Tokenomics: Dynamic Adoption and Valuation*

Lin William Cong† Ye Li§ Neng Wang‡

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Abstract

We develop a dynamic asset-pricing model of cryptocurrencies/tokens that facilitate peer-to-peer transactions on digital platforms. The equilibrium value of tokens is determined by users' transactional demand rather than cashflows as in standard valuation models. Endogenous platform adoption exhibits an S-curve – it starts slow, becomes volatile, and eventually tapers off. Users' adoption generates positive network externality, which leads to endogenous token-price risk and boom-bust price dynamics. Tokens allow users to capitalize on platform growth, inducing an intertemporal feedback between user adoption and token price that accelerates platform adoption, reduces user-base volatility, and improves welfare.

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†University of Chicago Booth School of Business. E-mail: will.cong@chicagobooth.edu

§The Ohio State University Fisher College of Business. E-mail: li.8935@osu.edu

‡Columbia Business School and NBER. E-mail: neng.wang@columbia.edu

1 Introduction

Blockchain-based applications and cryptocurrencies have recently taken a center stage among technological breakthroughs in finance. The global market capitalization of cryptocurrencies has grown to hundreds of billions of US dollars. Nevertheless, academics, practitioners, and regulators have divergent views on how cryptocurrencies derive their value.

In this paper, we provide a fundamentals-based dynamic valuation model for cryptocurrencies. We focus on the endogenous formation of a digital marketplace or network ("the platform") where its native cryptocurrency settles transactions and derives its value from underlying economic activities on the platform. To distinguish from cryptocurrencies as general-purpose medium of exchange (e.g., Bitcoin), we shall refer to platform-specific currencies as tokens.[1] In contrast to financial assets that derive value from cash flows, the value of tokens arises from a form of convenience yield that is specific to the platform.

Our model captures two key features shared by a majority of tokens. First, they are the means of payment on platforms that support specific economic transactions. For example, Filecoin is a digital marketplace that allows users to exchange data storage space for its tokens (FIL). Another example is Basic Attention Token (BAT): advertisers use BATs to pay for their ads, publishers receive BATs for hosting these ads, and web-browser users are rewarded BATs for viewing these ads. Second, user adoption exhibits network effects. In both examples, the more users the platform has, the easier it is for any user to find a transaction counterparty, and the more useful the tokens are. Our model also applies to tokens used on centralized platforms such as the cryptocurrency Facebook is developing.[2]

Consequently, the market price of tokens and the platform size (active users) naturally arise as two key endogenous variables in our model. Our equilibrium token pricing formula exhibit three desirable features. First, the platform's productivity captures the value of particular economic activities that the platform supports. Second, user base enters positively

[1] Tokens in our paper should also be distinguished from security tokens that represent claims on issuers' cash-flows or rights to redeem products and services.

[2] See "Facebook Building Cryptocurrency-Based Payments System", *The Wall Street Journal*, May 2nd 2019. Other examples include online social networks (e.g., QQ coins on Tencent's messaging platform) and online games (e.g., Linden dollar for *Second Life* and WoW Gold for *World of Warcraft*).

into the pricing formula, capturing the positive network externality of user adoption. Third, user heterogeneity matters for platform adoption and token pricing.

Moreover, we clarify the roles of tokens in platform adoption by comparing token-based platforms and platforms without tokens. Introducing tokens encourages early adoption of platforms with improving productivity, because agents expect token price appreciation. Such investment motive also stabilizes adoption in the presence of temporary productivity shocks. Finally, by jointly examining the dynamics of token price and user adoption, our analysis sheds light on the cross-sectional variation of token values, token price volatility, and the recent run-up and crash of leading cryptocurrencies.

Specifically, we consider a continuous-time economy with a continuum of agents who differ in their transaction needs on the platform. We broadly interpret transactions as including value transfers (e.g., Filecoin and BAT) and smart contracting (e.g., Ethereum). Accordingly, we model agents' gain from platform transactions as a flow utility from token holdings. As a form of convenience yield specific to the platform, such flow utility naturally depends on agents' transaction needs, user base, and the platform's productivity. The flow utility increases with the size of user base, capturing the network effects, and the productivity reflects the functionality of the platform and technological and regulatory factors.

In our model, agents make a two-step decision on (1) whether to incur a participation cost to join the platform, and if so, (2) how many tokens to hold, which depends on both blockchain trade surplus ("transaction motive") and the expected future token price ("investment motive"). A key insight of our model is that users' adoption decision not only exhibits static complementarity through the flow utility of token holdings (the transaction motive), but also an inter-temporal complementarity via the investment motive.

For illustration, consider a promising platform with a positive productivity drift. The prospective growth in productivity leads agents to expect more users to join the community in future, inducing a stronger future demand for tokens, which in turn generates a current expectation of token price appreciation. The investment motive increases the demand for tokens today and accelerates user adoption.

We characterize the Markov equilibrium with platform productivity being the state variable. The model has two key endogenous variables, the user base and token price. The equilibrium outcome features an S-curve of user adoption – as the platform productivity grows, the user base expands slowly, and then the expansion speeds up before eventually tapering off near full adoption. We derive an equilibrium token pricing formula that incorporates endogenous network effects in an otherwise canonical Gordon growth formula. The pricing formula accounts for platform productivity, user base, agents' expectations of future token price, and user heterogeneity. Token valuation boils down to solving an ordinary differential equation subject to intuitive boundary conditions. After analytically characterizing important equilibrium properties, we use data on token price and user base to guide our choice for our choice of parameter values and then provide numerical illustrations of results.

Our model features rich interactions between financial markets and the real economy: the financial side operates through the endogenous determination of token prices, whereas the real side manifests itself in the user adoption and utility flows from platform's economic transactions. Tokens affect user adoption through the expected price appreciation, i.e., agents' investment motive, while user base affects token prices by entering into the flow utility of tokens and increasing the token demand. This two-way feedback naturally prompts a question: given that platforms can settle transactions in other currencies (e.g., dollars), how does a platform with local means of payment (tokens) differ from one without?

To answer this question, we compare the endogenous S-curve in our token-based economy with adoption curves in two benchmark economies: the first-best economy (i.e., the planner's solution) and the tokenless economy where agents use the numeraire good as the media of exchange. Without tokens, under-adoption of promising platforms (modelled as having positive productivity drift) arises because a user does not internalize the positive externality from her adoption on others. In contrast, introducing tokens can improve welfare by inducing more adoption through agents' expectation of token price appreciation and investment motive. So far we have focused on platforms with growing productivity. For those with declining productivity, tokens precipitate the abandonment: agents forecast a smaller user base in the future, anticipating token price depreciation, so they shun away from holding tokens. In sum, embedding tokens on a platform front-loads the prospect of the platform and the resulting investment motive together the direct usage motive affects user adoption.

Introducing tokens can also reduce user-base volatility, making it less sensitive to productivity shocks. The key driver is again the agents' investment motive. Consider a platform with growing yet stochastic productivity. A negative productivity shock directly reduces the user's flow utility and thus lowers user adoption. However, this negative effect is mitigated by an indirect effect through the expected token price appreciation. A lower current adoption level implies that more users can be brought onto the platform in the future and therefore a higher expected token price appreciation. The investment motive sustains user adoption. Similarly, a positive productivity shock directly increases adoption by increasing the flow utility. But as the pool of potential newcomers shrinks, the expected token price appreciation declines, discouraging agents from adoption. Overall, productivity shocks, when translated to user-base fluctuations, are dampened by the endogenous dynamics of token price.

Just as tokens affect the user-base dynamics, endogenous user adoption is critical for understanding major asset-pricing issues surrounding tokens. First, the network effects generate a large cross-sectional variation in token price among platforms in early stages of adoption, in line with empirical observations. Moreover, such adoption externality also amplifies the impact of platform productivity shocks on token price, creating "excess volatility." The amplification effect is even stronger when we allow the productivity drift to increase with the user base — a form of community bootstrapping that practitioners emphasize. Finally, by allowing the productivity beta (systematic risk) to increase with platform adoption, our model generates an initial rise of token price followed by a decline and eventual stabilization, broadly consistent with the observed "bubbly" price dynamics.

Related Literature. Among early economics studies on blockchain games and consensus generation mechanisms, Biais, Bisière, Bouvard, and Casamatta (2017) and Saleh (2017) analyze mining/minting games in Proof-of-Work- and Proof-of-Stake-based public blockchains; Easley, O'Hara, and Basu (2017), Huberman, Leshno, and Moallemi (2017), and Cong, He, and Li (2018) study miners' compensation, organization, and market structure; Abadi and

Brunnermeier (2019) compares blockchain-based ledgers with traditional centralized ledgers; Cong and He (2018) examine informational issues in generating decentralized consensus with implications on industrial organization. We differ by taking as given the operational and technical aspects of blockchains, such as the formation of decentralized consensus, and thereby, focusing on users' trade-off and the resulting dynamic interaction between platform adoption and token pricing.

Among contemporary theories featuring token valuation in static settings, Sockin and Xiong (2018) studies tokens as indivisible membership certificates for agents to match and trade with each other; Li and Mann (2018) argues that initial token offering allows agents to coordinate by costly signaling through token acquisition; Pagnotta and Buraschi (2018) studies Bitcoin pricing on exogenous user networks; Catalini and Gans (2018) examines developers' pricing of tokens to fund projects and aggregate information; Chod and Lyandres (2018) contrast security token offerings with traditional financing. Our paper is the first to clarify the role of tokens in aligning the usage and investment motives and shaping the platform-adoption dynamics, in addition to delivering a token pricing formula as an equilibrium outcome and of practical relevance.

In dynamic settings, Athey, Parashkevov, Sarukkai, and Xia (2016) emphasize the role of learning in agents' decisions to use Bitcoin absent stochastic platform productivity and user network externality; Biais, Bisière, Bouvard, Casamatta, and Menkveld (2018) emphasize the fundamental value of Bitcoin from transactional benefits; Fanti, Kogan, and Viswanath (2019) provide a valuation framework for Proof-of-Stake (PoS) payment systems. We differ by studying the joint determination of user adoption and token valuation in a framework that highlights user heterogeneity, network externalities, and most importantly, inter-temporal feedback effects. Moreover, our model is applicable to platforms owned by trusted third parties as well as permissioned blockchains.

We do not analyze the implications of blockchain technology on general-purpose currencies and monetary policies (e.g., Balvers and McDonald, 2017; Raskin and Yermack, 2016; Garratt and Wallace, 2018; Schilling and Uhlig, 2018). Instead, we focus on the endogenous interaction between token pricing and user adoption on platforms that serve niche markets with time-varying productivity. Our study should therefore be distinguished from the monetary literature. Our model also differs from standard valuation models in that the underlying payoff of tokens is utility flow, a form of convenience yield, rather than cash flows. The convenience yield of tokens should also be distinguished from that of other assets, such commodities (Gibson and Schwartz, 1990) and safe bonds (Krishnamurthy and Vissing-Jorgensen, 2012), because its dependence on platform-specific factors (i.e., platform productivity, user base, and agents' transaction needs).

We organize the remainder of the article as follows. Section 2 sets up the model; Section 3 solves the dynamic equilibrium and derives the token valuation formula; Section 4 presents the solutions for the tokenless and first-best economies; Section 5 highlights the impact of tokens on user adoption; Section 6 analyzes token price dynamics; Section 7 concludes. The appendix contains a theoretical foundation for the token transaction surplus (the utility flow), the proofs of propositions, and parameter choices in the quantitative analysis.

2 A Model of Platform Economy

Consider a continuous-time economy where a unit measure of agents conduct peer-to-peer transactions and realize trade surpluses on a blockchain-based platform. A generic good serves as the numeraire. We first set up and solve the model under the risk-neutral measure. In Appendix C, we calibrate the model under the physical measure.[3]

2.1 Platform and Agents

The platform allows agents to conduct peer-to-peer transactions. These transactions are settled via a medium of exchange, which can either be the numeraire good or the local currency (token) on the platform. We use $x_{i,t}$ to denote the value of agent i's holdings of transaction medium in the unit of the numeraire good. These holdings facilitate transactions

[3]The typical no-arbitrage condition implies a probability measure—the risk-neutral measure—under which agents discount future cash flows using the risk-free rate. Note that the difference between risk-neutral measure and physical measure accounts for any risk premium (see chap 6 in Duffie, 2001).

on the platform and generate a flow of utility over dt given by

$$x_{i,t}^{1-\alpha}\left(N_t A_t e^{u_i}\right)^{\alpha} dt, \tag{1}$$

where N_t is the platform user base, A_t measures platform productivity, u_i captures agent i's specific needs for platform transactions, and $\alpha \in (0,1)$ is a constant. Appendix A contains a theoretical foundation for this reduced-form flow utility.

We choose this utility flow specification for the following considerations. First, the utility flow increases with user base N_t, i.e., the total measure of agents on the platform (with $x_{i,t} > 0$). This specification captures the user network effect, as it is easier to find a transaction counterparty in a larger community. Second, the marginal utility decreases with $x_{i,t}$, captured by $\alpha > 0$. The exponents of $x_{i,t}$ and $N_t A_t e^{u_i}$ sum up to one for analytical convenience. Note that holding tokens over the dt period is necessary for the agent to realize this utility flow.

The platform productivity, A_t, evolves according to a geometric Brownian motion:

$$\frac{dA_t}{A_t} = \mu^A dt + \sigma^A dZ_t^A, \tag{2}$$

where Z_t^A is a standard Brownian motion under the risk-neutral measure. We focus on the case of a promising yet risky platform, i.e., $\mu^A > 0$ and $\sigma^A > 0$. We interpret A_t broadly. A positive shock to A_t can reflect technological advances, favorable regulatory changes, growing users' interests, and increasing variety of activities feasible on the platform.

We assume that agents' transaction needs, u_i, are heterogeneous. Let $G(u)$ and $g(u)$ denote the cross-sectional cumulative distribution function and the density function of u_i that is assumed to be continuously differentiable over a finite support $[\underline{U}, \overline{U}]$. u_i can be broadly interpreted. For payment blockchains (e.g., Ripple), a high value of u_i reflects agent i's urge to conduct an international remittance. For smart-contracting blockchains (e.g., Ethereum), u_i captures agent i's project productivity. For decentralized computation (e.g., Dfinity) and data storage (e.g., Filecoin) applications, u_i corresponds to the need for secure and fast access to computing power and data.

To join the platform and realize the transaction surplus, an agent incurs a flow cost ϕdt.

For example, transacting on the platform takes effort and attention. At any time t, agents may choose not to participate and then collect no utility. Therefore, agents with sufficiently high u_i choose to join the platform, while agents with sufficiently low u_i do not participate. The flow cost serves as a modeling device for users' endogenous threshold of participation.

2.2 Tokens, Agents' Problem, and Equilibrium

Tokens and endogenous price. In what follows, we focus on platforms with native tokens that users hold to conduct transactions, i.e.,

$$x_{i,t} = P_t k_{i,t}, \tag{3}$$

where P_t is the unit price of token in terms of the numeraire good and $k_{i,t}$ is the units of token.[4]

We conjecture and later verify that the equilibrium token price follows a diffusion process,

$$dP_t = P_t \mu_t^P dt + P_t \sigma_t^P dZ_t^A \,, \tag{4}$$

where μ_t^P and σ_t^P are endogenously determined. Throughout the paper, we use upper-case letters for aggregate and price variables that individuals take as given, and lower-case letters for individual-level variables.

Agent's problem. Let $y_{i,t}$ denote agent i's (undiscounted) *cumulative* payoff from platform activities. Agent i then maximizes life-time payoff under the risk-neutral measure,

$$\mathbb{E}\left[\int_0^\infty e^{-rt} dy_{i,t}\right] \,, \tag{5}$$

[4]The numeraire value ($P_t k_{i,t}$), instead of $k_{i,t}$ alone, shows up in the surplus flow to facilitate the comparison between platforms with and without tokens. It is also motivated by the fact that the economic value of blockchain trades depends on the numeraire value of real goods and services that are transacted. Our results are qualitatively similar if $k_{i,t}$ replaces $P_t k_{i,t}$ in the utility flow given by Equation (1).

where we can write the utility flow $dy_{i,t}$ as follows:

$$dy_{i,t} = \max\left\{0, \max_{k_{i,t}>0}\left[(P_t k_{i,t})^{1-\alpha}\left(N_t A_t e^{u_i}\right)^{\alpha} dt + k_{i,t}\mathbb{E}_t\left[dP_t\right] - \phi dt - P_t k_{i,t} r dt\right]\right\}. \quad (6)$$

Here, the outer "max" operator reflects agent i's option to leave the platform and obtain zero profit, and the inner "max" operator reflects agent i's optimal choice of $k_{i,t}$.

Inside the inner max operator are four terms that add up to give the net flow payoff from platform participation. The first term corresponds to the transaction surplus given in (1). The second term is the expected capital gains from holding $k_{i,t}$ units of tokens, where $\mathbb{E}_t\left[dP_t\right] = P_t\mu_t^P dt$. As typical in asset pricing models, the user's incremental payoff over dt is equal to the sum of the contemporaneous payoff and the price fluctuation, given by the first two terms in (6). The third term is the participation cost. Finally, because holding $k_{i,t}$ units of tokens for the transaction purpose comes with a forgone opportunity cost of not being able to invest $x_{i,t}$ and earn interest for dt time, the financing cost is then $P_t k_{i,t} r dt$. This is essentially the user-cost-of-capital argument in Jorgenson (1963).[5]

It is worth emphasizing that in our token-based economy, agents must hold tokens for at least an instant dt to complete transactions and derive utility flows. This holding period exposes users to token price change over dt. For blockchain-based tokens, forging the ledger of transactions takes time.[6] This confirmation period is necessary for the finality of transactions as shown by Chiu and Koeppl (2017). Another example is smart contracting, which often requires holding tokens as collateral in escrow account and therefore exposes the collateral owners to token price fluctuations. A third example involves the locking-up of "staking tokens" or "work tokens" in order to compete for service provision in many blockchain applications. Appendix A elaborates further through simple theoretical foundations.

The Markov equilibrium. We study a Markov equilibrium with A_t, the only source of exogenous shocks in the economy, as the state variable whose dynamics generate the information filtration under which agents make decisions under rational expectation. For

[5] Alternatively, we can derive the same result using the standard dynamic programming approach.

[6] For example, the Bitcoin blockchain requires 10-11 minutes to generate consensus on transactions.

simplicity and to isolate user adoption and demand, we fix the token supply to a constant M.[7] The market clearing condition is

$$M = \int_{i \in [0,1]} k_{i,t} di, \tag{7}$$

where for those who do not participate, $k_{i,t} = 0$.

Definition 1. *A Markov equilibrium with state variable A_t is described by agents' decisions and equilibrium token price such that the token market clearing condition given by Equation (7) holds and agents optimally decide to participate (or not) and choose token holdings.*

3 Dynamic Equilibrium of Adoption and Valuation

We now solve for the Markov equilibrium, where the user base, N_t, users' token holdings, $k_{i,t}$, and token price P_t, are functions of the state variable A_t. First, we analyze agents' decision to participate and hold tokens, given A_t and agents' expectation of token price change μ_t^P. Then we complete the solution by solving the token price dynamics (and in particular, μ_t^P as a function of A_t). Each step ends with a summarizing proposition.

Token demand and user base. Conditioning on joining the platform, agent i chooses the optimal token holdings, $k_{i,t}^*$, using the first order condition,

$$(1-\alpha)\left(\frac{N_t A_t e^{u_i}}{P_t k_{i,t}^*}\right)^{\alpha} + \mu_t^P = r, \tag{8}$$

which states that the sum of *marginal* transaction surplus on the platform and the expected token price change is equal to the required rate of return, r. Rearranging this equation, we

[7] This is the case with many ICOs that fix the supply of tokens. More generally, the blockchain technology allows supply schedules to be based on explicit rules independent of any endogenous variables, and can be accommodated in the model by adding in the resulting token inflation or deflation that are beyond the price fluctuation due to time-varying token demands.

obtain the following expression for the optimal token holdings:

$$k_{i,t}^{*}=\frac{N_tA_te^{u_i}}{P_t}\left(\frac{1-\alpha}{r-\mu_t^P}\right)^{\frac{1}{\alpha}}. \tag{9}$$

$k_{i,t}^{*}$ has several properties. First, agents hold more tokens when the common productivity, A_t, or agent-specific transaction need, u_i, is high, and also when the user base, N_t, is larger because it is easier to conduct trades on the platform. Equation (9) reflects an investment motive to hold tokens, that is $k_{i,t}^{*}$ increases in the expected token appreciation, μ_t^P.

Using $k_{i,t}^{*}$, we obtain the following expression for the agent's profit conditional on participating on the platform:

$$N_tA_te^{u_i}\alpha\left(\frac{1-\alpha}{r-\mu_t^P}\right)^{\frac{1-\alpha}{\alpha}}-\phi. \tag{10}$$

Agent i will only particpate when the preceding expression is non-negative. That is, only those agents with sufficiently large u_i will particpate. Let $\underline{u}_t$ denote the type of user who is indifferent between participating on the platform or not.

Setting the expression (10) to zero gives the user-type cutoff threshold:

$$\underline{u}_t=\underline{u}\left(N_t;A_t,\mu_t^P\right)=-\ln\left(N_t\right)+\ln\left(\frac{\phi}{A_t\alpha}\right)-\left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right). \tag{11}$$

Because only agents with $u_i\geq\underline{u}_t$ will particpate, the user base is then given by

$$N_t=1-G\left(\underline{u}_t\right). \tag{12}$$

The adoption threshold $\underline{u}_t$ is decreasing in A_t because a more productive platform attracts more users. The threshold also decreases when agents expect a higher token price appreciation (i.e., higher μ_t^P).

Equations (11) and (12) jointly determine the user base N_t given A_t and μ_t^P. First, we note that zero adoption is always a solution. Next, we focus on the non-degenerate case, i.e., $N_t>0$. Fixing A_t and μ_t^P, we consider a *response* function $R\left(n;A_t,\mu_t^P\right)$ that maps a

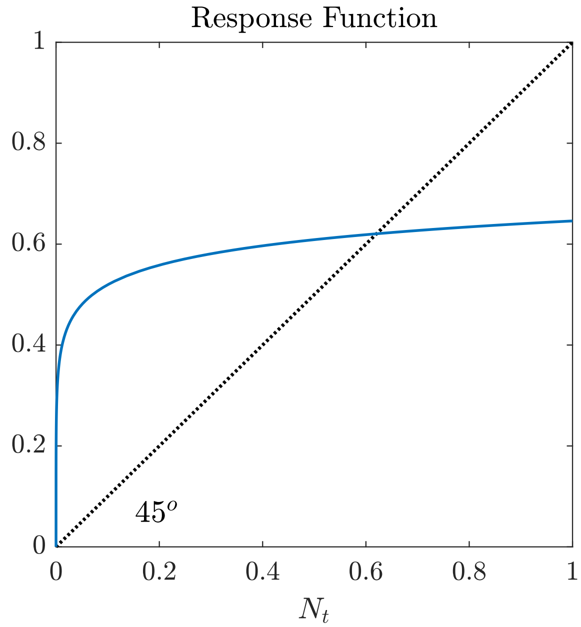

Figure 1: **Determining User Base.** This graph shows the aggregate response of users' adoption decision, $R\left(n; A_t, \mu_t^P\right)$, to different levels of $N_t = n \in [0, 1]$, given A_t and μ_t^P.

hypothetical value of N_t, say n, to the measure of agents who choose to participate after knowing $N_t = n$. As depicted in Figure 1, the response curve originates from zero (the degenerate case). In the Appendix, we first show that given μ_t^P, there exists a threshold $\underline{A}\left(\mu_t^P\right)$ such that for $A_t < \underline{A}\left(\mu_t^P\right)$, a non-degenerate solution does not exist, because the response curve never crosses the 45^o line. Then we prove that when $A_t \geq \underline{A}\left(\mu_t^P\right)$, the response curve crosses the 45^o line exactly once (and from above) under the assumption that the hazard rate for $g\left(u\right)$ is increasing.[8] Later in our numerical solution, we verify that such inequalities hold at all values of A_t.

Proposition 1 (**Token Demand and User Base**). *Given μ_t^P and a sufficiently high productivity, i.e., $A_t > \underline{A}\left(\mu_t^P\right)$, we have a unique non-degenerate solution, N_t, for Equations (11) and (12) under the increasing hazard-rate assumption. The user base, N_t, increases in μ_t^P and A_t. Agent i participates when $u_i \geq \underline{u}_t$, where $\underline{u}_t$ is given by Equation (11). Conditional on participating, Agent i's optimal token holding, $k_{i,t}^*$, is given by Equation (9). The token holding, $k_{i,t}^*$, decreases in P_t and increases in A_t, μ_t^P, u_i, and N_t.*

[8]The hazard rate, $\frac{g(u)}{1-G(u)}$, is increasing in u if and only if $1 - G(u)$ is log-concave. This assumption is common in the theory literature, for example, to avoid the complicated "ironing" of virtual values.

Token Pricing. First, we define the *participants' aggregate transaction need* as

$$S_t := \int_{\underline{u}_t}^{\overline{U}} e^u g(u)\, du, \tag{13}$$

the integral of e^{u_i} of participating agents. Substituting optimal holdings in Equation (9) into the market clearing condition in Equation (7), we obtain the *Token Pricing Formula*:

$$P_t = \frac{N_t S_t A_t}{M} \left(\frac{1-\alpha}{r - \mu_t^P} \right)^{\frac{1}{\alpha}}. \tag{14}$$

The token price increases in N_t – the larger the user base is, the higher trade surplus individual participants can realize by holding tokens, and stronger the token demand. The price-to-user base ratio, a natural valuation metric in our setting and practice, increases in the platform productivity, the expected price appreciation, and the network participants' aggregate transaction need, while it decreases in the token supply M.

Industry practices broadly corroborate the formula, for exmaple, by incorporating DAA (daily active addresses) and NVT Ratio (market cap to daily transaction volume) in token valuation framework. But instead of heuristically aggregating such inputs into a pricing formula, we derive *both* token pricing and user adoption as endogenous equilibrium outcomes.

Since $\underline{u}_t$ decreases in μ_t^P, the RHS of Equation (14) increases in μ_t^P, so A_t and P_t uniquely pin down μ_t^P, which contains the first and second derivatives of P_t to A_t by Itô's lemma. Therefore, the equation implies a unique mapping from A_t, $P(A_t)$, and $P'(A_t)$ to $P''(A_t)$. Specifically, the token pricing formula given in Equation (14) implies the following differential equation that characterizes $P(A_t)$ as a function of state variable A_t.

$$\mu^A A_t \left(\frac{dP_t}{dA_t} \right) + \frac{1}{2} \left(\sigma^A \right)^2 A_t^2 \frac{d^2 P_t}{dA_t^2} + (1-\alpha) \left(\frac{N_t S_t A_t}{M P_t} \right)^{\alpha} P_t - r P_t = 0. \tag{15}$$

We solve the preceding ODE for $P(A_t)$ with the following boundary conditions. The first is

$$\lim_{A_t \to 0} P(A_t) = 0, \tag{16}$$

which means that the token price is zero when the platform is permanently unproductive ($A_t = 0$ is an absorbing state).

Next, we discuss remaining boundary conditions using token price under full adoption. As $N_t = 1$, the aggregate transaction demand, S_t, is equal to $\overline{S}$, where

$$\overline{S} \equiv \int_{\underline{U}}^{\overline{U}} e^u g(u)\, du, \tag{17}$$

is the sum (integral) of all agents' e^{u_i}. Let $\overline{P}(A_t)$ denote token price under full adoption. As we focus on the fundamentals, the token price dynamics is fully determined by the underlying productivity growth, i.e., $\mu_t^P = \mu^A$. Therefore, we obtain the following Gordon Growth Formula for token price under full adoption:

$$\overline{P}(A_t) = \frac{\overline{S} A_t}{M} \left(\frac{1-\alpha}{r - \mu^A} \right)^{\frac{1}{\alpha}}, \tag{18}$$

where $\overline{S}$ is given by Equation (17). Let $\widetilde{A}$ denote the lowest value of A_t that induces full adoption. The value-matching and smooth-pasting conditions hold at $\widetilde{A}$:

$$P(\widetilde{A}) = \overline{P}(\widetilde{A}) \quad \text{and} \quad P'(\widetilde{A}) = \overline{P}'(\widetilde{A}). \tag{19}$$

The next proposition follows directly.

Proposition 2 (**Markov Equilibrium**). *With A_t being the state variable, the equilibrium token price $P(A_t)$ solves the ODE from Equation (14) subject to boundary conditions given by Equations (16) and (19). Given the token price dynamics, agents' optimal token holdings and participation decisions together with the user base are as described in Proposition 1.*

We numerically solve the ODE and further characterize the equilibrium in Sections 5 and 6. Figure 2 summarizes the key economic mechanism discussed thus far, where the blue dotted, black solid, and red dash arrows show respectively the user-base externality, the transaction motive of token holdings, and the investment motive of token holdings. Tokens allow users to capitalize on the future growth of user base, and thereby, encourages early

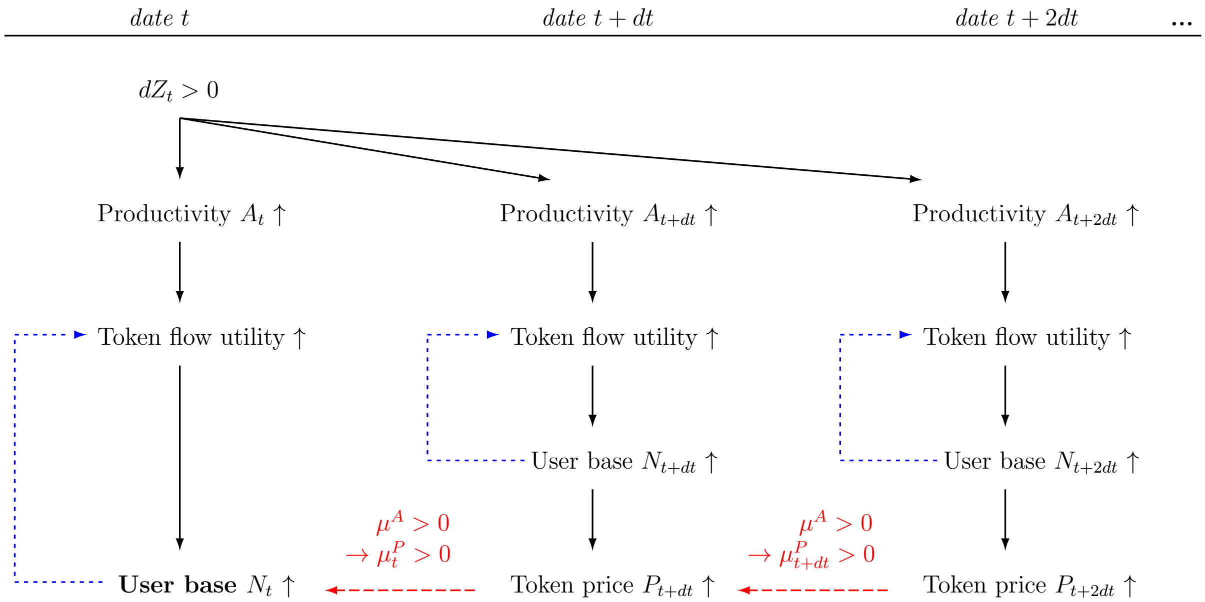

Figure 2: **The Economic Mechanism.** The black solid arrows point to the increases of the current and future (expected) levels of productivity A, which lead to higher flow utilities of tokens, and in turn, larger user bases N. The blue dotted arrows show that increases in user base result in even higher flow utility due to the contemporaneous user-base externality. Finally, more users push up the token prices P in future dates, which feed into a current expectation of price appreciation and greater adoption (red dash arrows).

adoption — a point we elaborate in the next two sections.

Remark: token and platform competition. Many blockchain platforms accommodate not only their native tokens but also other cryptocurrencies. For example, any ERC-20 compatible cryptocurrencies are accepted on the Ethereum blockchain.[9] To address this issue in the current framework, we may consider an alternative upper boundary of A_t. Define ψ as the cost of creating a new cryptocurrency that is a perfect substitute with the token we study because it functions on the same platform and therefore faces the same productivity and agents' need for transactions. This creates a reflecting boundary at $\overline{A}$ characterized by a value-matching condition and a smooth-pasting condition:

$$P\left(\overline{A}\right) = \psi \text{ and } P'\left(\overline{A}\right) = 0. \tag{20}$$

[9]ERC-20 defines a common list of rules that all tokens or cryptocurrencies should follow on the Ethereum blockchain. Gandal and Halaburda (2014) consider competition among cryptocurrencies from perspectives that differ from our focus on platform productivity.

When token price increases to ψ, entrepreneurs outside of the model will develop a new cryptocurrency that is compatible with the rules of our blockchain system. So, the price level never increases beyond this value.[10] Similarly, we may consider potential competing platforms, and interpret ψ as the cost of creating a new blockchain-based platform and its token, which together constitute a perfect substitute for our current system. This creates the same reflecting boundary for token price.

4 Benchmark Economies

This section analyzes two benchmark economies to help us understand the roles of tokens. The first is the tokenless economy, which features a platform where the medium of exchange is the numeraire good. By comparing our token-based economy with this benchmark, we highlight how introducing tokens affects user adoption. The second benchmark is the first-best economy, the solution to a central planner's problem. It helps us understand the welfare consequences of introducing tokens, and in particular, how tokens alleviate the lack of internalizing user network effects and the resulting under-adoption.

4.1 Tokenless Economy

In our tokenless economy, the numeraire good is the medium of exchange and agents only have transactional motives. The agent's profit is given by

$$dy_{i,t} = \max \left\{ 0, \max_{x_{i,t}>0} \left[(x_{i,t})^{1-\alpha} \left(N_t A_t e^{u_i} \right)^{\alpha} dt - \phi dt - x_{i,t} r dt \right] \right\}. \tag{21}$$

Unlike Equation (6) for the tokenized economy, as there is no native token, the token price fluctuation, μ_t^P, no longer appears in the agent's profits.

Conditional on joining the platform (i.e., $x_{i,t} > 0$), the agent chooses $x_{i,t}$ as follows:

$$x_{i,t}^* = N_t A_t e^{u_i} \left(\frac{1-\alpha}{r} \right)^{\frac{1}{\alpha}}. \tag{22}$$

[10] The requirement that $P'(A_t)$ is equal to zero rules out jumps of token prices at the reflecting boundary.

The maximized profit when joining the platform is then

$$N_t A_t e^{u_i} \alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi. \tag{23}$$

An agent joins the platform only when Expression (23) is positive. That is, agent i participates if and only if $u_i \geq \underline{u}_t^{NT}$, where $\underline{u}_t^{NT}$ is the endogenous threshold given by

$$\underline{u}_t^{NT} = -\ln(N_t) + \ln\left(\frac{\phi}{A_t \alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right) \ln\left(\frac{1-\alpha}{r}\right). \tag{24}$$

Here, the superscript "NT" refers to the "no-token" case. The user base is thus given by

$$N_t^{NT} = 1 - G\left(\underline{u}_t^{NT}\right). \tag{25}$$

Equations (24) and (25) jointly determine $\underline{u}_t^{NT}$ and N_t^{NT} as functions of A_t. Additionally, the user base, N_t^{NT}, increases in A_t, which can be shown as a special case covered by of Proposition 1. We define $\underline{A}^{NT}$ by imposing $\mu_t^P = 0$ in Proposition 1 such that for $A_t > \underline{A}^{NT}$, there exists a unique non-degenerate solution of N_t^{NT} if the hazard rate of $g(u)$ is increasing.

Next, we consider the social planner's problem by internalizing network externalities.

4.2 The First-best (FB) Economy

Given a user base N_t, the socially optimal holdings of transaction medium (goods) is still

$$x_{i,t}^* = N_t A_t e^{u_i} \left(\frac{1-\alpha}{r}\right)^{\frac{1}{\alpha}}. \tag{26}$$

Let $\mathcal{U}_t$ denote the set of participating users and in equilibrium the mass is equal to N_t. The total transaction surplus (if positive) is given by

$$\int_{i\in\mathcal{U}_t} \left[\alpha N_t A_t e^{u_i} \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi\right] di = N_t \left[\alpha \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} A_t \int_{i\in\mathcal{U}_t} e^{u_i} di - \phi\right]. \tag{27}$$

To maximize this welfare flow, the planner optimally sets $N_t = 1$, i.e., $\mathcal{U}_t$ being the full set of agents, unless given $N_t = 1$, the objective (27) is negative, in which case it is socially optimal to have zero adoption. The switching from zero adoption to full adoption happens at

$$\underline{A}^{FB} = \phi \left[\alpha \left(\frac{1-\alpha}{r} \right)^{\frac{1-\alpha}{\alpha}} \overline{S} \right]^{-1}, \tag{28}$$

where $\overline{S}$ is given by Equation (17). Given that $\overline{S} < \infty$, welfare maximization has a bang-bang solution, resulting in full adoption if $A_t \geq \underline{A}^{FB}$ and zero adoption otherwise.

We argue that there are more agents participating on the platform in the FB economy than in our tokenless economy, which means the productivity thresholds for adoption in the two economies satisfies $\underline{A}^{FB} < \underline{A}^{NT}$ (proof in the appendix). This result follows from that the social planner internalizes the positive externality of an agent's adoption on other users.

How does the decentralized equilibrium of tokenized economy differ from the planner's solution? On the one hand, tokens induce an investment motive in agents' adoption decision, alleviating the under-adoption problem in the decentralized tokenless equilibrium. On the other hand, over-adoption may happen in the sense that $N_t > 0$ even when $A_t < \underline{A}^{FB}$.

In Appendix C, we use data on token pricing and adoption to discipline our choices of parameter values for numerical solutions. Next, we combine the analytical results and numerical analysis to discuss the roles of tokens and asset pricing implications of endogenous platform adoption.

5 The Roles of Tokens

In this section, we analyze the adoption dynamics and highlight the roles of tokens by comparing the token-based economy with the two benchmark economies in the previous section. We illustrate the adoption acceleration and user-base volatility reduction effects of tokens with the numerical solutions.

5.1 User-Adoption Acceleration

Among the purported reasons for this common practice of introducing tokens, entrepreneurs foremost believe that using tokens can "bootstrap" the community. Heuristically, practitioners have argued that tokens help grow the ecosystem and allow all participants to benefit from the growth prospect of platforms, although no formal analysis has been provided. We now examine this argument formally in our framework.

When tokens are introduced as the platform's medium of exchange, token prices reflect agents' expectations of future productivity growth and user adoption. Tokens therefore accelerate adoption because agents joining the community enjoy not only the trade surplus but also the investment return from token price appreciation.[11]

The solid line in Figure 3 shows that the user base N_t is an S-shaped function of $\ln(A_t)$.[12] When the platform's productivity A_t is low, the user base N_t barely responds to changes in A_t. In contrast, when A_t is moderately high, N_t responds much more to changes in A_t. The growth of user base feeds on itself – the more agents join the ecosystem, the higher transaction surplus each derives. User adoption eventually slows down when the pool of newcomers gets exhausted. We also plot the scattered data points. We provide details on sample construction in Appendix C.

Figure 3 also compares the user adoption in tokenized and tokenless (decentralized) economies. The former strictly dominates the latter. Both economies reach full adoption when A_t becomes sufficiently large. Notice that the adoption thresholds in Equations (11) and (24) differ by the μ_t^P term. When $\mu_t^P > 0$, $N_t > N_t^{NT}$ given A_t, where N_t is determined by Equation (12). In other words, the expected token price appreciation induces a higher level of adoption than the case without tokens.

Token price appreciation critically depends on the growth of A_t. When $\mu_A > 0$, agents forecast a higher token price and the investment motive accelerates user adoption. Without

[11] When Tencent QQ introduced Q-coin, a case to which our model is applicable, many users and merchants quickly started accepting them even outside the QQ platform, tremendously accelerating adoption and token price appreciation. Annual trading volume reached billions of RMB in the late 2000s and the government had to intervene. See articles *China bars use of virtual money for trading in real goods* and *QQ: China's New Coin of the Realm?* (WSJ).

[12] The curve starts at $\ln(A_t) = -48.35$ ($A_t = 1e-21$), a number that we choose to be close to zero, the left boundary. The curve ends at $\ln(A_t) = 18.42$ ($A_t = 1e8$), the touching point between $P(A_t)$ and $\overline{P}(A_t)$.

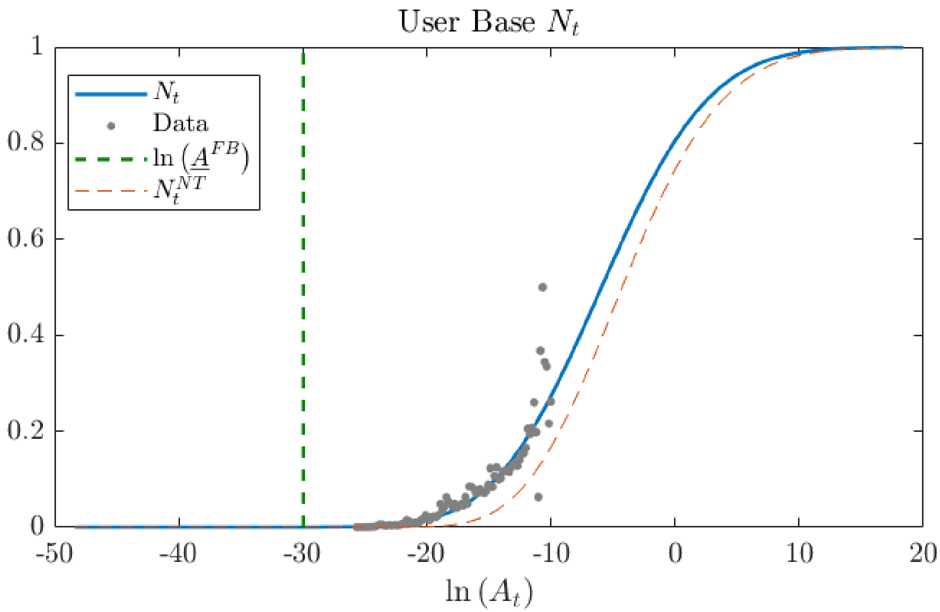

Figure 3: **Dependence of User Base on Platform Productivity.** This graph shows N_t, the user base of tokenized economy (blue solid curve), data of normalized active user addresses (gray scattered dots), and the user base of tokenless economy (red dash line) against $\ln(A_t)$, the productivity. The dotted vertical line marks the level of productivity, beyond which the planner chooses full adoption, and below which the planner chooses zero adoption.

tokens, this investment-driven demand is shut down. Therefore, introducing tokens help capitalize future productivity growth and grow promising platforms.[13] In contrast, when $\mu_A < 0$, the expected token depreciation ($\mu_t^P < 0$) precipitates user exits and the demise of the platform. Our numerical analysis focuses on the case where $\mu^A > 0$.

For comparison, we also plot in Figure 3 the first-best solution via the dotted vertical line at $\ln\left(\underline{A}^{FB}\right)$, which is given by Equation (28). Recall that the planner chooses full adoption if $A_t \geq \underline{A}^{FB}$ and zero adoption otherwise. Relative to the first-best economy, a tokenless economy features under-adoption and introducing tokens helps mitigate this inefficiency. Token-based economy may also lead to *over*-adoption because it is possible that $N_t > 0$ even when $A_t < \underline{A}^{FB}$. Under the current set of parameter values, over-adoption is not a severe

[13]We note that a predetermined token supply schedule is important. If token supply can arbitrarily increase ex post, then the expected token price appreciation is delinked from the the productivity growth and the resulting increase of user base and token demand. Pre-determinacy or commitment can only be credibly achieved through the decentralized consensus mechanism empowered by the blockchain technology. In contrast, traditional monetary policy has commitment problem (Barro and Gordon (1983)).

problem because N_t is extremely close to zero for $A_t < \underline{A}^{FB}$.

5.2 User-base Volatility Reduction

Next, we compare the user base volatility in tokenized and tokenless economies. Note that in the first-best economy, because the adoption is either zero or full, user base volatility is not an issue.

To derive the dynamics of N_t, we first conjecture the following equilibrium diffusion process:

$$dN_t = \mu_t^N dt + \sigma_t^N dZ_t^A. \tag{29}$$

In the appendix, we show that in the tokenless economy

$$\sigma_t^N = \left(\frac{g\left(\underline{u}_t^{NT}\right)}{1 - g\left(\underline{u}_t^{NT}\right)/N_t^{NT}} \right) \sigma^A \tag{30}$$

and in the tokenized economy

$$\sigma_t^N = \left(\frac{g\left(\underline{u}_t\right)}{1 - g\left(\underline{u}_t\right)/N_t} \right) \left[\sigma^A + \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{\sigma_t^{\mu^P}}{r - \mu_t^P} \right) \right], \tag{31}$$

where $\sigma_t^{\mu^P}$ is the diffusion of μ_t^P as defined below:

$$d\mu_t^P = \mu_t^{\mu^P} dt + \sigma_t^{\mu^P} dZ_t^A. \tag{32}$$

We know that N_t follows a reflected (or "regulated") diffusion process bounded in $[0, 1]$.

Comparing Equations (30) and (31), we see that introducing tokens alters the user-base volatility through $\sigma_t^{\mu^P}$, which is the volatility of expected token appreciation, μ_t^P, as defined in Equation (32). Embedding a native token may either amplify or dampen the shock effect on the user base, depending on the sign of $\sigma_t^{\mu^P}$. By Itô's lemma, $\sigma_t^{\mu^P} = \frac{d\mu_t^P}{dA_t}\sigma^A A_t$, so the sign of $\sigma_t^{\mu^P}$ depends on whether μ_t^P increases or decreases in A_t.

Intuitively, μ_t^P decreases in A_t (and thus, $\sigma_t^{\mu^P} < 0$), precisely because of the endogenous user adoption. Consider a positive shock to A_t, which has a direct effect of increasing N_t due

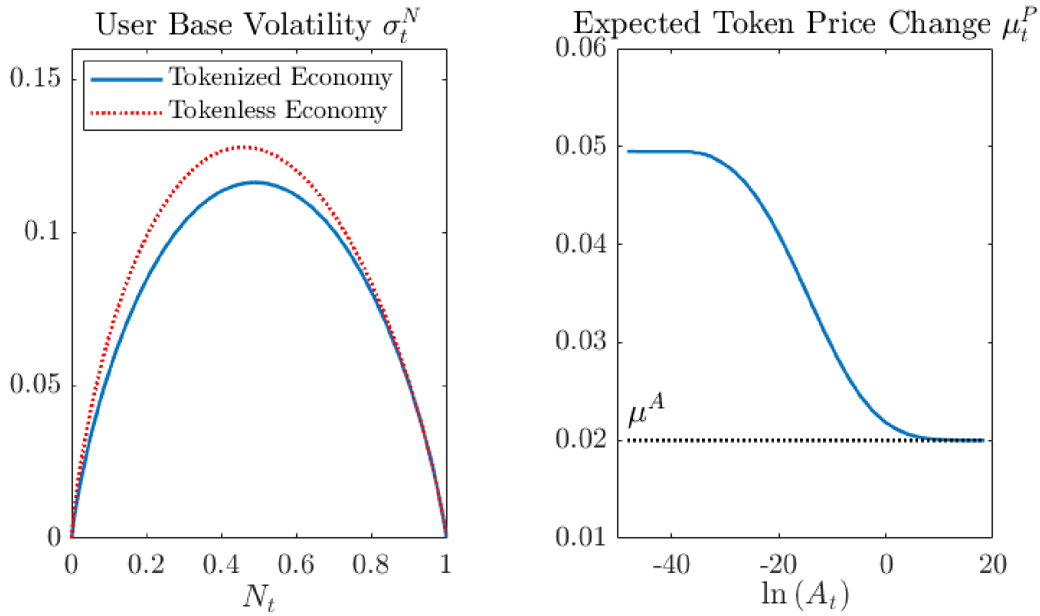

Figure 4: **User-Base Volatility Reduction Effect.** The left panel of this graph shows the volatility of user base, σ_t^N, in the tokenized (blue solid curve) and tokenless (red dotted curve) economies over adoption stages, N_t. The right panel shows the expected (risk-adjusted) token price change, μ_t^P, (i.e., under the risk-neutral measure) across different levels of blockchain productivity, $\ln(A_t)$. The black dotted line marks the expected growth rate of blockchain productivity.

to higher transaction surplus. For a large N_t, the potential for N_t to grow decreases, so does the expected token appreciation (i.e., μ_t^P) which discourages user adoption. The endogenous token price dynamics therefore moderate N_t's increase. Similarly, consider a negative shock to A_t. The token price channel mitigates the decrease in N_t, because given $\mu^A > 0$, a lower current level of adoption implies a stronger potential for the user base to grow in the future. Overall, introducing token can *reduce* the user-base sensitivity to shocks.

Next, we illustrate in Figure 4 how tokens reduce user-base volatility. The left panel plots σ_t^N, and compares the cases with and without token across different stages of adoption. Note that by fixing N_t, we also fix the adoption thresholds, i.e., $\underline{u}_t$ and $\underline{u}_t^{NT}$, so the first brackets in Equations (30) and (31) have the same value. Therefore, the only difference between the two curves of user-base volatility arises from $\sigma_t^{\mu^P}$, the volatility of expected token appreciation. Both curves start and end at zero, consistent with the S-shaped development in Figure 3 in which both curves start flat and end flat. This volatility reduction effect is more prominent in the early stage of development when A_t and N_t are low.

The right panel of Figure 4 plots μ_t^P, the risk-adjusted expected token price appreciation, against $\ln(A_t)$. It shows their negative relation that causes $\sigma_t^{\mu^P} < 0$, which generates the volatility reduction effect. When A_t is low and N_t is low, token price is expected to increase fast, reflecting both the future growth of A_t and N_t. As A_t and N_t grow, the pool of agents who have not adopted $(1 - N_t)$ shrinks and there is less potential for N_t to grow. As a result, the expected token appreciation declines.

Given the roles of the tokens in accelerating and stabilizing user adoption as our model reveals, entrepreneurs may want to introduce them in a platform. For example, suppose the platform can collect a fee from the users, greater adoption would increase the revenue of the platform and stabler user base means stable revenue.

6 Token Price Dynamics with User Adoption

In this section, we discuss how endogenous user adoption leads to nonlinear price dynamics that are broadly consistent with empirical observations. In particular, we examine three key asset-pricing issues concerning tokens: the cross section of token pricing, excess volatility, and "bubbly" price patterns (run-ups followed by crashes in token prices).

Token price over adoption stages. As inputs in our model such as $\ln(A_t)$ are difficult to measure, in Figure 5 we relate two key observables, the logarithmic token price $\ln(P_t)$ and the user base N_t, both functions of A_t in equilibrium. Token price increases quickly with adoption in the early stage, changes gradually in the intermediate stage, and speeds up again once the user base reaches a sufficiently high level. The two price run-ups in the initial and final stages of adoption correspond to the slow user base growth in these two stages relative to token price changes.

This figure helps us understand the cross-sectional differences in token pricing. To exposit the key result, we first sort blockchain platforms into three categories in term of their adoption stages: early, intermediate, and late. For two blockchain platforms in the early stage, a small difference of N_t between them can generate a very large difference in the mar-

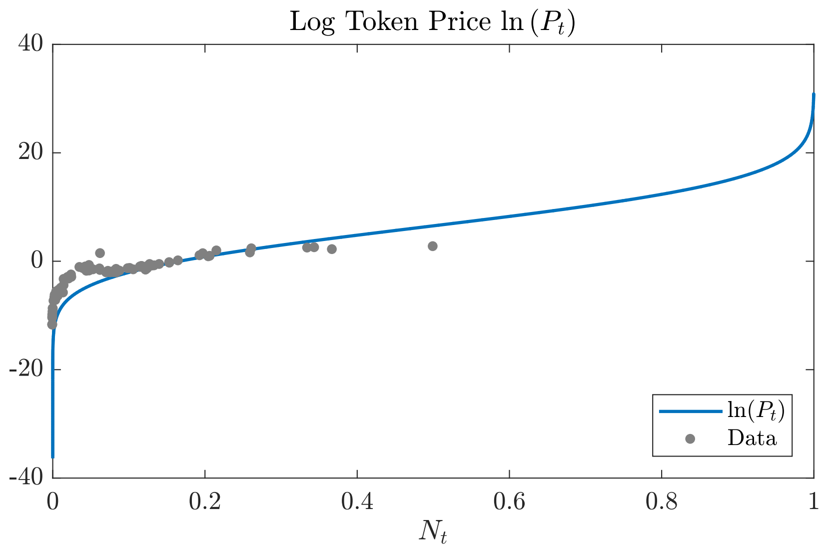

Figure 5: **Token Price Dynamics over Adoption Stages.** This graph shows the log token price across adoption stages, N_t (blue solid curve), and data as scattered dots.

ket capitalization of tokens ($P_t M$), as seen in Figure 5. Essentially the same result holds in the late stage. In contrast, in the intermediate stage, even a large difference of N_t between the two platforms only yields a small difference of $\ln(P_t)$.

Price volatility dynamics. The shocks to platform productivity are transmitted to token price through users' decision on adoption and token holdings. In fact, token-price volatility σ_t^P is generally larger than σ^A, the productivity volatility, as Figure 6 illustrates. To see the intuition, consider a positive shock to A_t that directly increases the utility flow of token holdings. User adoption increases as a consequence, which leads to an even higher utility flow (as N_t enters into the utility flow). This feedback effect amplifies the shock's impact on token price, which implies that endogenous user adoption amplifies volatility.

Importantly, our model features a new form of endogenous risk that is unique to platform economics. The volatility of token price is larger than the productivity volatility (exogenous risk) in equilibrium due to the endogenous formation of user base. The amplification constitutes an endogenous asset-price risk that is distinct from the fire-sale risk triggered by the balance-sheet channel in the macro-finance literature (e.g., Brunnermeier and Sannikov, 2014). Note that under full adoption $N_t = 1$, Equation (18) reveals that the ratio of P_t to A_t

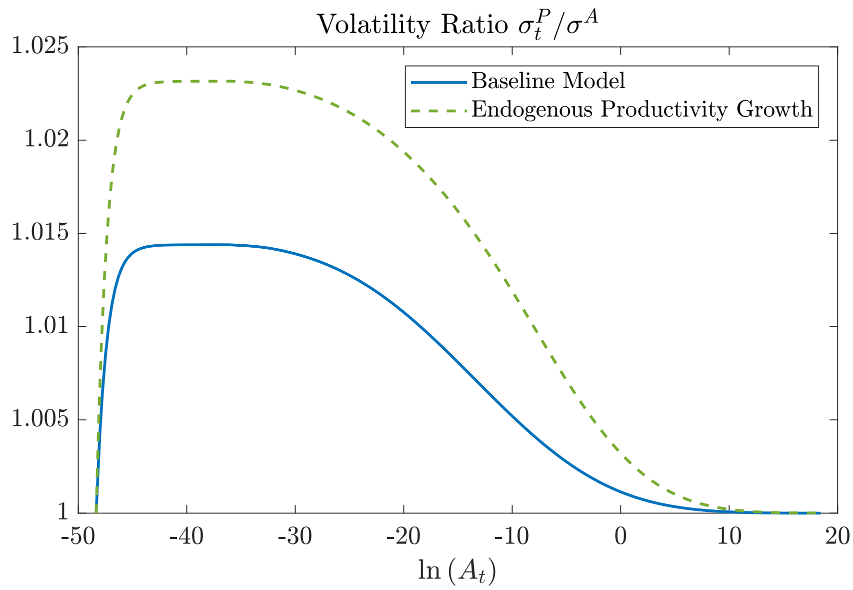

Figure 6: **Endogenous Productivity and Volatility Amplification.** This graph shows the ratio of token price volatility, σ_t^P, to productivity volatility, σ^A, which quantifies the strength of volatility amplification by the endogenous user adoption. The solid line shows the baseline model. The dash line shows the model with endogenous platform productivity.

is a constant, so $\sigma_t^P = \sigma^A$ and the endogenous risk disappears. The network effect induces strategic complementarity in agents' adoption decision, and thereby, amplifies the impact of fundamental shock. This mechanism is related to the literature on strategic complementarity and fragility (e.g., Goldstein and Pauzner, 2005). Here the endogenous risk from strategic complementarity manifests itself in the equilibrium asset (token) price.

Figure 6 also shows that the token price volatility exhibits non-monotonic dynamics as the platform productivity grows: σ_t^P shoots up in the early stage, gradually declines, and eventually converges to σ^A as N_t approaches one. This dynamics is broadly consistent with the following observations: token price volatility for a nascent platform is large and the cross-sectional variation of token price volatility for nascent platforms (differing in productivity) tends to be large.

Endogenous platform productivity. Our analysis thus far has taken the platform productivity process as exogenous. In reality, many token and cryptocurrency applications feature an endogenous dependence of platform productivity on the user base.

A defining feature of blockchain technology is the provision of consensus on decentralized ledgers. In a "proof-of-stake" system, the consensus is more robust when the user base is large and dispersed because no single party is likely to hold a majority stake; in a "proof-of-work" system, more miners deliver faster and more reliable confirmation of transactions, and miners' participation in turn depends on the size of user base (for example, through transaction fees). More broadly, the more users on the platform, the more economic activities taking place (i.e., higher A_t). Moreover, a greater user base attracts more resources and research onto the platform, accelerating the technological progress on the platform and creating a positive feedback loop.

The endogeneity of blockchain productivity and its dependence on the user base highlight the decentralized nature of this new technology. To capture this positive feedback feature between productivity and user base, we generalize the process of A_t as follows:

$$\frac{dA_t}{A_t} = \left[\mu^A (1-\omega) + \mu^A \omega N_t\right] dt + \sigma^A dZ_t^A, \tag{33}$$

where $\omega > 0$. Under this specification, the expected growth rate of A_t is μ^A under full adoption and below μ^A when $N_t < 1$. Solving the Markov equilibrium under endogenous productivity only requires to replace μ^A in Equation (15) with $\left[\mu^A (1-\omega) + \mu^A \omega N_t\right]$.

Endogenous productivity has critical implications on how fundamental shocks are transmitted to token price fluctuations. The growth rate of A_t is no longer i.i.d. and the shock impact on A_t becomes more persistent. Consider a positive shock. Not only the current A_t increases, but through N_t, the growth *rate* of A_t increases, propagating the shock impact into the future. This amplifies the volatility of token price, which is a forward-looking measure of productivity and adoption. Figure 6 shows that the ratio of σ_t^P to σ^A is higher with endogenous productivity than with exogenous productivity.[14] In summary, the decentralized nature of blockchain technology and the associated endogenous platform productivity amplify the transmission of volatility of fundamentals into token price volatility.

[14]The parameter ω is set to 0.9 for illustrative purpose.

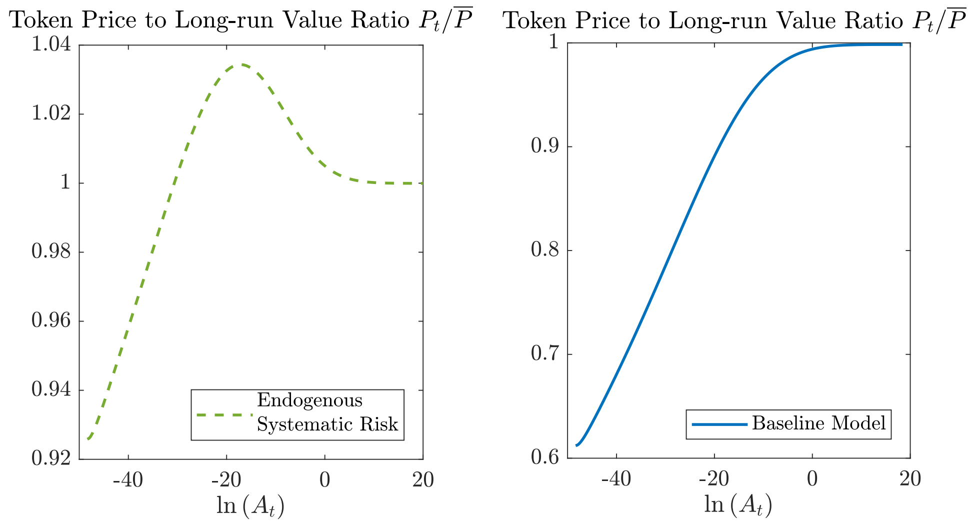

Figure 7: **The "Bubbly" Behavior of Token Price.** This graph plots the ratio of token price, $P(A_t)$, to the long-run (full-adoption) value of token, $\overline{P}(A_t)$, which shows how endogenous adoption shapes the token price dynamics. The solid line shows the baseline model. The dash line shows the model with endogenous systematic risk.

"Bubbly" token price. In the last two years, the prices of several prominent cryptocurrencies experienced significant run-up followed by crash and subsequent stabilization. We show that such price dynamics can arise in a rational model with endogenous adoption.

So far we have set up and analyzed the model under the risk-neutral measure. Next, we explicitly model risk premium by postpulating a geometric Brownian motion for A_t under the *physical* measure,

$$\frac{dA_t}{A_t} = \widehat{\mu}^A dt + \sigma^A d\widehat{Z}_t^A, \tag{34}$$

where $\widehat{\mu}^A$ is a constant drift and $\widehat{Z}_t^A$ is a standard Brownian motion.[15] We assume that the stochastic discount factor (SDF), Λ_t, evolves as

$$\frac{d\Lambda_t}{\Lambda_t} = -rdt - \eta d\widehat{Z}_t^\Lambda, \tag{35}$$

where $\widehat{Z}_t^\Lambda$ is the standard Brownian motion under the physical measure and η is the constant market price of risk.

Let ρ_t denote the instantaneous correlation coefficient between the productivity shock,

[15]By diffusion invariance theorem, the volatility parameter is equal to σ^A.

$d\widehat{Z}_t^A$, and the SDF shock, $d\widehat{Z}_t^\Lambda$. To model the endogenous beta of platform productivity, we allow ρ_t to depend on N_t. Suppose that $d\rho(N_t)/dN_t > 0$, which means that the productivity beta increases as the user base grows. As the technology becomes more "mainstream," shocks to it become increasingly systematic. This assumption is inspired by the adoption-dependent beta of new technologies in Pástor and Veronesi (2009). By using Girsanov's theorem, we obtain the following productivity process under the risk-neutral measure,

$$\frac{dA_t}{A_t} = \left[\widehat{\mu}^A - \eta\rho(N_t)\sigma^A\right]dt + \sigma^A dZ_t^A. \tag{36}$$

When the productivity shock becomes more correlated with the SDF shock, investors demand a higher risk premium, which lowers the risk-adjusted growth of productivity. In other words, the risk-neutral expected growth rate of A_t is $\widehat{\mu}^A - \eta\rho(N_t)\sigma^A$, is lower. To solve the Markov equilibrium, we simply replace μ^A in Equation (15) with this risk-adjusted drift of A_t.

There are thus two opposing forces that drive P_t as A_t grows. On the one hand, the mechanism that increases P_t is still present: when A_t directly increases the flow utility of token, or indirectly through its positive impact on N_t, token price increases. On the other hand, the risk premium increases as N_t increases, so the risk-adjusted growth of A_t declines, which in turn decreases P_t. The former channel could dominate in the early stage of adoption while the latter channel dominates in the late stage of adoption. Therefore, P_t first rises with N_t and then declines as N_t reaches a sufficiently high level, which resembles a bubble as shown in the left panel of Figure 7.[16]

7 Conclusion

We provide a tractable dynamic model of token pricing and platform adoption. Platforms create value by supporting special economic activities and their tokens derive value by facilitating transactions among platform users. As a result, token valuation reflects users' endogenous participation and the associated network externalities. User base also plays a critical role in explaining the cross-section variation of token pricing, the dynamics of token

[16]In the numerical solution, we set $\widehat{\mu}^A = 4\%$ and $\rho(N_t) = \widehat{\mu}^A N_t/3$ for illustrative purposes.

price volatility, and the run-up and crash of token prices.

By comparing platforms with and without tokens, our model demonstrates the effects of introducing tokens on agents' adoption decisions. For platforms with growing productivities, agents' expectation of token price appreciation encourages early adoption, and thus, enhances welfare by intertemporarily internalizing the user network externalities. Introducing tokens also reduces the volatility of user base because agents' expectation of long-term growth in token value weakens the impact of temporary productivity shocks on the user base.

Our model accommodates several extensions. This paper focuses on user activities, i.e., the demand side of platform infrastructure. On the supply side, agents contribute to the infrastructure in exchange for token-based rewards. Combining demand-side analysis with supply-side protocols can deliver new insights on platform design (e.g., Fanti, Kogan, and Viswanath, 2019). Another direction is the role of tokens in platform competition, which has been missing in the literature of platform economics (e.g., Rochet and Tirole, 2003). Platforms may serve similar economic activities. Users' adoption decision and token prices shall naturally reflect the platforms' competitive advantage.

References

Abadi, Joseph, and Markus Brunnermeier, 2019, Blockchain economics, Working paper Princeton University.

Alvarez, Fernando, and Francesco Lippi, 2013, The demand of liquid assets with uncertain lumpy expenditures, *Journal of Monetary Economics* 60, 753 – 770.

Athey, Susan, Ivo Parashkevov, Vishnu Sarukkai, and Jing Xia, 2016, Bitcoin pricing, adoption, and usage: Theory and evidence, *Working Paper*.

Balvers, Ronald J, and Bill McDonald, 2017, Designing a global digital currency, *Working Paper*.

Barro, Robert J., and David B. Gordon, 1983, A positive theory of monetary policy in a natural rate model, *Journal of Political Economy* 91, 589–610.

Baumol, William J., 1952, The transactions demand for cash: An inventory theoretic approach, *The Quarterly Journal of Economics* 66, 545–556.

Biais, Bruno, Christophe Bisière, Matthieu Bouvard, and Catherine Casamatta, 2017, The blockchain fold theorem, *Preliminary Work in Progress*.

——— , and Albert Menkveld, 2018, Equilibrium Bitcoin Pricing, Working paper Toulouse School of Economics.

Bolton, Patrick, Hui Chen, and Neng Wang, 2011, A unified theory of tobin's q, corporate investment, financing, and risk management, *The Journal of Finance* 66, 1545–1578.

Brunnermeier, Markus K, and Yuliy Sannikov, 2014, A macroeconomic model with a financial sector, *American Economic Review* 104, 379–421.

Catalini, Christian, and Joshua S Gans, 2018, Initial coin offerings and the value of crypto tokens, Discussion paper, National Bureau of Economic Research.

Chiu, Jonathan, and Thorsten V Koeppl, 2017, The economics of cryptocurrencies–bitcoin and beyond, *Working Paper.*

Chod, Jiri, and Evgeny Lyandres, 2018, A theory of ICOs: Diversification, agency, and asymmetric information, Working paper Boston University Questrom School of Business.

Cochrane, John H., 2018, The bitcoin market isn't irrational, *Chicago Booth Review.*

Cong, Lin William, and Zhiguo He, 2018, Blockchain disruption and smart contracts, *Forthcoming, Review of Financial Studies.*

——— , and Jiasun Li, 2018, Decentralized mining in centralized pools, *Working Paper.*

Duffie, Darrell, 1990, Chapter 3 money in general equilibrium theory, *Handbook of Monetary Economics* 1, 81–100.

——— , 2001, *Dynamic Asset Pricing Theory* (Princeton University Press).

Easley, David, Maureen O'Hara, and Soumya Basu, 2017, From mining to markets: The evolution of bitcoin transaction fees, *Working Paper.*

Fanti, Giulia, Leonid Kogan, and Pramod Viswanath, 2019, Economics of proof-of-stake payment systems, Working paper.

Gandal, Neil, and Hanna Halaburda, 2014, Competition in the cryptocurrency market, *Working Paper.*

Garratt, Rodney, and Neil Wallace, 2018, Bitcoin 1, bitcoin 2,....: An experiment in privately issued outside monies, *Economic Inquiry* 56, 1887–1897.

Gibson, Rajna, and Eduardo S. Schwartz, 1990, Stochastic convenience yield and the pricing of oil contingent claims, *The Journal of Finance* 45, 959–976.

Goldstein, Itay, and Ady Pauzner, 2005, Demanddeposit contracts and the probability of bank runs, *The Journal of Finance* 60, 1293–1327.

Hu, Albert, Christine Parlour, and Uday Rajan, 2018, Cryptocurrencies: Stylized facts on a new investible instrument, Working paper University of California Berkeley.

Huberman, Gur, Jacob Leshno, and Ciamac C. Moallemi, 2017, Monopoly without a monopolist: An economic analysis of the bitcoin payment system, working paper 17-92 Columbia Business School.

Jorgenson, Dale W., 1963, Capital theory and investment behavior, *The American Economic Review* 53, 247–259.

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, 2012, The aggregate demand for treasury debt, *Journal of Political Economy* 120, 233–267.

Li, Jiasun, and William Mann, 2018, Initial coin offering and platform building, *Working Paper*.

Liu, Yukun, and Aleh Tsyvinski, 2018, Risks and returns of cryptocurrency, Working Paper 24877 National Bureau of Economic Research.

Pagnotta, Emiliano, and Andrea Buraschi, 2018, An equilibrium valuation of bitcoin and decentralized network assets, Working paper Imperial College.

Pástor, Luboš, and Pietro Veronesi, 2009, Technological revolutions and stock prices, *The American Economic Review* 99, 1451–1483.

Pissarides, Christopher A, 2000, *Equilibrium unemployment theory* (MIT press).

Raskin, Max, and David Yermack, 2016, Digital currencies, decentralized ledgers, and the future of central banking, Working Paper 22238 National Bureau of Economic Research.

Rochet, Jean-Charles, and Jean Tirole, 2003, Platform competition in two-sided markets, *Journal of the European Economic Association* 1, 990–1029.

Saleh, Fahad, 2017, Blockchain without waste: Proof-of-stake, Discussion paper, working Paper.

Schilling, Linda, and Harald Uhlig, 2018, Some simple bitcoin economics, Discussion paper, National Bureau of Economic Research.

Sockin, Michael, and Wei Xiong, 2018, A model of cryptocurrencies, *Working Paper*.

Tobin, James, 1956, The interest-elasticity of transactions demand for cash, *The Review of Economics and Statistics* 38, 241–247.

Appendix A – Transaction Surplus and Flow Utility

In this section, we provide theoretical foundations for our specification of platform transaction surplus in Equation (1). We first present a general model based on transaction costs that applies to all platform tokens (not necessarily blockchain-based), which essentially captures a form of convenience yield (see, e.g., Cochrane, 2018), and then discuss a case that is specific to blockchain platform. Our goal here is not to microfound every application scenario, but to illustrate potential settings that support our specification.

A model of convenience yield. Agents have investment opportunities that occur at Poisson arrival times, $\{T_n\}_{n=1}^{+\infty}$, with time-varying and agent-specific intensity, $\lambda_{i,t}$. At a given Poisson time, T_n, agent i is endowed with a technology, $\omega_i F(\cdot)$, that transforms labor into goods, and is matched with another agent who can supply the required labor input. Agent-specific productivity is captured by ω_i. To simplify the exposition, we assume that the labor supply has a constant marginal cost of one, and the supplier breaks even, so the full trade surplus is enjoyed by agent i. This setup of uncertain lumpy transactions follows the Baumol-Tobin model of Alvarez and Lippi (2013).

Agent i's labor demand, denoted by h, is not restricted by the real balance of token holdings, $P_t k_{i,T_n-}$, where k_{i,T_n-} denotes the units of tokens carried to T_n. Since the focus of this paper is not on financial constraints, we allow the agent to borrow dollars (an instantaneous loan) at zero cost, so h may exceed agent i's wealth at the moment. The production is done immediately, and the loan is repaid immediately by the goods. So, given a competitive credit market, the loan rate is zero.

The lumpy payment for labor incurs a transaction cost that is proportional to the total payment value, δh ($\delta > 0$), but using tokens as means of payment save the transaction cost by $U(P_t k_{i,T_n-})$ ($U' > 0$, $U'' < 0$) because agent i does not need to exchange dollars for tokens, the required means of payment on the platform.

Agent i maximizes the investment profit, which is a jump in wealth,

$$\max_{h} \omega_i F(h) - h - \left(\delta h - U\left(P_t k_{i,T_n-}\right)\right), \tag{37}$$

where the last term is the transaction cost. The optimal labor demand, h^*, is given by

$$\omega_i F'(h^*) = 1 + \delta, \tag{38}$$

so the marginal value of production is equal to the marginal cost of labor plus the transaction cost, δ. We can substitute the constant h^* into the investment profit to have

$$\omega_i F\left(h^*\right)-(1+\delta) h^*+U\left(P_t k_{i,T_n-}\right). \tag{39}$$

We assume that ω_i is sufficiently high so $h^* \geq P_t k_{i,T_n-}$. The conversion between the local currency (token) and other assets can be costly, especially when a lumpy transaction is required within a short period of time. By holding tokens, agents save such costs. Linking transaction costs to the monetary value of assets has a long tradition in economics (Baumol, 1952; Tobin, 1956; Duffie, 1990).

Therefore, at time t, agent i has an expected gain of $\lambda_{i,t} U\left(P_t k_{i,t}\right) dt$ by holdings $k_{i,t}$ units of tokens for dt. To obtain a tighter analytical characterization of the equilibrium, we specify $\lambda_{i,t}=\left(N_t A_t e^{u_i}\right)^\alpha$, $\alpha \in (0,1)$. A larger community (N_t) makes it easier to find transaction counterparties. A higher platform quality (A_t) reflects a more efficient matching mechanism or the fact that the economic transactions supported by the platform are more popular. And u_i captures agent-specific transaction needs. We specify $U\left(P_t k_{i,t}\right)=\chi\left(P_t k_{i,t}\right)^{1-\alpha}$, so the expected transaction costs saved are

$$\lambda_{i,t} U\left(P_t k_{i,t}\right) dt=\left(N_t A_t e^{u_i}\right)^\alpha\left(P_t k_{i,t}\right)^{1-\alpha} \chi dt. \tag{40}$$

In the following we set $\chi=1$ because its scaling effect can be subsumed by the level of A_t.

We may reinterpret h as goods or services other than labor, and the investment profit as a burst of consumption or utility value from transactions. The foundation of this specification of trade surplus is two-fold: (1) the arrival of transaction opportunities depends on the user base, the platform quality, and agent-specific factors; (2) holding tokens on the tokenized platform save transaction costs for lumpy payments.

Note that the same setup can also generate the trade surplus for a tokenless economy where transaction is settled using the numeraire good, so in the main text, we assume the same functional form of transaction surplus. In a tokenless economy, agents hold assets that yield a risk-adjusted return of r and dollars worth of $x_{i,t}$. The transaction cost, δh, is thus the cost of immediately exchanging a lumpy chuck of assets for cash. Holding cash saves this cost by $U\left(x_{i,t}\right)$. If external financing is required, the per unit cost of transaction, δ, also captures the difficulty to raise funds in lumpy amounts. The concavity of $U(\cdot)$ can be motivated by models of cash holdings that recognize cash carry costs and external financial constraints (e.g., Bolton, Chen, and Wang, 2011).

Staking tokens. While the above transaction-cost based model applies to platform tokens that serve as means of payment, we next give another theoretical foundation to illustrate: (i) although we focus on tokens that serve explicitly as means of payment on platforms, our theory applies more generally to all tokens that provide users utilities specific to the underlying platform technology; (ii) blockchain-based platforms provide novel forms of transaction surplus from holding tokens on platforms and further motivate our specification of token flow utility.

Many blockchain-based platforms feature users providing service to peers to make a profit. For example, Filecoin, Golem, Storj, Elastic, etc., all have "storage miners" who help clients store digital files for a profit paid in native tokens. Oftentimes, storage miners have to "stake" native tokens (i.e., post it as collateral) in order to win the chance to service clients. In fact, staking is a common practice on blockchain platforms to encourage value-creating activities among their users. It goes beyond validating transactions and producing blocks within consensus mechanisms such as Proof-of-Stake (POS) and Distributed Proof-of-Stake (DPOS). In general, holding/staking tokens may enable network participants to potentially receive access to exclusive features of the platform, partake of business activities, or receive status recognition. For example, OmiseGO (OMG), the first ERC20 tokens on Ethereum sold via an ICO to reach unicorn status (US$1 billion market cap) in August 2017 (coinmarketcap.com), has validators deposit OMGs in staking contracts to validate transactions. OmiseGo selects the validator based on who has staked the highest token to validate the transaction and it performs the task. Depending on the performance of the validator, the validator will either receive rewards or penalties. Filecoin, VentureFusion, Numerai, etc., all feature some forms of staking.

Now suppose a storage miner on the platform has a realized storage space $e^{u_i\alpha}$, and is waiting to be matched with clients demanding decentralized storage (similar to a labor-market search-and-matching scenario). Potential client demand (analogous to the number of job seekers) is proportional to N_t, the user base of the platform, whereas the matching effort of the storage miner is proportional to staking amount x_i (similar to job vacancies).

Then if the platform has a matching efficiency of A^α, a matching function with constant return to scale (e.g., Pissarides, 2000) would yield the storage miner a payoff of

$$e^{u_i\alpha}A^\alpha N_t^\alpha(P_t k_{i,t})^{1-\alpha} = (P_t k_{i,t})^{1-\alpha}(AN_t e^{u_i})^\alpha, \tag{41}$$

which exactly gives the surplus flow in Equation (1). The storage miner then takes back the same number of tokens staked after providing the service for dt.

In the case above, it is crucial that a service provider (storage miner in the case of Filecoin) needs to stake/hold native tokens to have a chance of being matched with a customer. If she can match with a customer and instantaneously exchange tokens for the numeraire good, the the velocity of native tokens can be infinite, resulting in price indeterminacy.

Other tokens. While our focus is on a majority of tokens whose value derives from the productivity of underlying platforms and user network effects, we acknowledge that in reality, there exists a variety of tokens that serve purposes other than facilitating platform-specific transactions. Digital currencies developed by central banks serve general payment purposes. They are typically not tied to specific user networks, and their adoption is driven by policy and legal decisions. There are also tokens that enable the sharing of future corporate revenues or the distribution of products and services. Such security tokens can be valued using traditional discounted cash-flow models, and are therefore not our focus.

Moreover, our model does not capture the more complex interdependence of platforms and their tokens. For example, Litecoin and Dogecoin are "altcoins" — variants of the original open-sourced Bitcoin protocol to enable new features. Therefore, the productivity of their platforms inherits significantly from that of the Bitcoin blockchain. Other examples include the "AppCoins", what entrepreneurs often sell through the initial coin offerings (ICOs), that are developed for specific applications (e.g., Gnosis and Golem) and are built on existing blockchain infrastructures (e.g., Ethereum or Waves).

Appendix B - Proofs

B1. Proof of Proposition 1

Figure 1 illustrates the determination of N_t given A_t and μ_t^P, which we take as a snapshot of the dynamic equilibrium with time-varying productivity and expectation of price change. The proof below takes the following steps. First, we show that given μ_t^P, there exists a $\underline{A}$

such that for $A_t = A > \underline{A}$, the corresponding response curve,

$$R\left(n; A, \mu_t^P\right) = 1 - G\left(\underline{u}\left(n; A, \mu_t^P\right)\right) \tag{42}$$

$$= 1 - G\left(-\ln\left(n\right) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right)\right), \tag{43}$$

crosses the 45^o line at least once in $(0, 1]$, and for any value of $A_t = A < \underline{A}$, the response curve never crosses the 45^o line in $(0, 1]$. After proving the existence of $N_t > 0$ for $A_t \in [\underline{A}, +\infty)$, we prove the uniqueness given the increasing hazard rate of $g\left(u\right)$. Finally, we prove that N_t increases in μ_t^P. Before we start, for any $A_t = A > 0$, we define the value of its response function at $n = 0$: $R\left(0; A, \mu_t^P\right) = 0$. This is consistent with that given a zero user base, each agent derives zero transaction surplus from token holdings and chooses not to participate. Note that $\lim_{n\downarrow 0} R\left(n; A, \mu_t^P\right) = 0$, so the response function is continuous in n.

Given μ_t^P, we define a mapping, $A(n)$, from any equilibrium, non-zero value of user base, $n \in (0, 1]$, to the corresponding value of A_t, i.e., the unique solution to

$$1 - G\left(-\ln\left(n\right) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right)\right) = n, \quad \forall n \in (0, 1]. \tag{44}$$

This mapping is a continuous mapping on a bounded domain $\subseteq (0, 1]$. Then by the Least-Upper-Bound-Property of real numbers, the image set of this mapping, $\{A(n), n \in (0, 1)\}$, has an infimum, which we denote by $\underline{A}$. Now, for $A_t = \underline{A}$, consider a $n\left(\underline{A}\right) \in (0, 1]$ such that Equation (44) holds. For any $A > \underline{A}$, the LHS of Equation (44) is higher than the RHS, i.e., $R\left(n\left(\underline{A}\right); A, \mu_t^P\right) > n\left(\underline{A}\right)$, so that the response curve of $A_t = A$ is above the 45^o line at $n\left(\underline{A}\right)$. Next, because the response function $R\left(n; A, \mu_t^P\right)$ is continuous in n and $R\left(1; A, \mu_t^P\right) \leq 1$ by definition in Equation (43), i.e., it eventually falls to or below the 45^o line as n increases, there must exist a $n\left(A\right) \in (0, 1]$ such that when at $A_t = A$, Equation (44) holds by the Intermediate Value Theorem. Therefore, we have proved that for any $A_t = A > \underline{A}$, there exists a non-zero user base. Throughout the proof, we fix μ_t^P, so $\underline{A}$ is a function of μ_t^P.

Next, given $\frac{g(u)}{1-G(u)}$ is increasing, we show that the response curve crosses the 45^o line exactly once when $A_t \in [\underline{A}, +\infty)$. First note that $R\left(n; A_t, \mu_t^P\right) - n$ either has positive derivative or negative derivative at $n = 0$. If it has positive derivative (i.e., the response curve shoots over the 45^o line), then at n', *the first time* the response curve crosses the 45^o line again, the derivative of $R\left(n; A_t, \mu_t^P\right) - n$ must be weakly negative at n', i.e., the response curve crosses the 45^o from above,

$$g\left(\underline{u}\left(n';A_t,\mu_t^P\right)\right)\frac{1}{n'}-1\leq 0. \tag{45}$$

Now suppose the response curve crosses the 45^o line for the second time from below at $n^{''}>n'$, so the derivative of $R\left(n;A_t,\mu_t^P\right)-n$ at n'' must be weakly positive, and is equal to

$$\begin{aligned}
g\left(\underline{u}\left(n'';A_t,\mu_t^P\right)\right)\frac{1}{n''}-1 &= \frac{g\left(\underline{u}\left(n'';A_t,\mu_t^P\right)\right)}{1-G\left(\underline{u}\left(n'';A_t,\mu_t^P\right)\right)}-1 \\
&< \frac{g\left(\underline{u}\left(n';A_t,\mu_t^P\right)\right)}{1-G\left(\underline{u}\left(n';A_t,\mu_t^P\right)\right)}-1 \\
&= \frac{g\left(\underline{u}\left(n';A_t,\mu_t^P\right)\right)}{n'}-1 \\
&< 0,
\end{aligned} \tag{46}$$

where the first inequality comes from the increasing hazard rate and the fact that $\underline{u}\left(n;A_t,\mu_t^P\right)$ is decreasing in n for $n\in(0,1]$, and the second inequality follows from (45) and the fact that the response curve crosses the 45^o line at n' (i.e., $n'=R\left(n';A_t,\mu_t^P\right)=1-G\left(\underline{u}\left(n';A_t,\mu_t^P\right)\right)$). This contradicts the presumption that the response curve reaches the 45^o line from below (and the derivative of $R\left(n;A_t,\mu_t^P\right)-n$ is weakly positive). Therefore, we conclude that for $A_t\in[\underline{A},+\infty)$, there exists a unique adoption level n. Now if $R\left(n;A_t,\mu_t^P\right)-n$ has negative derivative at $n=0$, then in the previous argument, we can replace n' with 0 and show that there does not exist another intersection between the response curve and the 45^o line beyond $n=0$. Therefore, only if $R\left(n;A_t,\mu_t^P\right)-n$ has positive derivative at $n=0$, do we have a positive (non-degenerate) adoption level.

Finally, we show that the non-degenerate adoption level, N_t, is increasing in μ_t^P. Consider $\tilde{\mu}_t^P>\mu_t^P$. Suppose the contrary that their corresponding adoption levels satisfy $\tilde{N}_t\leq N_t$. Because we have proved that the response curve only crosses the 45^o line only once and from above, given N_t, we have

$$1-G\left(-\ln(n)+\ln\left(\frac{\phi}{A_t\alpha}\right)-\left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right)\right)\geq n,\quad\forall n\in(0,N_t]. \tag{47}$$

We know that by definition,

$$\begin{aligned}
\tilde{N}_t &= 1 - G\left(\underline{u}\left(\tilde{N}_t; A_t, \tilde{\mu}_t^P\right)\right) \\
&= 1 - G\left(-\ln\left(\tilde{N}_t\right) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\tilde{\mu}_t^P}\right)\right) \\
&> 1 - G\left(-\ln\left(\tilde{N}_t\right) + \ln\left(\frac{\phi}{A_t\alpha}\right) - \left(\frac{1-\alpha}{\alpha}\right)\ln\left(\frac{1-\alpha}{r-\mu_t^P}\right)\right) \\
&\geq \tilde{N}_t, \qquad (48)
\end{aligned}$$

where the first inequality uses $\tilde{\mu}_t^P > \mu_t^P$ and the second inequality uses the fact that $\tilde{N}_t \in (0, N_t]$ and the inequality (47). This contradiction implies that the adoption level N_t has to be increasing in μ_t^P.

B2. Derivation of the User-base Volatility

First, we consider the case without token. Using Itô's lemma, we can differentiate Equation (25) and then, by matching coefficients with Equation (29), derive μ_t^N and σ_t^N:

$$dN_t = -g\left(\underline{u}_t^{NT}\right) d\underline{u}_t^{NT} - \frac{1}{2}g'\left(\underline{u}_t^{NT}\right)\left\langle d\underline{u}_t^{NT}, d\underline{u}_t^{NT}\right\rangle, \qquad (49)$$

where $\left\langle d\underline{u}_t^{NT}, d\underline{u}_t^{NT}\right\rangle$ is the quadratic variation of $d\underline{u}_t^{NT}$. Using Itô's lemma, we differentiate Equation (24)

$$\begin{aligned}
d\underline{u}_t^{NT} &= -\frac{1}{N_t}dN_t + \frac{1}{2N_t^2}\left\langle dN_t, dN_t\right\rangle - \frac{1}{A_t}dA_t + \frac{1}{2A_t^2}\left\langle dA_t, dA_t\right\rangle \\
&= -\left(\frac{\mu_t^N}{N_t} - \frac{\left(\sigma_t^N\right)^2}{2N_t^2} + \mu^A - \frac{\left(\sigma^A\right)^2}{2}\right)dt - \left(\frac{\sigma_t^N}{N_t} + \sigma^A\right)dZ_t^A. \qquad (50)
\end{aligned}$$

Substituting this dynamics into Equation (49), we have

$$\begin{aligned}
dN_t = &\left[g\left(\underline{u}_t^{NT}\right)\left(\frac{\mu_t^N}{N_t} - \frac{\left(\sigma_t^N\right)^2}{2N_t^2} + \mu^A - \frac{\left(\sigma^A\right)^2}{2}\right) - \frac{1}{2}g'\left(\underline{u}_t^{NT}\right)\left(\frac{\sigma_t^N}{N_t} + \sigma^A\right)^2\right]dt \\
&+ g\left(\underline{u}_t^{NT}\right)\left(\frac{\sigma_t^N}{N_t} + \sigma^A\right)dZ_t^A, \qquad (51)
\end{aligned}$$

By matching coefficients on dZ_t^A with Equation (29), we can solve for σ_t^N.

Next, we consider the tokenized economy. Once tokens are introduced, N_t depends on the expected token price appreciation μ_t^P. which is also a univariate function of state variable A_t because by Itô's lemma, μ_t^P is equal to $\left(\frac{dP_t/P_t}{dA_t/A_t}\right)\mu^A + \frac{1}{2}\frac{d^2P_t/P_t}{dA_t^2/A_t^2}\left(\sigma^A\right)^2$. In equilibrium, its law of motion is given by a diffusion process

$$d\mu_t^P = \mu_t^{\mu^P} dt + \sigma_t^{\mu^P} dZ_t^A. \tag{52}$$

Now, the dynamics of $\underline{u}_t$ becomes

$$\begin{aligned} d\underline{u}_t = & -\frac{1}{N_t}dN_t + \frac{1}{2N_t^2}\langle dN_t, dN_t\rangle - \frac{1}{A_t}dA_t + \frac{1}{2A_t^2}\langle dA_t, dA_t\rangle \\ & -\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{r-\mu_t^P}\right)d\mu_t^P - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{2\left(r-\mu_t^P\right)^2}\right)\langle d\mu_t^P, d\mu_t^P\rangle \end{aligned} \tag{53}$$

Let $\sigma_t^{\underline{u}}$ denote the diffusion of $\underline{u}_t$. By collecting the coefficients on dZ_t^A in Equation (53), we have

$$\sigma_t^{\underline{u}} = -\frac{\sigma_t^N}{N_t} - \sigma^A - \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\sigma_t^{\mu^P}}{r-\mu_t^P}\right), \tag{54}$$

which, in comparison with Equation (50), contains an extra term that reflects the volatility of expected token price change. Note that, similar to Equation (49), we have

$$dN_t = -g\left(\underline{u}_t\right)d\underline{u}_t - \frac{1}{2}g'\left(\underline{u}_t\right)\langle d\underline{u}_t, d\underline{u}_t\rangle, \tag{55}$$

so the diffusion of N_t is $-g\left(\underline{u}_t\right)\sigma_t^{\underline{u}}$. Matching it with the conjectured diffusion coefficient σ_t^N gives σ_t^N.

B3. Proof of $\underline{A}^{FB} < \underline{A}^{NT}$

To prove this inequality, consider the agent whose type is $\underline{u}^{NT}$, i.e., the type whose flow profit is equal to zero when $A_t = \underline{A}^{NT}$ in the tokenless economy. Therefore, we have the following

$$0 = N^{NT}\underline{A}^{NT}e^{\underline{u}^{NT}}\alpha\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi < \alpha\underline{A}^{NT}\overline{S}\left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi, \tag{56}$$

where we use

$$N^{NT}e^{\underline{u}^{NT}} = \left[1 - G\left(\underline{u}^{NT}\right)\right] e^{\underline{u}^{NT}} < \int_{\underline{u}^{NT}}^{\overline{U}} e^{\underline{u}^{NT}} dG\left(u\right) + \int_{\underline{U}}^{\underline{u}^{NT}} e^{u} dG\left(u\right) \tag{57}$$
$$< \int_{\underline{u}^{NT}}^{\overline{U}} e^{u} dG\left(u\right) + \int_{\underline{U}}^{\underline{u}^{NT}} e^{u} dG\left(u\right) \equiv \overline{S}.$$

Recall that in the FB economy, we have

$$0 = \alpha \underline{A}^{FB} \overline{S} \left(\frac{1-\alpha}{r}\right)^{\frac{1-\alpha}{\alpha}} - \phi \,. \tag{58}$$

By comparing the right expressions in the two preceding inequalities, we conclude $\underline{A}^{NT} > \underline{A}^{FB}$.

Appendix C - Parameter Choices

We choose the model parameters under the physical measure so that the model generates patterns that are broadly consistent with user adoption and token price dynamics.

We assume that capital markets are perfectly competitive. For simplicity, we price all assets including tokens via the following stochastic discount factor ("SDF"):

$$\frac{d\Lambda_t}{\Lambda_t} = -r dt - \eta d\widehat{Z}_t^{\Lambda}, \tag{59}$$

where r is the risk-free rate and η is the market price of risk for $\widehat{Z}_t^{\Lambda}$ under the physical measure. Let ρ denote the correlation coefficient between the SDF shock and productivity shock. With these assumptions, under the physical measure, A_t follows a GBM process, where the drift coefficient, $\widehat{\mu}^A$, is equal to $\mu^A + \eta\rho\sigma^A$ and the volatility coefficient is σ^A.

We use token price and blockchain user-base dynamics from July 2010 and April 2018. We normalize one unit of time in the model to be one year. Since we fix the token supply at M, the token price P_t completely drives the market capitalization ($P_t M$). We map P_t to the *aggregate* market capitalization of major cryptocurrencies.[17] Since we study a representative

[17]We include all sixteen cryptocurrencies with complete market cap and active address information on bitinfocharts.com: AUR (Auroracoin), BCH (Bitcoin Cash), BLK (BlackCoin), BTC (Bitcoin), BTG (Bitcoin Gold), DASH (Dashcoin), DOGE (DOGEcoin), ETC (Ethereum Classic), ETH (Ethereum), FTC (Feathercoin), LTC (Litecoin), NMC (Namecoin), NVC (Novacoin), PPC (Peercoin), RDD (Reddcoin), VTC (Vertcoin). They represent more than 2/3 of the entire crypto market.

token economy, focusing on the aggregate market averages out idiosyncratic movements due to specificalities of token protocols.

We collect the number of active user addresses for these cryptocurrencies and map the aggregate number to N_t. We map the data to early stage of adoption in the model (i.e., $N_t \leq 0.5$). We normalize the maximum number of active addresses (in December 2017) to $N_t = 0.5$ and scale the number of addresses in other months by that of December 2017. For each month, we also need a value of $\ln(A_t)$. Since we cannot observe the platform quality, we assign December 2017 the value of $\ln(A_t)$ in our model that corresponds to $N_t = 0.5$. With December 2017 as a reference point, we calculate the values of $\ln(A_t)$ for other months by applying forward and backward the expected growth rate of A_t under the physical measure. As a result, we focus on the stage of adoption, i.e., $N_t \in [\underline{N}, 0.5]$, where $\underline{N} = 0.0001$.

Next, we choose parameter values such that the model generates data patterns in Figures 3 and 5. We set the annual risk-free rate, r, to 5% and choose $\mu^A = 2\% < r$ to satisfy the no-arbitage restriction. As we have previously discussed, we interpret A_t as a process that broadly captures technological advances, regulatory changes, and the variety of activities feasible on the platform, all of which suggest a fast and volatile growth of A_t. This consideration motivates us to choose $\sigma^A = 200\%$.

As shown in Figure 6, the model does generate a close link between the technology volatility and that of token returns, likely to due the fact that we focus on fundamental aspects of adoption and valuation while do not fully capture the behavioral and, in general, speculative factors in the model. That said, our choice of $\sigma^A = 200\%$ leads to a token return volatility that is close to the median cryptocurrency's return volatility in Hu, Parlour, and Rajan (2018). They document that the median cryptocurrency's daily return volatility is 14.6%, which is annualized to 232%.

This choice of $\sigma^A = 200\%$ gives us both a high volatility for A_t but also much of the growth for A_t under the physical measure, as the physical-measure drift of A_t is $\widehat{\mu}^A = \mu^A + \eta\rho\sigma^A$ (Girsanov's theorem). To match the growth of N_t in the data, we set $\eta\rho = 1$, so that $\widehat{\mu}^A = 202\%$ using the preceding equation. As a result, the user base N_t grows from $\underline{N} = 0.0001$ to 0.5 during the eight-year period of our data sample and the growth rate for the model-implied N_t matches that in data. One way to generate $\eta\rho = 1$ is to set η to 1.5, which is roughly the Sharpe ratio of ex-post efficient portfolio in the U.S. stock market (combining various factors) and ρ to 0.67, a sensible choice of betas for the technology sector (Pástor and Veronesi, 2009).

By no arbitrage, the drift of A_t, μ^A, is smaller than r under the risk-neutral measure, because after full adoption, $\mu_t^P = \mu^A$ as implied by the boundary condition. Therefore, for the model to generate the high growth of user base in data, we need the drift of A_t to be high under the physical measure, which requires, first, a high volatility of A_t and, second, a high enough $\eta\rho$. Setting ρ to 0.67 seems at odd with the existing studies on the returns of cryptocurrencies that show their correlations with the returns of traditional assets and macroeconomic factors are low (Hu, Parlour, and Rajan, 2018; Liu and Tsyvinski, 2018). However, we argue that in our model, dZ_t^A captures the shocks to the underlying technology or platform quality instead of direct return shocks. Moreover, the returns of cryptocurrencies can be driven by factors outside of our model, and such factors can add noise orthogonal to the SDF and reduce the correlation between cryptocurrency returns and the SDF.

We use the normalized distribution for u_i by truncating the Normal density function $g(u) = \sqrt{\frac{1}{2\pi\theta^2}} e^{-\frac{u^2}{2\theta^2}}$ within six-sigma on both sides. As the dispersion of u_i determines how responsive N_t is to the change of A_t, we match the curvature of N_t with respect to A_t by setting $\theta = 10/\sqrt{2}$, which implies that thee cross-section variance of u_i is 50.

We set α to 0.3 so that the senstivity of $\ln(P_t)$ with repect to N_t matches the data in the region where $N_t \in [\underline{N}, 0.5]$ as we show in Figure 5.

The remaining parameters quantitatively do not affect much the equilibrium dynamics. We set the participation cost, ϕ, to one and normalize M to 10 billion. As our model features monetary neutrality, P_t is halved when M is doubled but importantly the equilibrium dynamics is invariant.