

Managing Stablecoins: Optimal Strategies, Regulation, and Transaction Data as Productive Capital

Ye Li* Simon Mayer†

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Abstract

In a dynamic model of stablecoins, we show that even with over-collateralization, a pledge of one-to-one convertibility to a reference currency is not sustainable in a stochastic environment. The distribution of states is bimodal – a fixed exchange rate can persist, but debasement happens with a positive probability and recovery is slow. When negative shocks drain the reserves that back stablecoins, debasement allows the issuer to share risk with users. Collateral requirements cannot eliminate debasement, because risk sharing is ex-post efficient under any threat of costly liquidation, whether it is due to reserve depletion or violation of regulation. Optimal stablecoin management requires a combination of strategies commonly observed in practice, such as open market operations, transaction fees or subsidies, re-pegging, and issuance and repurchase of “secondary units” that function as stablecoin issuers’ equity. The implementation varies with user-network effects and is guided by Tobin’s q of transaction data as productive capital.

*The Ohio State University Fisher College of Business. E-mail: li.8935@osu.edu

†Erasmus University Rotterdam. E-mail: mayer@ese.eur.nl.

1 Introduction

More than a decade ago, Bitcoin heralded a new era of digital payments. Cryptocurrencies challenge the bank-centric payment systems by offering fast and round-the-clock settlement, anonymity, low-cost international remittances, mobile payment for unbanked population, and smart-contract-based programmable money (Duffie, 2019). However, the substantial volatility exhibited by first-generation cryptocurrencies limits their utility as a means of payment (Stulz, 2019). Stablecoins aim to maintain a stable price against a reference currency, or a basket of currencies by pledging to hold a reserve of fiat currencies or other assets against which stablecoin holdings can be redeemed.¹

This paper provides the first dynamic model of stablecoins that offers guidance to practitioners and lends itself to an evaluation of regulatory proposals. In our model, optimal stablecoin management features a rich set of strategies that are commonly observed in practice (Bullmann, Klemm, and Pinna, 2019), such as over-collateralization, dynamic reserve management and open market operations, transaction fees or subsidies, targeted price band, re-pegging, and issuance and repurchase of “secondary units” that function as equity shares of stablecoin platforms.

Our model addresses the fundamental questions on the credibility and sustainability of a fixed exchange rate. Bank deposits mediate payments in the traditional payment system with one-to-one convertibility to a fiat currency safeguarded by deposit insurance and regulatory supervision. For stablecoins, the issuer may find it desirable to fix the exchange rate through open market operations but does not face legal consequences for debasement. In the presence of shocks to reserves, such as fluctuations of collateral value and operational risk (e.g., cyber-attacks), commitment to redemption at par is only credible when reserves are sufficiently above the redemption value of outstanding stablecoins. Below this critical threshold, the issuer optimally debases the stablecoin.

In the debasement region, the redemption value comoves with reserves, effectively allowing the issuer to share risk with users and thereby to avoid liquidation. The issuer can also avoid liquidation by raising funds through the issuance of equity shares that are backed by future transaction fees charged to users (called “secondary units” among practitioners). However, as long as the financing costs exist, debasement happens before recapitalization. Moreover, we show that equity issuance must be accompanied by a jump in the stablecoin supply and a downward re-pegging; otherwise, arbitrage opportunities exist and the stablecoin issuer leaves money on the table.

The system exhibits a bimodal distribution of states. In states of high reserves, the stablecoin

¹An alternative is to use algorithmic supply rules to stabilize price. Success has been limited in this area.

issuer honors the fixed exchange rate, so stablecoin demand is strong and transaction volume is high. Through open market operations and transaction fees, the issuer collects revenues that further grow its reserves. The level of reserves is capped by an endogenous upper bound, beyond which the stablecoin issuer pays out dividends, or, equivalently, repurchase equity shares. In states of low reserves, the issuer has to off-load risk to users. The depressed stablecoin demand and transaction volume imply low revenues and slow accumulation of reserves. Therefore, a fixed exchange rate can last for a long time without any hint of instability, but once debasement happens, recovery is slow.

Stablecoins became the subject of heated debate after the technology giant Facebook and its partners announced their own stablecoin, Libra (now “Diem”), in June 2019.² Leveraging on their existing customer networks, global technology or financial firms are able to rapidly scale the reach of their stablecoins.³ In our model, stronger network effects make a fixed exchange rate more sustainable and allow the stablecoin to recover more quickly from debasement. Stablecoin initiatives sponsored by companies with global customer networks attract attention from regulators for not only its potential of wide adoption but also concerns over monopoly power. In our model, stronger network effects allow the stablecoin issuer to earn more transaction fees and revenues from open market operations. However, because individual users do not internalize the positive network externalities, the stablecoin issuer has incentives to stimulate users’ transactions by lowering fees and stabilizing the redemption value of the stablecoin. As a result, the split of welfare between the stablecoin issuer and users is rather insensitive to the degree degree of network effects.

The enormous transaction data brought by stablecoin systems offers a strong incentive for digital platforms to venture into payment services. Data enables new revenue sources, for example, advertisement targeting. In an extension, we model data as a productive asset that accumulates through users’ transactions. A stablecoin platform faces a trade-off between reserve management and data acquisition. To preserve and grow reserves, the platform relies on transaction fees and, possibly, open market operations that off-load shocks to users (i.e., debasement). To acquire data, the platform needs to raise transaction volume by reducing fees (or even offering subsidies to users) and maintaining a stable redemption value of stablecoins. The optimal strategies then depend on the ratio of marginal value of reserves to marginal value of data (the “data q”).

Finally, we evaluate two types of stablecoin regulations. The first is a standard capital require-

²The announcement triggered a globally-coordinated response under the umbrella of the G7. From then on, the G20, the Financial Stability Board (FSB), and central banks around the world have also embarked on efforts to address the potential risks while harnessing the potential of technological innovation.

³Another example is JPM Coin, a blockchain-based digital coin for fast payment settlement that is being developed by JP Morgan Chase and was announced in February 2019.

ment that stipulates the minimal degree of over-collateralization (equity). The capital requirement fails to eliminate debasement. As long as the threat of liquidation (or equity issuance costs) exists, whether it is due to reserve depletion or the violation of regulation, it is optimal for the stablecoin issuer and users to share risk through debasement. Therefore, the second type of regulation that forces a fixed exchange rate only hurts welfare by destroying the economic surplus from risk sharing.

Next, we provide more details on the model setup and mechanisms. The model is built to be technology-neutral so that it applies to stablecoins issued by central entities or on distributed ledgers.⁴ In a continuous-time economy, a digital platform issues stablecoins (“tokens”). Users derive a flow utility from token holdings, which captures the transactional benefits, and network effect is modelled by embedding the aggregate holdings in individuals’ utility (Cong, Li, and Wang, 2020).⁵ Users can redeem token holdings for numeraire goods (“dollars”). Redemption value can be continuously adjusted by the platform. The lack of commitment to a fixed exchange rate requires redemption value at any time to be optimal for the platform. Users are averse to the fluctuation of redemption value, so a volatile token value dampens token demand and transaction volume.

On the platform’s balance sheet, the liability side has tokens and equity. On the asset side, it holds dollar reserves that earn a constant interest rate and load on Brownian shocks. The shocks capture operational risk and unexpected fluctuation of reserve value. As standard in models of dynamic liquidity management (Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Hugonnier, Malamud, and Morellec, 2015), shareholders’ discount rate is above the interest rate of reserves, so the platform pays out reserves to shareholders when it has accumulated a sufficient amount as risk buffer. Beyond the interests on reserves, the platform earn fees charged to users and token-issuance proceeds. We allow open market operations in both directions, so the platform can also reduce token supply by selling dollars and buying back (and burn) tokens.

Through open market operations, the platform can implement any process of dollar price of token and preclude arbitrage between the secondary market and token redemption/issuance. Users are free to deposit dollars with the platform in exchange for tokens or redeem tokens for dollars under rational expectation of token price dynamics. Besides the token price process, the platform can also choose any processes of fees charged to users and payouts to shareholders.

For the platform’s dynamic optimization, the amount of excess reserves (equity) is the state

⁴At the current stage of stablecoin developments, policy makers take a technology-neutral approach that emphasizes economic insights over technological aspects of implementation ECB Crypto-Assets Task Force (2019).

⁵This money-in-utility approach follows the macroeconomics literature (Ljungqvist and Sargent, 2004). The modelling of network effect is in the traditional of social interaction (Glaeser, Sacerdote, and Scheinkman, 1996).

variable. The recursive formulation through Hamilton-Jacobi-Bellman (HJB) equation significantly simplifies the problem. First, the optimality condition on payouts to shareholders implies an endogenous upper bound on the state variable (the payout boundary). The natural lower bound is zero, the bankruptcy threshold, but is never reached because the platform can debase its token liabilities and will optimally do so to avoid costly and irreversible liquidation.

Then the choice of token price process boils down to choosing instantaneous drift and diffusion. Beyond the transactional benefits, users only care about the *net* appreciation or depreciation, i.e., drift minus fees that are proportional to token holdings, when choosing token demand. Therefore, the impact of drift and fees cannot be separately identified. The optimal set of fees, token-price drift and diffusion can thus be implemented through directly setting the aggregate token demand and choice of diffusion. Recovering the implied process of token redemption value can be formulated as a differential equation problem with the optimal token demand and token-price diffusion as inputs. The recovery delivers the optimal drift, which then implies optimal fees given the token demand.

In spite of positive equity (over-collateralization), the stablecoin issuer cannot always credibly promise redemption at par. To avoid costly liquidation, the platform opts for debasement whenever equity falls below a threshold. Debasement triggers a vicious cycle as the depressed token demand leads to a reduction in fee revenues, which causes a slow recovery of equity and persistent debasement. However, debasement is a necessary evil and a valuable option, as it allows the platform to share risk with users. When negative shocks decrease equity, debasement causes token liabilities to shrink. Above the debasement threshold, the platform credibly institutes token redemption at par. Then a strong token demand allows the platform to collect revenues to grow equity, which further strengthens the one-to-one convertibility to dollar. The virtuous cycle implies persistent expansion of platform equity until it reaches the payout boundary. The stationary distribution of platform equity is thus bimodal with two peaks near zero and the payout boundary, respectively.

Next, we allow the platform to raise equity. Under a fixed cost of equity issuance, the platform first resorts to debasement when equity falls below the threshold and only issues equity when equity falls to zero and when equity issuance leads to a higher shareholders' value than further debasement does. Once the fixed cost is paid, the platform raises equity all the way up to the payout boundary, where the marginal value of equity eventually falls to one. The jump in platform equity implies an immediate restoration of fixed exchange rate and, accordingly, a jump in the aggregate token demand. To preclude a predictable jump in token price (a secondary-market arbitrage opportunity), the platform must expand token supply at the payout boundary with the proceeds distributed to

shareholders, an operation akin to issuing debts for share repurchase. Then the exchange rate is re-pegged. Therefore, the exchange rate starts at one, and every time after recapitalization (which follows debasement), the exchange rate is re-pegged downward to the pre-issuance level.

We further extend our model to incorporate data as a productive asset for the platform. Data improves the quality of the platform and thereby increases the users' flow utility from token holdings (the transactional benefits). Data accumulates through transactions. Under a constant money velocity, i.e., a constant ratio of transaction volume to users' token holdings, a feedback loop emerges — transactions generate more data, which improves the platform and leads to a stronger token demand and even more transactions. As a result, data accumulates exponentially over time. The platform has to balance between acquiring data and preserving reserves. The former requires lower fees and a more stable token while the latter calls for higher fees and risk-sharing with users through debasement. A key result is that the amount of reserves is no longer the key state variable driving the platform's decisions. The state variable is now the ratio of reserves to data stock. Data enters into the platform's decisions through a sufficient statistic, the data q , which is the marginal contribution of data to shareholders' value in analogy to Tobin's q of productive capital.

The model allows us to conduct several experiments that shed light on the heated debates surrounding stablecoins. Data has become a major asset for payment platforms. An increase of data productivity captures the revolutionary progress in data science that enables the processing and analysis of enormous transaction data.⁶ In response, the stablecoin issuer becomes more aggressive in stimulating transactions for data acquisition, especially by lowering fees or even offering subsidies to users, at the expense of preserving reserves. A larger transaction volume also amplifies operational risk, i.e., the shock loading of reserves. As a result, debasement is more likely to happen. Therefore, a paradox exists — stablecoins built primarily for the acquisition and utilization of transaction data can become increasingly unstable precisely when data becomes more valuable.

To understand the advantages of well-established digital networks in the stablecoin space, we compare platforms with different degrees of network effects. A stronger network effect is indeed associated with stabler tokens. The frequency of debasement declines because a stronger network effect allows the platform to accumulate fee revenues faster when equity runs out and, through a higher franchise (continuation) value, incentivizes the platform to build up a large equity position. As previously discussed, two counteracting forces limit the share of economic surplus seized by

⁶Alternative payment-service providers also benefit from regulatory initiatives that facilitate data sharing. A new European Union directive, PSD2, requires banks to provide non-bank service providers with data that would allow those providers to offer payment and other services to the banks' customers (Duffie, 2019).

the platform. Under a stronger network effect, the platform can extract more rents from its users through fees or risk sharing, but it is also more eager to stimulate token demand by lowering fees and stabilizing tokens given that individual users do not internalize the positive network externalities.

As in our model, stablecoins are typically over-collateralized in practice (Bullmann, Klemm, and Pinna, 2019).⁷ The issuer optimally maintains a risk buffer so that it does not need to immediately debase tokens following negative shocks. Even under voluntary over-collateralization, capital requirement can still add value. To avoid violating the regulation, the issuer accumulates more reserves that earn an interest rate below shareholders' discount rate, so shareholders' value declines. However, users' welfare increases because, even though debasement still happens, its frequency declines. The increase of users' welfare dominates the decrease of platform shareholders' value as the capital requirement rises up until an optimum level, which maximizes the total welfare. The optimal capital requirement increases in the strength of network effects, which amplifies the positive response of users to the regulation. In contrast, when data becomes more productive, the optimal capital requirement declines. By favoring reserve preservation over data acquisition, capital requirement stems the exponential growth of data that drives the long-run trajectory of welfare.

The current policy debate surrounding stablecoins largely adopts the “same business, same risk, same rules” principle (Financial Stability Board, 2020). Therefore, the conceptual framework aims to adopt banking regulations and analyzes stablecoin-enabled payment platforms from the perspective of systemically important financial institutions. Our model, albeit partial-equilibrium in nature, reveals unique features that distinguish stablecoins from traditional securities. Due to the debasement option, stablecoins share similarities with contingent convertible bonds (CoCos) that automatically share risk between equity owners and debt (or stablecoin) holders. Unlike CoCos, stablecoins do not pre-specify events that trigger debasement and leave the option to the issuer.

Policy initiatives that aim to force a fixed exchange rate and regulate stablecoins as deposits ignore the ex-post efficiency of debasement option. Admittedly, our model omits several elements and thus can underestimate the value of a perfectly stable token. For example, debasement invites speculation that in turn amplifies price fluctuation and triggers a vicious cycle (Mayer, 2020). To guard against speculation, stablecoin issuers often require users to post collateral themselves as the first line of defense before the issuers taps into their own reserves. Future research may explore the efficacy of double-deck scheme in stabilizing exchange rates in the presence speculative trading.

⁷The concern over bank run and under-collateralization is partly justified by the inadequate auditing of reserves of certain stablecoin initiatives (Calle and Zalles, 2019).

2 A Model of Stablecoins

Consider a continuous-time economy where a continuum of agents (“users”) of unit measure conduct peer-to-peer transactions on a digital platform. The platform facilitates transactions by introducing a local currency (“token”). The generic consumption goods (“dollars”) are the numeraire in this economy. The platform sets the exchange rate between tokens and dollars. Let P_t denote the token price in units of dollars (i.e., the exchange rate between tokens and dollars). At time t , users can redeem their token holdings for dollars or buy more tokens from the platform at the dollar price P_t . By no arbitrage, users also trade tokens among themselves at the dollar price P_t . The platform manages a stock of dollar reserves to meet potential token redemption and/or to manage token supply. Next, we first introduce users and then set up the platform’s problem.

Users. We use $u_{i,t}$ to denote the numeraire (dollar) value of user i ’s holdings of tokens, so user i holds $k_{i,t} = u_{i,t}/P_t$ units of tokens. The aggregate value of token holdings is $N_t \equiv \int_{i \in [0,1]} u_{i,t} dt$.

A representative user i derives the following utility from token holdings

$$\frac{1}{\beta} N_t^\alpha u_{i,t}^\beta A^{(1-\alpha-\beta)} dt, \quad (1)$$

where $\alpha, \beta \in (0, 1)$ with $\alpha + \beta < 1$ and $A > 0$. We model the utility from holding means of payment following the classic models of monetary economics (e.g., [Baumol, 1952](#); [Tobin, 1956](#); [Feenstra, 1986](#); [Freeman and Kydland, 2000](#)) and related empirical studies (e.g., [Poterba and Rotemberg, 1986](#); [Lucas and Nicolini, 2015](#); [Nagel, 2016](#)). In this literature, agents derive utility from the real value of holdings, i.e., $u_{i,t}$.⁸ Following [Rochet and Tirole \(2003\)](#), we introduce network effect via N_t^α . As in [Cong, Li, and Wang \(2020\)](#), it captures the fact that when tokens are more widely used as means of payment, each individual user’s utility from using tokens is higher.⁹ The quality of payment system is captured by parameter A which we will endogenize in Section 5.

User i pays a proportional fee on her token holdings, $u_{i,t} f_t dt$, where f_t is set by the platform. In practice, fees are often charged on transactions, in that f_t can be interpreted as the transaction fee per dollar transaction. Note that as long as the money (token) velocity is constant within a small time interval (dt), transaction volume is proportional to token holdings. There exists a technical

⁸We refer readers to the textbook treatments (e.g., [Galí, 2015](#); [Ljungqvist and Sargent, 2004](#); [Walsh, 2003](#)). For the nominal value of means of payment (i.e., $k_{i,t}$) to affect agents’ decisions, additional frictions, such as nominal illusion (e.g., [Shafir et al., 1997](#)) or sticky prices (e.g., [Christiano et al., 2005](#)), have to be introduced.

⁹For instance, when there are more people use tokens, it becomes easier to find a transaction counterparty that accepts tokens, so token holders expect more token usage of means of payment.

upper bound on the volume of transactions that the platform can handle per unit of time. Without loss of generality, we model the bound as follows

$$N_t \leq \bar{N}. \quad (2)$$

In sum, we define tokens' transaction utility from an ex ante perspective and (1) can be viewed as the utility from *expected* transaction needs in dt . We do not model the ex post circulation of tokens following the aforementioned literature on money-in-utility and cash-in-advance constraint.

The Platform. Let S_t denote the total units of tokens outstanding. The token market clearing condition is given by $S_t = \int_{i \in [0,1]} k_{i,t} dt$, or equivalently, in the numeraire (dollar) value:

$$N_t = S_t P_t. \quad (3)$$

The platform decides on the fees and controls the dollar price of tokens, P_t , by adjusting the token supply. This is akin to central banks using open market operations to intervene in the foreign exchange markets (e.g., [Calvo and Reinhart, 2002](#)). When the platform issues more tokens ($dS_t > 0$), it collects dollar revenues as users buy tokens with dollars. When the platform retires tokens ($dS_t < 0$), it loses dollars to users. Stablecoin platforms often claim a fixed exchange rate. However, we will show that optimal exchange rate depends on the platform's reserves.

Let M_t denote the platform's reserves (dollar holdings), which has a law of motion

$$dM_t = rM_t dt + P_t dS_t + N_t f_t dt + N_t \sigma dZ_t - dDiv_t. \quad (4)$$

The first term is the interests earned on the reserve balance, and r is the constant interest rate. The second term is the revenues (losses) from issuing (buying back) tokens at price P_t (which is the exchange rate between dollars and tokens). The third term is the fee revenues. The fourth term deserves more attention. Z_t is a standard Brownian motion, and its increment, dZ_t , captures the shocks to the net revenues, which can stem from operating expenses, risks involved in liquidity management, and activities beyond fees and token management.¹⁰ This shock is the only source

¹⁰Examples include the revenues and costs associated with users' advertisement on the platform and loans to users, which are often enabled by the platform's possession of transaction data as will be discussed in Section 5. The risk of reserve portfolio may also arise from the platform holding different currencies and the fluctuation of these currencies' dollar exchange rates. Providers of global stablecoins (GSC) typically accept deposits in different currencies, and hold a portfolio of these currencies as backing (e.g., [Libra Association, 2020](#)).

of uncertainty. Let Div_t denote the *cumulative* dividend process. The platform's reserves decrease when the platform pays its owners dividends, $dDiv_t$, which is non-negative under limited liabilities.

The platform maximizes the expected discounted value of dividend payouts to its owners:

$$V_0 \equiv \max_{\{f_t, dS_t, dDiv_t\}} \mathbb{E} \left[\int_0^\infty e^{-\rho t} dDiv_t \right] \quad \text{subject to (4) and } dDiv_t \geq 0. \quad (5)$$

We assume that the platform's shareholders are impatient relative to other investors, $\rho > r$.¹¹

Stability Preference. In equilibrium, the dollar price of token has a law of motion

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t, \quad (6)$$

which the atomic users take as given. In the next section, we will show how μ_t^P and σ_t^P depend on the platform's strategy. Let $R_{i,t}$ denote user i 's (undiscounted) *cumulative* payoff from platform activities. The instantaneous payoff depends on user i 's choice of $u_{i,t}$ and is given by

$$dR_{it} \equiv \frac{1}{\beta} N_t^\alpha u_{it}^\beta A^{(1-\alpha-\beta)} dt + u_{it} \left(\frac{dP_t}{P_t} - r dt - f_t dt \right), \quad (7)$$

where the first term is the flow utility (1) and the second term includes the return from token price change net off the forgone interests and fees.

A representative user i chooses $u_{i,t} \geq 0$ to maximize

$$\max_{u_{i,t} \geq 0} \mathbb{E}_t [dR_{it}] - \eta |\sigma_t^P| u_{i,t} dt = \max_{u_{i,t}} \frac{1}{\beta} N_t^\alpha u_{it}^\beta A^{(1-\alpha-\beta)} dt + u_{it} \left(\mu_t^P - \eta |\sigma_t^P| - r - f_t \right) dt. \quad (8)$$

First note that the classic Inada condition rules out $u_{i,t} \geq 0$. The user's aversion to token volatility is captured by the parameter η (> 0), and is defined on the absolute value of σ_t^P to capture the fact that users are averse to token price fluctuation no matter whether the price moves with ($\sigma_t^P > 0$) or against ($\sigma_t^P < 0$) the platform's dollar revenue shock dZ_t . The disutility from token volatility can be motivated by risk-averse preference or users' aversion to exchange-rate shocks that cause losses of net worth when assets and liabilities are denominated in different currencies (tokens and dollars) (Doepke and Schneider, 2017; Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller, 2020).

¹¹This impatience could be preference based or could arise indirectly because shareholders have other attractive investment opportunities. From a modeling perspective, impatience motivates the platform to pay out because, otherwise, the expected return on M_t is greater than r (due to revenues from the fees and token issuance), which then implies that the platform never pays out dividends.

Liquidation. The platform liquidates if it can no longer guarantee the fulfillment of users’ potential redemption en masse, i.e., $M_t < S_t P_t$.¹² In liquidation, token holders receive the dollar reserves pro rata and the platform owners obtain zero value. This criterion can be a legally binding commitment. It can also be motivated by the classic run mechanism (Diamond and Dybvig, 1983). As users redeem tokens on a first-come, first-served basis, a coordination failure can happen if $M_t < S_t P_t$ – an individual user i optimally chooses to redeem all of her token holdings when the rest of users do so. A run on stablecoins is a prominent concern of policy makers (Brainard, 2019; G7 Working Group on Stablecoins, 2019; ECB Crypto-Assets Task Force, 2019). A run does not necessarily happen when M_t falls right below $S_t P_t$. We may allow reserves to run down below $S_t P_t$ to a certain extent. As noted by Calle and Zalles (2019) and Duffie (2019), there has not been standard practice of reserve disclosure and auditing. Stablecoin users are likely to receive noisy signals about M_t . In a global-game framework, the run threshold can then be uniquely determined (Morris and Shin, 1998; Goldstein and Pauzner, 2005). As long as there exists a threshold of reserves below which liquidation happens, our analysis below carries through.

3 Equilibrium

In this section, we characterize the analytical properties of the dynamic equilibrium and, to sharpen the economic intuition, we also provide graphical illustrations based on the numerical solutions.

3.1 Managing the Stablecoin Platform

User Optimization. A representative user i solves a static problem in (8) and the solution is

$$u_{it} = \left(\frac{N_t^\alpha A^{(1-\alpha-\beta)}}{r + f_t - \mu_t^P + \eta|\sigma_t^P|} \right)^{\frac{1}{1-\beta}}. \quad (9)$$

Users’ choices exhibit strategic complementarity as $u_{i,t}$ increases in the aggregate value N_t . In equilibrium $N_t = u_{i,t}$ under user homogeneity, which, through (9), implies

$$N_t = \frac{A}{(r + f_t - \mu_t^P + \eta|\sigma_t^P|)^{\frac{1}{1-\beta}}}, \quad (10)$$

¹²To avoid liquidation, the platform can potentially reset the redemption price P_t , but to make up the deficit, $S_t P_t - M_t$, the token price has to jump, causing users to expect an infinite rate of change and thus refrain from holding any tokens under their preference for token price stability.

where, to simplify the notations, we define $\xi \equiv \alpha + \beta (< 1)$. Aggregate token demand decreases in the fees charged by the platform, f_t , and depends on the token price dynamics, which the platform controls. The capacity constraint (2) on N_t has to be satisfied when the platform sets its strategy.

Platform Optimization. To solve the platform's optimal strategy, we first note that, given the token price dynamics (i.e., μ_t^P and σ_t^P), the platform can directly set N_t through the fees f_t . Rearranging (10), we can back out the fees implied by the platform's choice of N_t :

$$f_t = \left(\frac{A}{N_t} \right)^{1-\xi} - r + \mu_t^P - \eta|\sigma_t^P|. \quad (11)$$

Using (11), we substitute out f_t in the law of motion of reserves (4) and obtain

$$dM_t - P_t dS_t = rM_t dt + N_t^\xi A^{1-\xi} dt - rN_t dt + N_t (\mu_t^P - \eta|\sigma_t^P|) dt + N_t \sigma dZ_t - dDiv_t. \quad (12)$$

Next, we show the state variable for the platform's dynamic optimization is the *excess reserves*,

$$C_t \equiv M_t - P_t S_t. \quad (13)$$

To derive the law of motion of C_t , we first note that

$$dC_t = dM_t - d(S_t P_t) = dM_t - P_t dS_t - S_t dP_t = dM_t - P_t dS_t - N_t (\mu_t^P dt + \sigma_t^P dZ_t). \quad (14)$$

The last equality follows (6).¹³ From a balance-sheet perspective, the reserves, M_t , are the platform's assets and the outstanding tokens, $P_t S_t$, are the liabilities. The excess reserves constitute the equity. Thus, equation (14) is essentially the differential form of balance-sheet identity. Using (12), we substitute out $dM_t - P_t dS_t$ the right side of (14) and obtain the following law of motion of C_t :

$$dC_t = \left(rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P| \right) dt + N_t (\sigma - \sigma_t^P) dZ_t - dDiv_t. \quad (15)$$

Note that $N_t \mu_t^P$ disappears. As shown in (12), the platform receives more fee revenues (see (11)) when users expect tokens to appreciate ($N_t \mu_t^P$), but such revenues do not increase the platform's equity (excess reserves) as they are cancelled out by the appreciation of token liabilities. Thus, the

¹³The second equality follows $d(S_t P_t) = P_t dS_t + S_t dP_t$, which is essentially the standard self-financing portfolio condition - dS_t cannot load on the future information dZ_t . Accordingly, in our model, the platform can set dS_t to influence the token price but cannot condition its action on the future information.

drift term, $rC_t + N_t^\xi A^{1-\xi} - N_t \eta |\sigma_t^P|$, is the expected appreciation of the platform's equity position.

We characterize a Markov equilibrium with the platform's excess reserves, C_t , as the state variable. In the following, we solve the platform's control variables, $dDiv_t$, σ_t^P , and N_t , as functions of C_t , and thereby, show that (15) is an autonomous law of motion of the state variable.

The platform owners' value function at time t is given by

$$V_t = V(C_t) = \max_{\{N, \sigma^P, Div\}} \mathbb{E} \left[\int_{s=t}^{\infty} e^{-\rho(s-t)} dDiv_s \right]. \quad (16)$$

The platform pays dividends when the marginal value of excess reserves is equal to one, i.e., one dollar has the same value either held within the platform or paid out,

$$V'(\bar{C}) = 1. \quad (17)$$

The next proposition states that the value function is concave. The declining marginal value of excess reserves implies that \bar{C} in (17) is an endogenous upper bound of the state variable C_t . At any $C_t \in (0, \bar{C})$, the platform does not pay dividends to its owners because the marginal value of excess reserve, $V'(\bar{C})$, is greater than one, i.e., the owners' value of dividend. And the optimality of payout at \bar{C} also requires the following super-contact condition (Dumas, 1991),

$$V''(\bar{C}) = 0. \quad (18)$$

Proposition 1 (Value Function Concavity and Over-Collateralization). *There exists $\bar{C} > 0$ such that $C_t \in (0, \bar{C}]$. For $C_t \in (0, \bar{C})$, the value function is strictly concave, and $V'(C_t) > 1$ so the platform maintains excess reserves. At $C_t = \bar{C}$, $V'(\bar{C}) = 1$ and the platform pays dividends.*

The platform's token liabilities are over collateralized. The intuition can be understood through the wedge between the liquidation value (zero) and the strictly positive value of platform as an ongoing concern. At $C_t = 0$, even a tiny negative shock triggers a downward jump of platform value to zero. By holding excess reserves, even by a small amount, the platform can prevent this. Next we confirm that a platform as ongoing concern always has a positive value. In the interior region $C \in (0, \bar{C})$, $dDiv_t = 0$ and we obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V(C) = \max_{\{N \in [0, \bar{N}], \sigma^P\}} \left\{ V'(C) \left(rC + N^\xi A^{1-\xi} - \eta N |\sigma_t^P| \right) + \frac{1}{2} V''(C) N^2 (\sigma - \sigma^P)^2 \right\}, \quad (19)$$

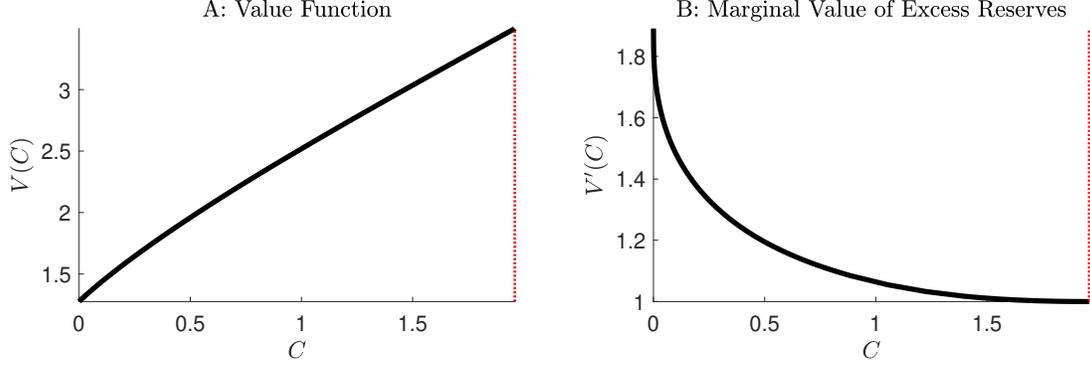


Figure 1: Value Function. This figure illustrates the level and first derivative of the platform's value function. The red dotted lines in both panels mark \bar{C} (defined in Proposition 1). The parameters are $r = 0.05$, $\rho = 0.06$, $\sigma = 0.1$, $\bar{N} = 5$, $\eta = 0.15$, $\alpha = 0.45$, $\beta = 0.05$, and $A = 0.0025$.

Setting first $\sigma^P = \sigma$ and then $N = 0$ is feasible in the HJB equation, which implies

$$V(C) \geq \frac{V'(C)}{\rho} \left(rC + \max_{\{N \in [0, \bar{N}]\}} \left\{ N^\xi A^{1-\xi} - \eta N \sigma \right\} \right) \geq \frac{V'(C)rC}{\rho} > 0. \quad (20)$$

Next, we analyze the platform's optimal choice of token exchange-rate process and transaction volume. Because μ_t^P disappears from (15), the platform's choice of redemption price, P_t (dollar exchange rate), boils down to the choice of $\sigma_t^P = \sigma^P(C_t)$. First, we consider how the choice of σ_t^P and N_t when C_t approaches zero. In the limit, σ_t^P must converge to σ to mute the shock exposure,

$$\lim_{C \rightarrow 0^+} \sigma^P(C) = \sigma; \quad (21)$$

otherwise, dC_t 's loading on dZ_t in (15) is positive and a negative shock can trigger liquidation. Equation (21) implies that, when taking the right-limit on both sides of (19), we obtain

$$\lim_{C \rightarrow 0^+} \frac{V(C)}{V'(C)} = \frac{1}{\rho} \max_{\{N \in [0, \bar{N}]\}} \left\{ N^\xi A^{1-\xi} - \eta N \sigma \right\} = \frac{A}{\rho} \left(\frac{\xi}{\eta \sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta \sigma, \quad (22)$$

where the second equality follows from plugging in the optimal N_t given by

$$\underline{N} \equiv \lim_{C \rightarrow 0^+} N(C) = \arg \max_{N \in [0, \bar{N}]} \left\{ N^\xi A^{1-\xi} - \eta N \sigma \right\} = A \left(\frac{\xi}{\eta \sigma} \right)^{\frac{1}{1-\xi}}. \quad (23)$$

Therefore, (21) and (23) characterize respectively the limiting behavior of σ_t^P and N_t as C_t approaches zero. In the process, we find the value of $\lim_{C \rightarrow 0^+} V(C)$, a boundary condition for the

HJB equation. As an interim summary, the next proposition summarizes the value function solution as solution to an ordinary differential equation (ODE) problem with an endogenous boundary. Figure 1 plots the numerical solution of value function (Panel A) and the decreasing marginal value of excess reserves with the red dotted line marking the payout boundary \bar{C} .

Proposition 2 (Solving Value Function). *The value function, $V(C)$, and the boundary \bar{C} are solved by the ordinary differential equation (19) under the boundary conditions (17), (18), and (22).*

Next, we fully characterize the platform's optimal choices of σ_t^P and N_t as functions of the state variable, C_t (via the derivatives of $V(C)$). First, we define the platform's *effective risk aversion*:

$$\gamma(C) \equiv -\frac{V''(C)}{V'(C)}. \quad (24)$$

This definition is analogous to the classic measure of absolute risk aversion of consumers (Arrow, 1965; Pratt, 1964). From Proposition 1, $\gamma(C) \geq 0$ and, in $(0, \bar{C})$, $\gamma(C) > 0$. The *endogenous* risk aversion arises from the concavity of value function, which is in turn due to the gap between liquidation value and continuation value as previously discussed. The next proposition states the monotonicity of $\gamma(C)$ in C and summarizes the optimal $\sigma_t^P = \sigma^P(C_t)$ and $N = N(C_t)$.

Proposition 3 (Risk Aversion, Token Volatility, and Transaction Volume). *The platform's effective risk aversion, $\gamma(C)$, strictly decreases in the level of reserve holdings, C . There exists $\tilde{C} \in (0, \bar{C})$ such that, at $C \in (0, \tilde{C})$, $N(C) = \underline{N}$ and $\sigma^P(C)$ strictly decreases in C , given by,*

$$\sigma^P(C) = \sigma - \frac{\eta}{\gamma(C)\underline{N}} \in (0, \sigma), \quad (25)$$

and at $C \in [\tilde{C}, \bar{C}]$, $\sigma^P(C) = 0$ and $N(C)$ increases in C , given by

$$N(C) = \min \left\{ \left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2} \right)^{\frac{1}{2-\xi}}, \bar{N} \right\}. \quad (26)$$

When the platform's reserves are low, i.e., $C \in (0, \tilde{C})$, it is the ratio of users' risk aversion to the platform's risk aversion that determines token volatility. Equation (25) shows that, in this region, when the platform accumulates more reserves and becomes less risk-averse, it absorbs risk from users by tuning down σ_t^P , and when the platform exhausts its reserves, it off-loads the risk in its dollar revenues to users.¹⁴ The platform and its users engage actively in risk-sharing in $C \in (0, \tilde{C})$.

¹⁴Equation (25) implies that the condition (22) is equivalent to $\gamma(C)$ (or $-V''(C)$) approaching infinity in the limit.

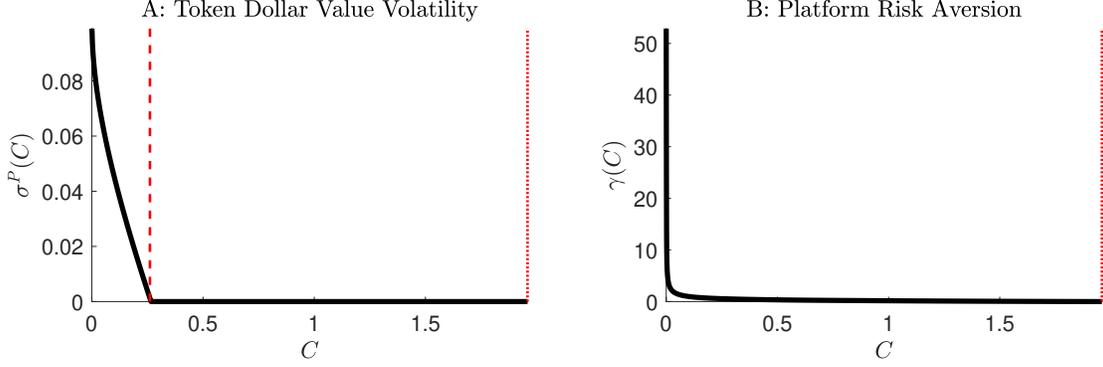


Figure 2: **Token Volatility and Platform Risk Aversion.** This figure plots token return volatility $\sigma^P(P)$ in Panel A and the platform’s risk aversion $\gamma(C)$ in Panel B. In Panel A, the red dashed line marks \tilde{C} (in Proposition 3). The red dotted lines in both panels mark \bar{C} (in Proposition 1). The parameterization follows Figure 1.

This is illustrated by the numerical solution in Panel A of figure 2 with \tilde{C} marked by the dashed line. In Panel B, we show that the platform’s risk aversion declines in C . In this region of low reserves, the transaction volume, which is simply the dollar value of users’ token holdings under our assumption of constant velocity, is pinned to the lowest level given by \underline{N} in (23).

Once the platform’s reserves surpass the critical threshold \tilde{C} , its risk aversion becomes sufficiently low and it optimally absorbs all the risk in its dollar revenues, setting $\sigma^P(C)$ to zero which also implies that in this region $\mu^P(C) = 0$.¹⁵ As a result, the transaction volume on the platform starts to rise above the “hibernation level”, \underline{N} , as illustrated by Panel A of Figure 3. Therefore, reserves are absolutely essential for stimulating economic activities on a stablecoin platform.

Interestingly, even though the platform shelters its users from risk at any $C > \tilde{C}$, its risk aversion still shows up in N_t given by (26). As shown in (11), the platform’s choice of N_t is implemented through fees. Therefore, the intuition can be more easily explained when we substitute (26), the optimal N_t , and the optimal $\sigma_t^P = 0$ (as well as $\mu_t^P = 0$) into (11) to solve f_t : when $\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}} < \bar{N}$,

$$f_t = \left(\frac{A\gamma(C)\sigma^2}{\xi}\right)^{\frac{1-\xi}{2-\xi}} - r, \quad (27)$$

and when $\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}} \geq \bar{N}$, i.e., C is sufficiently high such that $\gamma(C)$ falls below $\frac{\xi A^{1-\xi}}{\sigma^2 \bar{N}^{2-\xi}}$,

$$f_t = \left(\frac{A}{\bar{N}}\right)^{1-\xi} - r. \quad (28)$$

¹⁵This result arises because we express the equilibrium token price as a function of C , in that $P_t = P(C_t)$. Thus, token volatility and token returns can be expressed as functions of C too, in that $\sigma_t^P = \sigma^P(C_t)$ and $\mu_t^P = \mu^P(C_t)$. Since $\sigma^P(C) = 0$ for $C > \tilde{C}$, token price $P(C)$ must be constant for $C > \tilde{C}$, implying $\mu^P(C) = 0$ for $C > \tilde{C}$.

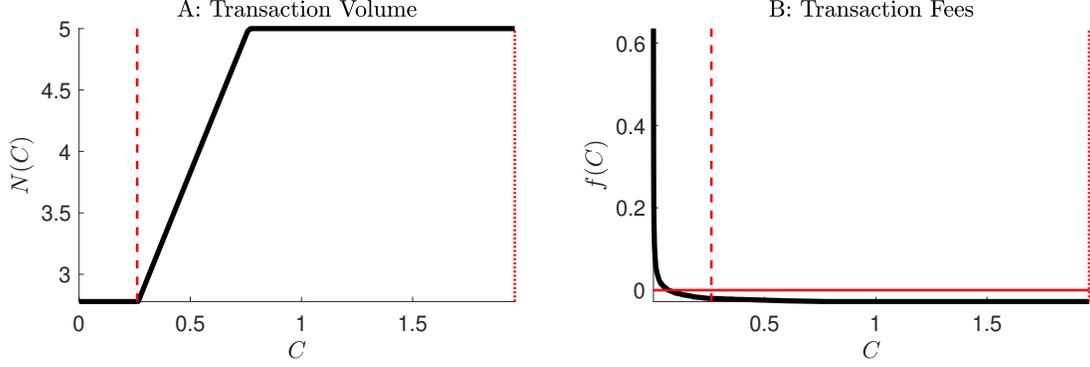


Figure 3: **Transaction Volume and Fees.** This figure plots transaction volume $N(C)$ in Panel A and fees per dollar of transaction $f(C)$ in Panel B. The red dotted lines mark \bar{C} (in Proposition 1). In Panel A, the red dashed line marks \tilde{C} (in Proposition 3). In Panel B, the red solid line marks zero. The parameterization follows Figure 1.

As a platform accumulates reserves, its risk aversion declines, which, through (27), implies low fees charged on users and in turn a larger token demand N_t (in (26)).

The platform faces a risk-return trade-off. The fees serve as a compensation for risk exposure but discourages users from participation. So when the platform's risk aversion rises, it charges users more per dollar of transaction at the expense of a smaller volume. When the platform's risk aversion declines, the fees per dollar of transaction decline while the total transaction volume increases. Once reserves are sufficiently high such that $\gamma(C) \leq \frac{\xi A^{1-\xi}}{\sigma^2 N^{2-\xi}}$, the fees no longer declines with the platform's risk aversion, as the platform has maxed out its transaction capacity, i.e., $N_t = \bar{N}$, and it becomes impossible to further stimulate user participation. Likewise, when the platform's reserves are below \tilde{C} and $\sigma^P(C) > 0$, $N_t = \underline{N}$, and the fees are given by

$$f_t = \left(\frac{A}{\bar{N}} \right)^{1-\xi} + \mu^P(C) - \eta \sigma^P(C) - r. \quad (29)$$

Even though the platform's risk aversion is high, it can no longer sacrifice transaction volume for higher fees because user participation already falls to the lowest level.

Panel B of Figure 3 plots the numerical solution of optimal fees that decrease in excess reserves. Depending on the parameters, fees can actually turn into user subsidies (i.e., fall below zero) when excess reserves are sufficiently high.¹⁶ The next corollary summarizes the results on fees.

Corollary 1 (Optimal Transaction Fees). *Transaction fees, $f(C)$, decreases in excess reserves, C . At $C \in (0, \tilde{C})$, where \tilde{C} is defined in Proposition 3, transaction fees are given by (29). At*

¹⁶Specifically, under the particular parameterization, the condition is for fees to turn into subsidies near \bar{C} is that $\frac{A^{1-\xi}}{\bar{N}^{1-\xi}} < r$ where we use (28) and the fact that $\mu^P(C) = 0$ for $C \in (\tilde{C}, \bar{C})$ (to be discussed later in this section).

$C \in [\tilde{C}, \tilde{C}')$, where \tilde{C}' is defined by $\gamma(\tilde{C}') = \frac{\xi A^{1-\xi}}{\sigma^2 \bar{N}^{2-\xi}}$, transaction fees are given by (27). At $C \in [\tilde{C}', \bar{C}]$, where \bar{C} is defined in Proposition 1, transaction fees are given by (28).

Another interesting implication of the optimal fees is that the platform charges (compensates) users the expected appreciation (depreciation) of tokens over risk-free rate, i.e., $\mu_t^P - r$ in f_t . To fully solve the fees, we need to know both $\gamma(C_t)$ and the function $\mu_t^P = \mu^P(C_t)$. In fact, the platform's choice of $\sigma_t^P = \sigma^P(C_t)$ already pins down the function of token price, $P_t = P(C_t)$, so the function $\mu^P(C_t)$ can be obtained from Itô's lemma. Next, we solve $P_t = P(C_t)$ from the function $\sigma^P(C_t)$. By Itô's lemma,

$$\sigma^P(C) = \frac{P'(C)}{P(C)} N(C) (\sigma - \sigma^P(C)), \quad (30)$$

where $N(C) (\sigma - \sigma^P(C))$ is the diffusion of state variable C_t . Rearranging the equation, we solve

$$\frac{P'(C)}{P(C)} = \frac{1}{N(C)} \left(\frac{\sigma^P(C)}{\sigma - \sigma^P(C)} \right). \quad (31)$$

Using Proposition 2, we solve the value function $V(C)$ and obtain the function $\gamma(C)$. Then using Proposition 3, we obtain the functions $\sigma^P(C)$ and $N(C)$. Plugging $\sigma^P(C)$ and $N(C)$ into (31), we obtain a first-order ODE for the function of dollar price of token, $P(C)$.

To uniquely solve the function $P(C)$, we need to augment the ODE (31) with a boundary condition. In our model, both the platform and users are not concerned with the level of token price and only care about the expected token return, μ_t^P , and return volatility, σ_t^P . Therefore, we have the liberty to impose the following boundary condition:

$$P(\bar{C}) = 1. \quad (32)$$

i.e., the platform sets an exchange rate of one dollar for one token when C_t reaches \bar{C} . The next corollary states the solution of token price as solution to a first-order ODE problem.

Corollary 2 (Solving Token Price). *Given the solutions of $V(C)$ from Proposition 2 and $\sigma^P(C)$ and $N(C)$ from Proposition 3, the dollar price of token, $P(C)$, is solved by the ordinary differential equation (31) under the boundary condition (32).*

Proposition 3 states that, once C crosses above the critical threshold \tilde{C} , $\sigma^P(C) = \mu^P(C) = 0$, which, by Itô's lemma (i.e., (30)), implies that $P'(C) = 0$. Therefore, if the platform's reserves are sufficiently high, it optimally fixes the dollar price (i.e., redemption value) of token at $P(C) = 1$.

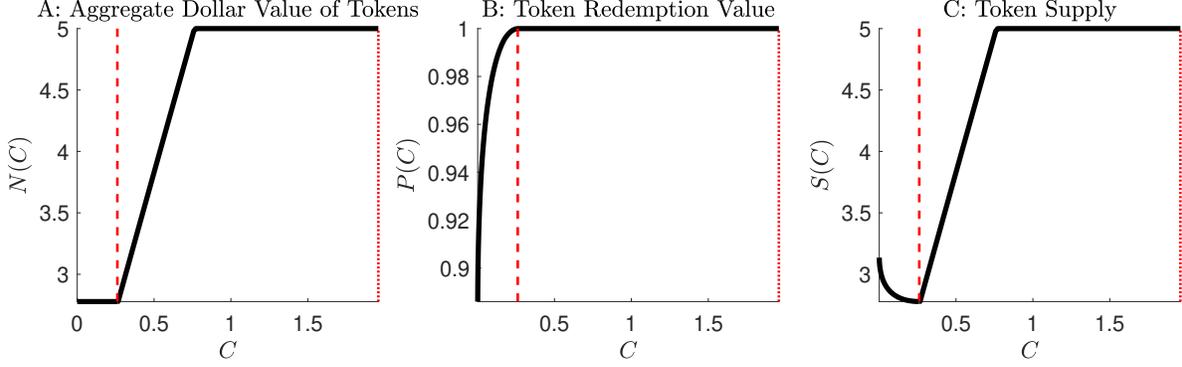


Figure 4: Token Price and Quantity Dynamics. This figure plots the aggregate dollar value of tokens $N(C)$ in Panel A, the token redemption value or dollar price $P(C)$ in Panel B, and token supply $S(C)$ in Panel C. The red dotted lines in all panels mark \bar{C} (defined in Proposition 1). The red dashed lines in all panels mark \tilde{C} (defined in Proposition 3). The parameterization follows Figure 1.

When C falls below \tilde{C} , (30) implies that $P'(C) > 0$ (because $\sigma^P(C) \in (0, \sigma)$ in Proposition 3) so the token redemption value comoves with the platform's excess reserves.

The endogenous transition between redemption at par and redemption below par happens as the platform accumulates or depletes reserves through various activities laid out in (4) (and then (15)), including the platform's issuance of new tokens, users' token redemption, fee revenues, and shocks to the dollar reserve flow. The platform's choice of redemption price is optimally chosen and thus credible in the sense that the platform does not have incentive to deviate.

Proposition 4 (Credible Redemption Value Regimes). *At $C \in [\tilde{C}, \bar{C}]$, where \tilde{C} is defined in Proposition 3, the platform credibly commits to redeem token at par, i.e., $P(C) = 1$. At $C \in (0, \tilde{C})$, the redemption value of token comoves with the platform's excess reserves (i.e., $P'(C) > 0$).*

Proposition 4 states that redemption at par is credible if and only if the platform holds a sufficiently large amount of *excess* reserves ($C > \tilde{C}$). When excess reserves fall below \tilde{C} , the platform optimally debases its tokens. By allowing the redemption value to comove with its excess reserves, the platform off-loads the risk in its dollar revenues to users and thereby prevents liquidation.

Figure 4 plots the numerical solutions of aggregate token value, $N(C) = P(C)S(C)$ (Panel A), the redemption value of one token optimally set by the platform $P(C)$ (Panel B), and the total quantity of tokens $S(C)$ implied by $N(C)$ and $P(C)$ (Panel C). The dashed line marks \tilde{C} . The platform implements the optimal token redemption value through the manipulation of token supply. When the platform has enough reserves to credibly sustain redemption at par (i.e., $C > \tilde{C}$), token supply comoves with demand so that $P(C)$ is fixed. Below \tilde{C} , a decrease of excess reserves triggers

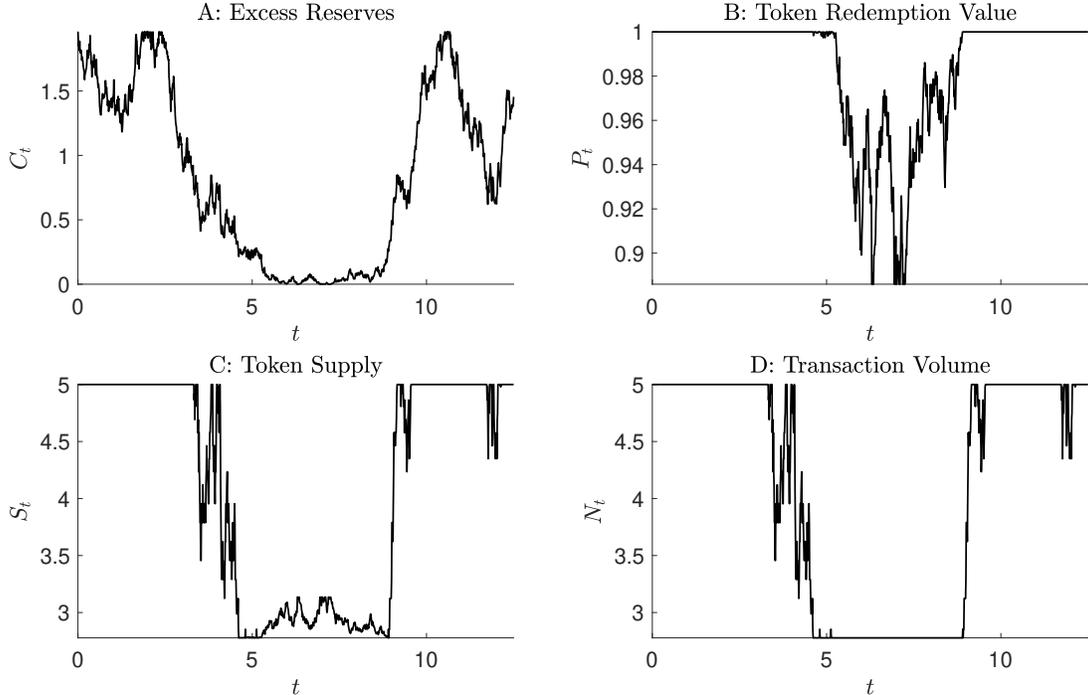


Figure 5: **Simulation.** Using the numerical solutions, we simulate a path of excess reserves (Panel A), token redemption value (Panel B), token supply (Panel C), and transaction volume on the platform (Panel D). The horizontal axis records the number of years. The parameterization follows Figure 1.

the platform to supply more tokens in exchange for dollars that replenishes reserves. The users respond to token debasement by reducing demand to \underline{N} , which in turn reinforces the debasement.

In practice, stablecoin platforms often claim commitment to redemption at par and substantiate this commitment by holding reserves that just cover their token liabilities. However, our analysis so far has drawn two conclusions that challenge such conventional wisdom. First, as long as there exists uncertainty in a platform’s dollar revenues, over-collateralization is not only necessary but also optimal from the platform’s perspective. Second, redemption at par in every state of world is not credible or “incentive-compatible”. As shown by Proposition 4, it is always in the platform’s interest to debase its tokens once its excess reserves falls below the critical threshold. The credibility of commitment to redemption at par is thus contingent on the reserve level.

Simulation and Long-Run Dynamics. Using the parameters in Figure 1 and the numerical solutions, we simulate in figure 5 a path of excess reserves C_t (Panel A), token redemption value P_t (Panel B), token supply S_t (Panel C), and transaction volume N_t (Panel D). The horizontal axis records the number of years. In the first three years, in spite of the volatility in C_t , the platform manages to sustain redemption at par, and with the transaction volume (or token demand) at the

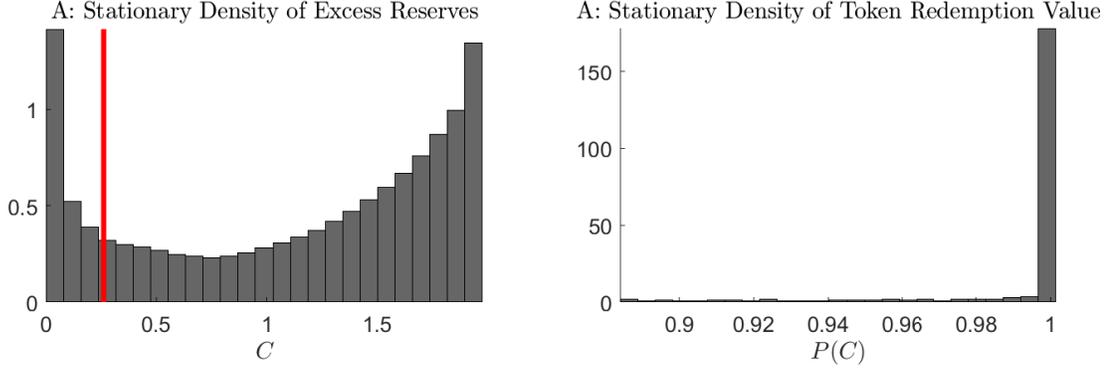


Figure 6: **Long-Run Dynamics and Stationary Density.** We plot stationary probability densities of excess reserves C (Panel A) and token value $P(C)$ (Panel B) in numerical solutions. The parameterization follows Figure 1.

full capacity at \bar{N} , a fixed dollar price of token implies a fixed token supply. Following a sequence of negative shocks between the third and fourth years, the platform raises fees. Users respond by reducing their token demand N_t , so the platform reduces token supply, maintaining redemption at par. The platform optimally trades off replenishing dollars reserves by raising fees and using dollar reserves in token buy-back. As more negative shocks hit between the fourth and ninth years, the platform gives up the peg and off-loads risk to users through the fluctuation of token redemption price. Users' token demand hits \underline{N} , and the platform starts actively expanding token supply in exchange for dollar revenues. Then following a sequence of positive shocks, the recovery started in the ninth year, and by the tenth year, the platform restores redemption at par.

We demonstrate the long-run dynamics of the model in Figure 6. Panel A plots the stationary probability density of excess reserves. It shows how much time over the long run the platform spends in different regions of C . The distribution is bimodal. The concentration of probability mass near $C = 0$ is due to the fact that, when the transaction volume (or token demand) gets stuck at the hibernation level \underline{N} , the platform can only grow out of this region very slowly by accumulating reserves through fee revenues and proceeds from expanding token supply. The platform also spends a lot of time near the payout boundary \bar{C} as this is a stable region where, given a sufficiently high level of reserve buffer, shocks' impact is limited. In Panel B, we show that, even though redemption at par seems to be the norm, the system exhibits significant risk of token debasement ($P(C) < 1$).

Discussion: Liquidation. In our model, the platform never liquidates. As C declines and approaches zero, the platform gradually off-loads risk to users (see (21)). As a result, the platform avoids liquidation and gradually accumulates reserves through the interests on reserve holdings,

the fee revenues, and, in case users' token demand increase, through the issuance of new tokens. As C increases either through the various sources of revenues or positive shocks, the platform can escape the low- C region where user participation is the lowest at the hibernation level \underline{N} .

In reality, stablecoin platforms can enter into liquidation, which is a major concern of practitioners and policy makers. Our model can be easily extended to make liquidation a positive probability event. The platform's dollar revenues can be subject to both small shocks (i.e., the diffusive, Brownian shock dZ_t) and large negative shocks that arrive following a Poisson process. When the level of reserves is sufficiently low, the arrival of a large negative shock triggers liquidation. Before the arrival of the large shock, the behavior of the extended model is akin to that of the current model except that the expectation of liquidation increases the concavity of value function and thereby induces more reserve holdings and risk-sharing between the platform and users.

3.2 The Role of Network Effects

After Facebook announced its stablecoin project Libra (recently renamed to Diem), the interest in stablecoins among practitioners and regulators skyrocketed. Different from other stablecoin issuers, Facebook has the unique advantage of strong network effects brought by its comprehensive infrastructure covering social network, social media, and e-commerce (Facebook Shop). For individual users, the benefit of adopting Diem is enormous if other users on Facebook adopts Diem, because a great variety of activities can potentially be enabled by a universal means of payment for users around the globe. In our model, strong network effects are captured by a large value of α (see (1)).

Next, we compare stablecoin platforms with different degrees of network effects. Panel A of Figure 7 plots the payout boundary \bar{C} as a measure of voluntary over-collateralization over different values of α . On the one hand, stronger network effects make the platform more profitable, which stimulates more precautionary savings to protect the franchise value. On the other hand, stronger network effects imply a higher level of user activities near $C = 0$, i.e., \underline{N} in (23), which in turn implies a faster recovery out of the low- C region (through fee and token-issuance revenues) and thereby a weaker incentive to hold excess reserves. The two counteracting forces lead to the hump-shaped relationship between \bar{C} and α in Panel A of Figure 7.

In Panel B of Figure 7, we show the long-run probability of $C > \tilde{C}$ (based on stationary probability distribution of C) increases in α . Stronger network effects imply that, over the long run, the system spends more time in states with $P(C) = 1$. Under stronger network effects, recovery out of the low- C region is faster due to a higher level of user activities and the resultant

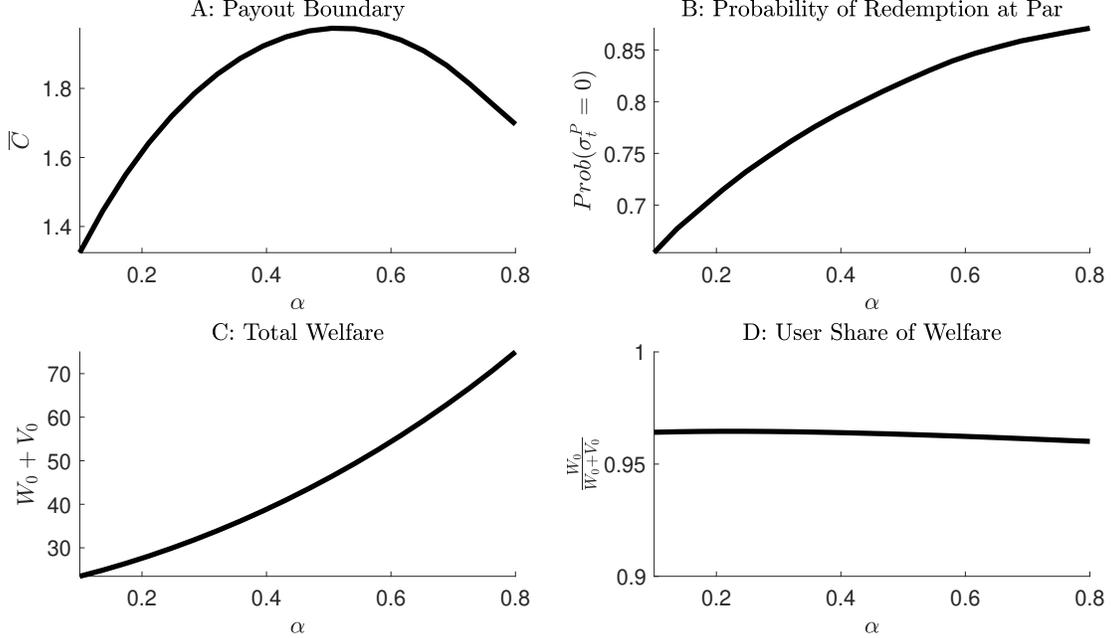


Figure 7: **Network Effects.** We plot the payout boundary (Panel A), the long-run probability of $C > \tilde{C}$ based on stationary distribution (Panel B), the sum of platform value and users' welfare (Panel C), and users' share of total welfare (Panel D) over different values of α (degree of network externality). The parameterization follows Figure 1.

faster replenishment of reserves via fees and token-issuance proceeds. Our paper sheds light on why stablecoins issued by Facebook and other technology giants with strong network infrastructure are regarded as more promising than those issued by start-up payment service providers. Stronger network effects lead to stabler tokens and a lower likelihood of token debasement.

Finally, we examine the impact of network effects on welfare. In Panel C of Figure 7, we show that total welfare of the platform and its users increases in the degree of network effects. This explains why it is particularly beneficial for technology giants with strong network infrastructure to introduce stablecoins as common means of payment among their customers. Network infrastructure is not limited to social network and e-commerce. Financial network is another example. JPMorgan Chase introduces JPM Coin to facilitate transactions among institutional clients.

Interestingly, as we gradually increase the degree of network effects in Panel D of Figure 7, the split of total welfare between the platform and its users is rather stable. Under stronger network effects, the monopolistic platform can extract more rents from its users through fees or off-loading risk in distress. However, precisely due to the network effects, individual users do not internalize the positive effect of their adoption on other users, so the platform has incentive to internalize the network externality by stimulating user activities through fee reductions (or subsidies) and

token stability. These two counteracting forces imply that, as network effects become stronger, the platform’s share of total surplus does not necessarily increase. This result alleviates the concern over technology giants abusing network effects in stablecoin projects.

3.3 Recapitalizing the Stablecoin Platform

At the end of this section, we take an excursion to analyze platform recapitalization. So far, the platform recovers from the low- C region through the accumulation of internal funds. We now we allow the platform to raise dollar funds by issuing equity shares to outside investors subject to a fixed financing cost, χ .¹⁷ To characterize the optimal recapitalization policy, we first note that when issuing equity, the platform raises enough funds so that C jumps to \bar{C} where $V'(\bar{C}) = 1$. Once the fixed cost χ is paid, raising one more dollar does not incur further costs, so as long as the marginal value of reserves, $V'(C)$, is greater than one, the platform keeps raising funds.¹⁸

Moreover, the platform raises external funds only when C falls to zero. First, it is not optimal to issue equity at the payout boundary, \bar{C} , because newly raised funds will be paid out immediately and thus the issuance cost is incurred without any benefits. Therefore, let \underline{C} denote the recapitalization boundary and we have $\underline{C} < \bar{C}$. Consider the *change* of existing shareholders’ value after equity issuance: $[V(\bar{C}) - (\bar{C} - \underline{C}) - \chi] - V(\underline{C})$. To arrive at the existing shareholders’ value, we deduct the issuance cost, χ , and competitive investors’ equity value (equal to the funds raised), $(\bar{C} - \underline{C})$, from the total platform value post-issuance, $V(\bar{C})$. To calculate the change, we subtract $V(\underline{C})$, the value without issuance. Taking the derivative with respect to \underline{C} , we obtain $-V'(\underline{C}) + 1 < 0$ for $\underline{C} < \bar{C}$ because $V'(\underline{C}) > V'(\bar{C}) = 1$ under value function concavity.¹⁹ Therefore, the platform prefers \underline{C} to be as low as possible and thus optimally sets it to zero.

Finally, as in the baseline model, the platform can avoid liquidation by off-loading risk to users, as shown in (21), and obtain the value given by (22). Therefore, as C approaches zero, the platform

¹⁷Firms face significant financing costs due to asymmetric information and incentive issues. A large literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction in response to the announcement of a new issue. Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs) of equity, the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), the direct costs average 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was -3% and the loss in equity value as a percentage of the size of the new equity issue was as high as -31% (see Eckbo, Masulis, and Norli, 2007, for a survey).

¹⁸If a proportional cost of equity issuance is introduced, fund raising stops where $V'(C)$ is equal to one plus the proportional cost. For simplicity, we only introduce the fixed cost. Our analysis below on the token supply policy at the recapitalization boundary will not change in the presence of proportional cost of issuance.

¹⁹To prove the concavity of value function stated in Proposition 1, we only need the HJB equation (19) and the upper boundary conditions (17) and (18), so recapitalization does not affect value function concavity.

only opts for recapitalization at $C = 0$ if recapitalization generates a higher value. Accordingly, the lower boundary condition (22) for the value function is modified to

$$\lim_{C \rightarrow 0^+} V(C) = \max \left\{ \lim_{C \rightarrow 0^+} V'(C) \frac{A}{\rho} \left(\frac{\xi}{\eta\sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta\sigma, V(\bar{C}) - \bar{C} - \chi \right\}, \quad (33)$$

The first term in the max operator is the value obtained from off-loading risk to users, given by (22). The second term is the post-issuance value for existing shareholders. The results in Proposition 1 to 2, 3 and Corollary 1 still hold except that the boundary condition (22) is replaced by (33).

Proposition 5 (Optimal Recapitalization). *The platform raises external funds through equity issuance only if $V(\bar{C}) - \bar{C} - \chi > \lim_{C \rightarrow 0^+} V'(C) \frac{A}{\rho} \left(\frac{\xi}{\eta\sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta\sigma$ (see (33)), where \bar{C} is given by Proposition 1, and only when excess reserves fall to zero. The amount of funds raised is \bar{C} .*

When recapitalization happens, C_t jumps from zero to \bar{C} . Therefore, according to Proposition 3, the aggregate token demand, N_t , jumps from \underline{N} to $N(\bar{C})$ because, as shown in Corollary 1, the platform lowers fees $f(C)$ when it holds more reserves and becomes less risk-averse.²⁰ If the platform does not adjust the token supply, S_t , there will be an upward *predictable* jump in the dollar price of token at $C = 0$, which implies an arbitrage opportunity.

To preclude arbitrage, the platform must expand token supply when it adjusts fees right after recapitalization so that the dollar price of token stays at its pre-issuance level. Since C_t already reaches the payout boundary, \bar{C} , the dollar proceeds from supplying new tokens are immediately paid out to shareholders. From a balance-sheet perspective, total assets (reserves) stay at \bar{C} while, on the liability side, token liability increases and equity decreases (through payout). This liability-structure adjustment is akin to corporations issuing debts for share repurchase.

Corollary 3 (Post-Recapitalization Adjustment of Liability Structure). *To preclude arbitrage in the token market, the platform adjusts its liability structure immediately after capitalization, supplying new tokens and paying out the dollar proceeds to the platform's shareholders.*

Finally, we revisit the results on the token price level in Corollary 2 and Proposition 4. Let $P_j(C)$ denote the token price function after the j -th recapitalization. According to Corollary (3), when recapitalization happens for the j -th time and C_t jumps from zero to \bar{C} , the platform sets

$$P_j(\bar{C}) = P_{j-1}(0), \quad (34)$$

²⁰As stated in Corollary 1, such jump is induced by the optimal resetting of fees.

which replaces (32) as the boundary condition for the price-level ODE (31). Token price level before the first recapitalization, $P_0(C)$ is still solved under the boundary condition (32), i.e., $P_0(\bar{C}) = 1$.

Corollary 4 (Recapitalization and Token Price Level). *Token price level after the j -th recapitalization is solved by the ordinary differential equation (31) under the boundary condition (34).*

In the baseline model without recapitalization, the debasement of token is temporary: Token price level falls below 1 when C falls below \tilde{C} due to negative shocks and it recovers back to 1 when the platform accumulates sufficient amount of dollar revenues so that C crosses above \tilde{C} (Proposition 4). When recapitalization happens, the debasement is permanent. After the j -th recapitalization, token price level starts anew at a lower peg, $P_j(\bar{C}) = P_{j-1}(0)$, and if negative shocks deplete the platform’s reserves and triggers another recapitalization, token price level declines along the process and, right after recapitalization, stabilizes at an even lower peg, $P_{j+1}(\bar{C}) = P_j(0)$.

Discussion: Financial Frictions. In our model, financial frictions play a key role. If costless recapitalization is possible, i.e., $\chi = 0$, then the platform will never allow the marginal value of reserves to exceed one because, when $V'(C) > 1$, it is profitable to raise funds from competitive investors that cost 1 per dollar and generates a value of $V'(C) > 1$. The constant marginal value of reserves implies that the platform is no longer risk-averse, i.e., $\gamma(C) = 0$, and thus, will absorb all risk, setting σ_t^P to zero. Tokens will always be redeemed at a fixed dollar value.

4 Regulating Stablecoins

In this section, we apply our model to analyze two types of stablecoin regulations. The first type, which is of our focus, stipulates a minimum level of excess reserves held by the platform (“capital requirement”). Violating such a regulation triggers liquidation. The rationale behind is to guarantee a sufficient risk buffer so that the platform is likely to sustain stable redemption value. The second type is a more direct intervention that targets the fluctuation of token value. It requires the platform to keep token volatility below a certain level (“volatility regulation”). Our conclusion is that capital requirement delivers desirable equilibrium outcome, and if carefully designed, can achieve Pareto improvement for the platform and its users. Volatility regulation, in contrast, destroys the economic surplus from risk-sharing between the platform and its users.

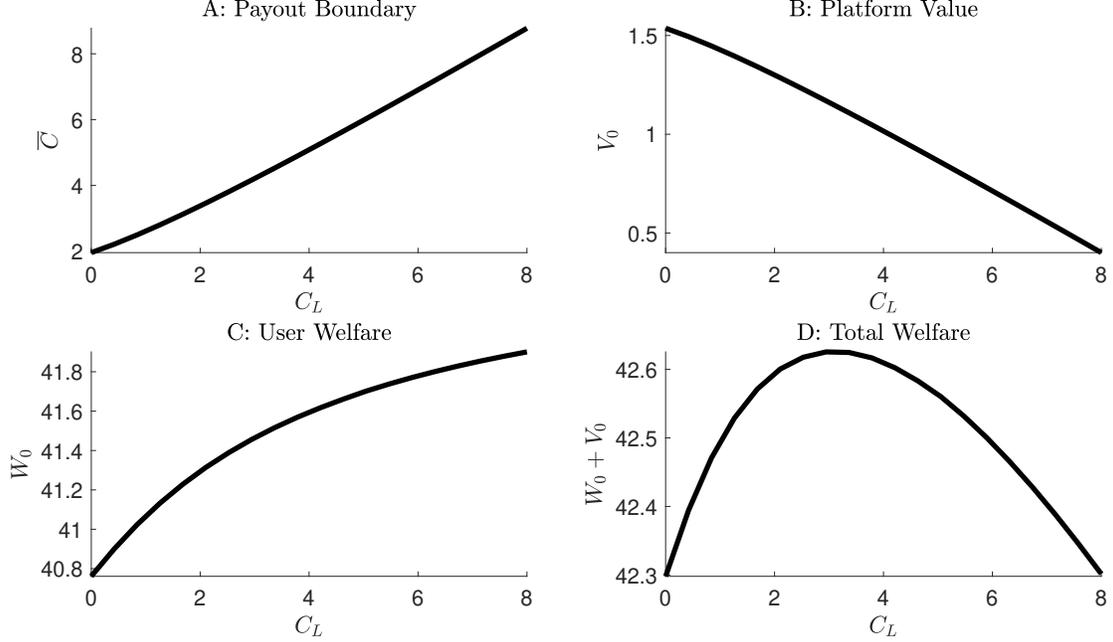


Figure 8: **Capital Requirement and Welfare.** We plot the numerical solutions of payout boundary \bar{C} (Panel A), the platform shareholders' value at $t = 0$, V_0 (Panel B), users' welfare at $t = 0$, W_0 (Panel C), and total welfare $W_0 + V_0$ (Panel D). The parameterization follows Figure 1.

4.1 Capital Requirement

The regulator imposes a capital requirement, $C \geq C_L$, on the stablecoin platform and forces the platform to liquidate if the regulation is violated. Therefore, C_L replaces zero as the lower bound of excess reserves. In Figure 8, we plot the payout boundary \bar{C} (Panel A), which is a measure of voluntary over-collateralization, and the welfare measures for different values of C_L . Not so surprisingly, when the capital requirement tightens, the whole region of excess reserves is pushed to the right, resulting in a higher payout boundary \bar{C} in Panel A. Because reserves earn an interest rate r that is below the shareholders' discount rate ρ , the platform shareholders' value, V_0 , declines in C_L , as shown in Panel B. Panel C shows that users' welfare is improved by the capital requirement but there exists a significant degree of decreasing return as the regulator pushes up C_L .

What is interesting is that, in Panel D of Figure 8, the total welfare is non-monotonic in C_L . When the regulator increases C_L from zero, the increase of users' welfare overwhelms the decrease of platform value, but as the capital requirement is tightened, the loss of platform value eventually dominates. This suggests the existence of an optimal level of C_L that maximizes the total welfare.

As long as the users' welfare increases faster than the platform value decreases, the regulator can administer a transfer from users to the platform, making the regulation Pareto-improving. For

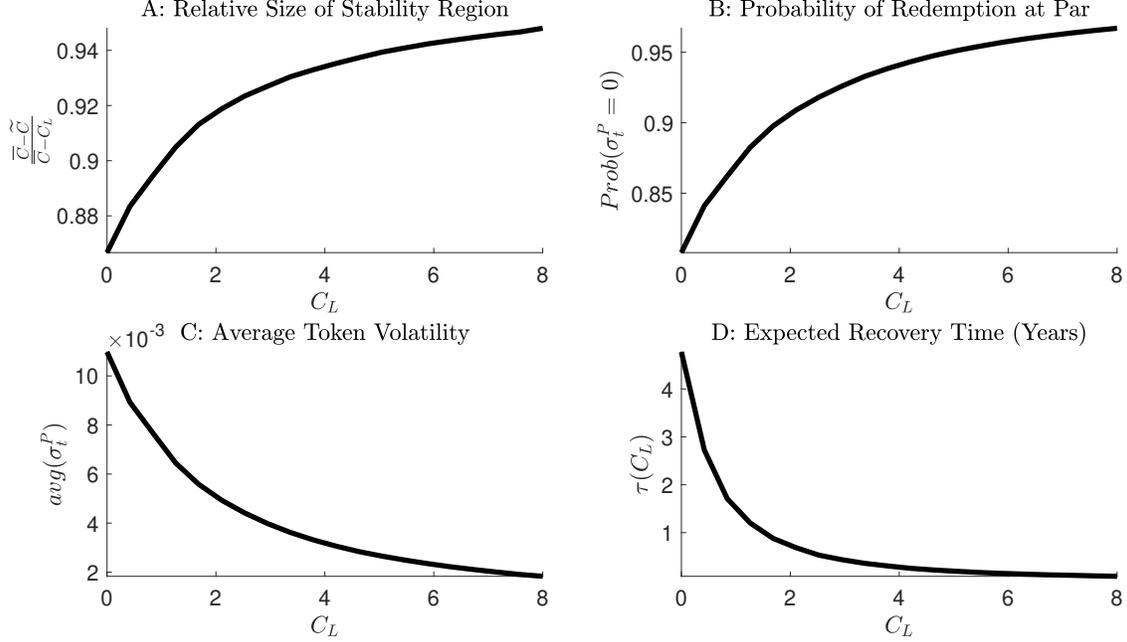


Figure 9: **Capital Requirement and Token Stability.** Using numeric solutions under different values of C_L , we plot the fraction of state space with redemption at par $\frac{\bar{C} - \tilde{C}}{\bar{C} - C_L}$ (Panel A), stationary probability of zero token volatility (Panel B), the long-run average (based on stationary probability density) of token volatility (Panel C), and the expected time to reach \tilde{C} from C_L (Panel D). The parameterization follows Figure 1.

example, the regulator can allow the platform to charge users a membership fees, i.e., a fixed cost of access, and imposes a cap on such fees. This type of access fees is commonly seen in the literature on regulation of utility networks (Laffont and Tirole, 1994; Armstrong, Doyle, and Vickers, 1996).

In Figure 9, we further demonstrate the stabilization effects of capital requirement. In Panel A, we plot the ratio of $\bar{C} - \tilde{C}$ to $\bar{C} - C_L$ that measures the size of the stable subset of C where the platform maintains token redemption at par (i.e., $P(C) = 1$). As C_L increases, the stable region enlarges. In Panel B, we plot the probability of $C > \tilde{C}$ (i.e., $\sigma^P(C) = 0$) based on the stationary distribution of C , which shows that over the long run the platform spends more time in the stable region when C_L increases. In Panel C, we plot the long-run average value of σ_t^P using the stationary probability distribution. A declining pattern emerges, indicating that capital requirement is indeed effective in reducing the token volatility. In Panel D, we plot the expected number of years it takes to reach \tilde{C} from C_L (denoted by $\tau(C_L)$). This recovery time decreases when the capital requirement is tightened. The intuition is that, as C_L increases, the platform near C_L still has abundant cash that self-accumulates over time by earning the interest rate r .

Lastly, Figure 10 demonstrates how network effects, as captured by α , drive the optimal capital

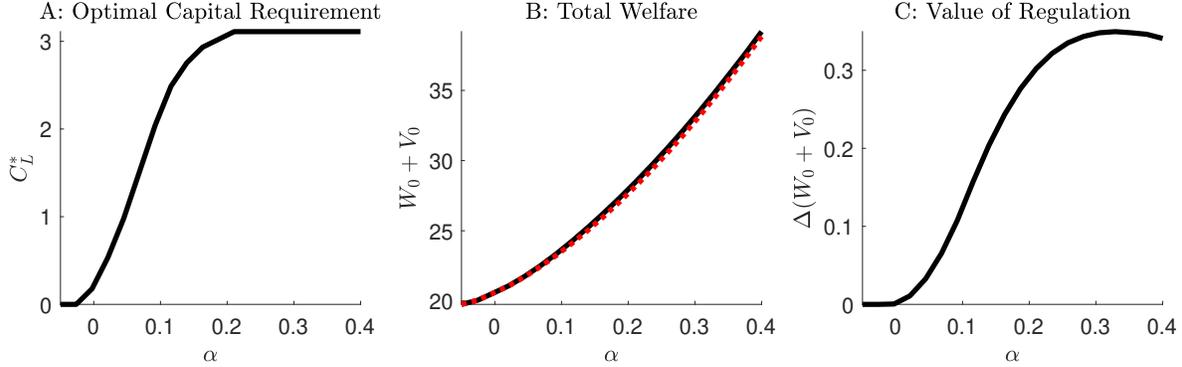


Figure 10: **Capital Requirement and Network Effects.** We calculate the optimal capital requirement C_L^* that maximizes total welfare (Panel A), total welfare both with capital requirement C_L^* (solid black line) and without capital requirement (dotted red line) (Panel B), and the welfare wedge between the optimally regulated equilibrium and the laissez-faire equilibrium (Panel C) over different values of α . Note that Panel C depicts the difference between the solid black line and the dotted red line from Panel B. The rest of parameterization follows Figure 1.

requirement. Panel A plots the capital requirement C_L^* that maximizes total welfare. Panel B plots total welfare both with optimal capital requirements (solid line) and without capital requirements (dotted line). Panel C plots the welfare wedge between the optimally regulated equilibrium and laissez-faire equilibrium. Importantly, it is optimal to raise capital requirement as network effects strengthen. In fact, platforms with no network effects ($\alpha = 0$) do not benefit at all from the regulation. The key to this result is the positive externality of individual users' token holdings, which explains the deviation of laissez-faire equilibrium from social optimum. The platform internalizes such externality in its decision to preserve reserves and stabilize tokens, but the internalization is not perfect. As shown in Figure 7, the platform cannot seize the full surplus as its share of total welfare is rather stable in α and always below 100%. Therefore, as α increases, the total welfare increases together with the component that is not internalized by the platform. This calls for a tighter capital requirement that moves the overall level of reserves closer to social optimum.

4.2 Volatility Regulation

The concern over token volatility and debasement may also motivate more direct regulatory intervention. A volatility regulation imposes a cap on σ_t^P and requires the platform to implement a rule of token supply to achieve this goal. In Figure 11, we compare the baseline model (solid line) and the model under regulation that forces $\sigma_t^P = 0$ (dotted line) over a range of values of users' risk aversion η .²¹ As shown by the HJB equation (19), under $\sigma_t^P = 0$, the platform's value function and

²¹Because the platform can no longer off-load risk to users as C approaches zero, liquidation can happen and $C = 0$ becomes an absorbing state. The boundary condition $V(0) = 0$ implies $-V''(C)$ approaches infinity, which is the

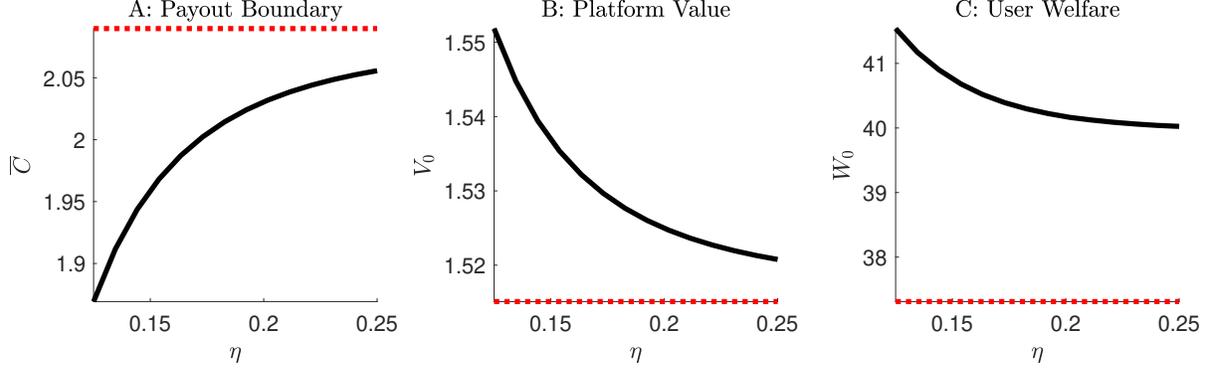


Figure 11: **Risk-Sharing, Volatility Regulation, and Welfare.** Using the numerical solutions, we calculate the payout boundary \bar{C} (Panel A), the platform shareholders’ value at $t = 0$, V_0 (Panel B), users’ welfare at $t = 0$, W_0 (Panel C), and the long-run average fees based on stationary probability density (Panel D) over different values of users’ risk aversion η for both the baseline model (solid line) and the model under zero-volatility regulation (red dotted line). The parameterization follows Figure 1.

control variables no longer depends on η , so the (dotted) lines are flat in all panels.

In Panel A of Figure 11, we show that under the zero-volatility requirement, the platform has to maintain a higher level of excess reserves to reduce the likelihood of liquidation because the option of off-loading risk to users is no longer available. Holding more reserves with an interest rate below the shareholders’ discount rate leads to a lower platform value, as shown in Panel B of Figure 11.

An interesting finding is that imposing the volatility regulation even decreases users’ welfare (Panel C of Figure 11) across all values of η . This seems to contradict the intuition that, by forcing the platform to maintain a perfectly stable token value, users will benefit, especially when they are more risk-averse. However, the argument ignores that, unable to off-load risk to users, the platform can compensate its risk exposure with higher fees. Volatility regulation is counterproductive because it limits the risk-sharing between the platform and its users. When the platform is close to liquidation, its effective risk aversion can be higher than η , so there is economic surplus created from users’ absorbing risk from the platform. Volatility regulation shuts down this insurance market.

5 Payment and Big Data

User-generated data is now a major asset of digital platforms. Social networks, such as Facebook and Twitter, utilize such data to target users for advertisement. Being able to utilize the enormous amount of transaction data has become a critical advantage of digital platforms relative to traditional payment service providers such as banks (Bank for International Settlements, 2019).

same boundary condition that we use to solve the baseline model (see Footnote 14).

Leading players, such as PayPal and Square, have become data centers and provide services beyond facilitating payments, for example, extending loans to consumers and businesses based on credit analysis that is enabled by transaction data. In this section, we follow [Veldkamp \(2005\)](#), [Ordoñez \(2013\)](#), [Fajgelbaum, Schaal, and Taschereau-Dumouchel \(2017\)](#), and [Jones and Tonetti \(2020\)](#) to model big data as a by-product of user activities.²² Our analysis focuses how data as a productive asset affects the optimal strategies of stablecoin platforms and the efficacy of regulations.

5.1 Big Data as a Productive Asset

In the baseline model, the quality parameter A is constant. We now interpret A as a measure of effective data units that enhance platform quality and assume the following law of motion

$$dA_t = \kappa A_t^{1-\xi} N_t^\xi dt. \quad (35)$$

Users' transactions generate a flow of raw data, $\kappa N_t^\xi dt$, where the parameter κ captures the technological efficiency of data processing and storage. To what extent the raw data contributes to the effective data units depends on the current amount of effective data via $A_t^{1-\xi}$. The complementarity between the old and new data captures the fact that the value of new data increases in the quality of statistical algorithms, which in turn depends on the amount of existing data that are needed to select and train the algorithms.²³ The Cobb-Douglas form is chosen for analytical convenience.²⁴

As the platform quality improves, we assume the transaction capacity increases accordingly, i.e., $\bar{N}_t = \bar{n}A_t$, where $\bar{n} > 0$ is constant. User optimization is static and follows the baseline model. As shown in (10), the dollar transaction volume (or token demand) $N_t \equiv n_t A_t$ where

$$n_t = \frac{1}{(r + f_t - \mu_t^P + \eta|\sigma_t^P|)^{\frac{1}{1-\xi}}} \wedge \bar{n}. \quad (36)$$

As in the baseline model, the platform sets n_t through the fees, f_t , and set the dynamics of token redemption price through its choice of σ_t^P . The model now has three natural state variables, reserves M_t , token supply S_t , and data stock A_t . Similar to the baseline model, $C_t = M_t - S_t P_t$ and A_t summarize payoff-relevant information, driving the platform value, $V_t = V(C_t, A_t)$, and the dollar value of token, $P_t = P(C_t, A_t)$. To simplify the notations, we will suppress the time subscripts.

²²[Veldkamp and Chung \(2019\)](#) provide an excellent survey of the literature of data and aggregate economy.

²³Related, in [Farboodi, Mihet, Philippon, and Veldkamp \(2019\)](#), data have increasing return to scale.

²⁴This data accumulation process is inspired by the specification of knowledge accumulation in [Weitzman \(1998\)](#).

We conjecture that the system is homogeneous in A , and in particular, the platform's value function and dollar value of token are given by $V(C, A) = v(c)A$ and $P(C, A) = P(c)$, respectively, where the excess reserves-to-data ratio is the key state variable for the platform's optimal strategies:

$$c \equiv \frac{C}{A}. \quad (37)$$

We will confirm the conjecture as we solve the platform's optimization problem in the following. First, to derive the law of motion of c_t , we follow the derivation of the baseline model to obtain

$$dC_t = \left(rC_t + A_t n_t^\xi - \eta A_t n_t |\sigma_t^P| \right) dt + A_t n_t (\sigma - \sigma_t^P) dZ_t - dDiv_t. \quad (38)$$

Given (35) and (38), the law of motion of c_t is given by

$$dc_t = \left(rc_t + n_t^\xi - \eta n_t |\sigma_t^P| - \kappa n_t^\xi c_t \right) dt + n_t (\sigma - \sigma_t^P) dZ_t - \frac{dDiv_t}{A_t}. \quad (39)$$

Under the value function conjecture, $V(C, A) = v(c)A$, and the laws of motion of A (35) and c (39), the HJB equation for $v(c)$ in the interior region (where $dDiv_t = 0$) is given by

$$\rho v(c) = \max_{n \in [0, \bar{n}], \sigma^P} \left\{ [v(c) - v'(c)c] \kappa n^\xi + v'(c) \left(rc + n^\xi - \eta n |\sigma^P| \right) + \frac{1}{2} v''(c) n^2 (\sigma - \sigma^P)^2 \right\}, \quad (40)$$

When the marginal value of reserves, $V_A(C, A) = v'(c)$, falls to one, the platform pays out dividends. We define the payout boundary as \bar{c} through $v'(\bar{c}) = 1$. The optimality of \bar{c} also implies $v''(\bar{c}) = 0$. Note that as in the baseline model, when C (or c) approaches zero, the platform can avoid liquidation by setting $\sigma^P(c) = \sigma$ to off-load risk to its users and gradually replenish reserves.²⁵ For simplicity, we do not consider recapitalization (equity issuance). In sum, the platform's excess reserves, C_t , move in $(0, \bar{c}A]$. As data grows, the platform accumulates more excess reserves.

Proposition 6 (Data Growth and Over-Collateralization). *The amount of excess reserves, C_t , varies in $(0, \bar{c}A_t]$ where the upper bound increases with A_t , the effective data units.*

The intuition behind Proposition 6 is that data growth provides another channel through which the continuation value appreciates, as shown on the right side of (40), which makes the platform more patient in distributing excess reserves to shareholders. The first term on the right side contains

²⁵The boundary condition for $v(c)$ is that as c approaches zero, $-v''(c)$ approaches infinity (see footnote 14).

the marginal value of data (which we call “data q”)

$$q(c) = \frac{\partial V(C, A)}{\partial A} = v(c) - v'(c)c. \quad (41)$$

Retaining more reserves allows the platform to sustain a wider region of c with credible token redemption at par. A more stable token in turn stimulates transactions and thereby allows the platform to accumulate more data and earn the data q , $q(c)$. Data as a productive asset and by-product of user activities enhances the platform’s incentive to reserve for tokens.

Next, we characterize the optimal transaction volume and token volatility. Following our analysis of the baseline model, we define the effective risk aversion based on $v(c)$:

$$\Gamma(c) = -\frac{v''(c)}{v'(c)}. \quad (42)$$

The following proposition summarizes the optimal choices of $n(c)$ and $\sigma^P(c)$.

Proposition 7 (Data q , Token Volatility, and Transaction Volume). *At c where the platform maintains the redemption of token at par, i.e., $\sigma^P(c) = 0$, the optimal transaction volume is*

$$N = n(c)A = \left[\frac{\xi}{\Gamma(c)\sigma^2} \left(1 + \frac{\kappa q(c)}{v'(c)} \right) \right]^{\frac{1}{2-\xi}} A \wedge \bar{n}A; \quad (43)$$

otherwise, the optimal token volatility is

$$\sigma^P(c) = \sigma - \frac{\eta}{\Gamma(c)n(c)} \in (0, \sigma), \quad (44)$$

and the optimal transaction volume is

$$N = n(c)A = \left[\frac{\xi}{\eta\sigma} \left(1 + \frac{\kappa q(c)}{v'(c)} \right) \right]^{\frac{1}{1-\xi}} A \wedge \bar{n}A. \quad (45)$$

The optimal transaction volume is proportional to A , the effective data units. Therefore, as data grows following (35), the transaction volume grows too. With data as a productive asset, the platform faces a new trade-off. It can accumulate more reserves through higher fee revenues or, by reducing fees, boost the transaction volume to accumulate more data. Therefore, the ratio of marginal value of data (the data q) and marginal value of reserves, $q(c)/v'(c)$, emerges in both (43) and (45). When the data q is higher relative to the marginal value of reserves, the platform imple-

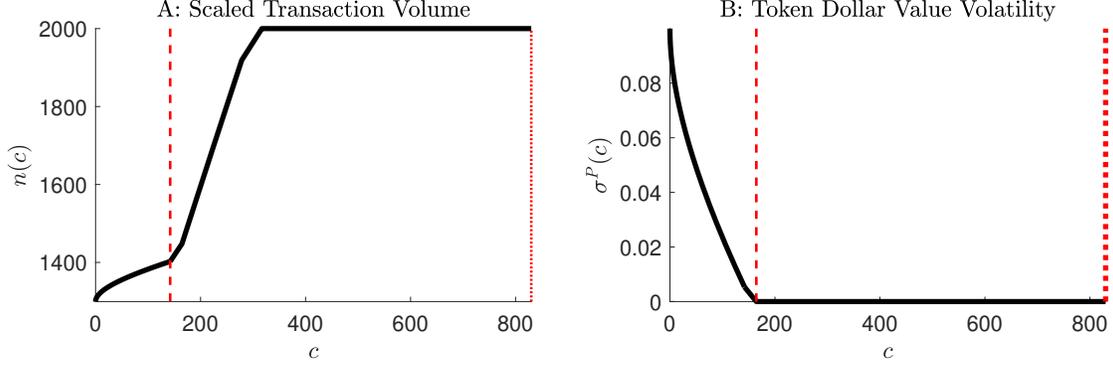


Figure 12: **Transaction Volume and Token Volatility.** This figure plots the A_t -scaled transaction volume $n(c)$ in Panel A and token return volatility $\sigma^P(c)$ in Panel B. In both panels, the red dotted lines mark the payout boundary \bar{c} , and the red dashed line marks \tilde{c} , the threshold that separates the regions of volatile and constant token prices. The parameterization follows Figure 1 with the additional parameters $\bar{n} = 2000$ and $\kappa = 0.00025$. Note that $\bar{n} = 2000$ implies that for $A_0 = 0.0025$, $\bar{N}_t = \bar{n}A_t = 5$ as under the parameterization in the baseline (see Figure 1).

ments a higher transaction volume through lower fees. Note that given the token price dynamics, the monotonic relationship between transaction volume and fees is given by (36).

The optimal choice of token volatility resembles that in the baseline model. In the region where $\sigma^P(c) > 0$, it is the ratio of users' risk aversion to the platform's risk aversion that drives $\sigma^P(c)$. And in this region, the optimal transaction volume in (45), even scaled by A , is no longer the constant as in the baseline model but depends $q(c)/v'(c)$ instead, showing the trade-off between investing in data and accumulating reserves. Moreover, the optimal transaction volume depends on users' risk aversion η as η determines the cost of obtaining insurance from users (losing transaction volume after off-loading risk to users). When the platform absorbs all risk (i.e., $\sigma^P(c) = 0$), the optimal transaction volume varies with its own risk aversion $\Gamma(c)$ (43) because $\Gamma(c)$ drives the required risk compensation through higher fees that causes the transaction volume to decline.

Panel A of Figure 12 reports the optimal transaction volume. In contrast to Panel A of Figure 3 where the transaction volume is constant in the region where $\sigma^P(c) > 0$, the A -scaled transaction volume now increases in c . The intuition is that as reserves become more abundant relative to data, the platform is more willing to lower fees, so it acquires more data through more active transactions at the expense of lower dollar revenues added to the reserve buffer. Panel B of Figure 12 shows a similar token volatility dynamics as Panel A of Figure 2 from the baseline model.

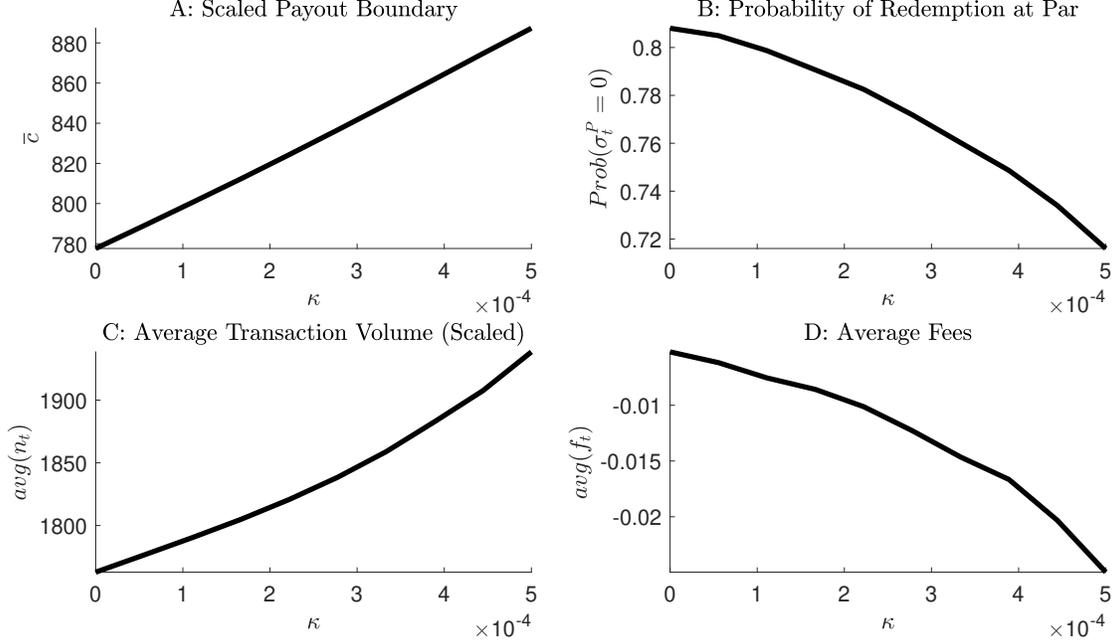


Figure 13: **Data Technology Progress and Platform Operation.** We plot the A -scaled payout boundary (Panel A), the probability of token redemption at par (Panel B), the average transaction volume (Panel C), and the average fees per dollar of transactions (Panel D) over κ (the efficiency of data technology). The moments in Panel B, C, and D are based on the stationary distribution of c . The parameterization follows Figure 1 with $\bar{\pi} = 2000$.

5.2 Data Technology Revolution and Stablecoin Platform Strategies

The last few decades have witnessed enormous progress in data science. In our model, such technological advance can be captured by an increase of the parameter κ . In Figure 13, we examine the impact of big data technology on the operation of stablecoin platforms. In Panel A, we show that in response to an increase in κ , the platform optimally raises the (A -scaled) payout boundary, \bar{u} , which suggests a greater degree of over-collateralization. The intuition of such response can be understood jointly with the platform’s decision on token volatility and fees.

To accumulate transaction data, the platform would increase the transaction volume. This can be achieved through lower fees. As shown in Panel C and D of Figure 13, the average fees (calculated from the stationary distribution of c) decline and the transaction volume increases in κ . The average fees per dollar of transaction even dips into the negative territory, becoming subsidies to users. This prediction is consistent with the practice that large digital platforms offer subsidies and fee services to retain and grow their customer base (Rochet and Tirole, 2006; Rysman, 2009).

However, a higher n implies a large exposure to operation risk as shown in (39). The platform responds by delaying payout, i.e., raising the boundary \bar{c} , to increase the reserve buffer, which

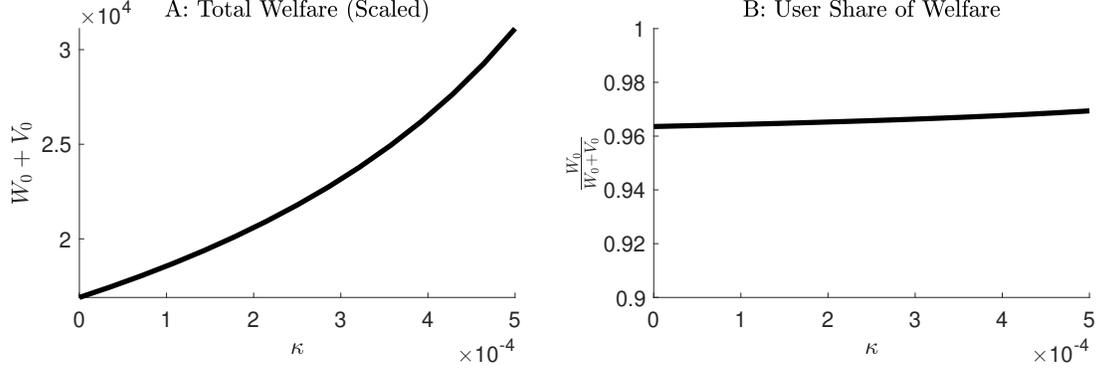


Figure 14: **Data Technology Progress and Welfare.** We plot the sum of platform value and users’ welfare (Panel A), and users’ share of total welfare (Panel B) against κ (the efficiency of data technology). The parameterization follows Figure 1. The parameterization follows Figure 1 with $\bar{n} = 2000$ and all quantities are scaled by data units A .

explains why the payout boundary \bar{c} increases in κ in Panel A of Figure 13. The platform can also respond by off-loading more risk to users. As shown in Panel B, the stationary distribution of c implies a smaller probability of $\sigma^P(c) = 0$ and a higher average $\sigma^P(c)$ when κ increases.

In sum, when transaction data can be better utilized, the platform becomes more aggressive in raising transaction volume through fee reduction (or subsidy). Accordingly, the platform maintains more reserves to buffer the resultant increase in operation risk. Part of the increased risk is shared with users through token price fluctuation. In Figure 14, we show that the improving efficiency of data technology increases the total welfare (Panel A) while the platform’s share is rather stable and always below 100%. Therefore, even though the platform has monopolistic power as a unique marketplace where users transact with each other using tokens, the platform cannot possess the full economic surplus created by big data technology. Data originates from user activities, so to obtain data, the platform must share the economic surplus with users. These outcomes also suggest that regulations targeting and limiting the use of transaction data undermine the platform’s incentives to accumulate liquidity reserves and are detrimental for both user and total welfare.

5.3 Data Technology Revolution and Stablecoin Regulation

Because the transaction volume is proportional to A , equation (35) implies an exponential growth of effective data units that scales up the platform value and users’ welfare. The improving efficiency of data technology causes the exponential growth to be increasingly steeper. In such an environment, how should the optimal capital requirement adjust? In this subsection, we address this question.

As previously discussed, a larger transaction volume N amplifies the shock exposure of reserves,

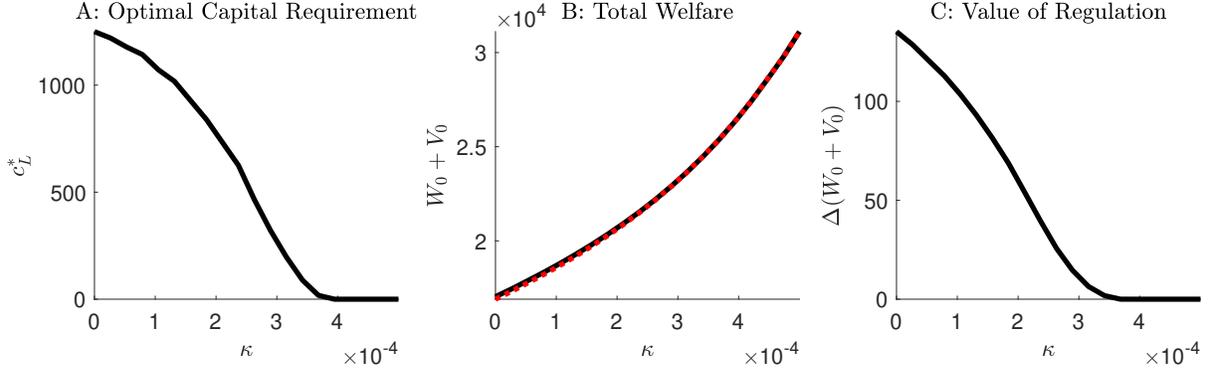


Figure 15: Data Technology Progress and Capital Requirements. We calculate the optimal scaled capital requirement $c_L^* \equiv C_L^*/A$ that maximizes total welfare (Panel A), scaled total welfare both with scaled capital requirement c_L^* (solid black line) and without capital requirement (dotted red line) (Panel B), and the welfare wedge between the optimally regulated equilibrium and laissez-faire equilibrium (Panel C) over different values of κ . Note that Panel C depicts the difference between the solid black line and the dotted red line from Panel B. The parameterization follows Figure 1 with $\bar{n} = 2000$ and all quantities are scaled by data units A .

and to achieve a larger transaction volume, the platform has to lower fees, sacrificing the growth of dollar reserves. Therefore, there exists tension between precautionary management of reserves and data acquisition through users' transactions. Capital requirement favors preserving reserves over stimulating transaction volume for data acquisition. Therefore, as data becomes more productive, i.e., κ increases, capital requirement becomes more burdensome.

Panel A of Figure 15 confirms the intuition. The optimal requirement of excess reserves (scaled by A) declines in κ . We study the scaled capital requirement to preserve the homogeneity property of the system and keep the solution in one-dimensional space of $c = C/A$. Indeed, tightening capital requirement causes the platform to build up reserves at the expense of data acquisition. However, given the self-reinforcing growth of data in (35), such regulatory measure hurts the long-run exponential growth of both platform value and users' welfare. In Panel B of Figure 15, we compare the total welfare under *optimal* capital requirement with that from the laissez-faire equilibrium, and in Panel C we plot the wedge. The increase of welfare in κ is not surprising. What is interesting is that the benefit of capital requirement dwindles as κ increases in Panel C.

Moreover, as we show in Figure 13, data as a self-accumulating productive asset offers a new opportunity for shareholders' equity to growth over time. This effectively makes the platform more patient in paying out dividends. Therefore, the voluntary build-up of reserves is strengthened as κ increases. As a result, capital requirement is less needed for the internalization of user-network effects. In sum, as data becomes more productive, the role of capital requirement weakens.

6 Conclusion

The first-generation cryptocurrencies, such as Bitcoin and Ethereum, were built to serve as transaction medium, but the enormous volatility in price compromises such functionality. For this reason, stablecoins — cryptocurrencies or tokens whose price is designed to be stable — have become increasingly popular. Typically, stablecoins are issued by private entities who promise to maintain price stability by holding collateral against which stablecoin holdings can be redeemed. However, as these private stablecoin issuers maximize their own payoffs rather than the total welfare, conflicts of interests between the issuers and users of stablecoins may arise, making room for welfare-enhancing regulations of stablecoin issuance. However, to this date, there has not been a systematic theoretical analysis of stablecoins, in spite of the enormous attention from regulators. In this paper, we fill this gap and develop a dynamic model of optimal stablecoin management and regulation.

In our model, users hold redeemable stablecoins to transact on a digital platform, and stablecoins can be exchanged for dollars, or vice versa, at an exchange rate set by the platform under its own discretion. The platform maintains liquidity reserves to meet token redemption and controls transaction volume through fees. When reserves are high, users can redeem tokens at par and pay low transaction fees charged by the platform, so that transaction volume is high. When reserves are low, the platform charges high fees and optimally debases token to share liquidity risks with users, in which case, user can only redeem tokens below par and transaction volume suffers. We then discuss optimal regulation of stablecoins. We find that regulation precluding token debasement is detrimental to welfare. In contrast, capital requirement, which stipulates a minimal level of reserve holdings, adds value by incentivizing the platform to internalize user-network effects.

The accumulation and utilization of transaction data launches self-reinforcing growth of transaction volume, platform value, and user welfare. By enhancing the platform’s incentive to reserve for stablecoins, data as a productive asset weakens the role of capital requirement. Moreover, capital requirement imposed on a data-enabled stablecoin platform can have the unintended consequence of favoring reserve preservation over data acquisition, stemming the data-driven exponential growth.

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A Appendix

A.1 Value Function Concavity

We prove the concavity of value function in Proposition 1. Recall the HJB equation (19), that is,

$$\rho V(C) = \max_{\{N \in [0, \bar{N}], \sigma^P\}} \left\{ V'(C) \left(rC + N^\xi A^{1-\xi} - \eta N |\sigma^P| \right) + \frac{1}{2} V''(C) N^2 (\sigma - \sigma^P)^2 \right\}.$$

Using the envelope theorem, we differentiate both sides of the HJB equation (evaluated under the optimal controls) with respect to C to obtain

$$\rho V'(C) = rV'(V) + V''(V) \left(rC + N^\xi A^{1-\xi} - \eta N |\sigma_t^P| \right) + \frac{1}{2} V'''(C) N^2 (\sigma - \sigma^P)^2.$$

We can solve for

$$V'''(C) = \frac{2}{N^2 (\sigma - \sigma^P)^2} \left[(\rho - r) V'(C) - V''(V) \left(rC + N^\xi A^{1-\xi} - \eta N |\sigma_t^P| \right) \right]$$

Using the smooth pasting condition, $V'(\bar{C}) = 1$, and the super-contact condition, $V''(\bar{C}) < 0$, it follows that $V'''(\bar{C}) > 0$. As $V''(\bar{C}) = 0$, it follows that $V''(C) < 0$ in a left-neighbourhood of \bar{C} .

We show that $V''(C) < 0$ for all $C \in [0, \bar{C})$. Suppose to the contrary that there exists $\hat{C} < \bar{C}$ with $V''(\hat{C}) \geq 0$ and set without loss of generality

$$\hat{C} = \sup\{C \geq 0 : V''(C) \geq 0\}. \quad (46)$$

As $V''(C) < 0$ in a left-neighbourhood of \bar{C} and the value function is twice continuously differentiable, it follows that $V''(\hat{C}) = 0$ and therefore $\sigma^P(\hat{C}) < \sigma$. In addition, $V'(\hat{C}) \geq 1$, so that $V'''(\hat{C}) > 0$. Thus, there exists $C' > \hat{C}$ with $V''(C') \geq 0$, a contradiction. Therefore, the value function is strictly concave on $[0, \bar{C})$.

A.2 Optimal Control Variables

In this section, we characterize the optimization in (19) and solve for the optimal control variables $N = N(C)$ and $\sigma^P = \sigma^P(C)$ in Proposition 3. To start with, we define

$$\underline{N} = \arg \max_{N \leq \bar{N}} N^\xi A^{1-\xi} - \eta N \sigma,$$

which yields

$$\underline{N} = \min \left\{ \left(\frac{\xi A^{1-\xi}}{\eta \sigma} \right)^{\frac{1}{1-\xi}}, \bar{N} \right\}.$$

Now, first optimize the HJB equation over σ^P or equivalently over $N\sigma^P$.

If interior, the choice of σ^P satisfies the first order optimality condition. The first-order-

condition in (19) with respect to σ^P is

$$-\eta V'(C) - V''(C)(N\sigma - N\sigma^P) = 0$$

Thus,

$$N\sigma^P = \max \left\{ 0, \frac{-\eta V'(C) + N\sigma V''(C)}{-V''(C)} \right\} = \max \left\{ 0, -\frac{\eta V'(C)}{-V''(C)} + N\sigma \right\}$$

We distinguish between two different cases

1. $\sigma^P > 0$, in which case we can insert σ^P into (19) to get

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C) \left[rC + N^\xi A^{1-\xi} - \eta N\sigma - \frac{\eta^2 V'(C)}{V''(C)} \right] + \frac{1}{V''(C)} \left[\frac{(\eta V'(C))^2}{2} \right] \right\}. \quad (47)$$

(48)

Thus, $N = \underline{N}$ is the optimal choice of N , so that

$$\sigma^P = \max \left\{ 0, -\frac{\eta V'(C)}{-V''(C)\underline{N}} + \sigma \right\}$$

2. $\sigma^P = 0$, so the HJB becomes

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C)[rC + N^\xi A^{1-\xi}] + V''(C) \left[\frac{N^2 \sigma^2}{2} \right] \right\}, \quad (49)$$

and

$$N(C) = \min \left\{ \left(\frac{A^{1-\xi} \xi V'(C)}{-V''(C)\sigma^2} \right)^{\frac{1}{2-\xi}}, \bar{N} \right\} \quad (50)$$

is the optimal choice of N .

It follows that

$$N(C) = \underline{N} \iff -\frac{\eta V'(C)}{-V''(C)\underline{N}} + \sigma > 0,$$

as desired.

A.3 Effective Risk Aversion

We prove $\gamma'(C) < 0$, i.e., $\frac{d(-V''(C)/V'(C))}{dC} < 0$, in Proposition 3. Consider the following two cases:

1. $\sigma^P > 0$, $N = \underline{N}$. The HJB equation can be simplified to

$$\rho \frac{V(C)}{V'(C)} = rC + \underline{N}^\xi A^{1-\xi} - \eta \underline{N}\sigma - \frac{\eta^2}{2} \frac{V'(C)}{V''(C)}. \quad (51)$$

Differentiating the equation above with respect to C we obtain that in $(0, \bar{C})$

$$\rho \left(1 - \frac{V''(C)V(C)}{V'(C)^2} \right) = r - \frac{\eta^2}{2} \frac{d(V'(C)/V''(C))}{dC}. \quad (52)$$

which implies $\frac{d(V'(C)/V''(C))}{dC} < 0$ (because $V''(C) < 0$ and $\rho > r$), i.e., $\frac{d(-V''(C)/V'(C))}{dC} < 0$.

2. $\sigma^P = 0$, so the HJB becomes

$$\rho V(C) = \max_{N \in [0, \bar{N}]} \left\{ V'(C)[rC + N^\xi A^{1-\xi}] + V''(C) \left[\frac{N^2 \sigma^2}{2} \right] \right\}, \quad (53)$$

In this case, we further consider two cases:

a. $N(C) < \bar{N}$ and $N = \left(\frac{A^{1-\xi} \xi V'(C)}{-V''(C) \sigma^2} \right)^{\frac{1}{2-\xi}}$. In this case, the HJB can be simplified to

$$\rho \frac{V(C)}{V'(C)} = rC + \frac{1}{2} \left(\frac{\xi A^{1-\xi}}{\sigma^\xi} \right)^{\frac{2}{2-\xi}} \left(\frac{2-\xi}{\xi} \right) \left(\frac{V'(C)}{-V''(C)} \right)^{\frac{\xi}{2-\xi}}. \quad (54)$$

Differentiating the equation above with respect to C , we obtain

$$\rho \left(1 - \frac{V''(C)V(C)}{V'(C)^2} \right) = r - \frac{1}{2} \left(\frac{\xi A^{1-\xi}}{\sigma^\xi} \right)^{\frac{2}{2-\xi}} \left(\frac{V'(C)}{-V''(C)} \right)^{\frac{2\xi-2}{2-\xi}} \frac{d(-V'(C)/V''(C))}{dC}, \quad (55)$$

implying $\frac{d(V'(C)/V''(C))}{dC} < 0$ (because $V''(C) < 0$ and $\rho > r$), i.e., $\frac{d(-V''(C)/V'(C))}{dC} < 0$.

b. $N(C) = \bar{N}$. In this case, the HJB can be simplified to

$$\rho \frac{V(C)}{V'(C)} = rC + \bar{N}^\xi A^{1-\xi} + \frac{\bar{N}^2 \sigma^2}{2} \frac{V''(C)}{V'(C)}. \quad (56)$$

Differentiating the equation above with respect to C , we obtain

$$\rho \left(1 - \frac{V''(C)V(C)}{V'(C)^2} \right) = r - \frac{\bar{N}^2 \sigma^2}{2} \frac{d(-V''(C)/V'(C))}{dC}, \quad (57)$$

which implies $\frac{d(-V''(C)/V'(C))}{dC} < 0$ (because $V''(C) < 0$ and $\rho > r$).

A.4 Calculating User Welfare

To start with, recall that any users' utility flow is

$$d\hat{R}_{it} \equiv N_t^\alpha A^{1-\xi} \frac{u_{it}^\beta}{\beta} dt + u_{it} \left(\frac{dP_t}{P_t} - rdt - f_t dt - \eta |\sigma_t^P| \right)$$

As such,

$$\mathbb{E} d\hat{R}_{it} = N_t^\alpha A^{1-\xi} \frac{u_{it}^\beta}{\beta} dt + u_{it} \left(\mu_t^P - rdt - f_t dt - \eta |\sigma_t^P| \right).$$

Inserting $u_{it} = N_t$ and (11) and using $\xi = \alpha + \beta$ yields

$$\begin{aligned}\mathbb{E}d\hat{R}_{it} &= \frac{N_t^\xi A^{1-\xi}}{\beta} dt + N_t \left(\mu_t^P - r dt - (N_t^{\xi-1} A^{1-\xi} + \mu_t^P - r - \eta |\sigma_t^P|) dt - \eta |\sigma_t^P| \right) \\ &= \frac{N_t^\xi A^{1-\xi}}{\beta} dt - N_t^\xi A^{1-\xi} dt = \frac{(1-\beta)A^{1-\xi}}{\beta} N_t^\xi dt.\end{aligned}$$

As a next step, define the user welfare from time t onward, i.e.,

$$W_t := \mathbb{E} \left[\int_t^\infty e^{-r(s-t)} (dR_{is} - \eta u_{is} |\sigma_s^P| ds) \right]. \quad (58)$$

As C is the payoff-relevant state variable, we can express user welfare as function of C , in that $W_t = W(C_t)$. The dynamic programming principle implies that user welfare solves on $[0, \bar{C}]$ the ODE

$$rW(C_t)dt = \mathbb{E}d\hat{R}_{it} + \mathbb{E}dW(C_t).$$

We can rewrite the ODE as

$$rW(C) = \frac{(1-\beta)A^{1-\xi}}{\beta} N(C)^\xi + W'(C)\mu_C(C) + \frac{W''(C)\sigma_C(C)^2}{2}, \quad (59)$$

whereby

$$\begin{aligned}\mu_C(C) &= rC + N(C)^\xi A^{1-\xi} - \eta N(C) |\sigma^P(C)| \\ \sigma_C(C) &= N(C)(\sigma - \sigma^P(C))\end{aligned}$$

are drift and volatility of net liquidity C respectively.

The ODE (59) is solved subject to the boundary conditions

$$W'(\bar{C}) = 0$$

and

$$\lim_{C \rightarrow 0} W(C) = \frac{1}{r} \lim_{C \rightarrow 0} \left(\frac{(1-\beta)A^{1-\xi}}{\beta} N(C)^\xi + W'(C)\mu_C(C) \right).$$

A.5 Calculating the Expected Arrival Time

Note that there exists $\tilde{C} \in (0, \bar{C})$ such that $\sigma^P(C) = 0$. Given $C_t = C$ at time t , we calculate

$$\tau(C_t) = \mathbb{E}[\tau^* - t | C_t = C] \quad \text{with} \quad \tau^* = \inf\{s \geq t : C_s \geq \tilde{C}\},$$

which is the expected time until net liquidity reaches \tilde{C} and token price volatility vanishes.

We can rewrite $\tau(C_t)$ as

$$\tau(C_t) = \mathbb{E}_t \left[\int_t^{\tau^*} 1 dt \right]. \quad (60)$$

By definition, it holds that when $C_t = C \geq \tilde{C}$, then $t^* = t$ and

$$\tau(C_t) = \tau(C) = 0.$$

By the integral expression (60) and the dynamic programming principle, it follows that For $C \leq \tau(C)$, the function $\tau(C)$ solves the ODE

$$0 = 1 + \tau'(C)\mu_C(C) + \frac{\sigma_C(C)^2\tau''(C)}{2}, \quad (61)$$

where

$$\begin{aligned} \mu_C(C) &= rC + N(C)^\xi A^{1-\xi} - \eta N(C)|\sigma^P(C)| \\ \sigma_C(C) &= N(C)(\sigma - \sigma^P(C)) \end{aligned}$$

are drift and volatility of net liquidity C respectively. The ODE (61) is solved subject to the boundary condition

$$\tau(\tilde{C}) = 0 \quad (62)$$

at $C = \tilde{C}$. At $C = C_L$, the lower boundary of the state space, the boundary condition

$$\lim_{C \rightarrow C_L} [1 + \tau'(C)\mu_C(C)] = 0.$$