The Distortionary Effects of Central Bank Direct Lending on Firm Quality Dynamics *

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Abstract

Bypassing the banking systems, central banks around the world lent to nonfinancial firms on an unprecedented scale during the Covid-19 crisis. Effective and necessary as it is, direct lending is subject to credit mispricing given central banks' lack of information on individual borrowers. Our dynamic model characterizes a downward bias in firm quality distribution that is self-perpetuating: Direct lending in the current crisis necessitates a greater scale of interventions in future crises, which in turn cause more severe distortion of firm quality distribution. Such effects are amplified by firms' forward-looking investment decisions in normal times. Low-quality firms overinvest to take advantage of underpriced central bank credit in future crises while, on a relative basis, high-quality firms underinvest. The distortionary effects can be mitigated by central banks' use of market information, collaboration and regulation of informed banks, and coordination of direct lending and conventional monetary policy.

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1 Introduction

Central banks have become the lenders of last resort not only for banks but for the whole domestic economy. European Central Bank started purchasing nonfinancial firms' debts in the global financial crisis. Bank of Japan has a long tradition of investing in both debts and equities. During the Covid-19 crisis, the Federal Reserve made a historic move by setting up facilities that bypass the banking system and provide liquidity directly to nonfinancial firms. Primary and Secondary Market Corporate Credit Facilities (PMCCF and SMCCF) made loans to and purchase bonds issued by large corporations. Main Street Lending Program (MSLP) allows the Federal Reserve to directly lend to small and medium enterprises with risk-bearing capital provided by the U.S. Treasury.

Central bank direct lending (CBDL) significantly expands the scope of policy responses in crises. The conventional wisdom of injecting liquidity through the banking system ignores frictions in the transmission mechanism (Trichet, 2013) but it is based upon a realistic information hierarchy – as regulators, central banks have information on banks' creditworthiness (Tucker, 2014), while, as delegated monitors, banks have information on firms' quality (Diamond, 1984). The lack of information on nonfinancial firms causes credit mispricing in CBDL that benefits low-quality firms at the expense of high-quality firms. Building on the two-sector model of Eberly and Wang (2008), we provide the first analysis of the distortionary effects of CBDL on firm quality distribution.

Our model features a cleansing effect of crisis (Caballero and Hammour, 1994) – low-quality firms' have a higher attrition rate under perfect credit markets. When credit markets break down, CBDL salvages production capacity, but credit mispricing mutes the cleansing effect. Low-quality firms with negative firm values – "zombies" (Laeven, Schepens, and Schnabel, 2020) – survive while high-quality firms with positive values exit. Such distortion leads to a slow recovery. Moreover, the impact is self-perpetuating. A downward-biased quality distribution necessitates a greater scale of CBDL in future crises, which in turn distorts the firm quality distribution more severely.

The self-perpetuating nature of CBDL distortion is amplified by the expectation effects. Tobin's Q, which drives firms' investment (Hayashi, 1982), incorporates the expectation of future crises and central bank credit. Low-quality firms' Q is biased upward, so they overinvest to take advantage of underpriced CBDL. In contrast, high-quality firms' Q is biased downward, so they underinvest. The distortionary effects of CBDL take place not only in crises but also in normal times through firms' forward-looking investment decisions. The unprecedented CBDL during the Covid-19 crisis is likely to reshape firms' expectation of future intervention and thus, strongly influences the growth trajectory over both short and long horizons (Gormsen and Koijen, 2020).

The distortionary effects can be mitigated by several measures observed in practice: (1) utilizing market information; (2) lending through informed banks; (3) coordinating CBDL and conventional monetary policy. The commercial paper and bond purchase programs (e.g., PMCCF and SMCCF) take advantage of information production in financial markets. Inefficiencies still exist among firms that do not have ratings or publicly traded securities. We show that low-quality firms' overinvestment dominates high-quality firms' underinvestment as the main source of inefficiency.

The market power of informed banks over high-quality, relationship borrowers is strong in crises as borrowers face difficulty convincing non-relationship banks of their types. When injecting liquidity through informed banks, a central bank can simultaneously set up a lending facility that improves high-quality borrowers' outside options. Such a facility should require the informed banks to participate and have skin in the game so that low-quality borrowers can be excluded (e.g., MSLP). High-quality firms benefit even when the facility is not utilized because CBDL changes the division of surplus between informed banks and borrowers.[1] Moreover, by reducing the expected costs of credit in future crises, CBDL increases high-quality firms' Tobin's Q and investments.

We study the coordination between CBDL and conventional monetary policy that targets interest rates and model policy transmission following Bigio and Sannikov (2019). Banks insure each other deposit redemption shocks through an interbank reserve market, and corridor rates affect market outcome (Bindseil, 2004; Friedman and Kuttner, 2010). When policy rates are unconstrained (for example, by zero lower bound), rate policy can release enough capacity of informed banks (Bernanke and Blinder, 1992; Kashyap, Stein, and Wilcox, 1993) to cover all high-quality firms so that only low-quality firms seek CBDL. The central bank then correctly price credit as firms are no longer pooled. We characterize the region of corridor rates that achieves such separation.[2]

Finally, we model firms' self-insurance through cash holdings (Fahlenbrach, Rageth, and Stulz, 2020). Low-quality firms choose not to hold liquidity as they expect underpriced CBDL. High-quality firms hold liquidity to rely less on overpriced CBDL but face a carry cost because the yield on cash holdings is below shareholders' required return. Therefore, when the interest rate on liquid assets increases, high-quality firms' precautionary savings increase, so the distortionary effects of CBDL are mitigated. Our findings offer a caution against ultra-low interest rates in the money markets, which echoes the insight of Quadrini (2020).

[1]This result speaks to the take-up of MSLP (Hanson, Stein, Sunderman, and Zwick, 2020).

[2]During two unscheduled meetings on March 3 and March 15, 2020, the Federal Open Market Committee (FOMC) lowered the target range for the federal funds rate in response to the Covid-19 crisis. Our results suggest that this action facilitates the implementation of CBDL by easing the stress in the interbank market.

Next, we summarize the model setup and elaborate on the main mechanisms. We follow the continuous-time formulation of the multi-sector model of Cox, Ingersoll, and Ross (1985), and in particular, adopt the two-sector setup of Eberly and Wang (2008). Both types of firms produce generic goods for consumption and investment but differ in the obsolescence rate of their capital.[3] Capital represents the production unit. High-quality firms' capital depreciates slower than that of low-quality firms' capital, which captures the distinction of sunrise and sunset industries. This is motivated by the drastic difference in the performances of technology sector and traditional sectors, such as energy and industrials, during the Covid-19 crisis, and within a sector, firms that adapt fast to digitization versus those that fail to do so. Both types of firms can invest in growing their capital. The forward-looking valuation of capital (Tobin' Q) drives the optimal investment (Hayashi, 1982; Abel and Eberly, 1994; Brunnermeier and Sannikov, 2014).[4]

The arrival of a liquidity crisis follows a Poisson process. In a crisis, firms' capital survives – if liquidity needs are met – or perishes, if not (Holmström and Tirole, 1998). A firm draws liquidity needs from a probability distribution, so when its liquidity needs exceed the value of its capital, it is optimal to exit. Because the value of high-quality capital (with low obsolescence rate) is greater than that of low-quality capital, the socially optimal attrition rate of high-quality firms is lower than that of low-quality firms. To highlight the role of CBDL, we assume that private financing is cut off, so firms borrow from the government. In return, the government receives shares of capital ownership as in the models of unconventional monetary policy (e.g., Gertler and Kiyotaki, 2010).

We compare the perfect- and imperfect-information equilibria. Under perfect information, the government correctly values the capital of different firms and offers break-even financing contracts. Therefore, this equilibrium features socially optimal attrition rates of high- and low-quality firms. As a result of the cleansing effect, the high-quality capital share jumps post-crisis, and the economy embarks on a growth path that is faster than its pre-crisis trajectory.

Under imperfect information, the government offers the same terms to both types of firms, as typically done in practice (English and Liang, 2020). The non-discriminatory lending terms can also be motivated by the political pressure against firms receiving differential treatment in crisis intervention. In this equilibrium, high-quality firms overpay for government credit while low-quality firms underpay. As a result, a fraction of high-quality firms with positive firm value exit,

[3]Moreira and Savov (2017) highlight the two-dimentional difference of capital in productivity and riskiness. For the transparency of mechanism, our setup features a simpler one-dimentional difference of capital in depreciation rate.

[4]While our model does not feature the fixed cost of investment in Abel and Eberly (1994), it captures the basic idea that Tobin'Q reflects future opportunities of expansion (Abel, Dixit, Eberly, and Pindyck, 1996).

while certain low-quality firms survive even when their liquidity needs exceed the value of their capital. The cleansing effect is muted. Therefore, even though CBDL helps preserve productive capacity, it distorts the firm's quality distribution and mutes the cleansing effect of crises. As a result, the post-crisis recovery is weaker than the perfect-information benchmark.

Importantly, the distortionary effects are cumulative. The firm quality distribution is biased downward in every future crisis, and when the subsequent crisis arrives, the economy will enter with more low-quality firms and thus experience a more significant output drop through attrition. Despite the underpriced credit support, low-quality firms' Q is still below that of high-quality firms, so more low-quality firms imply a higher overall attrition rate and a larger output drop as a result.

The distortionary effects are at work not only in crises but also in normal times. The expectation of underpriced credit support increases low-quality firms' Tobin's Q, which in turn stimulates their investment. In contrast, high-quality firms underinvest. The evolution of firm quality distribution depends on the relative investment and depreciation rates of low- and high-quality capital. Therefore, the expectation of distortionary CBDL biases downward the firm quality distribution in normal times. This again causes the economy to carry more low-quality firms into future crises.

The self-perpetuating nature of distortionary CBDL implies enormous welfare costs over the long run. Next, we extend our model in three directions to analyze mitigating measures. At the end of this paper, we extend our model to incorporate firms' liquidity management and draw the connection between the impact of CBDL and the level of interest rate in the money markets.

CBDL starts as an extension of quantitative easing – the purchase of secondary-market securities. This is based on the principle that markets aggregate information (Hayek, 1945; Grossman, 1976). When the types of a subset of firms are revealed in financial markets, the distortionary effects of CBDL are reduced. Interestingly, among the firms of unknown types, the dominant form of inefficiency is the overinvestment of low-quality firms rather than the underinvestment of high-quality firms. Among low-quality firms, those of hidden types receive underpriced credit while those of revealed types pay the fair price. The former outgrow the latter because in crises, more firms of hidden types are saved, and in normal times, their Tobin's Q and investment rates are higher. Among high-quality firms, the dynamics are exactly the opposite. Firms of revealed types drive out those of hidden types that overpay for CBDL. The expectation effects through Tobin's Q again act as the key reinforcing mechanism. Here we rediscover the principle that market-based invention feeds into market prices (Bond and Goldstein, 2015).[5]

[5]Goldstein and Yang (2017) review the literature on information production in financial markets.

What if central banks follow the conventional wisdom and inject liquidity solely through banks? Beyond the well-known frictions in the transmission mechanism (Trichet, 2013), Bank market power plays a distinct role in necessitating CBDL. High-quality borrowers are captured by their relationship banks as they are priced by other lenders as low-quality firms under asymmetric information (Santos and Winton, 2008). Injecting liquidity through informed banks save all firms of positive value but the economic surplus is seized by banks. While this does not create inefficiency in crises, it does depress high-quality firms' Q (and investment), which depends on the expectation of conceding value to banks in crises. CBDL adds value even under imperfect information. CBDL improves high-quality firms' outside options in their bargaining with relationship banks, which in turn boosts the firms' Q and investment. By requiring informed banks to have skin in the game, the central bank can exclude low-quality firms that seek underpriced credit. This design mimics MSLP in the U.S., and our result suggests that, despite the low take-up of MSLP, the program adds value by changing the division of economic surplus between banks and firms.

Next, we extend our model to include another key feature of banks – deposit redemption shocks (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005) – and analyze the coordination of CBDL and conventional monetary policy. Banks face a downward-sloping demand curve for deposits (Krishnamurthy and Vissing-Jorgensen, 2015; Drechsler, Savov, and Schnabl, 2017) and the uncertainty in deposit flows. Banks' deposit-risk exposure depends on the corridor-rate policy (Drechsler, Savov, and Schnabl, 2018) because corridor rates affect the outcome of interbank market (Bianchi and Bigio, 2014; Bigio and Sannikov, 2019).[6] We show that by adjusting the corridor rates, a central bank can reduce the effective cost of deposit withdrawal for banks, and thereby, incentivize banks to expand balance sheets and lend. When banks are informed and competitive, the resultant equilibrium can reach social optimum. High-quality firms borrow from informed banks at fair price, leaving only low-quality firms seeking CBDL. The separation allows the central bank to correctly price credit to low-quality firms. The set of optimal corridor rates depends on the output drop, which drives aggregate deposits, and thus, is a function of the firm quality distribution. Our results suggest that, beyond the New Keynesian rationale of stimulating aggregate demand, central

[6]Transmission depends on other factors such as banks' financial health (Bolton and Freixas, 2006; Drechsler, Drechsel, David Marques-Ibanez, and Schnabl, 2016), equity issuance costs (Repullo and Suarez, 2012; Goetz, Laeven, and Levine, 2020) and frictions that limit bank capital-structure adjustments (Dell'Ariccia, Laeven, and Marquez, 2014; Dell'Ariccia, Laeven, and Suarez, 2017). We focus on the interbank market as it is likely to respond most quickly to rate policies, and we focus on illiquidity or trading frictions of over-the-counter market (Ashcraft and Duffie, 2007; Afonso and Lagos, 2012, 2015) rather than interbank counterpart risks (Heider, Hoerova, and Holthausen, 2015).

banks should adjust their rate policies in crises to facilitate CBDL. Thus our paper furthers the line of research on coordinating different central bank policies (Repullo, 2000).

Finally, we augment the Q-theory of investment with liquidity management in the spirit of Bolton, Chen, and Wang (2011).[7] In crises, high-quality firms optimally follow a pecking order of financing, spending their liquidity holdings first and then tapping into the overpriced CBDL. In contrast, low-quality firms do not hold liquidity and rely fully on CBDL. Therefore, if high-quality firms were to carry enough liquidity that covers all needs in crises, the distortionary effects of CBDL disappear as once again the economy achieves a separation – high-quality firms self-finance their liquidity needs, and CBDL funds low-quality firms at a fair price. However, because the yield on liquidity holdings is below shareholders' required return, high-quality firms optimally choose to bear a positive probability of seeking CBDL. As a result, the distortionary effects of CBDL depends on the yield of liquid assets. A low interest rate implies less precautionary savings of high-quality firms and stronger distortionary effects of CBDL. [8]

The role of central banks as lenders of last resort constantly evolves throughout the history in response to crises, political struggles, and technological innovations (Goodhart, 1998; Calomiris, Flandreau, and Laeven, 2016). CBDL is a meaningful addition to the policy toolbox. When credit markets freeze (Stiglitz and Weiss, 1981; De Meza and Webb, 1987), the government can step in, effectively functioning as a financial intermediary (Bebchuk and Goldstein, 2011; Lucas, 2016). The Covid-19 crisis normalized the use of CBDL across the developed countries and will have a long-lasting effect on the expectation of nonfinancial firms and their investment and financing decisions. The existing models of unconventional monetary policy assume an exogenous dead-weight loss of CBDL (Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011; Gertler, Kiyotaki, and Queralto, 2012; Araújo, Schommer, and Woodford, 2015; Del Negro, Eggertsson, Ferrero, and Kiyotaki, 2017). We unpack the black box of CBDL costs and zoom into the endogenous and dynamic impact of CBDL on firm quality distribution.

As previously discussed, a key ingredient of our model is the Q-theory of investment (Hayashi, 1982). This is a rather standard way to model firms' forward-looking investment decisions in the

[7]We simplify the setup by assuming perfect capital markets outside of crises; otherwise solving the model is complicated by the tracking of firms' wealth distribution (e.g., Matsuyama, 2007; Moll, 2014).

[8]Our model takes the interest rate on liquid assets as exogenous, but in reality, it depends on demand from the private sector and the supply from both public and private sectors (Woodford, 1990; Holmström and Tirole, 1998; Farhi and Tirole, 2012a; Li, 2018; Li, Ma, and Zhao, 2019; Li, 2019; Kacperczyk, Perignon, and Vuillemey, 2020). Safe assets constitute the majority of firms' cash instruments. The literature on safe assets has explored interactions with various policy considerations (Brunnermeier, Merkel, and Sannikov, 2020). Our paper furthers this line of research.

literature (see Brunnermeier and Sannikov, 2014, among others). Historically, the evidence on the Q-investment relationship is mixed. However, more recently, after making significant progress on the measurement issues, the empirical literature has found meaningful support for the Q-investment relationship (Philippon, 2009; Peters and Taylor, 2017; Crouzet and Eberly, 2020).

The expectation of central bank intervention distorts firms' investment decisions, which contribute significantly to the long-run welfare cost of CBDL. The distortionary effects of expected government intervention has been studied extensively on both empirical and theoretical fronts (Calomiris, 1990; O'Hara and Shaw, 1990; Acharya and Yorulmazer, 2007; Acharya, 2009; Bond, Goldstein, and Prescott, 2009; Farhi and Tirole, 2012b; Gropp, Gruendl, and Guettler, 2013; Acharya and Mora, 2015; Gandhi and Lustig, 2015; Allen, Carletti, Goldstein, and Leonello, 2018). Our paper focuses on firms' expectation and investment decisions rather than banks' expectation and risk-taking behavior that have featured prominently in the existing literature triggered by financial crises. The rise of CBDL during the Covid-19 crisis is likely to put the role of firms' expectations front and center in the decades to come.

Broadly, our paper contributes to the literature on the costs of crisis intervention, such as risk cost (Lucas, 2012), tax distortions as a form of financing costs (Hanson, Scharfstein, and Sunderam, 2018), feedback loop between sovereign and private-sector risk (Acharya, Drechsler, and Schnabl, 2014; Brunnermeier, Garicano, Lane, Pagano, Reis, Santos, Thesmar, Van Nieuwerburgh, and Vayanos, 2016), and the costs of debt overhang and bankruptcy (Balloch, Djankov, Juanita Gonzalez-Uribe, and Vayanos, 2020; Brunnermeier and Krishnamurthy, 2020; Crouzet and Tourre, 2020; Greenwood, Iverson, and Thesmar, 2020; Wang, Yang, Iverson, and Kluender, 2020).[9]

2 Background: Central Bank Direct Lending

We review the responses of the Federal Reserve (Fed) and other central banks to the COVID-19 crisis. The lending facilities take advantage of market information or rely on banks' screening.

Traditionally, central banks provide liquidity through the banking system, relying on commercial banks to extend credit to the production sector. The standing facilities, for example the discount window of the Fed, effectively impose a ceiling rate in the interbank market to alleviate

[9] The recent contributions on the benefits of credit-market intervention focus on the positive externalities that cannot be internalized by private lenders (e.g., Bebchuk and Goldstein, 2011; Philippon and Schnabl, 2013; Liu, 2016; Giannetti and Saidi, 2019; Hanson, Stein, Sunderman, and Zwick, 2020).

financial stress.[10] During the COVID-19 crisis, the Fed's initial response was a 150bp decrease in the primary credit (discount-window) rate. The "stigma effect" of borrowing from the lender of last resort limits the utilization of such facilities (Armantier, Ghysels, Sarkar, and Shrader, 2015).

New facilities were established during the Global Financial Crisis.[11] For example, Term Auction Facility (TAF) was introduced to avoid the stigma effect (Hu and Zhang, 2019). Many of these facilities, such as Primary Dealer Credit Facility, Money Market Mutual Fund Liquidity Facility, and Term Asset-Backed Securities Loan, are active during the ongoing COVID-19 crisis.

The Paycheck Protection Program Liquidity Facility (PPPLF), introduced in April 2020, is another example of liquidity provision through the banking system. In the Paycheck Protection Program (PPP), banks lend to employers at a uniform rate of 1% and the loans are guaranteed by the Small Business Administration (SBA). PPPLF allows banks to pledge PPP loans as collateral to borrow from the Fed at a rate of 0.35%. Similar liquidity facilities were set up by the Bank of England and Bank of Japan during the same period.[12]

On March 23, 2020, the Federal Reserve made a historic move by announcing two credit facilities that bypass the banking system and aim at directly easing the credit conditions for nonfinancial firms (Boyarchenko, Kovner, and Shachar, 2020). Primary Market Corporate Credit Facility (PMCCF) makes loans to and purchase bonds from large companies. Secondary Market Corporate Credit Facility (SMCCF) purchases corporate bonds in the secondary markets. For both programs, eligible companies must be investment-grade or were investment-grade as of March 22, 2020.[13] Such facilities extend the scope of quantitative easing (QE) that initially targets long-term government bonds and mortgage-related securities and was mainly introduced in response to the global financial crisis. Direct credit facilities for nonfinancial firms were also introduced in Europe, Japan, and other countries.[14] These facilities take advantage of the information production in the financial markets or by the rating agencies when it comes to the heterogeneity of firms' credit-worthiness.

[10]Other examples include the operational standing lending facility at the Bank of England, the marginal lending facility at the European Central Bank (ECB), and the complementary lending facility at the Bank of Japan (BOJ).

[11]Examples include the Primary Dealer Credit Facility (PDCF) of the Federal Reserve, Term Purchase and Resale Agreement (PRA) Facility of the Bank of Canada (BOC), and Long-Term Refinancing Operations (LTRO) of ECB. For more details, please refer to "*Timeline of Policy Response to the Global Financial Crises*"

[12]The Term Funding Scheme with additional incentives for SMEs (TFSME) at the Bank of England accepts SME loans as collateral with a haircut, but different from PPPLF, the loans do not necessarily have the same rate. A facility similar to PPPLF at the Bank of Japn allows banks to borrow at rate of -0.1% using SME loans as collateral.

[13]The lending is conducted through a special purpose vehicle and the U.S. Treasury provided the equity capital.

[14]Dell'Ariccia, Rabanal, and Sandri (2018) review the unconventional monetary policies in the Euro Area, Japan, and the U.K. QE applies to corporate equities in Japan (Charoenwong, Morck, and Wiwattanakantang, 2019).

During the COVID-19 crisis, small and medium enterprises (SMEs) experienced signficant disruptions (Gourinchas, Kalemli-Özcan, Penciakova, and Sander, 2020). To cover the liquidity needs of SMEs, the Main Street Lending Program (MSLP) was introduced on April 9, 2020. It is a collaboration between the Fed and U.S. Treasury. The Fed will buy up to $600 billion in loans, with the U.S. Treasury contributing $75 billion as risk-bearing capital. The program targets small and medium-sized businesses and non-profit employers that are impacted by the COVID-19 pandemic. In contrast to PMCCF and SMCCF, in which the Fed bypasses the banks and directly engage the corporate credit markets, the Fed works with banks on MSLP, again relying on banks' expertise in screening firms. Federal Reserve will buy 95% of new or existing loans to qualified employers, while the loan-issuing bank will keep 5% as skin in the game. Similar to the PPP loans, all borrowers receive *same* interest rate of LIBOR plus 3%.

To sum up, liquidity facilities fall into three categories: (1) liquidity injection through banks (e.g., PPPLF); (2) direct purchases of debts of large companies that have credit ratings (e.g., PMCCF and SMCCF); (3) direct lending to small businesses in collaboration with banks (e.g., MSLP).

For PMCCF and SMCCF, the government relies on information production in the corporate bond market, which include the formation of fair prices through market participants' trading activities and the bond ratings.[15] The U.S. Government also has exposure individual firms' performances in PPPLF (through SBA) and MSLP (through the Treasury). Information production relies on banks in both cases. For PPPLF, potential loan forgiveness granted by SBA represents a form of subsidy (the willingness to take losses).[16] In all cases, credit quality stands out as a major issue. The lack of information on firm quality lies behind the non-discriminatory rate for all borrowers.

3 The Model Setup

Consider a continuous-time economy with a unit of mass of representative agents ("households") and a government. Households have risk-neutral utility with time discount rate r:

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt} dc_t\right], \tag{1}$$

[15] To maintain a well functioning market, the Fed limits itself to purchase no more than 10% of a single bond issue or 20% of an exchange-traded fund (ETF). See "*Secondary Market Corporate Credit Facility*".

[16] As part of broadly defined credit policies, loan subsidies are often introduced to support education, entrepreneurship, and home ownership with limited involvement of central banks (Stanton, Rhinesmith, and Easterly, 2017).

where c_t is the cumulative consumption. Households trade equity shares of firms that manage capital to produce non-durable numeraire goods and maximize shareholder values.

There are two types of firms, H and L, whose productive capital depreciate at different rates, $\delta^H < \delta^L$. The output of one unit of capital is normalized to one. Given the aggregate capital stocks, K_t^H and K_t^L, at time t, the total output of numeraire goods over dt is $\left(K_t^H + K_t^L\right) dt$.[17] Capital represents efficiency units. Its depreciation captures the obsolesce of firms' products and services, i.e., the shrinking capacity of generating basic consumption units. H firms face a slower obsolesce rate. Wherever necessary, superscripts denote type and subscripts denote time.

Let q_t^j, $j \in \{H, L\}$, denote the value of capital. It plays an important role in our analysis, as it incorporates the expectation of future growth path and disruption brought by liquidity crises. Firms can invest goods to create new units of capital. Given an investment rate i_t^j (i.e., the investment-to-capital ratio), the growth rate of capital is $F\left(i_t^j\right)$, $j \in \{H, L\}$ (increasing and strictly concave). As in Brunnermeier and Sannikov (2014), we obtain the following optimality condition for i_t^j. The marginal benefit of creating new capital is equal to the marginal cost

$$q_t^j F'\left(i_t^j\right) = 1\,. \tag{2}$$

Because $F(\cdot)$ is concave, the investment rate, i_t^j, is increasing in capital value, q_t^j, which captures the insight of Hayashi (1982). We consider an investment technology $F(\cdot)$ such that the optimal $i_t^j < 1$. As a result, investment can be self-financed out of the production flow, so we can focus on the problem of external financing only in the liquidity crises.

A liquidity crisis is modeled as a systematic Poisson shocks with intensity λ that hits all firms. The corresponding counting process is denoted by N_t. When $dN_t = 1$, a firm of type j, $j \in \{H, L\}$, temporarily loses its capital but, if it invests uk_t^j, capital is immediately restored, where k_t^j denotes a representative j-type firm's capital units. The liquidity need u is independently (across firms) drawn from a common distribution $G(u)$. Because capital is only destroyed temporarily, we call this shock a "liquidity shock" following Holmstrom and Tirole (1997). Firms' own production is of magnitude dt, so they must borrow lump sums of goods for investment when the liquidity shocks hit. Households are assumed to have sufficient amounts of endowed goods, but they cannot lend to firms as it is assumed that the financial markets break down in liquidity crises.[18]

[17] Firms differ only in the capital depreciation rate. To make the mechanism transparent, we do not assume that the survival of one type of firms affects the output of the other type (Caballero, Hoshi, and Kashyap, 2008).

[18] Our assumption of financial-market breakdown in crises is akin to the exogenous "financial shocks" in Jermann

The government steps in, effectively acting as a financial intermediary (Lucas, 2016). It finances lending to firms with lump-sum taxes on households and transfers the repayments to households. Following the models of unconventional monetary policy, we assume that the repayments are in the form of capital ownership.[19] Our focus is on the heterogeneous quality of firms' productive assets ("capital"). Therefore, we simplify the liability structure to be full equity.[20]

In the next section, we compare two equilibria with credit policies set under perfect and imperfect information about firm types, respectively. We show that credit support under perfect information results in a V-shape recovery while credit support under imperfect information results in an L-shape recovery by distorting the quality composition of firms in the economy.

4 Equilibrium

4.1 Perfect Information Benchmark

With perfect information on firm types, the government sets different repayment schedules for different types, $R^j_{G,t}(u)$, $j \in \{H, L\}$, using the break-even condition,

$$q^j_t R^j_{G,t}(u) = u\,, \tag{3}$$

for $j \in \{H, L\}$. Per unit of capital, a firm's liquidity needs are given by u. It is assumed that the government only lends the exact amounts needed, i.e., u per unit of capital.[21] Once capital is restored, a type-j firm that borrows u repays the government with a fraction $R^j_{G,t}(u)$ of capital.

and Quadrini (2012) that tightens agents' financial constraints. Credit markets break down for various reasons, such as lenders' lack of capital (Bernanke and Lown, 1991), the delay of information in booms (Gorton and Ordoñez, 2014; Asriyan, Laeven, and Martin, 2018), foreign investors' withdrawal (Van Nieuwerburgh and Veldkamp, 2009; Koijen, Koulischer, Nguyen, and Yogo, 2020), and ambiguity in risk evaluation Boyarchenko (2012); Caballero and Simsek (2013); Drechsler (2013). In the U.S., credit markets were under serious stress during the Covid-19 pandemic before government intervention (Falato, Goldstein, and Hortaçsu, 2020; Haddad, Moreira, and Muir, 2020; Halling, Yu, and Zechner, 2020; Kargar, Lester, Lindsay, Liu, Weill, and Zúñiga, 2020). A similar market breakdown happened in the global financial crisis (Acharya, Schnabl, and Suarez, 2013; Brunnermeier, 2009; Gorton, Laarits, and Metrick, 2017; Kacperczyk and Schnabl, 2010; Krishnamurthy, 2010).

[19]Please refer to Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki, and Queralto (2012), Araújo, Schommer, and Woodford (2015), and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017).

[20]A liability structure with both equity and equity invites the questions of crisis intervention in the form of debt moratoria (Bolton and Rosenthal, 2002) or the debt-overhang effects resulting from government credit support (Crouzet and Tourre, 2020; Krishnamurthy and Brunnermeier, 2020) that are beyond the scope of this paper.

[21]This can be motivated by the fact that the taxation agency observes firms' costs of operation.

The repayments can potentially vary over time with q_t^j.

We solve a Markov equilibrium where total capital, $K_t = K_t^H + K_t^L$, and H capital share,

$$\omega_t \equiv \frac{K_t^H}{K_t^H + K_t^L}\,, \tag{4}$$

are the two state variables. K_t determines the level of aggregate output. Since investment rates differ by capital type due to potentially different values of capital (see (2)), the composition of firms varies over time, captured by ω_t, which essentially drives the composite growth rate of output.

We conjecture that the values of capital in equilibrium are constant. To confirm this conjecture, we derive firms' return on capital and the equilibrium condition on q_t^j, $j \in \{H, L\}$.

Let y_t^j denote the *cumulative* return of type-j capital holdings. To characterize the instantaneous return, dy_t^j, first note that individual firms' capital units have the following law of motion

$$\frac{dk_t^j}{k_{t-}^j} = \left(F\left(i^j\right) - \delta^j\right) dt - \min\left\{1, R_G^j\left(u\right)\right\} dN_t\,, \tag{5}$$

where the subscript "$t-$" denotes the pre-shock value. Under the conjecture of constant q^j, the investment rate i^j given by (2), and the share of capital repaid to the government, $R_G^j(u)$, are no longer time-varying. If $R_G^j(u) \geq 1$, the firm does not participate in the lending program and exits. If $R_G^j(u) < 1$, the firm loses a fraction $R_G^j(u)$ to the government. Let $\overline{u}^j$ denote the threshold:

$$\overline{u}^j = \min\left\{u,\ s.t.,\ R_G^j\left(u\right) \geq 1\right\}\ . \tag{6}$$

We obtain the following expression of total return on capital holdings:

$$dy_t^j = \left(\frac{1 - i^j}{q^j} + F\left(i^j\right) - \delta^j\right) dt - \min\left\{1, R_G^j\left(u\right)\right\} dN_t \tag{7}$$

The first term, $(1 - i^j)/q^j$, is dividend yield minus investment cost, i.e., the net output per unit of capital divided by the unit value, q^j. The second and third terms, $(F(i^j) - \delta^j)$, account for the net growth rate in normal times. The last term is from the liquidity shock.

The expected return on capital is equal to the shareholders' discount rate in equilibrium, i.e.,

$\mathbb{E}_t\left[dy_t^j\right] = rdt$, so we obtain the following equilibrium condition for q^j:

$$r = \frac{1-i^j}{q^j} + F\left(i^j\right) - \delta^j - \lambda\left[1 - G\left(\overline{u}^j\right) + \int_{u=0}^{\overline{u}^j} R_G^j\left(u\right) dG\left(u\right)\right]. \tag{8}$$

where the optimal investment rate, i^j, satisfies the optimality condition (2). The last term on the right side is from the liquidity shock. With probability $1 - G\left(\overline{u}^j\right)$, a firm draws $u \geq \overline{u}^j$ and loses capital; when $u < \overline{u}^j$, the firm loses a fraction $R_G^j\left(u\right)$ as loan repayments to the government. Rearranging (8), we obtain the Gordon growth formula for q^j:

$$q^j = \frac{1-i^j}{r - \left\{F\left(i^j\right) - \delta^j - \lambda\left[1 - G\left(\overline{u}^j\right) + \int_{u=0}^{\overline{u}^j} R_G^j\left(u\right) dG\left(u\right)\right]\right\}}. \tag{9}$$

The numerator is net output and the denominator is the discount rate minus the growth rate in $\{\cdot\}$.

Under (3), the participation threshold, $\overline{u}^j$, is given by

$$\overline{u}^j = q^j. \tag{10}$$

The participation threshold is exactly the zero-NPV cutoff: investing up to $\overline{u}^j$ to restore capital that is worth q^j. Substituting out $\overline{u}^j$ with q^j and the repayment schedule $R_G^j\left(u\right)$ with (3), we obtain an equation that solves q^j: for $j \in \{H, L\}$,

$$r = \frac{1-i^j}{q^j} + F\left(i^j\right) - \delta^j - \lambda\left[1 - G\left(q^j\right) + \int_{u=0}^{q^j} \frac{u}{q^j} dG\left(u\right)\right], \tag{11}$$

where i^j is a function of q^j given by (2). This equation solves q^j as a constant and thus confirms our conjecture of constant values of capital in equilibrium.

To sum up, the value of capital, q^j, is solved by (11). Given q^j, the investment rate, i^j, and type-j firms' exit threshold in the liquidity crisis, $\overline{u}^j$, are solved by (2) and (10), respectively.

Proposition 1 (Capital Value, Investment, and Attrition under Perfect Information) *H-type firms' capital value and investment rate are higher than L-type firms': $q^H > q^L$ and $i^H > i^L$. H-type firms' exit threshold is higher than L-type firms': $\overline{u}^H = q^H > \overline{u}^L = q^L$.*

Given the equilibrium q^j, i^j, and $\overline{u}^j$, we solve the laws of motion of K_t^j: for $j \in \{H, L\}$,

$$\frac{dK_t^j}{K_{t-}^j} = \left(F\left(i^j\right) - \delta^j\right) dt - \left[1 - G\left(\overline{u}^j\right)\right] dN_t \,, \tag{12}$$

Therefore, the aggregate output (per unit of time), $K_t = K_t^H + K_t^L$, has the law of motion:

$$\begin{aligned}\frac{dK_t}{K_{t-}} =& \underbrace{\left[\omega_{t-}\left(F\left(i^H\right) - \delta^H\right) + \left(1 - \omega_{t-}\right)\left(F\left(i^L\right) - \delta^L\right)\right]}_{\triangleq\ \mu^K\left(\omega_{t-}\right)} dt \\ & - \underbrace{\left[\omega_{t-}\left(1 - G\left(\overline{u}^H\right)\right) + \left(1 - \omega_{t-}\right)\left(1 - G\left(\overline{u}^L\right)\right)\right]}_{\triangleq\ \eta^K\left(\omega_{t-}\right)} dN_t \,.\end{aligned} \tag{13}$$

The distribution of firm quality, ω_t (H capital share), plays an important role in the output dynamics (13). Since $i^H > i^L$ according to Proposition 1, the growth rate of output in normal times, $\mu^K\left(\omega_{t-}\right)$, is higher when ω_{t-} increases, i.e., there are more H-type firms. Moreover, from $\overline{u}^j = q^j$ in (10), $q^H > q^L$ also implies $\overline{u}^H > \overline{u}^L$.Thus, when ω_{t-} increases, the percentage drop of output in the liquidity crisis, $\eta^K\left(\omega_{t-}\right)$, is smaller. The economy starts with one initial $\omega_0 \in (0, 1)$, and ω_t evolves between the two absorbing states, zero and one, as follows

$$\begin{aligned} d\omega_t =& \omega_{t-}\left(1 - \omega_{t-}\right) \underbrace{\left[\left(F\left(i^H\right) - \delta^H\right) - \left(F\left(i^L\right) - \delta^L\right)\right]}_{\triangleq\ \mu^\omega} dt \\ & + \underbrace{\left(\frac{\omega_{t-} G\left(\overline{u}^H\right)}{\omega_{t-} G\left(\overline{u}^H\right) + \left(1 - \omega_{t-}\right) G\left(\overline{u}^L\right)} - \omega_{t-}\right)}_{\triangleq\ \eta^\omega\left(\omega_{t-}\right)} dN_t \,.\end{aligned} \tag{14}$$

In normal times, ω_t increases, because $\mu^\omega > 0$ (which is in turn due to $i^H > i^L$ and $\delta^H < \delta^L$). When the Poisson shock hits, ω_t jumps upward because the loading on dN_t is positive, which is in turn due to $G\left(\overline{u}^H\right) > G\left(\overline{u}^L\right)$ (i.e., $\overline{u}^H = q^H > \overline{u}^L = q^L$).

Therefore, the quality of firms improves over time. In fact, $\lim_{t \to +\infty} \omega_t = 1$ in our model. Specifically, the liquidity crises have a cleansing effect – L firms exit at a higher rate than H firms ($\overline{u}^H > \overline{u}^L$), so ω_t jumps upward. The liquidity crises cause an output drop, as shown in (13), but the attrition of L firms boosts the post-crisis growth rate of output through a higher ω_t.

Therefore, our model with perfect information predicts a V-shape recovery. The post-shock growth rate higher than the pre-shock growth rate, so the economy gradually reverses the output loss and catches up the pre-shock trajectory of output growth.

Proposition 2 (Aggregate Dynamics and V-Shape Recovery under Perfect Information) *The H capital share jumps upward in a liquidity crisis, i.e., $\eta^{\omega}(\omega_{t-}) > 0$. Therefore, the post-crisis growth rate of output is higher than the pre-crisis growth rate, i.e., $\mu^{K}(\omega_t) > \mu^{K}(\omega_{t-})$.*

4.2 Direct Lending under Imperfect Information

When the government cannot differentiate H and L firms, it sets one repayment schedule to all firms. Motivated by the non-discriminatory interest rate of the liquidity facilities reviewed in Section 2, we consider a time-invariant linear repayment schedule, $R_G(u) = \gamma u$ and conjecture that the values of capital are constant in equilibrium. We are agnostic about how the government chooses γ except that $R_G(u)$ falls between the two break-even repayment schedules given by (3):

$$\frac{u}{q^L} > R_G(u) = \gamma u > \frac{u}{q^H}. \tag{15}$$

Given the fraction of capital as repayment to the government, $R_G(u)$, only firms that draw $u \leq \overline{u}$ participate in the government lending program, where the participation threshold, $\overline{u}$, is given by

$$\overline{u} = \min\{u,\ s.t.,\ R_G(u) \geq 1\} = \frac{1}{\gamma}. \tag{16}$$

In contrast to the thresholds given by (6) (and solved in (10)), which differ by firm types, $\overline{u}$ uniformly applies to all firms, so both H and L firms exit at the same rate in a liquidity crisis.

To solve the capital values, we use the pricing equation (8) from the previous subsection,

$$r = \frac{1-i^j}{q^j} + F\left(i^j\right) - \delta^j - \lambda\left[1 - G(\overline{u}) + \int_{u=0}^{\overline{u}} R_G(u)\, dG(u)\right], \tag{17}$$

except that, on the right side, the repayment schedule, $R_G(u)$, and the exit threshold, $\overline{u}$, no longer have the type superscript. As before, i^j is a function of q^j by (2), so (17) is an equation that solves q^j and thus confirms our conjecture of constant capital values in equilibrium.

Proposition 3 (Capital Value, Investment, and Attrition under Imperfect Information) *H-type* *firms' capital value and investment rate are higher than L-type firms': $q^H > q^L$ and $i^H > i^L$. All firms have the same exit threshold $\bar{u} = 1/\gamma$ given by (16).*

Alongside our analytical results, we provide graphical illustrations from our numeric solution based on the following parameterization: $\delta^H = 0.02$, $\delta^L = 0.06$, $r = 0.08$, $\lambda = 0.1$, $K_0 = 1$, and $F(i^j) = \frac{1}{10}\sqrt{i^j + 0.05}$. The repayment parameter under imperfect information, γ, is set such that $\gamma\left(\frac{1}{2}q^H + \frac{1}{2}q^L\right) = 1$, i.e., the government breaks even when there are equal amounts of H- and L-type firms. Our results hold under any γ that satisfies the condition (15).

In contrast to the perfect-information equilibrium where all and only positive-NPV projects are financed, some positive-NPV projects of H firms are not financed, and some negative-NPV projects of L firms receive the government credit support. First, consider an H firm at the threshold $\overline{u}$ with $R_G(\overline{u}) = 1$. The inequality in (15) implies $q^H > \overline{u}$, i.e., a positive-NPV project of spending $\overline{u}$ to restore capital worth q^H that is not financed. In fact, for any $u \in (\overline{u}, q^H)$, we have $q^H > u$ but the capital is abandoned. In contrast, for any $u \in (q^L, \overline{u})$, $q^L < u$ but the L-type capital is saved.

Proposition 4 (Inefficiency of Credit Support under Imperfect Information) *The inequalities (15) imply that $q^H > \overline{u} > q^L$. Therefore, H firms with $u \in (\bar{u}, q^H)$ (positive-NPV projects) exit, while L firms with $u \in (q^L, \bar{u})$ (negative-NPV projects) receive government support.*

More importantly, given that the same threshold $\overline{u}$ apply to both types, ω_t no longer jumps upward in a liquidity crisis, following a law of motion that differs from (14):

$$d\omega_t = \omega_t (1 - \omega_t) \left[\left(F\left(i^H\right) - \delta^H \right) - \left(F\left(i^L\right) - \delta^L \right) \right] \tag{18}$$

In the perfect-information equilibrium, a greater fraction of L firms exit than H firms. This cleansing effect causes an upward jump of ω_t, which in turn leads to an output growth rate that is higher than its pre-shock value, so the impact of liquidity crisis on output trajectory is temporary.

In contrast, credit support under imperfect information cause both types of firms to exit at the same rate. As a result, ω_t does not jump, and the post-shock growth rate, which depends on ω_t as shown by (19), is the same as the pre-shock growth rate. This implies that the impact of liquidity crisis on output trajectory is now permanent. With the same pre- and post-shock growth rates, the economy can never catch up the pre-crisis trend, exhibiting an L-shape recovery.

Proposition 5 (Distortionary Direct Lending) *Direct lending is distortionary precisely because it is distributional-neutral: under non-discriminatory lending terms, the H capital share, ω_t, does not change in liquidity crises, as the policy inefficiently subsidizes L type at the expense of H type.*

In the perfect-information equilibrium, a recovery is faster because of the cleansing effect of direct lending. Firms pay the discriminatory and fair price of credit, so H-type firms, whose capital is worth more, save more units of capital than L-type firms. The quality distribution is thus improved in the crisis, causing a catch-up effect on the growth trajectory. The impact of crisis is thus temporary. In the imperfect-information equilibrium, both types of firms face the same cost of credit, so they exit by the same rate and the quality distribution stays the same. After the crisis, the path of output never catches up the extrapolated path from pre-crisis growth. The impact of crisis is permanent. Next, we demonstrate the full equilibrium dynamics.

Under the same exit threshold for both types, the aggregate output (per unit of time), $K_t = K_t^H + K_t^L$, has the law of motion:

$$\frac{dK_t}{K_{t-}} = \left[\omega_{t-}\left(F\left(i^H\right) - \delta^H\right) + \left(1-\omega_{t-}\right)\left(F\left(i^L\right) - \delta^L\right)\right] dt - \left[1 - G\left(\overline{u}\right)\right] dN_t \,. \tag{19}$$

The drift term is a function of ω_{t-} as that of (13), but, given the same exit threshold for both types of firms, the percentage drop of output in a liquidity crisis no longer depends on ω_{t-}.

In comparison with the perfect-information benchmark, government credit support under imperfect information shows three types of inefficiencies. First, the cleansing effect is absent as ω_t no longer jumps upward. Panel A of Figure 1 compares the simulated paths of ω_t from the perfect-information (dash line) and imperfect-information equilibria (solid line). In our simulation, two liquidity crises were introduce at the 5th and 15th years, respectively.

Second, in crises, all and only positive-NPV projects are financed under perfect information, while, under imperfect information, a fraction of positive-NPV projects of H-type firms are abandoned while L-type firms obtain funding for negative-NPV projects (Proposition 4). As a result, the percentage drop of output in a liquidity crisis is constant under imperfect information, as shown in (19), and, in contrast, the percentage drop of output shrinks over time in the perfect-information equilibrium. Panel A of Figure 1 compares the simulated paths of log-output (i.e., $\ln\left(K_t\right)$) from two equilibria. In the perfect-information equilibrium (dash line), ω_t drifts upward in normal times and jumps upward in crises. As shown in (13), the percentage drop of output in crisis decreases in ω_t, so, over time, the impact of crisis on output becomes increasingly smaller. In the imperfect-

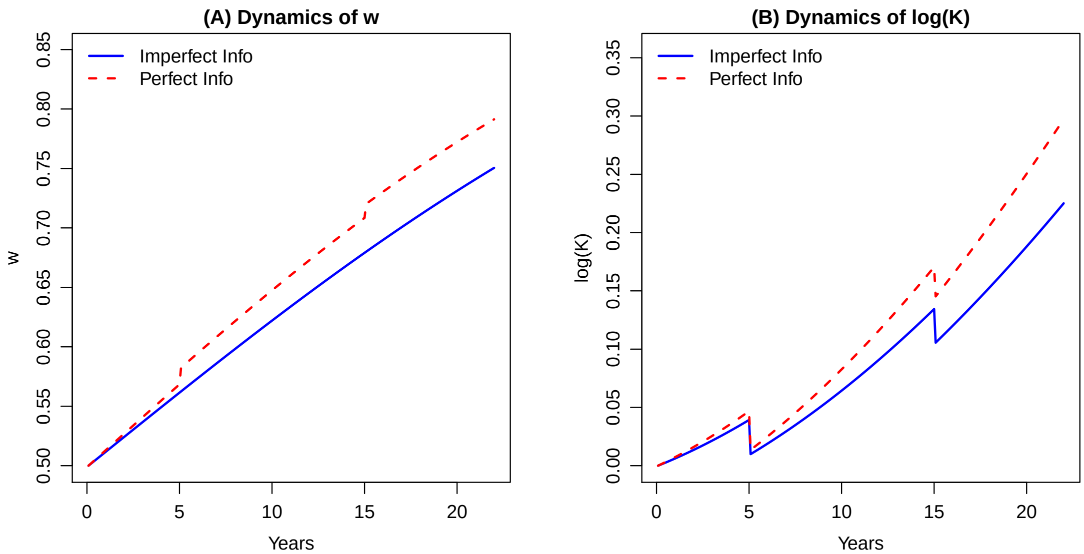

Figure 1: **Aggregate Dynamics.** We show the dynamics of states (ω_t, K_t) and the decomposition (K_t^H, K_t^L) by simulating the perfect- and imperfect information equilibria for 25 years. The starting state is $(w_0, K_0) = (0.5, 1)$. Two liquidity crisis shocks are introduced at Year 5 and 15, respectively. The parameterization follows Figure 1.

information equilibrium (solid line), even though ω_t still drifts upward, the growth over time is slower (due to the lack of cleansing effect in crisis) and does not translate into an increasingly smaller output drop. As shown in (19), the percentage of output in crises is always $1 - G(\overline{u})$, where $\overline{u} = 1/\gamma$ in (16) is the same exit threshold for both types of firms.

The third type of inefficiency is the distortionary effect of government credit support on firms' investment decisions. In this dynamic setting, capital value incorporates the expectation of future crises (see (17)). As shown in (2), capital values drive investments. Under imperfect information, L-type firms underpay for credit support while H-type firms overpay. The informational rent earned by L-type firms inflates the value of L capital, q^L (through a low exit rate) and, thereby, biases upward the investment of L-type firms. In contrast, H-type firms underinvest relative to the perfect-information benchmark as they overpay for credit support in crises. As a result, the capital composition is biased toward the L type, hurting output growth. In Figure 2, we compare the simulated accumulation of H-type capital (Panel A) and L-type capital (Panel B) in the two equilibria. The slope outside of crises reflects the investment rate (net off depreciation). For H type, capital accumulation is slower under imperfect information, while for L type, the opposite is true. The negative (positve) wedge for H (L) type widens over time as the investment effects accumulate.

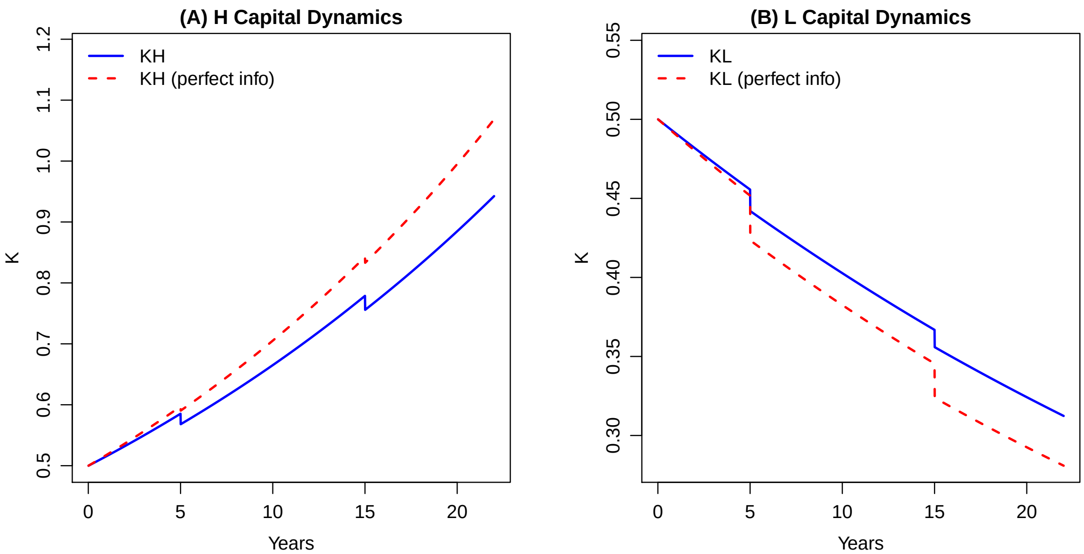

Figure 2: **Capital Accumulation.** In this figure, we show the dynamics of K_t^H and K_t^L by simulating the perfect- and imperfect information equilibria for 25 years. The starting state is $(w_0, K_0) = (0.5, 1)$. Two liquidity crisis shocks are introduced at Year 5 and 15, respectively. The parameterization follows Figure 1.

4.3 Welfare and Cumulative Distortions

We define welfare idefined as the present value of consumption flows:

$$\mathbb{E}_0[\int_0^\infty c\left(\omega_{t-}\right) K_{t-}e^{-rt}dt - \int_0^\infty U_G\left(\omega_{t-}\right) K_{t-}e^{-rt}dN_t], \tag{20}$$

where the consumption-to-output ratio in normal times,

$$c\left(\omega_{t-}\right) = \frac{K_{t-} - K_{t-}^H i^H - K_{t-}^L i^L}{K_{t-}} = 1 - \omega_{t-} i^H - \left(1-\omega_{t-}\right) i^L, \tag{21}$$

and under perfect information, the funds (divided by output K_t) lent through the government are

$$\begin{aligned} U_G\left(\omega_{t-}\right) =& \frac{\left(\int_0^{\overline{u}^H} ug(u)du\right) K_{t-}^H + \left(\int_0^{\overline{u}^L} ug(u)du\right) K_{t-}^L}{K_{t-}} \\ =& \omega_{t-}\left(\int_0^{\overline{u}^H} ug(u)du\right) + \left(1-\omega_{t-}\right)\left(\int_0^{\overline{u}^L} ug(u)du\right) \end{aligned} \tag{22}$$

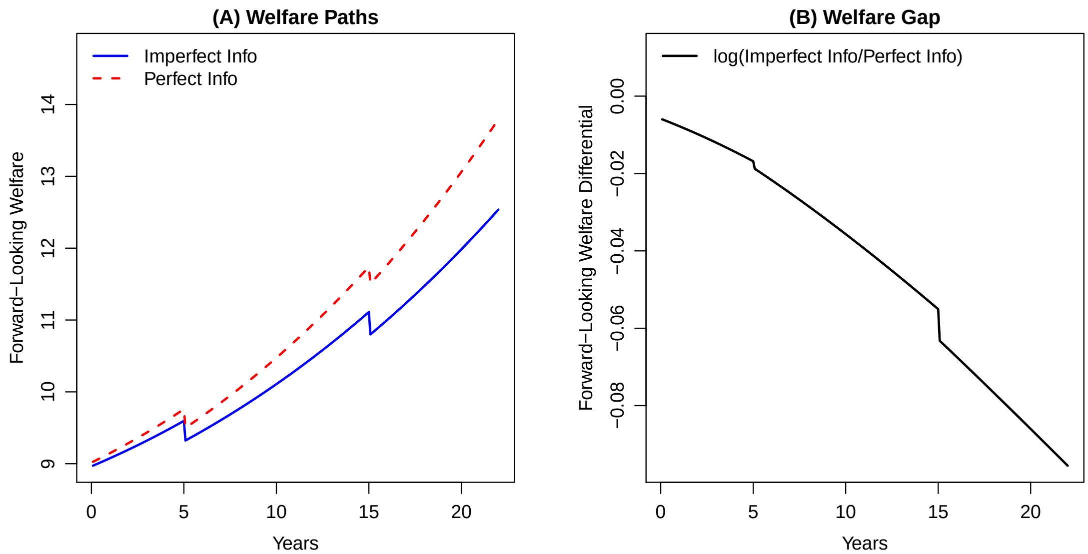

Figure 3: **Welfare Comparison.** In this figure, we plot the simulated paths of the forward-looking welfare defined in (20). The two liquidity crises happen in the 5th and 15th years, respectively.

while in the imperfect-information equilibrium, the credit support-to-output ratio becomes

$$U_G = \int_0^{\overline{u}} u g(u) du \,. \tag{23}$$

In crises, the government effectively acts as an intermediary that channels the funds from deep-pocket households to firms in exchange for capital ownership. The government holds capital ownership on behalf of households. Since households own firms, the transfer of capital ownership from firms to the government does not affect welfare. What affects welfare is the spending necessary in crises to save capital, which is only made possible by the government direct lending program.

Equation (20) defines the welfare at time 0. As the state variables vary over time, the welfare function, $W(\omega_t, K_t)$, varies, reflecting the discounted sum of future consumption paths as a *forward-looking* measure. We solve the welfare functions and plot the simulated paths of welfare in the perfect-information equilibrium (solid line) and imperfect-information equilibrium (dash line) in Panel A of Figure 3. The dynamics of K_t drives the geometric growth of welfare in both equilibria. We report in Panel B the log difference with the imperfect-information equilibrium as the base case. The divergence arises from the distortionary effects of government credit support: (1) in a crisis, the imperfect-information equilibrium features government inefficiently saving L-

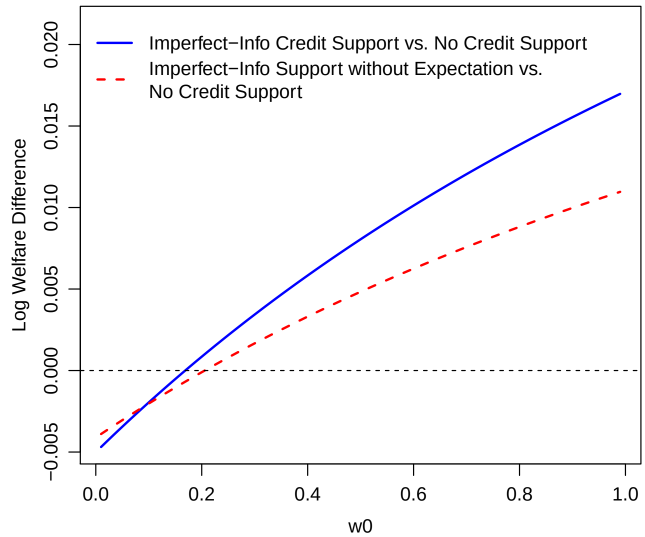

Figure 4: **Welfare Wedge over H Capital Share**. We plots the log welfare difference (over different values of H capital share) between the imperfect-information equilibrium and the benchmark equilibrium without credit support (solid line) and the log welfare difference between the hypothetical equilibrium with credit support but without expectation of support and the benchmark equilibrium (dashed line). The parameterization follows Figure 1.

firms but letting viable H-firm exit; (2) outside a crisis, the expectation effect of future distortion causes lower H-type capital value and investment, but higher L-type capital value and investment. As shown in Panel B, the welfare wedge between perfect- and imperfect-information equibilirium widens in crises, driven by the first force, and also in normal times, driven by the second force.

4.4 Welfare Loss and the Expectation Effect

Can direct lending can be counter-productive under imperfect information? To answer this question, we introduce an equilibrium where credit support is completely absent. In this equilibrium, all firms are priced as L-type firms in crises due to an extreme form of asymmetric information. Therefore, both types of firms face a repayment schedule given by $q^L R(u) = u$. Substituting this repayment schedule into the capital valuation (8), we obtain the capital values (and, via (2), the optimal investment) of the benchmark equilibrium. Figure 4 plots the log difference in welfare between the benchmark equilibrium and the imperfect-information equilibrium (solid line). The welfare wedge is plotted against different values of H capital share ω_0.

A key message from Figure 4 is that direct lending can indeed be counter-productive, resulting in lower welfare than the benchmark case when H capital share is low. Two forces contribute to this result given that credit support with $\gamma < 1/q^L$ is cheaper than the private-market credit in the benchmark equilibrium. First, in crises, the negative effect of financing inefficient L-type firms (with u greater than capital value) can overwhelm the positive effect of saving efficient H-type firms. Second, in normal times, the negative effect of inducing L-type firms' overinvestment can overwhelm the positive effect of alleviating H-type firms' underinvestment. Both forces are stronger when H-type firms account for a smaller share of all firms. When H capital share is high, the positive effects dominate and direct lending under imperfect information improves welfare.

The unprecedented scale of central bank direct lending during the Covid-19 pandemic is likely to reshape firms' expectation of future credit support. A key mechanism in our model is precisely the expectation effect. The expectation of future credit support and the associated distortions feeds into the current value of capital, which drives firms' forward-looking investment decisions. To demonstrate the expectation effect, we plot in Figure 4 the log welfare difference (over different values of ω_0) between the benchmark equilibrium and a hypothetical equilibrium where direct lending under imperfect information happens in crises but firms do not expect such interventions and are surprised in every crisis (dashed line).[22] Without the expectation effect, the welfare loss (in the low-ω_0 region) and gain (in the high-ω_0 region) are both more moderate and can be attributed to what happens solely in crises. Therefore, the expectation effect acts as an amplifier through firms' forward-looking investment decisions in normal times.

5 Mitigating the Distortionary Effects

In this section, we extend our model to capture the key features of liquidity facilities discussed in Section 2. Motivated by PMCCF and SMCCF, we allow the government to take advantage of information production in financial markets in Section 5.1. Section 5.2 evaluates the design of MSLP in our setting of firm quality dynamics. Section 5.3 shows that the coordination of monetary policy and lending programs is key to the success of PPPLP and similar lending programs that require banks to significantly expand the size of their balance sheets.

[22] To solve this hypothetical equilibrium, we obtain investment rates from the benchmark equilibrium and then have the laws of motion of ω_t and K_t feed on these investment rates in normal times and credit support in crises.

5.1 Information Production in Financial Markets

During the COVID-19 crisis, PMCCF and SMCCF allow the Federal Reserve to provide credit support through corporate bond purchases. Motivated by these programs and related programs of other central banks, we model separately firms with and without access to the public market, and accordingly, the government will set repayment schedules for different firms.[23] We analyze how price discovery in capital markets improves efficiency.

Let ω_t^H and ω_t^L denote respectively the fractions of H- and L-type capital belonging to firms that have access to the public credit market. In liquidity crises, these firms can obtain financing from competitive investors (households) at repayment schedules given by the break-even conditions (3). To capture the market breakdown in crises, it is assumed that the funds raised in from investors are negligible so credit support is still necessary. We still use ω_t to denote the H share of *all* capital.

For firms with credit priced in the public market, the government observes the market prices and simply sets the repayment schedules accordingly as $R^H(u)$ and $R^L(u)$ for H and L types of firms, respectively, which in turn imply the exit thresholds, $\overline{u}^H = q^H$ and $\overline{u}^L = q^L$. For firms without access to the public market, the repayment schedule is denoted by $R^G(u)$. As in Section 4.2, we consider a linear schedule, $R^G(u) = \gamma u$, so the exit threshold $\overline{u}^G = 1/\gamma$.

As in Section 4, we conjecture that capital values are constant. There are four types of capital, investment rates, and exit thresholds. Capital values are denoted by q^H (H type, publicly traded), q^h (H type, not publicly traded), q^L (L type, publicly traded), and q^l (L type, not publicly traded). The investment rates of different types are also differentiated by these superscripts. The derivation of equilibrium capital values, investment rates, and exit thresholds follows Section 4.1 and 4.2.

Proposition 6 (Capital Value, Investment, and Attrition under Market Information) *In equilibrium, the public market allows for more efficient valuation, investment and liquidity support: (1) $q^H > q^h > q^l > q^L$; (2) $i^H > i^h > i^l > i^L$; (3) $\overline{u}^H > \overline{u}^G > \overline{u}^L$.*

In Proposition 6, q^j with $j \in \{H, L\}$ is solved by (11). Given q^j, the investment rate i^j and exit threshold $\overline{u}^j$ are solved by (2) and (10), respectively. For $j \in \{h, l\}$, q^j, is solved by (17). Given q^j, (2) solves i^j. All firms of type $j \in \{h, l\}$ have the same exit threshold $\overline{u}^G$ given by (16).

[23] European Central Bank (ECB) announced its Corporate sector purchase programme (CSPP) in 2009. Bank of England (BoE) introduced Corporate Bond Purchase Scheme (CBPS) in August 2016.

Following the derivation in Section 4, the aggregate output has the following law of motion

$$\frac{dK_t}{K_{t-}} = \mu^K\left(\omega_{t-}, \omega_{t-}^H, \omega_{t-}^L\right) dt - \eta^K\left(\omega_{t-}, \omega_{t-}^H, \omega_{t-}^L\right) dN_t\,, \tag{24}$$

where the pre-shock growth rate of aggregate output is given by

$$\begin{aligned}\mu^K\left(\omega_{t-}, \omega_{t-}^H, \omega_{t-}^L\right) =& \omega_{t-}^H \omega_{t-}\left(F\left(i^H\right) - \delta^H\right) + \left(1-\omega_{t-}^H\right)\omega_{t-}\left(F\left(i^h\right) - \delta^H\right) + \\ & \omega_{t-}^L\left(1-\omega_{t-}\right)\left(F\left(i^L\right) - \delta^L\right) + \left(1-\omega_{t-}^L\right)\left(1-\omega_{t-}\right)\left(F\left(i^l\right) - \delta^L\right)\,,\end{aligned} \tag{25}$$

and the drop in aggregate output in the liquidity crisis is given by

$$\begin{aligned}\eta^K\left(\omega_{t-}, \omega_{t-}^H, \omega_{t-}^L\right) =& \omega_{t-}^H \omega_{t-}\left(1 - G\left(\overline{u}^H\right)\right) + \omega_{t-}^L\left(1-\omega_{t-}\right)\left(1 - G\left(\overline{u}^L\right)\right) + \\ & \left[\left(1-\omega_{t-}^H\right)\omega_{t-} + \left(1-\omega_{t-}^L\right)\left(1-\omega_{t-}\right)\right]\left(1 - G\left(\overline{u}^G\right)\right)\,.\end{aligned} \tag{26}$$

As previously discussed, ω_t plays an important role. The drop of output in crises decreases in the pre-shock value ω_{t-}, and the post-crisis growth of output increase in the post-shock value ω_t. The law of motion of ω_t now depends on how many firms have access to the public credit market:

$$d\omega_t = \omega_{t-}\left(1-\omega_{t-}\right)\mu^\omega\left(\omega_{t-}^H, \omega_{t-}^L\right) dt + \eta^\omega\left(\omega_{t-}, \omega_{t-}^H, \omega_{t-}^L\right) dN_t\,, \tag{27}$$

where $\mu^\omega\left(\omega_{t-}^H, \omega_{t-}^L\right)$ in the pre-shock drift is given by

$$\mu^\omega\left(\omega_{t-}^H, \omega_{t-}^L\right) = \left[\omega_{t-}^H F\left(i^H\right) + \left(1-\omega_{t-}^H\right) F\left(i^h\right) - \delta^H\right] - \left[\omega_{t-}^L F\left(i^L\right) + \left(1-\omega_{t-}^L\right) F\left(i^l\right) - \delta^L\right]\,, \tag{28}$$

and the jump in the liquidity crisis is given by

$$\begin{aligned}&\eta^\omega\left(\omega_{t-}, \omega_{t-}^H, \omega_{t-}^L\right) \\ =& \frac{\omega_{t-}\left[\omega_{t-}^H G\left(\overline{u}^H\right) + \left(1-\omega_{t-}^H\right) G\left(\overline{u}^G\right)\right]}{\omega_{t-}\left[\omega_{t-}^H G\left(\overline{u}^H\right) + \left(1-\omega_{t-}^H\right) G\left(\overline{u}^G\right)\right] + \left(1-\omega_{t-}\right)\left[\omega_{t-}^L G\left(\overline{u}^L\right) + \left(1-\omega_{t-}^L\right) G\left(\overline{u}^G\right)\right]} - \omega_{t-}\,.\end{aligned} \tag{29}$$

The fraction of H-type capital belonging to firms with capital-market access, ω_{t-}^H, evolves as

$$d\omega_t^H = \omega_{t-}^H\left(1-\omega_{t-}^H\right)\left(F\left(i^H\right) - F\left(i^h\right)\right) dt + \left(\frac{\omega_{t-}^H G\left(\overline{u}^H\right)}{\omega_{t-}^H G\left(\overline{u}^H\right) + \left(1-\omega_{t-}^H\right) G\left(\overline{u}^G\right)} - \omega_{t-}^H\right) dN_t\,.$$

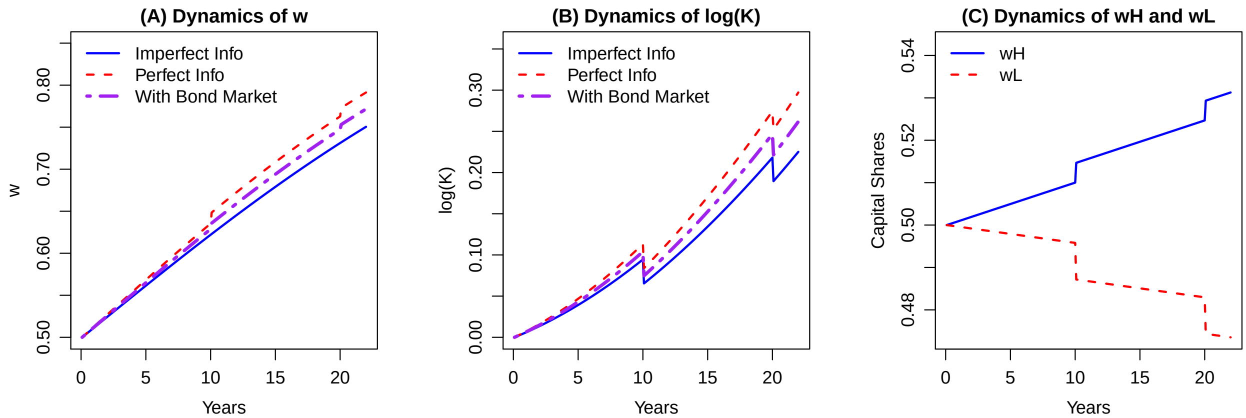

Figure 5: **Aggregate Dynamics under Market Information.** This figure compares the dynamics of state variables, ω_t (Panel A) and $\log(K_t)$ (Panel B), in the imperfect-information equilibrium (solid line), the perfect-information equilibrium (dash line), and the equilibrium with a subset of firms having capital-market access (dash-dot line). The simulation settings are the same as Figure 1 and ω_t^H and ω_t^L both start at 0.5. In Panel C, we plot ω_t^H and ω_t^L, the fractions of H-type and L-type firms with public market access.

The fraction of L-type capital belonging to firms with capital-market access, ω_{t-}^H, evolves as

$$d\omega_t^L = \omega_{t-}^L \left(1 - \omega_{t-}^L\right) \left(F\left(i^L\right) - F\left(i^l\right)\right) dt + \left(\frac{\omega_{t-}^L G\left(\overline{u}^L\right)}{\omega_{t-}^L G\left(\overline{u}^L\right) + \left(1 - \omega_{t-}^L\right) G\left(\overline{u}^G\right)} - \omega_{t-}^L \right) dN_t \,.$$

Proposition 6 implies that among H-type firms, those with capital-market access gradually dominate those without, because, ω_t^H drifts upward pre-crisis ($i^H > i^h$) and jumps upward in crisis ($\overline{u}^H > \overline{u}^G$). As ω_t^H increases, the distortionary effect of credit support via H-type firms' underinvestment weakens over time. However, among L-type firms, those without capital-market access gradually dominate because $i^L < i^l$ and $\overline{u}^L < \overline{u}^G$. Therefore, the distortionary effect of credit support via L-type firms' overinvestment strengthens over time.

Therefore, by revealing the types of a subset of firms, capital markets allow the government to correctly price credit support for these firms. The overall inefficiency of credit support under imperfect information is reduced. Figure 5 shows that public credit markets move the equilibrium dynamics of ω_t (Panel A) and log output (Panel B) closer to those of the perfect-information equilibrium. Moreover, public markets affect the sources of inefficiency. Inefficiency increasingly stems from the L-type firms. Panel C of Figure 5 shows the divergence of ω_t^H and ω_t^L. The fraction of L-type firms with their types revealed (ω_t^L) declines because they pay the fair price for credit while the firms with hidden types underpay for credit. The decline can be attributed to both the differential survival rates in crises and the differential investment rates in normal times (which are in turn driven by the differential capital valuations). In contrast, the fraction of H-type firms with their types revealed (ω_t^H) rises because publicly traded firms pay the fair price for credit while private firms overpay for credit. Therefore, over the long run, the overinvestment of L-type firms with hidden types becomes the dominant form of inefficiency relative to the underinvestment of H-type firms with hidden types.

5.2 Collaboration with Informed Banks

While PMCCF and SMCCF take advantage of information production by the financial markets, the Main Street Lending Program (MSLP) relies on banks for information production. As reviewed in Section 2, this program requires banks to have a skin in the game, which is intended to motivate banks' screening efforts. Motivated by this program, we analyze how banks' information advantage helps reduce the inefficiency induced by credit support under imperfect information. Importantly, we show that, in the presence of banks' credit market power, credit support improves efficiency by taking on a new role of improving H-type firms' outside option.

Traditionally, central banks inject liquidity through the banking system. A key rationale is that banks have the necessary information on firms' credit worthiness (Diamond, 1984; Ramakrishnan and Thakor, 1984; Heider and Inderst, 2012). In our model, introducing competitive banks with perfect information on firm types and access to central-bank liquidity backdrop leads to the perfect-information equilibrium of Section 4.1. The key assumption here is that banks are competitive, and thus, set the repayment schedule by the break-even condition given by (3). This leaves the full surplus of saving capital in liquidity crises to firms.

However, inefficiency arises if banks have market power over the borrowing firms. In crises when alternative sources of financing are limited, firms typically rely on banks with long-term relationships (Santos and Winton, 2008).[24] In our analysis, we consider a unit mass of banks that have unlimited access to households' deep pockets, serving as intermediaries. Each firm is paired with one (relationship) bank that knows the firm's type. Similar as before, we conjecture an equilibrium of constant values of capital and investment rate. We maintain the assumption on $F(\cdot)$ that implies $i^j < 1$, so there is no need for borrowing in normal times. Doing so allows us to

[24] Banks' credit market power has been well documented (e.g., Sunderam and Scharfstein, 2016; Cahn et al., 2017).

focus on the roles of bank lending and government intervention in liquidity crises.

When the Poisson shock hits, all firms can borrow from the non-relationship banks but will be treated as L-type firms due to severe asymmetric information discount.[25] Non-relationship banks are many and thus competitive. They repayment schedule they offer to non-relationship firms is given by the break-even condition: $q^L R_B^L(u) = u$.[26]

In liquidity crises, the relationship bank gives the paired H firm a take-it-or-leave-it offer with the following repayment schedule that extracts all surplus beyond the firm's outside options:

$$R_B^H(u) = \min\left\{\frac{u}{q^L}, 1\right\}. \tag{30}$$

If an H-type firms draws $u < q^L$, which implies that the share of capital repaid to non-relationship banks, $R_B^L(u) = u/q^L$ is less than one, the firm can always borrow from competitive non-relationship banks at $R_B^L(u)$, so its relationship bank offers the same. After repaying the loan, the H firm keeps capital worth $\left(1 - R_B^L(u)\right) q^H$. If an H firm draws $u \geq q^L$, borrowing from competitive non-relationship banks requires the firm to give up at least 100% of the restored capital (i.e., $R_B^L(u) = u/q^L \geq 1$), so the relationship bank charges 100% of the capital ownership, leaving zero surplus to the H firm. A relationship bank offers $R_B^L(u)$ to the paired L firm because L-type firms can always turn down the offer and borrow from competitive non-relationship banks.

Informed bank lending, even under market power, manages to fund all positive-NPV projects. H capital is saved as long as the cost of saving is not greater than the capital value, i.e., $u \leq q^H$. The market power only affects how the surplus is divided: if $u < q^L$, the firm keeps some surplus; if $u \geq q^L$, the relationship bank seizes all surplus.[27] Credit to L-type firms is fairly priced at the break-even condition, and L-type firms keep all the surplus.

In spite of the fact that all positive-NPV projects are financed, inefficiency exists due to the dynamic nature of our model. The value of L-type firms' capital is the same as the value in the perfect-information equilibrium in Section 4.1 (i.e., $q_B^L = q^L$, , where the subscript "B" denotes this equilibrium with banks), but the value of H capital is lower ($q_B^H < q^B$) because H-type firms foresee a loss of investment surplus to their relationship banks in liquidity crises. Replacing $R_G^j(u)$

[25]Rigorously speaking, the discount is endogenous to the pool of borrowers faced by the banks. We assume that L-type firms, whose identity is unknown to the non-relationship banks, can borrow small amounts and come back repeatedly. As a result, these banks effective face a pool of borrowers that is almost all L type firms.

[26]To differentiate from the repayment charged by the government in Section 4.1, we use the subscript "B" for banks.

[27]The intuition is akin to production efficiency under first-degree price discrimination.

with $R_B^j(u)$, $j \in \{H, L\}$, in (9), we obtain the capital evaluation equations with the investment rates given by (2), confirming our conjecture of constant values of capital and investment rates.

A lower capital value implies a lower investment rate for H-type firms, denoted by i_B^H. Therefore, the pre-crisis growth rate of aggregate output, $\mu_B^K(\omega_{t-})$, is below the perfect-information value because the quality distribution is tilted towards L capital (i.e., lower ω_{t-}). Replacing i^H with i_B^H in (13) and (14), we obtain the laws of motion for ω_t and K_t, respectively.

Proposition 7 (Equilibrium with Informed Banks and Market Power) *Informed lending allows all positive-NPV projects to be funded in liquidity crises. However, inefficiency arises from the depressed pre-crisis growth of output, because H-type firms optimally choose investment rate below the perfect-information value, foreseeing the investment surplus seized by informed banks.*

Next, we analyze the equilibrium impact of government credit support. Different from Section 4, we consider a collaboration between the informed banks and the government. Our setup follows the Main Street Lending Program (MSLP) that was introduced during the COVID-19 crisis. Two features of the program stand out. First, all borrowers receive the same interest rate. Therefore, we consider a linear repayment schedule, $R_G(u) = \gamma u$. As in Section 4.2, $\gamma \in \left(\frac{1}{q^H}, \frac{1}{q^L}\right)$, so under this repayment schedule, H-type firms overpays and L-type firms underpays for liquidity. Second, the loan-issuing banks retain 5% of the loans. Accordingly, we assumes that banks keep a positive fraction of loans they originate and the rest are financed by the government.

Because the repayment γq^L is smaller than one (per unit of borrowing), L-type firms would borrow from the facility. But they are denied by their relationship banks, because, under the loan retention requirement, the banks will take losses. L-type firms are also denied by the liquidity facilities operated by the non-relationship banks that view all non-relationship borrowers as L-type firms and deny them to avoid losses. As a result, L-type firms borrow in the private market and accept $R_B^L(u) = u/q^L$. This is an efficient outcome because L-type firms with $u < q^L$ (positive-NPV projects) receive financing while L-type firms with $u \geq q^L$ (negative-NPV project) do not.

While the facility does not affect L-type firms, it changes the outside option of H-type firms. For $u < 1/\gamma$, H-type firms that borrow from the facility keep capital worth $(1 - \gamma u)\, q^H$. Therefore, in the private market, their relationship banks have to match the offer: $R_B^H(u) = R_G(u)$. For $u \geq 1/\gamma$, the relationship banks still offer $R_B^H(u) = 1$, seizing the full surplus, because H-type firms have to give up the restored capital anyway if they borrow from the government ($\gamma u \geq 1$). Therefore, for $u < 1/\gamma$, H-type firms are indifferent between borrowing from their relationship banks or the facility, and for $u \geq 1/\gamma$, H-type firms borrow from their relationship banks and render the full investment surplus. To break the tie, we assume that when facing the same offer, firms choose their relationship banks over the government. As before, H-type firms never borrow from non-relationship banks whose credit is more expensive than the government's ($\gamma < 1/q^L$).

In comparison with the equilibrium without government credit support, the only difference is that, when deciding on the relationship banks' take-it-or-leave-it offer, H firm's outside option improves from $R_B^L(u) = u/q^L$ from $R_G(u) = \gamma u$. With or without government credit support, all positive-NPV projects are financed. However, by improving H-type firms' outside option, government credit support improves efficiency by increasing the equilibrium value of H capital, which incorporates the potential loss of investment surplus to relationship banks in future liquidity crises. A higher capital value in turn implies a higher investment rate, denoted by i_G^H (where the subscript "G" is for government credit support).[28] In fact, the more lenient the repayment is (i.e., lower γ), the better H-type firms' outside option is, which in turn implies greater efficiency gains from shifting the investment surplus in liquidity crisis from the informed banks to firms.

Proposition 8 (The Impact of Credit Policy under Bank Market Power) *Government liquidity support improves efficiency by improving H-type firms' outside option. The resultant higher value of H capital leads to high investment rates and, thus, tilts the quality distribution of firms towards H type, increasing the pre-crisis growth rate of output.*

In equilibrium, L-type firms never borrow from the government facility as they are denied by the banks that have skin in the game. H-type firms (with $u < 1/\gamma$) are indifferent between borrowing from the government or their relationship banks. This prediction speaks to the under-utilization of MSLP (Hanson, Stein, Sunderman, and Zwick, 2020).[29] The success of liquidity facility cannot be judged by its lending volume. Under imperfect information on firm types and banks' credit market power, liquidity facility improves H-type firms' outside option. Figure 6 compares the equilibrium with only informed banks (solid line), the equilibrium with informed banks and government credit support (dash-dot line), and the perfect-information equilibrium (dash line). Credit support moves the equilibrium dynamics closer to the perfect-information outcome.

[28] Replacing $R_G^j(u)$ with $R_B^j(u)$, $j \in \{H, L\}$, in (9), we obtain the capital evaluation equations with the investment rates given by (2). Replacing i^H with i_G^H in (13) and (14), we obtain the laws of motion for ω_t and K_t, respectively.

[29] See "As Washington scrambles for more bailout money, the Fed sits on mountain of untapped funds" by Rachel Siegel and Jeff Stein, The Washington Post October 19, 2020.

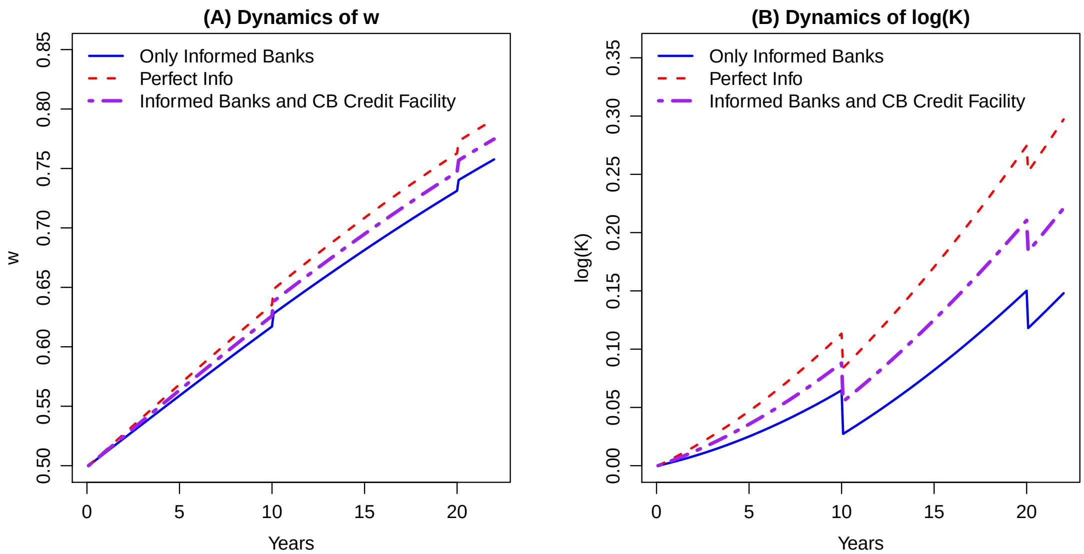

Figure 6: **Aggregate Dynamics with Informed Banks.** This figure compares the dynamics of state variables, ω_t (Panel A) and $\log(K_t)$ (Panel B), in the equilibrium with only informed banks (solid line), the equilibrium with informed banks and government credit support (dash-dot line), and the perfect-information equilibrium (dash line). The simulation settings are the same as Figure 1.

As one may have already noticed, the loan retention requirement plays a key role – it incentives banks to deny L-type firms, forcing them to pay the fair price of credit in the private market. Without the retention requirement, banks are no longer concerned about losses when accepting L-type firms into the lending facility. Because $\gamma < 1/q^L$, L-type firms always prefer borrowing from the facility over borrowing in the private market. Therefore, as in Section 4.2, two issues emerge in liquidity crises when the loan retention requirement is eliminated. First, L-type firms with negative-NPV projects (i.e., $u > q^L$ but $u < 1\gamma$) receive funding. Second, L-type firms underpay for credit, so the equilibrium value of L capital increases, stimulating L-type firms' investment. The resultant bias of firm quality distribution to the L type implies a lower pre-crisis growth rate of output and a larger output drop in future crises.

5.3 The Coordination of Credit and Monetary Policies

During two unscheduled meetings on March 3 and March 15, 2020, the Federal Open Market Committee (FOMC) lowered the target range for the federal funds rate in response to the Covid-19 crisis. This policy response eased the stress in the interbank market and allowed banks to expand balance sheets, taking more deposits and expanding credit provision to firms. These measures support the implementation of Paycheck Protection Program (PPP). As reviewed in Section 2, the Federal Reserve also set up specific liquidity facility for PPP that allows banks to borrow cheaply from the central bank. Our analysis in this section is motivated by these policies. We will show that, in liquidity crises, a combination of credit support under imperfect information and interest-rate policy responses from the central bank can achieve the perfect-information outcome.

We model a competitive banking sector that has perfect information on firms' type but is financially constrained due to frictions in the interbank market of borrowing and lending reserves. We focus on the interbank market because it is the most direct channel of monetary policy transmission. A central bank can adjust its interest-rate policy in liquidity crises to unleash bank credit (Bernanke and Blinder, 1992; Kashyap et al., 1993; Kashyap and Stein, 2000). We model the transmission of monetary policy following Bigio and Sannikov (2019). Our focus is on characterizing the interest-rate policy response that achieves the perfect-information outcome.

In normal times, firms can freely raise funds from households, so banks do not play any role. If the liquidity shock is realized (i.e., $dN_t = 1$), a unit mass of banks are set up and endowed with the information on firms' type. Banks are competitive, so they set the repayment schedules according to the break-even conditions (3) in Section 4.1, i.e., $q^j R_B^j(u)$, $j \in \{H, L\}$ where the subscript "B" is for banks. As in Section 4.2, the government does not have information on firms' type and offers the same repayment schedule $R_G(u) = \gamma u$ to all firms.

The bank can also hold reserve and earn the interest on reserve i_M (set by the central bank). Let a_t^B and m_t^B denote the values of bank lending and reserve holdings, respectively. The bank faces a budget constraint,

$$a_t^B + m_t^B \leq d_t^B \, . \tag{31}$$

where d_t^B is households' deposits. The deposit rate, i_t^D, will be endogenously determined.

After the bank chooses a_t^B, m_t^B, and d_t^B, a random variable, $\widetilde{\varepsilon}^D$, is drawn from a binary distribution (i.e., equal to ε^D and $-\varepsilon^D$ with equal probabilities). The bank's deposit liabilities become $\left(1+\widetilde{\varepsilon}^D\right) d_t^B$ and reserves become $m_t^B + \widetilde{\varepsilon}^D d_t^B$. This deposit-flow shock captures the fact that deposits serve as means of payment. If the bank draws $-\varepsilon^D$, its depositors make payments to depositors of other banks so, to settle these transactions, the bank has to disperse reserves to the payees' banks (on the asset side of balance sheet) and simultaneously loses deposits (on the liability side of balance sheet). The payees' banks receive reserves and credit deposits to the payees' accounts. If the bank draws ε^D, both its deposits and reserves increase. Since one bank's loss is another bank's gain, it is assumed that banks draw $\widetilde{\varepsilon}^D$ independently from the same binary distribution.

After $\widetilde{\varepsilon}^D$ is realized for each bank, the interbank market opens where banks can borrow and lend reserves. Banks in surplus of reserves become lenders. Their outside option is lending to the central bank and earn i_M. Banks in shortage of reserves (i.e., having a negative reserve position $m_t^B - \varepsilon^D d_t^B < 0$) end up borrowing reserves in the interbank market. Their outside option is borrowing from the central bank at i^E, the *emergency lending rate*.[30] Banks meet each other randomly in an over-the-counter (OTC) market, and the bargaining outcome depends on the outside options, i_M and i_E. The outcome also depends on "market tightness" that is driven by the *aggregate* reserve-to-deposit ratio, M_t^B/D_t^B. Bigio and Sannikov (2019) provide a tractable OTC formulation in the spirit of Afonso and Lagos (2015). We adopt this formulation.

The central bank affects the size of banks' balance sheet (the choice of d_t^B) and the composition of banks' assets (the choice of a_t^B and m_t^B) through the two policy rates, i_M and i_E, that affect the interbank market equilibrium ex post and thereby influence banks' trade-offs ex ante. A representative bank maximizes the following expected profits by choosing a_t^B, m_t^B, and d_t^B:

$$\pi_t^B \equiv \max_{\{a_t^B, m_t^B, d_t^B\}} a_t^B r + m_t^B i_M - d_t^B i_t^D + \mathbb{E}_t \left[\chi_S \left(m_t^B + \widetilde{\varepsilon}^D d_t^B \,;\; i_E,\, i_M,\, \frac{M_t^B}{D_t^B} \right) \right] . \tag{32}$$

The bank's problem is static. It liquidates in dt and its owners (households) realize profits or losses.[31] The first term is the expected return from lending. After the bank lends to firms, firms restore capital and immediately repay the loans with part of their capital. In equilibrium, the expected return on capital holdings is r, because firms own the remaining part of capital and have a discount rate r. The second and third terms are the interests earned on reserves and interest expenses paid to depositors, respectively. We consider $i_M \leq r$, because otherwise, the bank would hold an infinite amount of reserves. The bank is willing to hold reserves even when $i_M < r$ because reserves serve as a hedge against $\widetilde{\varepsilon}^D$ by reducing the expected costs from borrowing in the interbank market (the last term). The functional form of $\chi_S \left(m_t^B + \widetilde{\varepsilon}^D d_t^B \,;\; i_E,\, i_M,\, \frac{M_t^B}{D_t^B} \right)$ depends on the formulation of interbank market trading. We provide details in in Appendix A.10.

[30]Traditionally, i_E is the discount-window rate. However, given the variety of facilities that central banks established since the global financial crisis and have continued operating, we broadly interpret i_E as emergency lending rate.

[31]Because losses are covered by bank owners, deposits (also owned by households) are safe.

Following Bigio and Sannikov (2019), we obtain the optimality condition for a_t^B,

$$r = i_M + \chi_K \left(i_E,\, i_M,\, \frac{M_t^B}{D_t^B} \right) , \tag{33}$$

and the equilibrium deposit rate,

$$i_t^D = i_M + \chi_D \left(i_E,\, i_M,\, \frac{M_t^B}{D_t^B} \right) , \tag{34}$$

where the function $\chi_K(\cdot)$ and $\chi_D(\cdot)$ are defined in Appendix A.10. Equation (33) solves the aggregate reserve-to-deposit ratio as a function of the two policy rates:

$$\frac{M_t^B}{D_t^B} \equiv \chi_M(i_M,\, i_E) . \tag{35}$$

Then we substitute the solution of M_t^B/D_t^B into (34) and solve $i_t^D = i^D(i_M,\, i_E)$.

Next, we solve banks' aggregate lending, $A_t^B \equiv \int_{i\in[0,\,1]} a_t^B(i)\, di$. First, given the deposit rate, we pin down the aggregate quantity of deposits by modeling households' deposit demand following the tradition of money-in-utility (Sidrauski, 1967). The resultant demand demand, D_t^H is given by

$$D_t^H = K_t \zeta \left(\frac{r - i_t^D}{\xi_0} \right)^{-\frac{1}{\xi_1}} . \tag{36}$$

Intuitively, households are willing to accept i_t^D that is below their discount rate r, because of deposit-in-utility. Following Krishnamurthy and Vissing-Jorgensen (2015), we specify the utility from deposit holdings as function of the deposit-to-output ratio, D_t^H/K_t, so households' deposit demand is linear in K_t. In Appendix A.10, we provide more details. When the deposit market clears, $D_t^H = D_t^B$, so we obtain the equilibrium ratio of bank lending to output:

$$\begin{aligned} \chi_A(i_M,\, i_E) \equiv \frac{A_t^B}{K_t} =& \frac{D_t^B - M_t^B}{K_t} = \zeta \left(\frac{r - i_t^D}{\xi_0} \right)^{-\frac{1}{\xi_1}} \left(1 - \frac{M_t^B}{D_t^B} \right) \\ =& \zeta \left(\frac{r - i^D(i_M,\, i_E)}{\xi_0} \right)^{-\frac{1}{\xi_1}} (1 - \chi_M(i_M,\, i_E)) . \end{aligned} \tag{37}$$

The first term, $\zeta \left(\frac{r - i^D(i_M,\, i_E)}{\xi_0} \right)^{-\frac{1}{\xi_1}}$, is the deposit demand-to-output ratio from (36). It captures

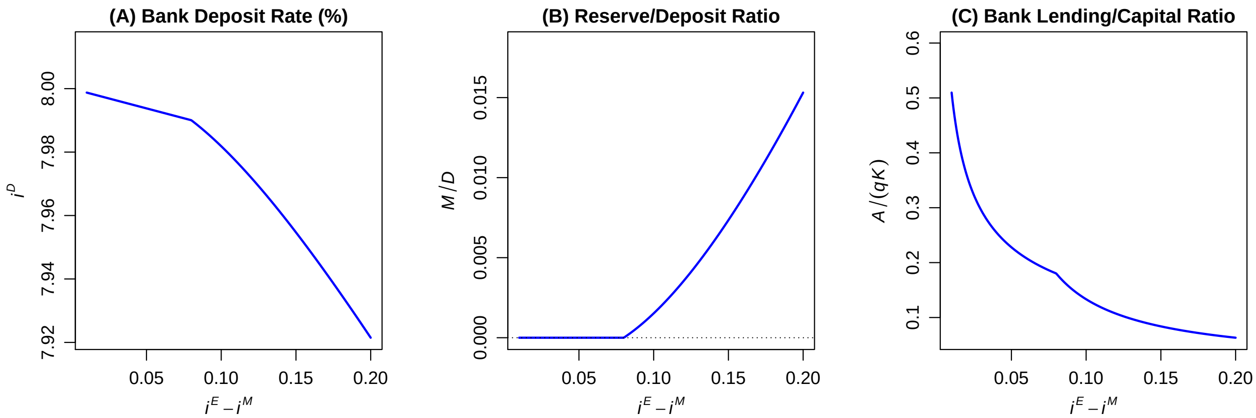

Figure 7: **Corridor Spread Policy.** This figure illustrates how deposit rate, reserve-to-deposit ratio, and bank lending-to-output ratio vary with the spread of policy rates, $i_E - i_M$ (with i_M fixed). The parameterization follows Section 4 and ε_D, ω_{t-}, and i_M are set to 0.05 (Bianchi and Bigio, 2014), 0.5 and 0.04, respectively.

banks' sources of funds and increases in the deposit rate. The second term, $(1 - \chi_M(i_M, i_E))$, is about banks' allocation of funds, i.e., the ratio of lending to deposits. The model is silent on how banks allocate A_t^B between H- and L-type firms as both types offer an expected return of r.

Banks' lending capacity is sufficient to cover H-type firms' needs if

$$\chi_A(i_M, i_E) K_t \geq K_{t-}^H \int_{u=0}^{\overline{u}^H} u\, dG(u)\,. \tag{38}$$

Aggregate bank credit on the left side depends on the post-shock output K_t, which drives households' deposits. H-type firms' liquidity needs on the right side depends on the pre-shock units of H-type capital K_{t-}^H. In (13), we introduced the percentage drop of output $\eta^K(\omega_{t-})$. Therefore, the pre- and post-shock outputs are connected via $K_t = K_{t-}\left(1 - \eta^K(\omega_{t-})\right)$. Dividing both sides by K_{t-} and using the decomposition of $\chi_A(i_M, i_E)$ in (37), we obtain

$$\zeta\left(\frac{r - i^D(i_M, i_E)}{\xi_0}\right)^{-\frac{1}{\xi_1}} (1 - \chi_M(i_M, i_E))\left(1 - \eta^K(\omega_{t-})\right) \geq \omega_{t-} \int_0^{\overline{u}^H} u\, dG(u)\,. \tag{39}$$

The central bank can set i_M and i_E to ease the stress in the interbank market, and thereby, incentivizes banks to raise more deposits via a higher $i^D(i_M, i_E)$ (the first term on the left side of (39)). By adjusting i_M and i_E, the central bank can also influence banks' allocation of funds between reserves and lending (the second term on the left side). Finally, a more severe output drop, i.e., a higher $\eta^K(\omega_{t-})$, calls for a more dramatic policy response (the third term on the left side). As shown in (13), the output drop depends on the exit thresholds, $\overline{u}^H$ and $\overline{u}^L$. When the monetary policy response is sufficiently strong and the inequality (39) holds, all H firms borrow from banks, which then implies that $\overline{u}^H = q^H$ (as credit is fairly priced) and all positive-NPV projects are financed. Given that only L firms borrow from the government, imperfect information on firms' type no longer matters and the government can simply set the repayment schedule for L-type firms according to the break-even condition (3) in Section 4.1, which then implies that $\overline{u}^L = q^L$.[32] The economy achieves the perfect-information outcome in Section 4.1.

Proposition 9 (Monetary Policy and Perfect-Information Outcome) *The economy achieves the perfect-information outcome if the central bank can set the interest on reserves, i_M, and the emergency lending rate, i_E, such that the inequality (39) is satisfied.*

In practice, the central bank adjusts the corridor spread, $i_E - i_M$. Figure 7 illustrates the effect. We fix i_M. Panel A shows that as the spread narrows (from the right to left), the equilibrium deposit rate increases, which then implies a higher deposit-to-output ratio in (36). Panel B shows that as the spread narrows, banks allocate more funds to lending and less to reserves. The intuition is that a narrower spread implies a smaller impact of the deposit-flow shock $\widetilde{\varepsilon}^D$ because the interbank borrowing and lending rates are bounded within the corridor of $[i_M, i_E]$. With $\widetilde{\varepsilon}^D$ less of a concern, banks are more willing to expand balance sheets by attracting more deposits and to lend more instead of hold reserves with a lower yield ($i_M < r$). The reserve-to-deposit ratio is bounded below at zero and bounded above at ε^D (above which, a bank is perfectly hedged against $\widetilde{\varepsilon}^D$). Panel C reports the overall effects of the rate corridor policy on bank lending-to-output ratio.

As shown in Figure 7, the marginal impact of narrowing the spread on the deposit rate (Panel B) and reserve-to-deposit ratio (Panel A) decreases, especially when the deposit rate is close to its upper bound, households' discount rate r, and banks' reserve holdings are close to zero. If adjusting the corridor spread cannot satisfy the inequality (39), the central bank can still adjust the interest on reserves, i_M. Figure 8 illustrates the effect. We fix $i_E - i_M$. Panel A shows that as i_M increases (from the left to right), banks hold more reserves. However, as the optimal allocation of funds tilts towards reserves with lower yields ($i_M < r$), banks' deposit-taking incentive weakens,

[32] Formally, banks set out different break-even schedules for both H and L types of firms, and the government offers only one repayment schedule, $q^L r_L(u) = u$, under which L-type firms break even and H-type firms incur losses. Therefore, H-type firms opt for banks, and L-type firms are indifferent. Given banks' capacity is only enough to cover H-type firms' financing needs, only L firms borrow from the government

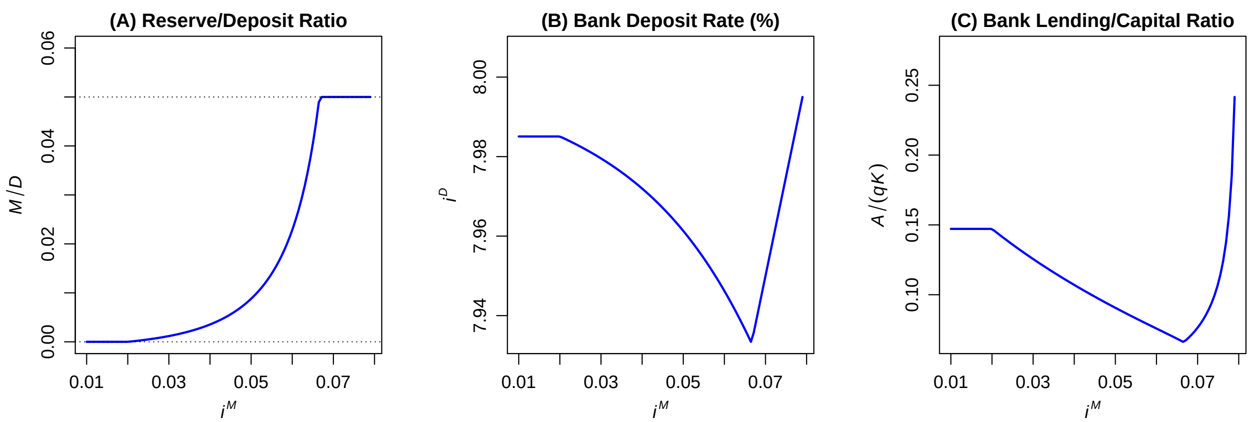

Figure 8: **Interest on Reserves.** This figure illustrates how reserve-to-deposit ratio, deposit rate, and bank lending-to-output ratio vary with interest on reserves, i_M (with $i_E - i_M$ fixed). The parameterization follows Figure 7.

resulting in a lower equilibrium deposit rate (Panel B) and a smaller amount of deposits. As banks shrink their balance sheets, holding more reserves and lending less, the bank credit-to-output ratio declines (Panel C). However, such impact is reversed when the interest on reserves is sufficiently high and banks hold enough reserves to cover any deposit-flow shock, i.e., $m_t^B > \varepsilon_D d_t^B$. The last term in (32) is shut down. The zero-profit, competitive banks pass along any further increase of i_M to depositors, so they raise more deposits and extend more credit to firms (Panel C).

Figure 9 reports the region of i_M and i_E where the economy achieves the perfect-information outcome (i.e., the inequality (39) is satisfied). The parameterization follows Section 4 with $\omega_{t-} = 0.5$. In practice, the central bank many have other considerations when choosing i_M and i_E that help pin down a particular choice of policy rates in the region of perfect-information outcome.

6 Corporate Liquidity Management

Let m_t^j denote the type-j firm's liquidity holdings per unit of capital, and r_m denote the interest rate on liquidity holdings, which we take as an exogenous parameter. In normal times, firms can accumulate liquidity holdings by raising equity from households who have the discount rate r. Therefore, if r_m is greater than or equal to r (the required rate of return), firms hold an infinite amount of liquidity. In the following, we consider $r_m \leq r$.

When the Poisson shock hits, a type-j firm maximizes the firm value (per unit of capital) by

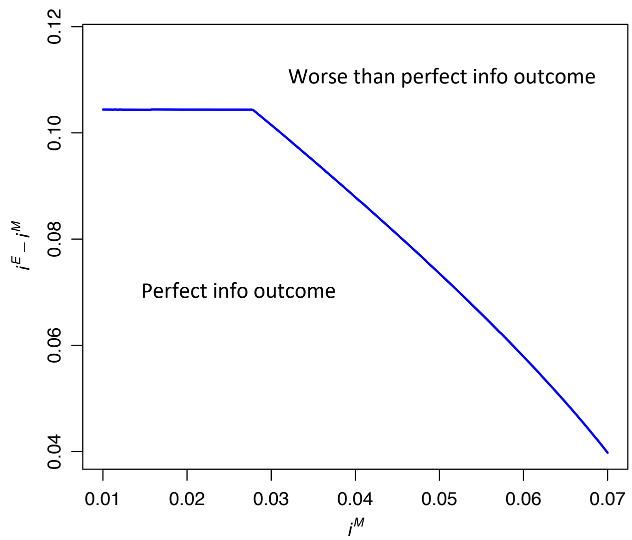

Figure 9: **Monetary Policy as a Tool to Achieve Perfect-Information Outcome.** This figure illustrates the region of policy rates, (i_M, i_E), where the economy achieves the perfect-information outcome in Section 4.1, i.e., the inequality (39) is satisfied. The parameterization is the same as Figure 7.

choosing the amount of liquidity to spend, z_t^j ($\leq m_{t-}^j$, where "$t-$" indicates the pre-shock value):

$$\max\left\{m_{t-}^j,\, q_t^j\left(1-R_G(u-z_t^j)\right)+\left(m_{t-}^j-z_t^j\right)\right\}. \tag{40}$$

The firm can abandon its capital and uses its liquidity as liquidation payout (the first term in the "$\max$" operator). If the firm decides to save its capital and spend z_t^j ($\leq u$), it still needs $u - z_t^j$ from the government and repays a fraction $R_G(u - z_t^j)$ of capital, ending up with a firm value per unit of capital equal to the second term in the "$\max$" operator.

To simplify the exposition, we consider a linear repayment schedule

$$R_G\left(u-z_t^j\right)=\gamma\left(u-z_t^j\right). \tag{41}$$

In equilibrium, capital values, q^j, and liquidity per unit of capital, m^j, are constant. We confirm this conjecture in the following. As in Section 4.2, to capture the inefficiency induced by credit policy under imperfect information, we consider the value of γ in $\left(1/q^H,\, 1/q^L\right)$, so, for one unit of credit support, an L firm repays a fraction γ of its capital, which is worth $\gamma q^L < 1$, while the repayment of an H firm is worth $\gamma q^H > 1$. In other words, L firms underpay and H-type overpay.

Because the cost of credit support, γq_t^L, is below 1, an L firm never spends its own liquidity in a crisis, so the benefits of holding liquidity is zero. But the costs of carrying low-yield assets is non-negative (i.e., $r_m - r \leq 0$). Therefore, $m_t^L = 0$. Intuitively, an L firm underpays for government credit support, so it does not hold any liquidity buffer in anticipation of crises.

To analyze an H firm's decision on liquidity holdings, we first note that $m^H \leq q^H$. If $m^H > q^H$, the extra amount of low-yield liquidity, $q^H - m^H$, incurs a non-negative marginal cost, $r - r_m$, but has zero marginal benefit because it will never used in crises: if $u \leq q^H$, the firm only needs to spend u at the maximum; if $u > q^H$, spending liquidity to save capital is a negative-NPV project.

Next, we solve the threshold $\overline{u}^H$ as a function of m^H and q^H. When a firm draws $u > \overline{u}^H$, it abandons its capital and exits with m^H. First, we have $\overline{u}^H \geq m^H$. Intuitively, the optimal liquidity holdings must be no greater than the largest possible liquidity needs, i.e., $m^H \leq \overline{u}^H$; otherwise, part of liquidity holdings will never be used yet still incurs a flow cost of $(r - r_m)\, dt$. The exit threshold is determined by equating the first and second terms in the $\max$ operator in (40):

$$\overline{u}^H = m^H + \frac{1}{\gamma}\left(1 - \frac{m^H}{q^H}\right). \tag{42}$$

When a firm draws $u > m^H$, it spends all of its own liquidity first before borrowing from the government, because spending its own liquidity a unit cost of 1 while government credit support costs $\gamma q^H > 1$. Therefore, when solving $\overline{u}^H$, we use the fact that $z_t^H = m^H$.

Therefore, the expected *net* value of an H firm's liquidity holdings *in a crisis* is given by

$$\int_0^{m^H} \left(q^H + m^H - u\right) dG(u) + \int_{m^H}^{\overline{u}^H} \left[q^H - \gamma\left(u - m^H\right) q^H\right] dG(u) + \left(1 - G\left(\overline{u}^H\right)\right) m^H - m^H$$

When $u < m_{t-}^H$, the firm does not need to seek credit support from the government, so, after covering its liquidity needs, it ends up with a firm value per unit of capital equal to $q^H + m_{t-}^H - u$. When $u \in \left(m_{t-}^H, \overline{u}^H\right)$, the firm spends all of its liquidity holdings and borrow $u - m_{t-}^H$ from the government to save its capital, ending up with a firm value per unit of capital equal to $q^H - \gamma\left(u - m_{t-}^H\right) q^H$ where the second term represents the fraction of capital ownership repaid to the government. When $u \geq \overline{u}^H$, the firm exits with m^H.

In equilibrium, the marginal value of liquidity in crises (derivative with respect to m^H) times

λ, which is the crisis probability, is equal to the carry cost, i.e.,

$$\lambda\left(q^H\gamma - 1\right)\left[G\left(m^H + \frac{1}{\gamma}\left(1 - \frac{m^H}{q^H}\right)\right) - G\left(m^H\right)\right] = r - r_m\,, \tag{43}$$

where $\overline{u}^H$ is substituted out by $m^H + \frac{1}{\gamma}\left(1 - \frac{m^H}{q^H}\right)$ according to (42). The equality holds when $m^H > 0$. When $m^H = 0$, the left side of (43) is less than or equal to the right side.

We assume that extreme liquidity needs are rare events, i.e., the probability density function, $g(\cdot)$, is decreasing in u. We differentiate equation (43) with respect to the carry cost and obtain

$$\lambda\left(q^H\gamma - 1\right)\left[\left(1 - \frac{1}{\gamma q^H}\right) g\left(m^H + \frac{1}{\gamma}\left(1 - \frac{m^H}{q^H}\right)\right) - g\left(m^H\right)\right] \frac{dm^H}{d\left(r - r_m\right)} = 1\,, \tag{44}$$

where the term in the square bracket is strictly negative because, first, given $m^H + \frac{1}{\gamma}\left(1 - \frac{m^H}{q^H}\right) \geq m^H$ (under $m^H \leq q^H$), we have $g\left(m^H + \frac{1}{\gamma}\left(1 - \frac{m^H}{q^H}\right)\right) \leq g\left(m^H\right)$, and, second, $1 - \frac{1}{\gamma q^H} < 1$ (under $\gamma q^H > 1$). Therefore, m^H is decreasing in the carry cost: when $m^H > 0$,

$$\frac{dm^H}{d\left(r - r_m\right)} < 0\,. \tag{45}$$

Proposition 10 (Carry Cost and Liquidity Holdings) *L-type firms do not hold liquidity in equilibrium, while H-type firms' liquidity holdings decrease in the carry cost, $r - r_m$.*

When the carry cost declines sufficiently, m^H can rise to q^H. In this case, $\overline{u}^H = m^H$ according to (42). Thus, in all scenarios where saving capital is a positive-NPV project, i.e., $u \leq q^H = m^H$, a firm saves capital by spending its own liquidity and does not need government support. To solve this threshold level of carry cost, we substitute m^H with q^H in (43) and obtain

$$r - r_m = 0\,. \tag{46}$$

When $r_m < r$, we have $m^H < q^H$, so equation (42) implies $\overline{u}^H < q^H$ – positive NPV-projects with $u \in \left(\overline{u}^H, q^H\right)$ are abandoned because H-type firms do not have enough liquidity to cover the liquidity needs and have to overpay for credit support from the government.

Corollary 1 (Carry Cost and Underinvestment in Crises) *When $r_m < r$, H-type firms' liquidity holdings do not cover all possible liquidity needs in crises (i.e., $m^H < q^H$), so $\overline{u}^H < q^H$ and* *positive-NPV projects with $u \in \left(\overline{u}^H,\, q^H\right)$ are abandoned. As the carry cost, $r - r_m$, increases, the exit threshold, $\overline{u}^H$, decreases, leaving more positive-NPV projects abandoned.*

Therefore, if and only if the carry cost is zero, H-type firms hold sufficient liquidity to self-finance all positive-NPV projects, and are no longer subject to the mispricing of credit by the government under imperfect information. In fact, when the carry cost is zero, the economy has a separation equilibrium and the inefficiency from imperfect information is completely eliminated. Since only L-type firms seek credit support, the government can fairly price credit, i.e., $\gamma = 1/q^L$, according to the break-even condition (3). Therefore, L-type firms no longer receive subsidy in the form of mispriced credit, so their capital value and investment are not biased upward. As a result, the dynamics of ω_t and K_t are identical to those in the perfect-information equilibrium.

Corollary 2 (Zero Carry Cost and Perfect-Information Outcome) *If and only if the carry cost is zero ($r_m = r$), the equilibrium outcome is the same as the perfect-information outcome. H-type firms hold enough liquidity to fund all positive-NPV projects in crises and never seek credit support. The government only faces L-type firms as borrowers and fairly prices credit.*

Firms hold liquidity as a hedge against the aggregate Poisson shocks, and the carry cost is essentially the price they pay for such insurance. Insuring against the Poisson shock adds value because the external financing becomes impossible in crises except for the overpriced credit support from the government. When the carry cost is zero, firms achieve free and perfect insurance.

Next, we derive the capital valuation equations for both types. As previously discussed, capital value is an important endogenous variable that drives firms' investment, the growth of output K_t, and the evolution of capital composition. Because L firms do not hold liquidity, the capital valuation equation for L-type capital is equation (17) in Section 4.2 when $r_m < r$ and L-type firms receive subsidy in the form of underpriced credit support. When $r_m = r$, only L-type firms seek credit support and the government fairly prices credit, so the capital valuation equation is equation (8) in Section 4.1. L-type firms' optimal investment rate is also given by (2) as in previous sections.

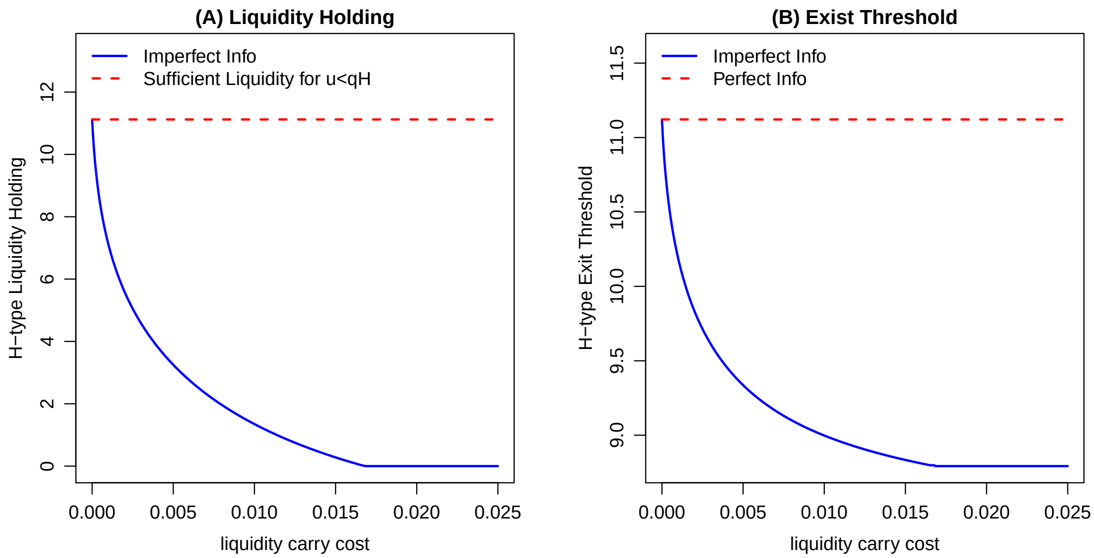

Figure 10: **Liquidity Carry Cost, Liquidity Holdings and Firm Exit for H-type Firms**. We illustrate how liquidity carry cost, $r - r_m$, affects the H-type firms' liquidity holding m^H and exit threshold $\bar{u}^H$.

The valuation equation for H-type differs from (17), reflecting the impact of liquidity holdings:

$$r = \underbrace{(1 - x^H)r_m + x^H \left(\frac{1 - i^H}{q^H} + F(i^H) - \delta_H \right)}_{\text{weighted average return without crises}} \\ - \underbrace{\lambda x^H \left(\int_0^\infty \min\{1, R_G(u - z^H(u)) + \frac{z^H(u)}{q^H}\} g(u) du \right)}_{\text{expected losses in crises}} \tag{47}$$

where the portfolio share of capital is $x^H = q^H/(q^H + m^H)$, and the liquidity spending strategy is $z^H(u) = u$ (cover all liquidity needs) if the liquidity shock is smaller than liquidity holding ($u < m^H$), and $z^H(u) = m^H$ (use up liquidity holding) if the liquidity shock is bigger than the liquidity holding ($u \geq m^H$).

The weighted average return of firm assets without crises shows up in the first two terms. The last term is the expected liquidity spent in crises *per dollar worth of capital* (i.e., every $1/q^H$ units of capital. For $u \in \left[0, m^H\right]$, u/q^H is spent. For $u \in \left(m^H, \bar{u}^H\right]$, the firm spends all of its liquidity holdings and then seeks credit support from the government. For $u > \bar{u}^H$, the firm abandons the capital exits with its liquidity holdings unspent. The third term is investment net off depreciation.

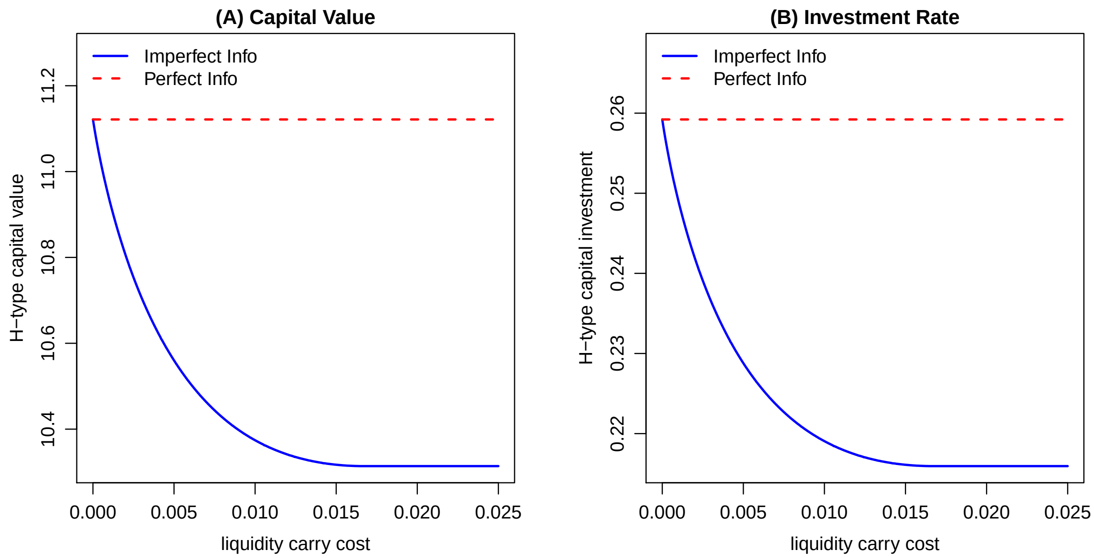

Figure 11: **Liquidity Carry Cost, Capital Value, and Investment for H-type Firms**. We illustrate how liquidity carry cost, $r - r_m$, affects the H-type firms' capital value q^H and investment rate i^H.

The last term records the percentage losses of capital. For $u \in \left[0,\, m^H\right]$, capital is intact because the firm has enough liquidity to cover its needs. For $u \in \left(m^H,\, \overline{u}^H\right]$, the firm seeks credit support, $u - m^H$, and repays the government a fraction of capital ownership. For $u > \overline{u}^H$, the capital is abandoned. H-type firms' investment, i^H, is given by (2), which only depends on q^H. The threshold of capital abandonment, $\overline{u}^H$, is given by (42), which depends on m^H and q^H. Therefore, when $m^H > 0$, equations (43) and (47) jointly solve m^H and q^H.[33] Therefore, we have confirmed that the conjecture of constant m^H and q^H is internally consistent.

To graphically illustrate the results, we solve the model numerically with the parameterization in Section 4 and consider different values of r_m (the only new parameter). In Panel A of Figure 10, we plot the equilibrium level of H-type' liquidity holdings against carry cost. When the carry cost is sufficiently high, H-type do not hold any liquidity. As we read the graph from the right to the left, the carry cost declines and H-type' liquidity holdings increase.

As the carry cost approaches zero, H-type' liquidity holdings approaches q^H, the level where the equilibrium outcome is identical to that of the perfect-information equilibrium. Panel B shows

[33] It is possible that multiple equilibria exist because low liquidity holdings result in low capital value, which in turn reduces firms' incentive to hold liquidity for saving their capital. However, our analysis so far carries through in any one of the equilibria. In our numeric illustrations, we do not find multiple equilibria under our parameterizations.

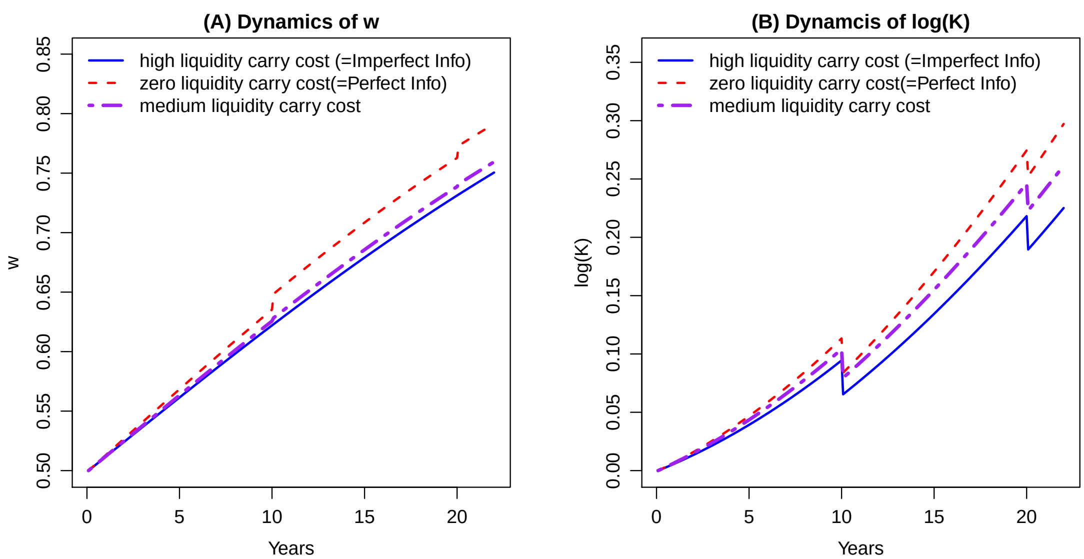

Figure 12: **Aggregate Dynamics with Corporate Liquidity Management.** This figure illustrates the dynamics of state variables ω_t and $\log(K_t)$ in the extension model with corporate liquidity market. Three cases are illustrated: (1) high liquidity carry cost, so that $m^H = m^L = 0$, which is the same as the imperfect information equilibrium; (2) zero liquidity carry cost, $r - r_m = 0$; (3) medium liquidity carry cost with $r - r_m = 0.4\%$. Simulation settings are the same as Figure 1. In particular, government credit support $R_G(\cdot)$ is the same as that in Figure 1.

that as H-type hold more liquidity, the exit threshold rises, all the way to the efficient threshold q^H, because holding liquidity helps H-type avoid the overpriced credit support. In Panel A of Figure 11, we show how H-type capital value increases as the carry cost decreases, and in Panel B, we show the investment rate. Both converge to the values in the perfect-information equilibrium as the carry cost approaches zero.

Once we have solved firms' investment rates and exit thresholds, we simulate the dynamics of ω_t (H-type capital share) and K_t (aggregate output) . For the laws of motion of ω_t and K_t, we use equation (14) and (13) in Section 4.1 that can be apply to any equilibrium with different investment rates and exit thresholds of the two types given the equilibrium values of capital. As in Section 4, our simulation have two Poisson shocks at the 10th and 20th years, respectively. Panel A and B of Figure 12 report, respectively, three cases of ω_t and K_t: (1) high carry cost, which implies $m^H = 0$; (2) medium carry cost, $r - r_m = 0.4\%$, which implies $m^H > 0$; (3) zero carry cost, which implies $m^H = q^H$. The first and third cases generate equilibrium outcomes that are identical to those of imperfect-information and perfect-information equilibria, respectively.

In Figure 12, both ω_t and K_t monotonically decrease in the carry cost of liquidity holdings.

An increase of carry cost reduces H-type firms' liquidity holdings, and thereby, decreases the exit threshold, $\overline{u}^H$, leaving more positive-NPV projects abandoned in crises. As a result, the drop in output is greater and the jump in ω_t is smaller (which in turn causes slower recovery post-crisis). The impact of a higher carry cost on the crisis dynamics feeds back into the normal times through firms' investment decisions. H-type firms' capital value declines as it incorporates the expectation of greater losses in crises, so, over time ω_t grows slower, resulting in a lower growth rate of K_t. A sufficiently high carry cost can discourage H-type firms from holding any liquidity at all, and thus, generates the imperfect-information equilibrium in Section 4.2.

Our analysis suggests a higher interest rate of money-market instruments (i.e., nonfinancial corporations' cash and cash equivalents) can be beneficial as it stimulates firms' self-insurance (via liquidity holdings) and thereby reduces the distortionary effects of mispriced credit support. In contrast, a low-rate environment hurts firms, because the yield on the money-market instruments decline, firms face an increasing cost of self-insurance against liquidity crises. This mechanism is related to Quadrini (2020) who emphasize that low interest rate hurts the aggregate output by suppressing producers' precautionary savings that are essential for buffering the idiosyncratic shocks in production process. The caution against low interest rate also echoes Brunnermeier and Koby (2018) who analyze the detrimental effects of low interest rate on banks' profitability.

Our analysis is partial-equilibrium, as we do not model the supply of saving instruments. In reality, there does not exist an infinitely elastic supply of storage technology at rate r_m. The rate is endogenously determined in the money market and driven by the supply of liquid assets from both the public and private sectors (Holmström and Tirole, 1998; Li, 2017). From the perspective of Friedman's rule, the government can and should play an active role in supply liquid assets (for example, in the form of Treasury bills) and thereby raising the equilibrium interest rate.[34]

Therefore, when dealing with systematic liquidity crises, the government can play two roles. First, it extends credit when crises happen. However, as demonstrated in our paper, imperfect information on firm quality induces various forms of inefficiencies. Therefore, the second role is also important, that is the government issues liquid securities that (high type) firms can hold to self-insure against crises and thereby avoid the mispriced credit support.

[34] The Friedman rule states that the opportunity cost of holding money faced by private agents should equal the social cost of creating liquidity (Friedman, 1969).

7 Conclusion

In this paper, we study how central bank direct lending (CBDL) affects firm quality dynamics in light of the recent policy developments in response to the Covid-19 crisis. The credit mispricing of CBDL distorts the firm quality distribution, which in turn affects long-term economic growth. The distortionary effects are active both in crises when CBDL is actually implemented and in normal times through firms' forward-looking investment decisions.

We then analyze several remedies observed in practice. First, we show that utilizing market information can reduce the distortionary effects but the inefficiency persists mainly in the form of low-quality firms' overinvestment. Second, when injecting liquidity through informed banks with market power, a central bank can set up a lending facility that improves the bargaining powers of high-quality firms and thereby raise these firms' Tobin's Q and investment. Therefore, even though the take-up of CBDL may be low, CBDL can still add value. Third, we study the coordination between conventional monetary policy and CBDL and characterize a region of corridor rates that achieve the perfect-information outcome when implemented jointly with CBDL under imperfect information. Finally, we show that the distortionary effects of CBDL decrease in the level of yield on cash instruments as a higher interest rate induces high-quality firms to self-insure against liquidity crises through more precautionary savings.

In our setting, firms only differ in the obsolescence rate of their capital. A future direction is to incorporate firms' difference in riskiness, following Moreira and Savov (2017), and study how CBDL affects not only the average quality of productive capital but also the risk exposure of the economy. Incorporating firms' difference in riskiness is also important for analyzing the coordination between CBDL and traditional monetary policy as interest-rate policy has been shown to affect banks' risk-taking both theoretically and empirically (e.g., Repullo, 2005; Maddaloni and Peydró, 2011; Martinez-Miera and Repullo, 2017).

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A Proofs and Algorithms

A.1 Solving the Welfare Function

We show how to solve for the welfare function for both the perfect information equilibrium and the imperfect information equilibrium.

Perfect Information Equilibrium

To solve the welfare function, we conjecture that

$$W\left(\omega, K\right) = h\left(\omega\right) K\,, \tag{A-1}$$

where, to simplify the notations, we suppress the time subscripts. Under this conjecture, we obtain

$$\begin{aligned} rW\left(\omega, K\right) dt = &\left[c\left(\omega\right) - \lambda U_G\left(\omega\right)\right] K dt + K h'\left(\omega\right) \omega \left(1-\omega\right) \mu^{\omega} + h(\omega)\mu^K\left(\omega\right) K dt \\ &- \lambda h(\omega)\eta^K\left(\omega\right) K dt - \lambda K\left[h\left(\omega + \eta^{\omega}\left(\omega\right)\right) - h\left(\omega\right)\right] dt\,, \end{aligned} \tag{A-2}$$

with

$$\mu^{\omega} = \left(F\left(i^H\right) - \delta^H\right) - \left(F\left(i^L\right) - \delta^L\right)$$

$$\eta^{\omega}\left(\omega\right) = \frac{\omega G\left(\bar{u}^H\right)}{\omega G\left(\bar{u}^H\right) + \left(1-\omega\right) G\left(\bar{u}^L\right)} - \omega$$

by definition. With conjecture (A-1), equation (A-2) implies an ordinary differential equation (ODE) for $h\left(\omega\right)$,

$$\begin{aligned} rh\left(\omega\right) = &c\left(\omega\right) - \lambda U_G\left(\omega\right) + h'\left(\omega\right)\omega\left(1-\omega\right)\mu^{\omega} + \mu^K\left(\omega\right) h\left(\omega\right) \\ &- \lambda h\left(\omega\right)\eta^K\left(\omega\right) - \lambda\left[h\left(\omega + \eta^{\omega}\left(\omega\right)\right) - h\left(\omega\right)\right]\,. \end{aligned} \tag{A-3}$$

which solves $h\left(\omega\right)$ and thus confirms of our conjecture of (A-1).

To solve the above the ODE for $h\left(\omega\right)$, we can start from the initial guess that ignores the derivative term and the jump term,

$$h^{(0)}(\omega) = \frac{c(\omega) - \lambda U_G(\omega)}{r - \mu^K(\omega) + \lambda\eta^K(\omega)}$$

Then we use the following "false time derivative" algorithm:

$$\Delta h^{(n)}(\omega) = rh^{(n-1)}(\omega) - \begin{pmatrix} (c(\omega) - \lambda U_G(\omega)) + \dfrac{dh^{(n-1)}(\omega)}{d\omega}\omega(1-\omega)\mu^{\omega} + \mu^K(\omega)h^{(n-1)}(\omega) \\ -\lambda\eta^K(\omega)\, h^{(n-1)}(\omega) - \lambda\left[h^{(n-1)}(\omega + \eta^{\omega}(\omega)) - h^{(n-1)}(\omega)\right] \end{pmatrix}$$

$$h^{(n)}(\omega) = h^{(n-1)}(\omega) - \Delta h^{(n)}(\omega) \cdot dt$$

The algorithm should converge if we pick a small enough dt. Another numerical trick is to set the time interval dt to be shrinking with n, e.g., dt_0/n.

Imperfect Information Equilibrium

To simplify the notations, we suppress the time subscripts. As in Section 4.1, we have

$$W(\omega, K) = h(\omega)\, K\,, \tag{A-4}$$

and obtain the following ordinary differential equation that solves $h(\omega)$,

$$rh(\omega) = c(\omega) - \lambda U_G + h'(\omega)\,\omega(1-\omega)\,\mu^{\omega} + \mu^K h(\omega) - \lambda h(\omega)\,(1 - G(\overline{u}))\,. \tag{A-5}$$

The algebra is simpler in this case, because there is no jump in the state ω, and government credit support is exactly the same for both H-type and L-type firms.

To solve the above the ODE for $h(\omega_t)$, we can start from the initial guess that ignores the derivative term,

$$h^{(0)}(\omega) = \frac{c(\omega) - \lambda U_G}{r - \mu^K + \lambda(1 - G(\overline{u}))}$$

Then we use the following "false time derivative" algorithm:

$$\Delta h^{(n)}(\omega) = rh^{(n-1)}(\omega) - \begin{pmatrix} (c(\omega) - \lambda U_G) + \dfrac{dh^{(n-1)}(\omega)}{d\omega}\omega(1-\omega)\mu^{\omega} \\ +\mu^K h^{(n-1)}(\omega) - \lambda(1 - G(\bar{u}))\, h^{(n-1)}(\omega) \end{pmatrix}$$

$$h^{(n)}(\omega) = h^{(n-1)}(\omega) - \Delta h^{(n)}(\omega) \cdot dt$$

The algorithm should converge if we pick a small enough dt.

A.2 Proof of Proposition 1

First, using (2), we replace q^j with $F'(i^j)$ in equation (11),

$$(1-i^j)F'(i^j) + F(i^j) - \lambda \int_0^\infty \min\{u \cdot F'(i^j), 1\} dG(u) = r + \delta^j \tag{A-6}$$

Denote

$$L(x) = L_1(x) - L_2(F'(x))$$

$$L_1(x) = F'(x)(1-x) + F(x)$$

$$L_2(z) = \lambda \int_0^\infty \min\{u \cdot z, 1\} dG(u) = \lambda \left(z \int_0^{1/z} u dG(u) + 1 - G(\frac{1}{z}) \right)$$

Then we get

$$L_1'(x) = F''(x)(1-x) < 0$$

$$L_2{}'(z) = \lambda \left(\int_0^{1/z} u dG(u) \right) > 0$$

Therefore,

$$L'(x) = L_1{}'(x) - L_2{}'(F'(x))F''(x)$$

$$= F''(x) \left((1-x) - \lambda \int_0^{1/F'(x)} u dG(u) \right)$$

$$< F''(x) \left((1-x) - \lambda \int_0^\infty u dG(u) \right)$$

To finish our proof, we need a sufficient condition

$$i^j < 1 - \lambda \int_0^\infty u dG(u) \tag{A-7}$$

Under this condition, we get $L'(x) < 0$ so that $L(x)$ is a decreasing function. By assumption, $\delta^L > \delta^H$. Therefore, $i^L < i^H$. Furthermore, $q^j = 1/F'(i^j)$ monotonically increases with i^j. This leads to $q^L < q^H$.

A.3 Proof of Proposition 2

By definition,

$$\mu^K(\omega) = \omega\left(F\left(i^H\right) - \delta^H\right) + (1-\omega)\left(F\left(i^L\right) - \delta^L\right)$$
$$= \omega\left(F\left(i^H\right) - F\left(i^L\right) + \delta^L - \delta^H\right) + F\left(i^L\right) - \delta^L$$

According to Proposition 1, we have $i^H > i^L$ so that $F(i^H) > F(i^L)$. Furthermore, by assumption, $\delta^L - \delta^H > 0$. Consequently, the capital growth rate $\mu^K(\omega)$ increases in the H-capital share ω.

Next, by definition,

$$\eta^\omega(\omega) = \frac{\omega G\left(\bar{u}^H\right)}{\omega G\left(\bar{u}^H\right) + (1-\omega)\,G\left(\bar{u}^L\right)} - \omega$$

$$= \omega(1-\omega)\left(\frac{G\left(\bar{u}^H\right) - G\left(\bar{u}^L\right)}{\omega G\left(\bar{u}^H\right) + (1-\omega)\,G\left(\bar{u}^L\right)}\right)$$

Since $\bar{u}^H > \bar{u}^L$ according to Proposition 1, we get $G(\bar{u}^H) - G(\bar{u}^L) > 0$, so that $\eta^\omega > 0$. As a result, after each crisis, under perfect information, the share of H capital jumps up. Combining this result with $\mu^K(\omega)$ increasing in ω, we get

$$\mu^K(\omega_t) = \mu^K(\omega_{t-} + \eta^\omega(\omega_{t-})) > \mu^K(\omega_{t-})$$

A.4 Proof of Proposition 3

The proof is similar to Appendix A.2 for Proposition 1.

First, using (2), we replace q^j with $F'(i^j)$ in equation (17),

$$(1-i^j)F'(i^j) + F(i^j) = r + \delta^j + \lambda\left[1 - G(\bar{u}) + \int_{u=0}^{\bar{u}} R_G(u)\,dG(u)\right] \tag{A-8}$$

We note that only the left-hand side of equation (A-8) involves i^j. The derivative of left-hand side over i^j is

$$F''(i^j)(1-i^j) < 0$$

Thus, the solution i^j to (A-8) decreases in the right-hand side. Given $\delta^H < \delta^L$, we get $i^H > i^L$, which implies $q^H = 1/F'(i^H) > q^L = 1/F'(i^L)$.

A.5 Proof of Proposition 4 and 5

Proofs are in the statements before each proposition in the main text.

A.6 Proof of Proposition 6

We note that under the conjecture of constant price of capital, the equation for $i^j, j \in \{L, H\}$ is exactly the same as the case of perfect information equilibrium in (A-6),

$$(1-i^j)F'(i^j)+F(i^j) = r+\delta^j+\lambda\left(1-G\left(\frac{1}{F'(i^j)}\right)+\int_{u=0}^{1/F'(i^j)} F'(i^j)\cdot u\,dG\left(u\right)\right), j \in \{L, H\} \tag{A-9}$$

The equation for $i^j, j \in \{l, h\}$ is exactly the same as the case of imperfect information equilibrium in (A-8)

$$(1-i^j)F'(i^j)+F(i^j) = r+\delta^j+\lambda\left(1-G\left(\frac{1}{\gamma}\right)+\int_{u=0}^{1/\gamma} \gamma u\,dG\left(u\right)\right), j \in \{l, h\} \tag{A-10}$$

From Proposition 1, we know that $i^H > i^L$. By definition,

$$F'(i^H) = \frac{1}{q^H} < \gamma < \frac{1}{q^L} = F'(i^L)$$

Define

$$L_1(x) = (1-x)F'(x) + F(x)$$

$$L_2(z) = \lambda\left(1 - G(\frac{1}{z}) + \int_0^{1/z} z\cdot u\,dG(u)\right)$$

Then we get

$${L_1}'(x) = (1-x)F''(x) < 0$$

$$L_2'(z) = \lambda\left(\int_0^{1/z} udG(u)\right) > 0$$

Therefore,

$$L_2(F'(i^H)) < L_2(\gamma) < L_2(F'(i^L))$$

Denote the inverse function of $L_1(x)$ as $L_1^{(-1)}(y)$, which is decreasing. Then we get

$$i^H = L_1^{(-1)}\left(r + \delta^H + L_2(F'(i^H))\right) > L_1^{(-1)}\left(r + \delta^H + L_2(\gamma)\right) = i^h$$

$$i^L = L_1^{(-1)}\left(r + \delta^L + L_2(F'(i^L))\right) < L_1^{(-1)}\left(r + \delta^L + L_2(\gamma)\right) = i^l$$

$$i^h = L_1^{(-1)}\left(r + \delta^H + L_2(\gamma)\right) > L_1^{(-1)}\left(r + \delta^L + L_2(\gamma)\right) = i^l$$

Taking the above together, we conclude

$$i^H > i^h > i^l > i^L$$

Since i^j monotonically maps to $q^j = 1/F'(i^j)$, we immediately get

$$q^H > q^h > q^l > q^L$$

The ranking of cutoff directly comes from the definition of γ,

$$\bar{u}^L = q^L < \bar{u}^G = \frac{1}{\gamma} < q^H = \bar{u}^H$$

A.7 Proof of Proposition 7

When only banks lend to firms in a crisis, the solution of q^H and q^L is:

$$r = \frac{1 - i^H}{q^H} + F\left(i^H\right) - \delta^H - \lambda \int_0^\infty \min\{\frac{u}{q^L}, 1\} dG\left(u\right) \tag{A-11}$$

$$r = \frac{1 - i^L}{q^L} + F\left(i^L\right) - \delta^L - \lambda \int_0^\infty \min\{\frac{u}{q^L}, 1\} dG\left(u\right) \tag{A-12}$$

Dynamics of the ω_t and K_t are in the same form as the baseline solution.

$$\frac{dK_t}{K_{t-}} = \left[\omega_{t-}\left(F\left(i^H\right) - \delta^H\right) + \left(1 - \omega_{t-}\right)\left(F\left(i^L\right) - \delta^L\right)\right] dt - \eta^K\left(\omega_{t-}\right) dN_t\,, \tag{A-13}$$

where the drop in aggregate output in the liquidity crisis is given by

$$\eta^K(\omega_{t-}) = \omega_{t-}\left((1 - G(\bar{u}^H))\right) + (1 - \omega_{t-})\left((1 - G(\bar{u}^L))\right) \tag{A-14}$$

As previously discussed, ω_t plays an important role, because the pre-shock output growth increases in ω_t and the drop in output decreases in ω_t. The law of motion of ω_t is given by

$$d\omega_t = \omega_t(1 - \omega_t)\left[\left(F\left(i^H\right) - \delta^H\right) - \left(F\left(i^L\right) - \delta^L\right)\right]dt + \left(\frac{\omega_{t-}G(\bar{u}^H)}{\omega_{t-}G(\bar{u}^H) + (1 - \omega_{t-})G(\bar{u}^L)} - \omega_{t-}\right)dN_t \tag{A-15}$$

Replacing q^H with investment rate i^H using (2) in equation (A-11), we get

$$(1 - i^H)F'(i^H) + F(i^H) = r + \delta^H + \lambda\left(1 - G\left(q^L\right) + \int_{u=0}^{q^L}\frac{u}{q^L}\,dG(u)\right) \tag{A-16}$$

Let $\bar{q}^L$ and $\bar{q}^H$ be the value of L and H capital in the perfect information equilibrium, respectively. Since the lending schedule for L firms is the same as the perfect information equilibrium, q^L is the same as the perfect information equilibrium, i.e., $q^L = \bar{q}^L$. We also denote $L_1(x)$, $L_2(z)$ the same as Section A.6. As proved by Section A.6, $L_1(x)$ is a decreasing function while $L_2(z)$ is an increasing function. Therefore,

$$i^H = L_1^{(-1)}\left(r + \delta^H + L_2(\frac{1}{q^L})\right) < L_1^{(-1)}\left(r + \delta^H + L_2(\frac{1}{q^H})\right) = \bar{i}^H$$

which implies that with bank market power, H capital investment rate is below the rate under perfect information.

The pre-crisis output growth rate is

$$\mu^K(\omega) = \omega(F(i^H) - \delta^H)) + (1 - \omega)(F(i^L) - \delta^L)$$

$$< \omega(F(\bar{i}^H) - \delta^H)) + (1 - \omega)(F(\bar{i}^L) - \delta^L) = \bar{\mu}^K(\omega)$$

where we use $\bar{\mu}^K$ to denote the output growth rate (because productivity is always 1, output growth is the same as capital growth) in the perfect information equilibrium. Therefore, with bank market power, the pre-crisis output growth is below that in the perfect information equilibrium.

Furthermore, we note that

$$i^L = L_1^{(-1)}\left(r + \delta^L + L_2(\frac{1}{q^L})\right) < L_1^{(-1)}\left(r + \delta^H + L_2(\frac{1}{q^L})\right) = i^H$$

so that L capital still has lower investment than H capital.

A.8 Proof of Proposition 8

When we introduce government liquidity facility, equation (A-16) that determines i^H is changed into

$$(1 - i^H)F'(i^H) + F(i^H) = r + \delta^H + \lambda\left(1 - G\left(\frac{1}{\gamma}\right) + \int_{u=0}^{1/\gamma} \gamma u \, dG\left(u\right)\right) \tag{A-17}$$

where

$$\gamma < \frac{1}{q^L}$$

Following the same definitions of $L_1(x)$ and $L_2(z)$ as Section A.6, we can compare i^H defined in (A-17) with that in (A-12) as

$$L_1^{(-1)}\left(r + \delta^H + L_2(\gamma)\right) > L_1^{(-1)}\left(r + \delta^H + L_2(\frac{1}{q^L})\right)$$

In other words, the H-capital investment rate is higher with government credit facility to curb bank market power. It is immediate that H-capital value is also higher with government credit facility.

Since capital growth rate

$$\mu^K(\omega) = \omega(F(i^H) - \delta^H)) + (1 - \omega)(F(i^L) - \delta^L)$$

is positively related to H-capital investment rate, government credit facility increases capital growth and thus output growth.

A.9 Corporate Liquidity Management and Proof of Proposition 10

A Formal Solution to the Corporate Liquidity Management Problem

In Section 6, we informally prove the solution to the corporate liquidity management problem. In this appendix section, we will provide a rigorous proof. We discuss the solution under $r - r_m > 0$ and solve for $r - r_m = 0$ as a limit case.

Let e_t be the equity of a type j firm, and define the value function as

$$V(e_t) = \max_{m^j_{t-},\, z^j_{t-} \leqslant m^j_{t-},\, dc_t,\, x^j_{t-}} E_t[\int_t^{\infty} e^{-r(s-t)} dc_s]$$

where dc_t is dividend payout, x^j_{t-} is fraction of equity used in holding capital, m^j_{t-} is the amount of liquidity holding *per unit of capital*, and z^j_{t-} is the amount of firm-held liquid asset used in a crisis *per unit of capital*. The above optimization is subject to the following budget constraint

$$\begin{aligned} de_t = &\left((1 - x^j_{t-})e_{t-}r_m + x^j_{t-}e_{t-}\left(\frac{1 - i^j_{t-}}{q^j} + F(i^j_{t-}) - \delta_j\right)\right) dt \\ &- x^j_{t-}e_{t-}E_{t-}[\min\{1, R_G(\tilde{u} - z^j_t) + \frac{z^j_t}{q^j}\}]dN_t - dc_t \end{aligned} \tag{A-18}$$

$$\frac{x^j_{t-}e_{t-}}{q^j}\left(q^j + m^j_{t-}\right) = e_{t-} \tag{A-19}$$

$$e_t \geqslant 0 \tag{A-20}$$

Equation (A-19) implies

$$x^j_{t-} = \frac{q^j}{q^j + m^j_{t-}} \tag{A-21}$$

Replacing all x^j_{t-} with m^j_{t-} in (A-18), we get

$$\begin{aligned} de_t = e_{t-}&\left(\frac{m^j_{t-}}{q^j + m^j_{t-}} r_m + \frac{q^j}{q^j + m^j_{t-}}\left(\frac{1 - i^j_{t-}}{q^j} + F(i^j_{t-}) - \delta_j\right)\right) dt \\ &- e_{t-}\frac{q^j}{q^j + m^j_{t-}} E_{t-}[\min\{1, R_G(\tilde{u} - z^j_t) + \frac{z^j_t}{q^j}\}]dN_t - dc_t \end{aligned}$$

Due to the immediate substitution between consumption and equity, we conjecture that the value function of the firm is $V(e_t) = e_t$. Then the HJB equation becomes

$$V(e_t) = \max_{m^j_{t-},\, z^j_{t-} \leqslant m^j_{t-},\, dc_t,\, x^j_{t-}} E_t[dc_t + E_t[(1 - rdt)V(e_{t+dt})]]$$

$$= \max_{m^j_{t-},\, z^j_{t-} \leqslant m^j_{t-},\, dc_t,\, x^j_{t-}} \left\{ \begin{array}{c} e_{t-}\left(\frac{m^j_{t-}}{q^j + m^j_{t-}} r_m + \frac{q^j}{q^j + m^j_{t-}}\left(\frac{1 - i^j_{t-}}{q^j} + F(i^j_{t-}) - \delta_j\right)\right) dt \\ -\lambda e_{t-} \frac{q^j}{q^j + m^j_{t-}} \int_0^\infty \min\{1, R_G(\tilde{u} - z^j_t) + \frac{z^j_t}{q^j}\} g(u) du dt + V(e_t) - rV(e_t) dt \end{array} \right\}$$

We notice that the consumption dc_t term drops out so it is undetermined. Then the decision problem of liquidity holding at $t-$ is

$$\max_{x^j_{t-} \in (0,1],\, z^j_{t-} \leqslant m^j_{t-}} x^j_{t-}\left(\frac{1 - i^j_{t-}}{q^j} + F(i^j_{t-}) - \delta_j - r_m\right) - \lambda x^j_{t-} \int_0^\infty \min\{1, R_G(u - z^j_t) + \frac{z^j_t}{q^j}\} g(u) du \tag{A-22}$$

with x^j_{t-} and m^j_{t-} constrained by (A-21).

Denote the optimal choice over z^j_t as $z^j_t(u; q^j_t, m^j_{t-})$. Consider the government credit policy

$$R_G(u) = \gamma \cdot \max\{u, 0\}$$

Then we get

$$z^j_t(u; q^j_t, m^j_{t-}) = \min\{m^j_{t-}, u\} \mathbf{1}\{q^j_t \gamma > 1\} \tag{A-23}$$

where we break the tie of $q^j_t \gamma = 1$ by assuming that the firm does not use any liquidity holding at all when it is indifferent.

The HJB equation also implies

$$r = (1 - x^j_{t-}) r_m + x^j_{t-}\left(\frac{1 - i^j_{t-}}{q^j} + F(i^j_{t-}) - \delta_j\right) - \lambda x^j_{t-}\left(\int_0^\infty \min\{1, R_G(u - z^j_t) + \frac{z^j_t}{q^j}\} g(u) du\right) \tag{A-24}$$

where the first term is the opportunity cost of holding liquid assets, while the second term is the benefits of holding liquid assets in terms of improving the left-over value in a liquidity crisis. This equation should be viewed as the equilibrium restriction on the value of capital q^j. From (A-24), we know that

$$\left(\frac{1-i^j_{t-}}{q^j}+F(i^j_{t-})-\delta_j\right)-\lambda\left(\int_0^\infty \min\{1,R_G(u-z^j_t)+\frac{z^j_t}{q^j}\}g(u)du\right)=\frac{r-(1-x^j_{t-})r_m}{x^j_{t-}}\geqslant r \tag{A-25}$$

where the equality is strict when $x^j_{t-}<1$.

Next, we discuss the solutions based on whether $q^j_t\leq 1/\gamma$.

1) When $q^j_t\leq 1/\gamma$, the objective function in (A-22) is simplified as

$$\max_{x^j_{t-}} x^j_{t-}\left(\frac{1-i^j_{t-}}{q^j}+F(i^j_{t-})-\delta_j-\lambda\left(\int_0^\infty \min\{1,R_G(u)\}g(u)du\right)-r_m\right)$$

Since the multiplier on x^j_{t-} is positive from (A-25), we get the optimal solution as

$$x^j_{t-}=1$$

Therefore, the optimal quantity of liquidity holding is $m^j_{t-}=0$.

2) When $q^j_t>1/\gamma$, the objective function in (A-22) is simplified as

$$S(m^j_{t-};q^j_t)=\frac{q^j}{q^j+m^j_{t-}}\left(\begin{array}{c}\frac{1-i^j_{t-}}{q^j}+F(i^j_{t-})-\delta_j-r_m\\ -\lambda\left(\int_0^\infty \min\{1,\gamma(u-\min\{u,m^j_{t-}\})+\frac{\min\{u,m^j_{t-}\}}{q^j}\}g(u)du\right)\end{array}\right) \tag{A-26}$$

Then the bankruptcy threshold can be solved via setting

$$\gamma(\bar{u}-\min\{\bar{u},m^j_{t-}\})+\frac{\min\{\bar{u},m^j_{t-}\}}{q^j}=1 \tag{A-27}$$

which says that the cost of saving one unit of type j capital is exactly one unit of capital. If $u\geq\bar{u}$, then the left-side will be larger than the right side so that exiting becomes a better strategy for the firm.

To explicitly solve for the bankruptcy cutoff, we need to discuss whether the cutoff is above or below m_{t-}^j. Suppose $\overline{u} < m_{t-}^j$, then we get

$$\bar{u} = q_t^j$$

Thus a necessary condition for $\overline{u} < m_{t-}^j$ is that $q_t^j < m_{t-}^j$. On the other hand, if $\overline{u} > m_{t-}^j$, then we get

$$\bar{u} = \frac{1}{\gamma}\left(1 - \frac{1}{q_t^j} m_{t-}^j\right) + m_{t-}^j$$

which requires $q_t^j > m_{t-}^j$. Therefore, $\bar{u} < m_{t-}^j$ is equivalent to $q_t^j > m_{t-}^j$.

As discussed in the main text, $m_{t-} > q_t^j$ is never optimal. On one hand, the extra liquidity holding $m_{t-} - q_t^j$ will not be used, given that for any firm with $u > q_t^j$, it is a negative NPV project to continue the project. On the other, the extra liquidity $m_{t-} - q_t^j$ has a positive carrying cost of forgoing higher capital returns as indicated by (A-25). Therefore, we know that $\bar{u} \geq m_{t-}^j$, which implies the default threshold is

$$\bar{u}(m, q) = \frac{1}{\gamma}\left(1 - \frac{1}{q} m\right) + m \tag{A-28}$$

Thus, (A-26) can be simplified as

$$S(m_{t-}^j; q^j) = \frac{q^j}{q^j + m_{t-}^j}\left(\begin{array}{c} \Delta\mu^j - \dfrac{\lambda}{q^j}\displaystyle\int_0^{m_{t-}^j} ug(u)du - \dfrac{\lambda}{q^j}\displaystyle\int_{m_{t-}^j}^{\bar{u}(m_{t-}^j, q^j)} \left(q^j\gamma(u - m_{t-}^j) + m_{t-}^j\right)g(u)du \\ -\dfrac{\lambda}{q^j}\left(1 - G(\bar{u}(m_{t-}^j, q^j))\right) \end{array}\right)$$

where we introduce the simplifying notation

$$\Delta\mu^j = \frac{1 - i_{t-}^j}{q^j} + F(i_{t-}^j) - \delta_j - r_m$$

which denotes the higher return by capital compared to liquid assets in normal times. We remove the sub- and super-scripts to get

$$S(m; q) = \frac{q}{q + m}\left(\Delta\mu - \frac{\lambda}{q}\int_0^m ug(u)du - \frac{\lambda}{q}\int_m^{\bar{u}(m,q)} (q\gamma(u - m) + m)g(u)du - \frac{\lambda}{q}\left(1 - G(\bar{u}(m, q))\right)\right)$$

The derivative over m is

$$S'(m;q) = -\frac{q}{(q+m)^2}\left(\Delta\mu - \frac{\lambda}{q}\int_0^m ug(u)du - \frac{\lambda}{q}\int_m^{\bar{u}(m,q)}(q\gamma(u-m)+m)g(u)du - \frac{\lambda}{q}(1-G(\bar{u}(m,q)))\right)$$

$$+\lambda\frac{1}{q+m}\left((q\gamma-1)\left(G(\bar{u}(m,q))-G(m)\right)\right)$$

To study the property of this derivative, we note that the restriction of (A-24) can simplify the derivative $S'(m;q)$ into

$$S'(m;q) = \frac{-(r-r_m)+\lambda(q\gamma-1)\left(G(\bar{u}(m,q))-G(m)\right)}{q+m}$$

The boundary value is

$$S'(m=q) = -\frac{1}{q+m}(r-r_m) < 0$$

$$S'(m=0) = \frac{1}{q+m}\left(-(r-r_m)+\lambda(q\gamma-1)G(\frac{1}{\gamma})\right)$$

We also note that

$$\frac{1}{\gamma}+(1-\frac{1}{\gamma q})m = m+\frac{1}{\gamma}(1-\frac{m}{q}) \leq m \text{ for } m \leq q$$

and the derivative over the numerator of $S'(m;q)$ is

$$g(\frac{1}{\gamma}+(1-\frac{1}{\gamma q})m)(1-\frac{1}{\gamma q}) - g(m) < 0$$

Therefore, there exists an $\bar{m}(q)$ such that for any $m \leq \bar{m}(q)$, $S'(m;q) > 0$, but for any $m > \bar{m}(q)$, $S'(m;q) < 0$. This implies a unique solution. When the liquidity spread is higher than the marginal gain in liquidity holding,

$$r - r_m \geq \lambda\left(q_t^j\gamma - 1\right)G(\frac{1}{\gamma}) \tag{A-29}$$

we have $\Leftrightarrow S'(0) < 0$ so the optimal liquidity holding is $m_{t-}^j = 0$.

When the liquidity spread is smaller than the marginal gain in liquidity holding,

$$0 < r - r_m < \lambda\left(q_t^j\gamma - 1\right)G(\frac{1}{\gamma}), \tag{A-30}$$

we have $S'(0) > 0$ so the optimal liquidity holding is the **unique** solution to the following equation:

$$r - r_m = \lambda \left(q_t^j \gamma - 1\right) \left(G(\frac{1}{\gamma} + (1 - \frac{1}{\gamma q_t^j})m) - G(m)\right) \tag{A-31}$$

In both cases, $m_{t-}^j < q_t^j$, and m_{t-}^j **decreases** with the liquidity spread $r - r_m$.

Combining case 1) and case 2), we conclude that $m_{t-}^j = 0$ if (A-29) holds, while $m_{t-}^j > 0$ and is the unique solution to (A-31) when (A-30) holds. Finally, as $r - r_m \to 0$, $m_{t-}^j \to q_t^j$. At the limit, firms will hold enough liquidity so that all efficient firms with $u \le q_t^j$ are saved during a crisis.

Proof of Proposition 10

The proof of Proposition 10 is an immediate application of the above discussions.

For $j = L$, we have $q^L \gamma < 1$, which falls in the category of (A-29). Therefore, $m^L = 0$ and L firms do not hold any liquidity.

For $j = H$, we have $q^L \gamma > 1$, which falls the category of (A-31) if $r - r_m > 0$. Therefore, $m^H > 0$ and decreases with the liquidity spread $r - r_m$. However, as long as $r - r_m > 0$, we get $m^H < q^H$ so that all H firms with $u \in [m^H, q^H]$ still need government liquidity support. Only when $r - r_m = 0$, H firms hold enough liquidity $m^H \ge q^H$ so that H firms do not need any government liquidity support.

Multiplicity of Equilibrium

For L capital, the equilibrium always feature zero liquidity holding. However, for H capital, it is possible that two equilibriums exist: (1) No liquidity holding and low capital price that sustain the no liquidity holding equilibrium; (2) Positive liquidity holding and positive capital price to sustain the liquidity holding equilibrium. The reason for this possibility is that when firms hold more liquidity, the endogenous capital value is also higher, which makes liquidity holding more valuable. Such complementarity may generate multiple equilibriums.

How to numerically check this? Do the following:

- First, check if
$$r - r_m \ge \lambda(q^H \gamma - 1) G(\frac{1}{\gamma})$$
is satisfied with q^H the price without corporate liquidity management. If so, we know that there is an equilibrium of zero corporate liquidity holding.

- Second, conjecture there is another $\tilde{q}^H$ so that

$$r - r_m < \lambda(\tilde{q}^H \gamma - 1) G(\frac{1}{\gamma}) \tag{A-32}$$

Then we can solve $\tilde{q}^H$ and $\tilde{m}^H$ together from equations (2), (A-21), (A-24), and (A-31). If the solution still satisfies (A-32), then we have found another equilibrium.

A.10 Interbank Market

A representative bank's profits are given by

$$\pi_t^B dt \equiv \max_{\{a_t^B, m_t^B, d_t^B\}} \mathbb{E}_t \left[dr_t^K \right] a_t^B + i_M m_t^B - i_t^D d_t^B \tag{A-33}$$

$$+ \left[\frac{1}{2} \chi_S \left(m_t^B + \varepsilon^D d_t^B ; \theta_t, i_E - i_M \right) + \frac{1}{2} \chi_S \left(m_t^B - \varepsilon^D d_t^B ; \theta_t, i_E - i_M \right) \right], \tag{A-34}$$

where

$$\chi_S \left(z; \theta_t, i_E - i_M \right) = \begin{cases} \chi^+ \left(\theta_t, i_E - i_M \right) z & \text{if } z > 0 \\ \chi^- \left(\theta_t, i_E - i_M \right) z & \text{if } z \leq 0 \end{cases}, \tag{A-35}$$

and the coefficient functions $\chi^+ (\cdot)$ and $\chi^- (\cdot)$ are given by

$$\chi^+ \left(\theta_t, i_E - i_M \right) = \left(i_E - i_M \right) \frac{\left(\theta_t \sqrt{\theta_t + (1 - \theta_t) \exp(\phi)} - \theta_t \right)}{(1 - \theta_t) \exp(\phi)}, \tag{A-36}$$

and

$$\chi^- \left(\theta_t, i_E - i_M \right) = \left(i_E - i_M \right) \frac{\left(\sqrt{\theta_t + (1 - \theta_t) \exp(\phi)} - \theta_t \right)}{(1 - \theta_t) \exp(\phi)}. \tag{A-37}$$

The parameter ϕ captures the trading efficiency of the interbank market, and θ_t, the interbank market tightness, is a function of $\frac{M_t^B}{D_t^B}$, the ratio of aggregate reserve holdings, M_t^B, to the aggregate deposit stock, D_t^B:

$$\theta_t = \theta \left(\frac{M_t^B}{D_t^B} \right) = \max \left\{ \frac{\varepsilon^D - \frac{M_t^B}{D_t^B}}{\varepsilon^D + \frac{M_t^B}{D_t^B}}, 0 \right\}. \tag{A-38}$$

Intuitively, the numerator of the first term inside the $\max$ operator is the liquidity shortage (scaled by aggregate deposit stock), while the denominator is the liquidity surplus.

In the main text, the deposit rate is given by

$$i_t^D = i_M + \chi_D\left(\frac{M_t^B}{D_t^B}, i_E - i_M\right). \tag{A-39}$$

(Bigio et al., 2019) provide a model of deposit payment flow uncertainty, interbank credit market, discount window lending, and reserve holdings. The function $\chi_D(\cdot)$ depends on the functioning of interbank market, the payment shocks, and the policy variables, such as the discount window rate, i_E, and the interest on reserve, i_M. It is given by,

$$\chi_D\left(\frac{M_t^B}{D_t^B}, i_E - i_M\right) = \frac{1}{2}\left(1+\varepsilon^D\right)\chi^+\left(\theta_t, i_E - i_M\right) + \frac{1}{2}\left(1-\varepsilon^D\right)\chi^-\left(\theta_t, i_E - i_M\right). \tag{A-40}$$

In the main text, the banks' required rate of return on capital holdings is given by

$$\mathbb{E}_t\left[dr_t^K\right] = i_M + \chi_K\left(\frac{M_t^B}{D_t^B}, i_E - i_M\right). \tag{A-41}$$

Similar to the function $\chi_D(\cdot)$ in the deposit rate, the function $\chi_K(\cdot)$ depends on the functioning of interbank market, the payment shocks, the discount window rate, i_E, and the interest on reserve, i_M. It is given by,

$$\chi_K\left(\frac{M_t^B}{D_t^B}, i_E - i_M\right) = \frac{1}{2}\chi^+\left(\theta_t, i_E - i_M\right) + \frac{1}{2}\chi^-\left(\theta_t, i_E - i_M\right). \tag{A-42}$$

Deposit Demand

Households have a demand for deposits as means of payment. The related literature takes a money-in-utility approach (Sidrauski, 1967). Households' utility from deposits is specified as a power utility function separable from the consumption utility (Poterba and Rotemberg, 1986; Nagel, 2016; Begenau and Landvoigt, 2018) and deposits are normalized by measures of income (Begenau, 2019; Krishnamurthy and Vissing-Jorgensen, 2015). Following the literature, a representative household's utility is given by

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt}\left(dc_t^W + \xi_0\frac{(d_t^W)^{1-\xi_1}(y_t^W)^{\xi_1}}{1-\xi_1}\right)\right], \tag{A-43}$$

where c_t^H is the cumulative consumption, d_t^W is the deposit holdings, and y_t^W is the wage. The "Cobb-Douglas" component over labor income and deposit holding is a convenient modeling technique that allows a linear aggregation. Because we have already used superscript "H" for high type capital, we use "W" for households (workers).

Consider a unit mass of households, $\mathbb{W} = [0, 1]$. It is assumed that the aggregate labor income is proportional to the output of tangible capital,

$$\int_{i \in \mathbb{W}} y_t^W(i)\, di = \zeta \left(K_t^H + K_t^L\right) . \tag{A-44}$$

This follows the models of output attribution to labor, physical capital, and intangibles (Lucas, 1978; Atkeson and Kehoe, 2005).

Let i_t^D denote the interest rate on deposits. Given the households' discount rate r, the optimality condition on d_t^W is

$$\xi_0 \left(\frac{d_t^W}{y_t^W}\right)^{-\xi_1} = r - i_t^D . \tag{A-45}$$

Rearranging the equation and aggregating across households, we obtain

$$D_t^W = \int_{i \in \mathbb{W}} d_t^W(i)\, di = \left(\int_{i \in \mathbb{W}} y_t^W(i)\, di\right) \left(\frac{r - i_t^D}{\xi_0}\right)^{-\frac{1}{\xi_1}} = \zeta \left(K_t^H + K_t^L\right) \left(\frac{r - i_t^D}{\xi_0}\right)^{-\frac{1}{\xi_1}} , \tag{A-46}$$

where the last step utilizes (A-44).

Boundary Cases

So far we have considered the interior case where banks hold positive amount of reserves and reserves are not too abundant to insure all liquidity shocks and make interbank market obsolete. In the following, we discuss the two corner cases when the above interior restrictions do not hold.

Case 1: $i^E - i^M$ is too small so that it is always optimal to borrow from the central bank instead of holding reserves. In this case, the numerical algorithm will tell us $M/D < 0$, which we set as $M/D = 0$. Then how shall we solve for the model?

$$\frac{A_t^B}{K_t} = \frac{D_t^B}{K_t} = \zeta \left(\frac{r - i_t^D}{\xi_0}\right)^{-\frac{1}{\xi_1}}$$

Note that in this case, the first-order condition for deposits is

$$i_t^D = r + \frac{1}{2}\varepsilon^D \left(\chi^+ - \chi^-\right)$$

which comes from the optimization

$$\max_{d_t^B} r d_t^B - i_t^D d_t^B + \left[\frac{1}{2}\chi_S \left(\varepsilon^D d_t^B; \theta_t, i_E - i_M\right) + \frac{1}{2}\chi_S \left(-\varepsilon^D d_t^B; \theta_t, i_E - i_M\right)\right],$$

Case 2: $i^E - i^M$ is too large so that all liquidity shocks are insured and $\theta_t = 1$.

In this case, the discount window is obsolete. Banks never use the discount window any more. Then banks will pick the minimum reserve holding, i.e., $M = \varepsilon^D D$. (note: it is important that $r > i^M$ since otherwise the model blows up) Then the deposit rate FOC is simply

$$i_t^D = (1 - \varepsilon^D) r + \varepsilon^D i_M$$

which comes from the optimization

$$\max_{d_t^B} r(1 - \varepsilon^D) d_t^B + i_M \varepsilon^D d_t^B - i_t^D d_t^B$$

Condition for the Existence of Equilibrium

A particular condition for the existence of an equilibrium is $r > i^D$ so that deposit demand is finite. For any $M > 0$, we have

$$r = i_M + \chi_K \left(\frac{M_t^B}{D_t^B}, i_E - i_M\right).$$

$$i_t^D = i_M + \chi_D \left(\frac{M_t^B}{D_t^B}, i_E - i_M\right).$$

Thus

$$r - i_t^D = \chi_K \left(\frac{M_t^B}{D_t^B}, i_E - i_M\right) - \chi_D \left(\frac{M_t^B}{D_t^B}, i_E - i_M\right)$$

$$r - i_t^D = \frac{1}{2}\varepsilon^D \left(\chi^- - \chi^+\right)$$

We note that $\theta < 1$, so that always

$$\chi^+ < \chi^-$$

Furthermore, at $\theta = 1$, the limit also implies $\chi^+ < \chi^-$. Consequently, any $M > 0$ is consistent with the requirement that $r > i^D$.