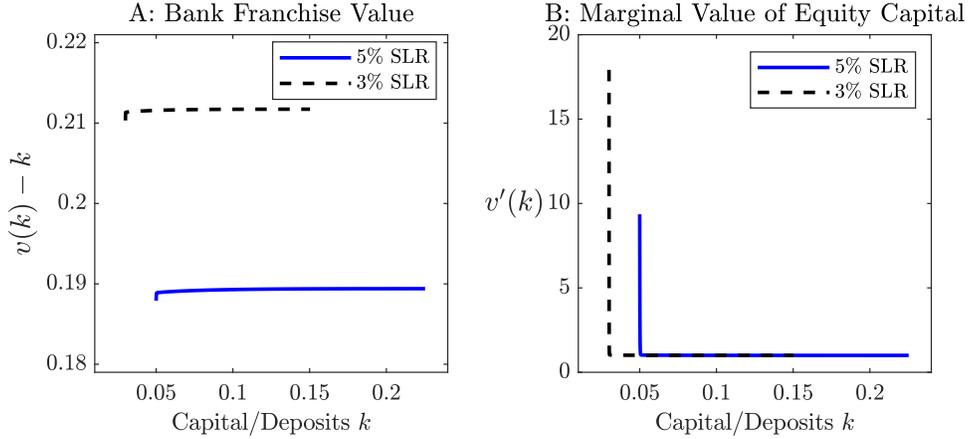


Figure 9: The Impact of Relaxing the SLR Requirement on Bank Valuation

$K)/X = v(k) - k$ , when the SLR requirement is reduced. A higher shareholder value implies that the bank is more eager to protect its continuation value, explaining why the marginal value of equity is higher near the equity issuance boundary, as shown in Panel *B*.

Tightening leverage requirements results in a sizeable loss of bank shareholder value across all values of  $k$ . Kashyap, Stein, and Hanson (2010) point out that the impact of tightening leverage requirements on bank shareholders' value is temporary because shareholders pay the equity issuance (dilution) costs once and then the bank will settle on a higher level of equity capital. This argument holds in a deterministic environment. In our model, uncertainty is the key. Either negative shocks to earnings due to loan losses ( $dW^A < 0$ ) or positive shocks to the stock of deposit liabilities ( $dW^X > 0$ ) can reduce  $k = K/X$  and trigger costly equity issuance when  $k$  hits  $\underline{k}$ . Therefore, in a risky environment, the impact of leverage requirements on bank shareholder value is no longer a one-time cost of raising equity. The cost is now recurring, and through shareholders' rational expectations, is reflected in bank valuations even when  $k$  is away from  $\underline{k}$ . Moreover, to reduce the likelihood of incurring the equity issuance cost, the bank has to retain a higher level of equity capital when the leverage requirement is tightened, which is also costly to shareholders because dividend payouts are delayed. Overall, our result contributes to the ongoing debate on the cost of equity capital regulations for banks (Admati, DeMarzo, Hellwig, and Pfleiderer, 2013).

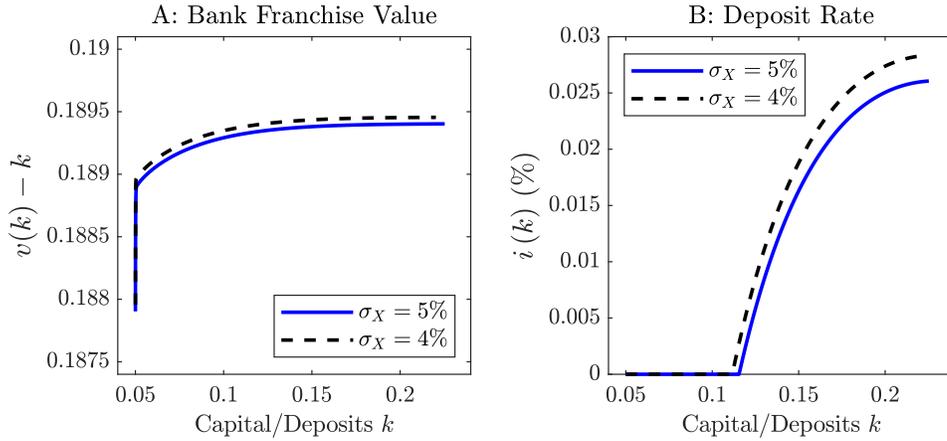


Figure 10: Deposit Risk and Bank Valuation

## 6 Deposit Risk, Interbank Market, and the Fintech Impact

The ratio of equity capital to deposits,  $k = K/X$ , drives risk management. It measures the bank's financial slack, i.e., the distance from costly equity issuance at  $\underline{k}$ . The uncertainty in  $k$  is from both loan-return shocks, which hits equity (the numerator  $K$ ), and deposit-flow shocks, which hits the denominator  $X$ . Incomplete market is key. If the bank were able to perfectly insure against shocks, effectively reducing  $\sigma_A$  and  $\sigma_X$  to zero, risk management would have not mattered at all even under financial frictions (equity issuance costs).<sup>25</sup> As an example, Figure 10 shows that a reduction in deposit-flow risk,  $\sigma_X$ , from 5% (baseline solid line) to 4% (dashed line) leads to higher franchise value (Panel A) and incentivizes deposit-taking via a higher deposit rate (Panel B).

The interbank market is often seen as the place where banks hedge deposit shocks with each other (Bhattacharya and Gale, 1987). Consider deposit flows that result from payments. When a depositor sends a payment to another depositor at a different bank, the payer's bank loses deposits while the payee's bank gains deposits. The payee's bank can then lend to the payer's bank. As a result, a shock to the deposit stock is offset by a simultaneous shock to *net interbank liabilities*. The payer's bank experiences a negative deposit shock but, through the interbank loan, its inter-

<sup>25</sup>To be specific, consider fairly priced hedging contracts with mean-zero payoffs that are correlated with the shocks. The bank will use such hedging contracts to fully unload the shocks to its risk-neutral counterparties (insurers) at zero cost following the standard hedging argument in Merton (1973).

bank liabilities increase. The payee's bank experiences a positive deposit shock and increases its interbank assets, so its position in net interbank liabilities decreases.

Consider the case where deposit shocks are purely idiosyncratic. Then banks can commit to insure each other against deposit shocks via an interbank arrangement that automatically offsets deposit shocks with commensurate changes in net interbank liabilities. In fact, we can reinterpret  $X$  as the sum of net interbank liabilities and deposits, which is subject to smaller shocks thanks to interbank hedging. In Figure 10, the reduction of deposit risk captures the effect of interbank hedging. This hedging mechanism is implemented automatically in deferred net settlement (DNS) systems (e.g., the Canadian payment system). In real-time gross settlement (RTGS) systems, the implementation relies on an overnight interbank market after payments are settled intraday (Furfine, 2000; Bech and Garratt, 2003; Ashcraft et al., 2011).

In reality, a significant component of deposit shocks is systematic. For example, during the Covid-19 pandemic, the deposit influx into the U.S. banking system is system-wide. Moreover, trading frictions in the over-the-counter interbank market make implementing this hedging mechanism costly (Afonso and Lagos, 2015; Bianchi and Bigio, 2014). Finally, the (implicit) commitment of banks to lend to each other can break down in crises. Nevertheless, it is meaningful to examine the response of our model to the reduction of deposit risk possibly as a result of improved interbank hedging (for example, due to technological advances (D'Andrea and Limodio, 2019)).

During the Covid-19 pandemic, banks' holdings of high-quality liquid assets (HQLAs) increased alongside with their deposit liabilities. Yet the deposit risk is still a concern. Holding safe assets, i.e., lowering  $\pi^A$ , reduces the bank's exposure to loan-return shocks, but does not help reduce the deposit risk. Interbank hedging works by netting off deposit shocks with commensurate changes in net interbank liabilities. Therefore, the interbank market plays an essential role in ameliorating the impact of deposit risk even when banks hold a large amount of HQLAs.

Finally, our results shed light on the impact of the entry of alternative payment service providers on banks. Fintech firms, such as PayPal and Square, are actively reshaping the topology of payment flows. Our model predicts that the resultant uncertainty in deposit flows (i.e., an increase in  $\sigma_X$ ) leads to lower bank franchise values and lower deposit rates as banks' concern over

deposit risk management heightens. This channel complements the standard narrative that focuses on banks losing customers. This narrative also suggests a decline of bank franchise value, but in contrast to our prediction of lower deposit rates following an increase in  $\sigma_X$ , the narrative of banks losing customers points to higher deposit rates that are necessary for banks to retain customers.

## 7 Banking in a Low Interest Rate Environment

When the bank finances lending with deposits, it expects to earn a net interest margin (NIM), i.e., the spread between the expected loan return,  $r + \alpha_A$ , and the deposit rate,  $i$ . Earning the NIM requires the bank to take on the loan-return risk and deposit flow risk, and risk management is crucial under the equity issuance costs. We decompose the net interest margin into two components,  $\alpha_A$  (lending expertise) and  $r - i$ .<sup>26</sup> Our model emphasizes the deposit spread,  $r - i$ . In this section, we show that the bank suffers in a low interest rate environment, because as  $r$  declines, it squeezes the NIM and makes the deposit-rate lower bound a more binding constraint. In our model, the NIM is not only a measure of profitability as the classic banking theories predict, but, more importantly, the NIM reflects the bank's flexibility in managing its deposit liabilities.

The bank increases the deposit rate when it is well-capitalized (i.e.,  $k$  is high). Given the deposit rate lower bound, the higher the bank can set its deposit rate in the high- $k$  region, the more flexibility it has to reduce deposit rate when  $k$  declines. However, raising the deposit rate increases interest expenses and hurts earnings. Therefore, the bank faces a trade-off. It can sacrifice its earnings in the high- $k$  region to gain flexibility of adjusting the deposit rate in the low- $k$  region. When the risk-free rate  $r$  is high, the bank can set a high deposit rate and still earn a positive deposit spread  $r - i$ . When the risk-free rate  $r$  is low, the bank has less room to manipulate the deposit rate without squeezing the deposit spread too much.

Therefore, the flexibility to adjust deposit rate and to regulate deposit flows depends on the distance between  $r$  and zero, the deposit-rate lower bound. When  $r$  is high, the bank has more

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<sup>26</sup>The deposit spread reflects the bank's deposit market power Drechsler, Savov, and Schnabl (2017) and the extent to which depositors value the convenience of deposit accounts for payment activities. Motivated by the role of deposits as means of payment, the deposit spread is also called money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2019; Begenau, 2019).

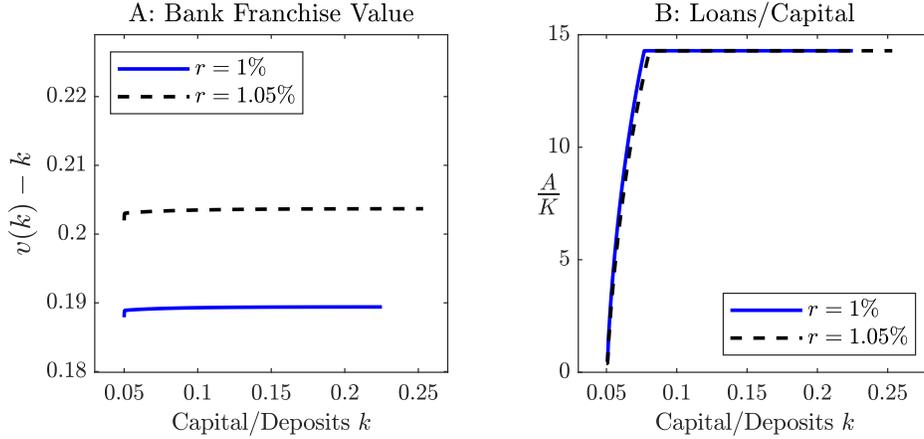


Figure 11: Interest Rate Level, Bank Valuation, and Risk-Taking

flexibility in setting its deposit rate and thus is more in control of the size of its deposit liabilities. In contrast, the bank in a low rate environment faces a greater challenge of managing its deposit liabilities. This mechanism is consistent with the empirical findings. For example, Heider, Saidi, and Schepens (2019) find that the distribution of deposit rates of euro-area banks is truncated at zero and more deposit rates bunch at zero once the ECB lowers the policy rate.

Panel A of Figure 11 compares the bank franchise value under different risk-free rates and shows that a higher  $r$  leads to a higher bank franchise value. In Panel B, we show that when  $r$  increases, the bank reduces its risk exposure per unit of equity capital.<sup>27</sup> The increase of franchise value under a higher  $r$  results from more flexibility to adjust deposit rate rather than more aggressive risk-taking to earn the loan spread,  $\alpha_A$ . Moreover, as shown in both Panel A and B, when  $r$  increases, the bank sets the optimal payout boundary,  $\bar{k}$ , at a higher value (i.e., the right ends of the curves extend). This results shows that under a higher  $r$ , a higher franchise value incentivizes the bank to retain more equity capital as a risk buffer. By the same logic, in a low rate environment, the bank's incentive to maintain equity capital is weaker and it pays out dividend at a lower  $\bar{k}$ .

Panel A of Figure 12 shows that when  $r$  is higher, the deposit  $q$  is higher at all levels of  $k$ .

<sup>27</sup>Note that when  $r$  increases, the expected return from risky lending,  $r + \alpha_A$  in (1), also increases. In other words, when we adjust the risk-free rate, we keep the loan spread constant in line with the evidence in Drechsler, Savov, and Schnabl (2020).

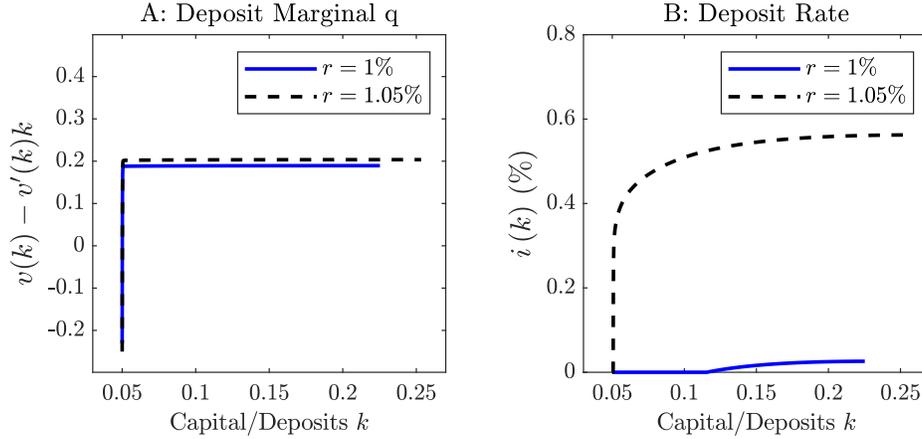


Figure 12: Interest Rate Level, Deposit marginal  $q$ , and Deposit-Taking

Deposits become more valuable when the bank can better control the deposit flows by adjusting deposit rate. In Panel B of Figure 12, we plot the deposit rate. When  $r$  is higher, the bank is more aggressive in raising deposit rate in the high- $k$  region to preserve more flexibility for rate reduction when  $k$  declines in response to negative earning shocks ( $d\mathcal{W}_t^A < 0$ ) or positive deposit shocks ( $d\mathcal{W}_t^X > 0$ ). Under a higher  $r$ , the deposit rate lower bound becomes less binding.

Our model provides a rationale that links bank profitability and franchise value to the level of interest rate. The mechanism is related to the channel of deposit market power in Drechsler, Savov, and Schnabl (2017). In their paper, a higher risk-free rate makes cash, the deposit substitute, becomes more expensive to hold, and this allows banks to raise deposit spreads,  $r - i$ , without losing deposits to cash. Our specification of deposit flow (2) captures deposit market power through the stickiness of deposit stock. When the bank adjusts deposit rate, the flow happens by the order of  $dt$ . Different from Drechsler, Savov, and Schnabl (2017), we highlight the risk in deposit flow and the fact that a higher risk-free rate offers the bank more flexibility to manage such risk.

As shown in Panel B of Figure 12, a lower  $r$  implies less flexibility to set deposit rate, and more importantly, a greater region of the state variable  $k$  where the deposit rate lower bound binds and the bank completely loses control of its deposit stock. The banking literature has largely focused on the positive effect of low interest rate on risk-taking, which we revisits in our setting

(Panel B of Figure 11). Our paper puts more emphasis on the management of deposit risk. Moreover, our model predicts that in a low interest rate environment, the bank is more eager to pay out to shareholders (i.e., set a lower  $\bar{k}$ ). This is consistent with the massive share repurchases done by banks in the last decade of a low interest rate environment.

## 8 Conclusion

Deposits allow banks to cheaply finance lending. Depositors accept a low rate for the convenience of freely moving funds in and out of deposit accounts. The wedge between the deposit rate and the prevailing risk-free rate is often termed as the money premium, because the freedom to transfer funds is essential for deposits to serve as means of payment. However, such commitment exposes banks to the uncertainty in deposit flows, so the value of deposits can be drastically different for well-capitalized banks and undercapitalized (risk-sensitive) banks. When a sequence of negative shocks deplete bank capital, the marginal  $q$  of deposits can turn negative, meaning that deposit inflows hurt bank shareholders. Our result stands in contrast with the existing literature that is mainly concerned about deposit outflows and bank runs. Our model shows that a key challenge of depository institutions is the lack of perfect control over the balance-sheet size and composition.

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## A Alternative Setup with Liquidity Requirement

In this appendix, we enrich the decision environment of the bank. On the asset side of balance sheet, the bank must hold assets that are more liquid than loans (Drechsler, Savov, and Schnabl, 2018). These assets can be reserves or other high-quality liquid assets (HQLA).

At time  $t$ , the bank chooses the value of liquidity holdings, denoted by  $R_t$ . Liquidity holdings pay an interest rate  $\iota$  that is below the risk-free rate  $r$ . The bank is willing to pay the carry cost for the benefits of having a more liquid asset portfolio, as shown in the law of motion of equity capital

$$\begin{aligned} dK_t = & A_t [(r + \alpha_A) dt + \sigma_A d\mathcal{W}_t^A] - B_t r dt - X_t i_t dt - C(n(i_t), X_t) dt \\ & - dU_t + dF_t + R_t \iota dt - S(R_t, X_t, A_t) dt. \end{aligned} \quad (\text{A.1})$$

In comparison to (9), the last two terms are new. The interest income from liquidity holdings is given by  $R_t \iota dt$ . The last term,  $S(R_t, X_t, A_t)$ , captures loss due to illiquidity of asset portfolio. This specification is isomorphic to the following microfounded setup: a Poisson-arriving withdrawal of a large amount of deposits can only be met by liquidity holdings and selling a large amount of loans in exchange for liquidity incurs a fire-sale cost (Moreira and Savov, 2017; Drechsler, Savov, and Schnabl, 2018). Accordingly, we assume  $S_R(R_t, X_t, A_t) < 0$ ,  $S_X(R_t, X_t, A_t) > 0$ , and  $S_A(R_t, X_t, A_t) > 0$ . Note that in the main text, we only consider small (diffusive) deposit shocks.

The bank has to meet the regulatory requirement of liquidity holdings:

$$R_t \geq \xi_R X_t. \quad (\text{A.2})$$

This regulatory constraint can be motivated by the traditional reserve requirement or more recent requirement on liquidity coverage ratio (Basel Committee on Banking Supervision, 2013). When  $B < 0$ , the bank holds risk-free assets that pay interest rate  $r$ . Note that these assets are not part of the liquidity holdings. Here we draw the distinction between liquid and illiquid safe assets in line with the evidence that these assets offer different yields (Krishnamurthy, 2002; Nagel, 2016).

The bank has long-term funding equal to  $X_t + K_t$ . As in the main text, let  $\pi_t^A$  denote the portfolio weight on loans, i.e.,  $\pi_t^A (X_t + K_t) = A_t$ , and  $\pi_t^R$  denote the portfolio weight on liquid assets, i.e.,  $\pi_t^R (X_t + K_t) = R_t$ , so the weight on bonds is  $(\pi_t^A + \pi_t^R - 1)$  because  $B_t =$

$A_t + R_t - (X_t + K_t)$ . We can rewrite the law of motion for  $K_t$  in (A.1) as

$$dK_t = (X_t + K_t) [r + \pi_t^A \alpha_A - \pi_t^R (r - \iota)] dt + (X_t + K_t) \pi_t^A \sigma_A d\mathcal{W}_t^A - X_t i_t dt - C(n(i_t), X_t) dt - S(\pi_t^R (X_t + K_t), X_t, \pi_t^A (X_t + K_t)) - dU_t + dF_t. \quad (\text{A.3})$$

Accordingly, the HJB equation in the interior region where  $dU_t = 0$  and  $dF_t = 0$  is

$$\begin{aligned} \rho V(X, K) = & \max_{\{\pi^A, \pi^R, i\}} V_X(X, K) X [-\delta_X + n(i)] + \frac{1}{2} V_{XX}(X, K) X^2 \sigma_X^2 \\ & + V_K(X, K) (X + K) [r + \pi^A \alpha_A - \pi^R (r - \iota)] + \frac{1}{2} V_{KK}(X, K) (X + K)^2 (\pi^A \sigma_A)^2 \\ & - V_K(X, K) [S(\pi^R (X + K), X, \pi^A (X + K)) + Xi + C(n(i), X)] \\ & + V_{XK}(X, K) X (X + K) \pi^A \sigma_A \sigma_X \phi. \end{aligned} \quad (\text{A.4})$$

**Risk-taking.** The first-order condition for  $\pi^A$  gives the following solution:

$$\pi^A = \min \left\{ \frac{\alpha_A + \epsilon(X, K) \sigma_A \sigma_X \phi - S_A(R, X, A)}{\gamma(X, K) \sigma_A^2 \left(\frac{X+K}{K}\right)}, \frac{K}{\xi_K(X + K)} \right\}. \quad (\text{A.5})$$

While setting up  $\pi^A = A/(X + K)$  as the control variable is convenient for solving the model, it is intuitive to express the solution in loan-to-capital ratio, i.e.,  $A/K = \pi^A (X + K)/K$ :

$$\frac{A}{K} = \min \left\{ \frac{\alpha_A + \epsilon(X, K) \sigma_A \sigma_X \phi - S_A(R, X, A)}{\gamma(X, K) \sigma_A^2}, \frac{1}{\xi_K} \right\}. \quad (\text{A.6})$$

In comparison with (30), the only difference is that the numerator is deducted by  $S_A(R, X, A)$ .

**Liquidity Holdings.** When the liquidity requirement (A.2) does not bind, the optimality condition for  $\pi^R$  equates the marginal cost of holding reserves, i.e., accepting the below- $r$  rate of return  $\iota$ , and the marginal benefit of holding reserves to reduce the payment settlement cost:

$$r - \iota = -S_R(\pi^R (X + K), X, \pi^A (X + K)). \quad (\text{A.7})$$

The reserve requirement can be rewritten as the following restriction on  $\pi^R$ :

$$\pi^R \geq \frac{\xi_R X}{(X + K)}. \quad (\text{A.8})$$

Next, we specify the functional form of  $S(R, X, A)$  that satisfies the properties that  $S(R, X, A)$  decreases in  $R$  and increases in  $X$  and  $A$ :

$$S(R, X, A) = \frac{1}{2} \frac{(\chi_1 X + \chi_2 A)^2}{R}. \quad (\text{A.9})$$

The numerator is convex in  $X$  and  $A$  while the denominator is linear in  $R$ . Therefore, to maintain the same level of  $S(R, X, A)$ , the bank will have to hold increasingly more liquidity as it expands its balance sheet (i.e., increases  $X$  and  $A$ ). This captures the decreasing marginal return to liquidity holdings that have been microfounded in various ways (Moreira and Savov, 2017).

Under this functional form of  $S(R, X, A)$ , we obtain

$$S_R(R, X, A) = -\frac{1}{2} \left( \frac{\chi_1 X + \chi_2 A}{R} \right)^2. \quad (\text{A.10})$$

Therefore, the optimality condition for  $\pi_t^R$  implies that  $r - \iota = \frac{1}{2} \left( \frac{\chi_1 X + \chi_2 A}{R} \right)^2$ , so rearranging the equation we obtain the following reserve holding policy

$$R = \frac{\chi_1 X + \chi_2 A}{\sqrt{2(r - \iota)}} \quad (\text{A.11})$$

This liquidity holding policy is in the spirit of Baumol (1952) and Tobin (1956) who show that the demand for liquidity is equal to the product of transaction costs (mapping to  $\chi_1$  and  $\chi_2$ ) and transaction needs (mapping to  $X$  and  $A$ ) divided by the square root of two times the carry cost. As previously discussed, a microfoundation can be built for  $S(R, X, A)$  where the transaction or liquidity needs of the bank arises from deposit withdrawal and depends the amount of relatively illiquid assets (loans) in the portfolio that are subject to fire-sale losses.

Given the functional forms of  $S(R, X, A)$  and deposit maintenance costs in the main text, the bank's problem is homogeneous in  $X$  and its value function  $V(X, K) = v(k) X$ , where

$$k = \frac{K}{X}. \quad (\text{A.12})$$

And, as in the main text, we simplify the expressions of the effective risk aversion in (31)

$$\gamma(k) = \frac{-V_{KK}(X, K) K}{V_K(X, K)} = -\frac{v''(k) k}{v'(k)}, \quad (\text{A.13})$$

and the elasticity of marginal value of capital to deposits

$$\epsilon(k) = \frac{V_{XK}(X, K) X}{V_K(X, K)} = -\frac{v''(k) k}{v'(k)}, \quad (\text{A.14})$$

which happens to be equal to  $\gamma(k)$ .

Next, we simplify the expression of loan-to-capital ratio, a measure of the bank's risk-taking. First, note that from (A.11), we obtain the marginal illiquidity cost of loans:

$$S_A(R, X, A) = \chi_2 \left( \frac{\chi_1 X + \chi_2 A}{R} \right) = \chi_2 \sqrt{2(r - \iota)}, \quad (\text{A.15})$$

Using (A.15) and  $\epsilon(k) = \gamma(k)$ , we simplify the optimal loan-to-capital ratio:

$$\frac{A}{K} = \min \left\{ \frac{\alpha_A - \chi_2 \sqrt{2(r - \iota)}}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \frac{1}{\xi_K} \right\}, \quad (\text{A.16})$$

The only difference from (30) is that in the numerator, we subtract  $\alpha_A$  by the marginal illiquidity cost  $\chi_2 \sqrt{2(r - \iota)}$ . To make lending profitable, we impose the parameter restriction

$$\alpha_A > \chi_2 \sqrt{2(r - \iota)}. \quad (\text{A.17})$$

Using these expressions, we can rewrite the HJB equation (A.4) as

$$\begin{aligned} \rho v(k) = & \max_{\pi^A, \pi^R, i} [v(k) - v'(k) k] (-\delta_X + \omega i) + \frac{1}{2} v''(k) k^2 \sigma_X^2 \\ & + v'(k) (1+k) [r + \pi^A \alpha_A - \pi^R (r - \iota)] + \frac{1}{2} v''(k) (1+k)^2 (\pi^A \sigma_A)^2 \\ & - v'(k) \left[ \frac{1}{2} \left( \frac{\chi_1}{1+k} + \chi_2 \pi^A \right)^2 \pi^R (1+k) + i + \theta_0 + \frac{\theta_1}{2} (\omega i)^2 \right] \\ & - v''(k) k (1+k) \pi^A \sigma_A \sigma_X \phi. \end{aligned} \quad (\text{A.18})$$

To show that (A.18) is an ODE for  $v(k)$ , we need to show that the control variables only depend on  $k$  and the level and derivatives of  $v(k)$ . First, by definition,  $\pi^A = A/(X + K)$ , so we obtain the following simplified expression for  $\pi^A$  from (A.16):

$$\pi^A = \left(\frac{A}{K}\right) \left(\frac{K}{K + X}\right) = \min \left\{ \frac{\alpha_A - \chi_2 \sqrt{2(r - \iota)}}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \frac{1}{\xi_K} \right\} \left(\frac{k}{1 + k}\right). \quad (\text{A.19})$$

Rearranging (A.11), we can solve  $\pi^R$  as a linear function of  $\pi^A$  and the state variable  $k$ :

$$\pi^R = \frac{\chi_2}{\sqrt{2(r - \iota)}} \pi^A + \frac{\chi_1}{(1 + k) \sqrt{2(r - \iota)}}, \quad (\text{A.20})$$

so it also only depends on  $k$  and the level and derivatives of  $v(k)$ . The deposit rate, still given by (36) in the main text, only depends on  $V_X(X, K) = v(k) - v'(k)k$  and  $V_K(X, K) = v'(k)$ .

After substituting the optimal control variables into the HJB equation, we obtained an ordinary equation with the same boundary conditions discussed in the main text. The determination of endogenous upper bound of  $k$  also follows the main text. The only difference is in the determination of endogenous lower bound of  $k$ , i.e., the equity issuance boundary.

Let  $\underline{k}_S$  denote the lower bound in (25) implied by the supplementary leverage ratio (SLR) requirement. The liquidity requirement implies another lower bound  $\underline{k}_L$ . Substituting (A.20) into the reserve requirement (A.8), we have

$$\frac{\chi_2}{\sqrt{2(r - \iota)}} \pi^A + \frac{\chi_1}{(1 + k) \sqrt{2(r - \iota)}} \geq \frac{\xi_R}{(1 + k)}, \quad (\text{A.21})$$

Using (A.19) to substitute out  $\pi^A$  and rearranging the equation, we have

$$\min \left\{ \frac{\alpha_A - \chi_2 \sqrt{2(r - \iota)}}{\gamma(k) \sigma_A^2} + \frac{\sigma_X}{\sigma_A} \phi, \frac{1}{\xi_K} \right\} k \geq \frac{\xi_R \sqrt{2(r - \iota)} - \chi_1}{\chi_2}. \quad (\text{A.22})$$

In our numeric solution, the right side increases in  $k$  (as  $\gamma(k)$  increases in  $k$ ). Therefore, (A.22) imposes a lower bound of  $k$ , denoted by  $\underline{k}_L$ . Therefore, we have

$$\underline{k} = \max \{0, \underline{k}_S, \underline{k}_L\}. \quad (\text{A.23})$$

To sum up, introducing the bank's needs to hold reserves or HQLA leads to three changes in the solution. First, the new control variable, optimal liquidity-holding policy, is given by the Baumol-Tobin style money demand (A.11). Second, in the optimal risk-taking policy (A.16),  $\alpha_A$  is subtracted by the marginal illiquidity cost of loans. Third, the equity issuance boundary is defined by (A.23) nesting considerations of liquidation, SLR requirement, and liquidity requirement.