Firm Quality Dynamics and the Slippery Slope of Credit Intervention *

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Abstract

Central banks and fiscal authorities around the world lent directly to nonfinancial firms on an unprecedented scale during the Covid-19 crisis. Credit support is subject to mispricing due to the potential lack of information on individual borrowers' credit worthiness or the political constraints on discriminatory credit pricing. In a dynamic model, we demonstrate that the mispricing of credit support generates a downward bias in the firm quality distribution that is self-perpetuating. As a result, intervention in the current crisis necessitates future interventions of greater scales, which in turn cause more distortions in firm quality dynamics. Such effects are amplified by firms' forward-looking investment decisions in normal times. Low-quality firms over-invest as they expect underpriced credit support in crises, while, on a relative basis, high-quality firms under-invest. The slippery slope of intervention is a necessary evil, as we show that when carefully designed, credit support still improves welfare.

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1 Introduction

Central banks around the world have become the lenders of last resort not only for banks but for their whole domestic economies. European Central Bank (ECB) started purchasing nonfinancial firms' debts in the global financial crisis. Bank of Japan has a long tradition of investing in both debts and equities. During the Covid-19 crisis, the Federal Reserve made an unprecedented move of directly lending to nonfinancial firms.[1] Moreover, the fiscal authorities have a long tradition of providing direct credit to small and medium-sized enterprises and sectors of strategic importance.

Direct credit support has the benefit of circumventing the traditional transmission mechanism of monetary stimulus that is subject to various frictions (Trichet, 2013). However, credit mispricing may arise when the government fails to differentiate firms of different credit worthiness under either its lack of information or the political considerations against discriminatory treatment of firms in crises (English and Liang, 2020). Carefully pricing credit may be infeasible when speedy implementation is required. Therefore, credit support pulls closer the costs of capital of high- and low-quality firms. Even when intermediated by banks, credit support still features non-discriminatory terms. When banks pledge loans to borrow from the central bank, the loans are broadly categorized when the margin requirements and refinancing rates are determined (Tamura and Tabakis, 2013). Building on the multi-sector model of Eberly and Wang (2008), we provide the first analysis of the distortionary effects of credit support on the dynamics of firm quality distribution.

Our model features cleansing effects of crises (Caballero and Hammour, 1994): Low-quality firms invest less than high-quality firms in a laissez-faire economy as they face tighter financial constraints and have lower Tobin's q. The liquidity provision by the government reduces the damage on the overall production capacity, but credit mispricing dampens the cleansing effects. Even under the dynamically optimized pricing of government funding, the low-quality firms are subsidized by the non-discriminatory pricing and conduct wasteful investment, while the high-quality firms invest more than the laissez-faire benchmark but pay a premium for government funding.

Therefore, the liquidity injection contains the decline of output, but by distorting downward the firm quality distribution, it reduces the total productivity, thus slowing down the recovery. Moreover, a downward-biased quality distribution necessitates interventions of greater scales in future crises, implying further distortions in the firm quality distribution. Yet this slippery slope of policy intervention is a necessary evil as it still improves welfare when carefully designed.

[1] Primary and Secondary Market Corporate Credit Facilities extended loans to and purchased bonds issued by large corporations. Main Street Lending Program directly lent to small and medium-sized enterprises.

The self-perpetuating nature of policy distortion is amplified by the expectation effects. Tobin's q, which drives firms' investment (Hayashi, 1982), incorporates the expectation of future crises and policy interventions. Tobin's q of the low-quality firms is biased upward by the expectation of mispriced central bank credit, so they over-invest even in normal times, while the high-quality firms may still under-invest with their Tobin's q below the first-best level. The unprecedented scale of direct lending by governments during the Covid-19 crisis is likely to reshape the expectation of policy interventions, thus influencing the firm quality dynamics both in and outside of crises.

We show that expanding the supply of liquid assets alleviates the distortionary effects of government funding. The incentive to accumulate a liquidity buffer is weak among the low-quality firms, as they expect underpriced government funding in crises. In contrast, the government funding is overpriced from the perspective of the high-quality firms. Therefore, the high-quality firms hold liquidity, but they incur a carry cost as the yield on liquidity holdings is below shareholders' required return.[2] When the supply of liquid assets increases and their yields rise, the high-quality firms' precautionary savings increase, investing more in crises, and the firm quality distribution improves as a result. Moreover, firms' savings allow the government to scale back its lending in crises, thus directly reducing the distortionary effects it has on the firm quality distribution.[3]

A difference between the Covid-19 pandemic and other crises is that banks in the U.S. were well-capitalized during the pandemic, which begs the question of whether government funding is needed at all. We extend our model to incorporate banks whose relationship lending channels funds from households to firms, but as in Santos and Winton (2008), banks hold up borrowers in crises, seizing the economic surplus created by the part of investment that can only be financed by relationship lending. Government funding serves as an outside option for firms. It helps firms seize the investment profits back from banks, thus boosting firms' franchise values. As a result, firms increase investment. Moreover, the financial constraints on non-relationship financing are relaxed by the higher (pledgeable) franchise values, which allows firms to rely even less on relationship banks. Therefore, the effectiveness of a lending facility cannot be judged by its lending volume.[4]

[2] During the Covid-19 crisis, firms with strong liquidity positions performed better (Fahlenbrach, Rageth, and Stulz, 2020). In aggregate, nonfinancial corporations hold an enormous amount of cash instruments (Bates, Kahle, and Stulz, 2009; Riddick and Whited, 2009; Quadrini, 2017). Our model builds up the existing theories of corporate liquidity management (Froot, Scharfstein, and Stein, 1993; Bolton, Chen, and Wang, 2011; Hugonnier, Malamud, and Morellec, 2015; He and Kondor, 2016; Li, 2018a; Nikolov, Schmid, and Steri, 2019).

[3] Our findings offer a caution against ultra-low interest rates in the money markets as a result of safe asset shortage, which echoes the insight of Brunnermeier and Koby (2018) and Quadrini (2020).

[4] Hanson, Stein, Sunderman, and Zwick (2020) document the under-utilization of MSLP.

Below we summarize the model setup and the main results. We follow the continuous-time formulation of the multi-sector models (Cox, Ingersoll, and Ross, 1985; Eberly and Wang, 2008) but simplify households' preferences to be risk-neutral. There are two types of firms that produce generic goods for consumption and investment but differ in their capital productivity.[5] In normal times, firms are not financially constrained and invest to grow their capital at the idiosyncratic Poisson times. The forward-looking valuation of capital (Tobin's q) drives the optimal investment (Hayashi, 1982; Abel and Eberly, 1994; Brunnermeier and Sannikov, 2014).[6]

The arrival of a systematic crisis follows a Poisson process. In a crisis, firms see a random fraction of their capital (collateral) destroyed. As a result, their financial constraints tighten, and a subset of firms now face binding financial constraints.[7] Therefore, even though firms can still invest to rebuild their capital, those with a binding financial constraint invests below the targeted levels implied by their Tobin's q. In a laissez-faire economy, the high-quality firms have higher collateral values and Tobin's q, so they invest more in aggregate than the low-quality firms in a crisis. Therefore, the economy comes out of the crisis with an improved firm quality distribution.

The binding financial constraints of the subset of firms leave room for policy intervention. The government effectively acts as a financial intermediary (Lucas, 2016). It finances lending with lump-sum taxes on deep-pocket households and transfers the instantaneous repayments to households.[8] Government funding differs from private funding because it is not subject to firms' limited commitment problem and thus unconstrained by the collateral values. Through its taxation agency and by the state power, the government has a superior ability to enforce repayments. The other difference is that unlike private investors (i.e., households), the government offers the same repayment schedule to all firms, as typically done in practice (English and Liang, 2020). There are two motivations. First, discriminatory credit pricing can be a politically challenging proposition. Second, the government may not have information on firms' types, in line with the long tradition in economics that emphasizes the informational disadvantage of central authorities (Hayek, 1945).

[5]Moreira and Savov (2017) highlight the two-dimentional difference of capital in productivity and riskiness. For the transparency of mechanism, our setup features a simpler one-dimensional difference of capital productivity.

[6]While our model does not feature the fixed cost of investment in Abel and Eberly (1994), it captures the basic idea that Tobin'Q reflects future opportunities of expansion (Abel, Dixit, Eberly, and Pindyck, 1996).

[7]We model a firm's financial constraint through a collateral constraint following the literature on limited commitment (Kehoe and Levine, 1993; Kiyotaki and Moore, 1997; Geanakoplos, 2010; Rampini and Viswanathan, 2010).

[8]In line with the models of unconventional monetary policy, the repayments are in the form of capital shares (equity ownership) just as firms' repayments to households in the private funding market. Please refer to Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki, and Queralto (2012), Araújo, Schommer, and Woodford (2015), and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017).

If the government funding is fairly priced for the high type, the low type faces extremely favorable terms and over-invests too much. If the government funding is fairly priced for the low type, the high type faces extremely unfavorable terms and under-invests. Therefore, the optimal repayment rate is set between the two scenarios, and, as a result, the high-quality firms overpay for government funding while the low-quality firms underpay. In a crisis, all low-quality firms seek government-funding. Only a subset of the high-quality firms do so, as a sufficiently high fraction of their capital is destroyed, causing their financial constraints to tighten significantly.

The cleansing effects of crises are dampened by the policy interventions. The economy emerges from a crisis with a higher fraction of capital being the low-quality (and a lower total productivity) than the laissez-faire economy. However, the government funding allows more investment, which implies a larger stock of capital than the laissez-faire benchmark. The government essentially faces a trade-off between capital quality and quantity. If the government tightens its supply of funding, the cleansing effect strengthens, so the economy has a higher total productivity post-crisis but has to climb out of a deeper decline of total output. If the government loosens its supply of funding, the decline of total output is contained but it takes longer for the economy to recover to the pre-crisis trajectory of growth given a lower total productivity.

The distortionary effects are also at work outside of crises. The expectation of underpriced credit support increases low-quality firms' Tobin's q, which in turn stimulates their investment in normal times. In contrast, high-quality firms under-invest out of the precaution that they may have to seek overpriced government funding should a crisis occur. The evolution of firm quality distribution depends on the relative investment rates of the low- and high-quality firms. Therefore, the expectation of mispriced government funding biases downward the firm quality distribution in normal times. This again causes the economy to carry more low-quality firms into future crises.

Importantly, the distortionary effects of policy intervention are cumulative. The firm quality distribution is biased downward in every future crisis and the run-up to it, so when a subsequent crisis arrives, the economy will enter with more low-quality firms (than the laissez-faire benchmark), and thus, for the government to contain the output drop to a certain level, more government funding is needed. Therefore, intervention begets more interventions of greater scales. Another factor that contributes to the slippery slope of policy intervention is the expectation effect. Even in normal times, the firm quality distribution is biased downward by the low-quality firms' over-investment under the expectation of underpriced government funding in future crises. Therefore, policy makers fall into a trap of their own making. Both the past interventions and agents' expectation of future interventions cause the government to spend more should a crisis occur. However, this policy trap is a necessary evil, as we show that when optimally designed, it improves welfare.

The social welfare function depends on both capital quantity (total units of capital from both types of firms) and capital quality (the share of capital being the high type). At any point in time, its value is the present value of households' life-time consumptions. Therefore, the welfare function reflects the trade-off between current consumption and investment (future consumption). We show that ultra-lenient funding provision by the government reduces welfare through the low-quality firms' over-investment at the expense of consumption. When the supply of government funding is tight in crises, the marginal improvement of welfare due to the high-quality firms' efficient investment is large, while the wasteful investment from the low-quality firms is still small. Therefore, a timid intervention almost guarantees a positive (but not necessarily great) outcome. This result favors gradualism in policy making, especially when the intervention cannot be discriminatory due to either the lack of information (on firms types, for example) or political constraints.

We extend our model to incorporate firms' precautionary savings.[9] The high-quality firms hold liquid assets whose values are immune to the systematic crisis. Liquidity holdings relax the financial constraint in crises and reduce the reliance on overpriced government funding. However, holding liquidity incurs a carry cost as the yield on liquidity holdings is below the prevailing interest rate. The wedge typically results from liquid asset shortage (Caballero, Farhi, and Gourinchas, 2008; Kiyotaki and Moore, 2019). In contrast, the low-quality firms do not hold liquidity, as they profit from the mispricing of government funding in crises. When an exogenous expansion of liquid-asset supply increases the yield of liquidity holdings, the high-quality firms save more and invest more.[10] Importantly, with less funding needs from the high-quality firms, the government can reduce its direct lending in crises, which implies less subsidies to the low-quality firms and thus alleviates their over-investment. Therefore, an increase in the supply of liquid assets improves the firm quality distribution. In practice, the government may optimally combine the two policy instruments, the issuance of liquid assets pre-crisis (e.g., Treasury bills) and direct lending in crises. The former alleviates the distortionary effects of the latter on the firm quality dynamics.

[9] We simplify the setup of Bolton, Chen, and Wang (2011) by assuming perfect capital markets outside of crises; otherwise solving the model involves tracking the firm savings distribution (e.g., Matsuyama, 2007; Moll, 2014).

[10] The supply of liquid assets is from both public and private sectors (Woodford, 1990; Holmström and Tirole, 1998; Farhi and Tirole, 2012a; Li, 2018b; Li, Ma, and Zhao, 2019; Li, 2019; Kacperczyk, Perignon, and Vuillemey, 2020; Ma, Xiao, and Zeng, 2020a,b). The literature on safe assets has explored interactions with various policy considerations (Brunnermeier, Merkel, and Sannikov, 2020). Our paper furthers this line of research.

Will government funding still be necessary if firms can borrow from banks? To address this question, we extend the model to incorporate banks. It is assumed that when firms borrow from banks, their collateral constraints become irrelevant, and banks do not face collateral constraints themselves. The setup is essentially an extreme case of Rampini and Viswanathan (2018). When banks are competitive, the first-best outcome is achieved. Following Santos and Winton (2008), we assume that the flexibility of bank financing comes from relationship and with the cost of a potential hold-up. The relationship bank seizes the full surplus of the part of investment that is above the collateral value. Within the collateral constraint, the bank has to compete with other investors (deep-pocket households) and break even. Therefore, relationship banks finance all profitable investments but the profits are not reflected in firms' Tobin's q. Government funding serves as firms' outside options. Specifically, by allowing firms to seize profits back from banks, it increases Tobin's q, thus boosting firms' investment targets and relaxing firms' financial constraints. The downside is still the over-investment of the low-quality firms, but when the high-quality firms have a dominant share of the economy, government funding improves welfare.

Literature. The role of central banks as lenders of last resort constantly evolves throughout the history in response to crises, political struggles, and technological innovations (Goodhart, 1998; Calomiris, Flandreau, and Laeven, 2016). Direct lending is a meaningful addition to the policy toolbox. When credit markets freeze (Stiglitz and Weiss, 1981; De Meza and Webb, 1987), the government can step in, effectively functioning as a financial intermediary (Bebchuk and Goldstein, 2011; Lucas, 2016). The Covid-19 crisis normalized the use of direct lending to nonfinancial firms across the developed countries and will have a long-lasting effect on the expectation of nonfinancial firms and their investment and financing decisions. The existing models of unconventional monetary policy assume an exogenous dead-weight loss of direct lending (Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011; Gertler, Kiyotaki, and Queralto, 2012; Araújo, Schommer, and Woodford, 2015; Del Negro, Eggertsson, Ferrero, and Kiyotaki, 2017). We unpack the black box of costs of direct lending (or asset purchases in general) and zoom into the endogenous and dynamic impact of direct lending on firm quality distribution.

As previously discussed, a key ingredient of our model is the Q-theory of investment (Hayashi, 1982). This is a rather standard way to model firms' forward-looking investment decisions in the literature (see Brunnermeier and Sannikov, 2014, among others). Historically, the evidence on the Q-investment relationship is mixed. However, more recently, after making significant progress on the measurement issues, the empirical literature has found meaningful support for the Q-investment relationship (Philippon, 2009; Peters and Taylor, 2017; Crouzet and Eberly, 2020).

The expectation of intervention distorts firms' investment decisions, which contribute significantly to the long-run welfare cost of direct lending. The distortionary effects of expected government intervention has been studied extensively on both empirical and theoretical fronts (Calomiris, 1990; O'Hara and Shaw, 1990; Acharya and Yorulmazer, 2007; Acharya, 2009; Bond, Goldstein, and Prescott, 2009; Farhi and Tirole, 2012b; Gropp, Gruendl, and Guettler, 2013; Acharya and Mora, 2015; Gandhi and Lustig, 2015; Allen, Carletti, Goldstein, and Leonello, 2018; Dávila and Walther, 2020). Our paper focuses on firms' expectation and investment decisions rather than banks' expectation and risk-taking behavior that have featured prominently in the existing literature triggered by financial crises. The unprecedented government intervention during the Covid-19 crisis is likely to put the role of firms' expectations front and center in the decades to come.

Broadly, our paper contributes to the literature on the costs of crisis intervention, such as risk cost (Lucas, 2012), tax distortions as a form of financing costs (Hanson, Scharfstein, and Sunderam, 2018), feedback loop between sovereign and private-sector risk (Acharya, Drechsler, and Schnabl, 2014; Brunnermeier, Garicano, Lane, Pagano, Reis, Santos, Thesmar, Van Nieuwerburgh, and Vayanos, 2016), and the costs of debt overhang and bankruptcy (Balloch, Djankov, Juanita Gonzalez-Uribe, and Vayanos, 2020; Brunnermeier and Krishnamurthy, 2020; Crouzet and Tourre, 2020; Greenwood, Iverson, and Thesmar, 2020; Wang, Yang, Iverson, and Kluender, 2020).[11]

2 The Model

2.1 Preferences and Technology

Consider a continuous-time economy with a unit of mass of representative agents ("households") and a government. Households have risk-neutral utility with time discount rate r:

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-rt} dc_t\right], \tag{1}$$

[11]The recent contributions on the benefits of credit-market intervention focus on the positive externalities that cannot be internalized by private lenders (e.g., Bebchuk and Goldstein, 2011; Philippon and Schnabl, 2013; Liu, 2016; Giannetti and Saidi, 2019; Hanson, Stein, Sunderman, and Zwick, 2020).

where c_t is the process of cumulative consumption. Households can trade equity shares of firms that maximize shareholder values by managing capital to produce non-durable numeraire goods.

There are two types of firms, H and L. The capital of an type-H firm produces A^H units of goods per unit of time, while the productivity of type-L capital is A^L ($A^H > A^L$). Both types of capital depreciate at the same rate, δ. Given the aggregate capital stocks of both types at time t, K_t^H and K_t^L, the total output of numeraire goods over dt is $\left(A^H K_t^H + A^L K_t^L\right) dt$.[12] Wherever necessary, we use superscripts to for firm type and subscripts for time. To represent the firm quality distribution, we introduce ω_t, the fraction of total capital that is of H type,

$$\omega_t \equiv \frac{K_t^H}{K_t^H + K_t^L} \tag{2}$$

The aggregate states, K_t^K and K_t^L, can thus be represented by ω_t and the total capital stock

$$K_t \equiv K_t^H + K_t^L. \tag{3}$$

Firms have investment opportunities that arrive at idiosyncratic Poisson time. Let N_t^I denote the corresponding Poisson counting process with intensity λ_I. When the opportunity arrives, a firm of type j can convert $x_t^j k_t^j$ units of goods into $F\left(x_t^j\right) k_t^j$ units of new capital.[13] Let q_t^j, $j \in \{H, L\}$, denote the value of capital. It plays an important role in our analysis, as it incorporates the expectation of future growth path and disruptions in crises. Given the time-t value of capital, q_t^j, the targeted investment level, denoted by $\bar{\iota}_t^j$, is given by the condition that equates the marginal benefit of investing, $q_t^j F'\left(\bar{\iota}_t^j\right)$, to the marginal cost, 1:

$$q_t^j F'\left(\bar{\iota}_t^j\right) = 1. \tag{4}$$

The function $F(\cdot)$ is increasing and strictly concave, so that $\bar{\iota}_t^j$ is increasing in q_t^j, which captures the insight of Q-theory of investment (Hayashi, 1982).

When making the investment, the firm obtains goods from households. Let x_t^j denote a type-j firm's investment per unit of capital that is funded by households. Households are competitive

[12]Firms differ only in the capital productivity. To make the mechanism transparent, we do not assume that the survival of one type of firms affects the output of the other type (Caballero, Hoshi, and Kashyap, 2008).

[13]Following Hayashi (1982), we specify an investment technology with homogeneity in the level of capital for analytical convenience.

investors with deep pockets, so in equilibrium, they break even, investing x_t^j and obtain an instantaneous repayment value of x_t^j in the form of shares of capital. Formally, let $R_{M,t}^j\left(x_t^j\right)$ denote the units of capital as repayment that a firm j promises its investors, where "M" represents "market" to distinguish from government funding that will be introduced later. Here capital units sold to households are essentially equity shares. Because we focus on the heterogeneous quality of firms, we simplify the liability structure to be full equity.[14] The households' break-even condition implies the repayment (on the left) is equal to the investment (on the right) in the following:

$$q_t^j R_{M,t}^j\left(x_t^j\right) = x_t^j. \tag{5}$$

The investment target, $\bar{\iota}_t^j$, may not be attainable because of a financial constraint. We model a firm's financial constraint through a collateral constraint following the literature on limited commitment (Kehoe and Levine, 1993; Kiyotaki and Moore, 1997; Geanakoplos, 2010; Rampini and Viswanathan, 2010; Ai et al., 2020). The amount of credible repayment is limited by the value of capital that can be repossessed by the investors. Specifically, per unit of capital, the investment is constrained by

$$x_t^j \leq \chi q_t^j\,, \tag{6}$$

where the parameter χ is smaller than one. For simplicity, χ does not vary with the firm's type. Moreover, the newly created capital cannot serve as collateral, so only the current k_t^j units of capital enter the right side of (6) when an investment opportunity arrives.[15] The assumption that $\chi < 1$ is motivated by the disruption of production, the loss of intangibles (e.g., organizational capital), and the fire-sale discount in piece-meal liquidation during default and investor repossession.

A systematic crisis arrives following a Poisson counting process, denoted by N_t, with intensity λ. In a crisis (i.e., $dN_t = 1$), a firm draws u_t from a cumulative distribution function $G(u)$ defined on the support $[0, \upsilon]$, where $\upsilon < 1$. A fraction u_t of capital is destroyed. The shock size, u_t, is independently (across firms) drawn from a common distribution. Capital represents efficiency

[14] A liability structure with both equity and equity invites the questions of crisis intervention in the form of debt moratorium (Bolton and Rosenthal, 2002). Moreover, the cost of debt overhang tends to be more prominent in crises (Chen and Manso, 2016), and government credit support can amplify debt overhang (Crouzet and Tourre, 2020; Krishnamurthy and Brunnermeier, 2020). Such inefficiencies brought by debt contracts are beyond the scope of this paper.

[15] If we allow newly created capital to be collateral, we need to solve a fixed point problem – given investment and the amount of new capital created, we solve the firm's financing capacity, and then, we solve the optimal investment under the new collateral constraint to obtain the updated amount of newly created capital until convergence.

units of production. The destruction of capital can be interpreted as a sudden decline of customers' demand, disruptions in production or supply chain, or government mandatory shut-down.

In a crisis, the firm can still invest through the technology $F(\cdot)$, and the investment output is proportional to the pre-crisis level of capital, k^j_{t-}. Specifically, a type-j firm with shock u_t that invests $x^j_t(u_t)k^j_{t-}$ will obtain $F\left(x^j_t(u_t)\right)k^j_{t-}$ units of new capital. The proportionality of investment outcome to pre-crisis capital is motivated by the fact that the firm, while in a crisis, may still possess the same customer base, technology, and organizational capital. In crisis of capital destruction, the firm faces a tighter financial constraint on investment than the one in normal times:

$$x^j_t(u_t)k^j_{t-} \leq \chi q^j_t k^j_t\,, \tag{7}$$

where the left side is the investment and the right side is the collateral value. Dividing both sides by k^j_{t-}, we obtain

$$x^j_t(u_t) \leq \chi q^j_t \frac{k^j_t}{k^j_{t-}} = \chi q^j_t(1-u_t). \tag{8}$$

By comparing (8) with (6), the collateral constraint on investment in normal times, we can see that per unit of capital, the pledgeable value declines by a fraction u_t.

Comparing the collateral constraint (6) in normal times and (8) in crises, we can see that if the collateral constraint binds in normal times, it always binds in crises in spite of the size of the capital destruction shock, u_t. To avoid this extreme case, we consider an equilibrium where the collateral constraint (6) does not bind in normal times, so that in crises, only firms with sufficiently high u_t face a binding constraint (8). The fact that firms' financial constraint does not bind in normal times is also consistent with our focus on government intervention only in crises.[16] Therefore, in normal times, firms achieve their investment target is defined in (4):

$$x^j_t = \bar{\iota}^j_t = F'^{(-1)}(1/q^j_t). \tag{9}$$

The investment target increases in q^j_t because $F(\cdot)$ is strictly concave.

Allowing firms to invest in a crisis captures the observation that firms can salvage their franchise value by restructuring the production and organizational processes (e.g., online sales during the Covid-19 pandemic). As the capital is destroyed but can be rebuilt, we may call the crisis a sys-

[16] We follow the literature on the cyclicality of firms' financing conditions (CHEN, 2010; Bolton et al., 2013; Eisfeldt and Muir, 2016; **?**).

tematic liquidity shock following Holmstrom and Tirole (1997). Capital destruction tightens the financial constraint on new investment, so the crisis also reminisces a financial shock (Jermann and Quadrini, 2012). We may consider other forms of financial constraint, but as long as the financing capacity is linked to the firm's capital, k_t^j, our qualitative results carry through.[17]

2.2 Credit Policy

The tightened financial constraint in a crisis leaves room for government intervention. Let $g_t^j(u_t)$ denote the funds that a type-j firm with shock u_t obtains from the government, so the total investment per unit of pre-crisis capital is given by

$$i_t^j(u) = x_t^j(u) + g_t^j(u). \tag{10}$$

with the additional funding, the firm now creates $F\left(i_t^j(u_t)\right) k_{t-}^j$ units of capital.

When the government intervenes, it acts as a financial intermediary (Lucas, 2016). It finances lending with lump-sum taxes on deep-pocket households and transfers the instantaneous repayments to households. In line with the models of unconventional monetary policy, the repayments are in the form of capital shares (equity ownership) just as firms' repayments to households are.[18]

Government funding differs from private funding because it is not subject to firms' limited commitment problem. Through its taxation agency and by the state power, the government has a superior ability in enforcing repayments. The other difference is that unlike private investors, the government cannot condition the repayment schedule on firms' types. There are two motivations. First, a differential treatment on firms can be a politically challenging proposition. Second, the government may not have information on firms' types, in line with the long tradition in economics that emphasizes the informational disadvantage of central authorities (Hayek, 1945). For a type-j

[17] Credit markets break down for various reasons, such as lenders' lack of capital (Bernanke and Lown, 1991), the delay of information in booms (Gorton and Ordoñez, 2014; Asriyan, Laeven, and Martin, 2018), foreign investors' withdrawal (Van Nieuwerburgh and Veldkamp, 2009; Koijen, Koulischer, Nguyen, and Yogo, 2020), and ambiguity in risk evaluation Boyarchenko (2012); Caballero and Simsek (2013); Drechsler (2013). In the U.S., credit markets were under serious stress during the Covid-19 pandemic before government intervention (Falato, Goldstein, and Hortaçsu, 2020; Haddad, Moreira, and Muir, 2020; Halling, Yu, and Zechner, 2020; Kargar, Lester, Lindsay, Liu, Weill, and Zúñiga, 2020). A similar market breakdown happened in the global financial crisis (Acharya, Schnabl, and Suarez, 2013; Brunnermeier, 2009; Gorton, Laarits, and Metrick, 2017; Kacperczyk and Schnabl, 2010; Krishnamurthy, 2010).

[18] Please refer to Gertler and Kiyotaki (2010), Gertler and Karadi (2011), Gertler, Kiyotaki, and Queralto (2012), Araújo, Schommer, and Woodford (2015), and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017).

firm with shock u_t that borrows $g_t^j(u_t)k_{t-}^j$, the repayment is

$$q_t^j R_{G,t}\left(g_t^j(u_t)\right)k_{t-}^j = q_t^j \gamma_t g_t^j(u_t) k_{t-}^j\,, \tag{11}$$

where for simplicity, we consider a linear repayment schedule, i.e., $R_{G,t}\left(g_t^j(u_t)\right) = \gamma_t g_t^j(u_t)$.

In practice, the government typically chooses the size of a rescue package. Let $\overline{g}_t$ denote the ratio of government funding to the pre-crisis level of aggregate capital, K_{t-}. The equilibrium cost of government funding, γ_t, is thus determined by equating the demand and supply:

$$K_{t-}^H \int_{u=0}^1 g_t^H(u,\gamma_t)dG(u) + K_{t-}^L \int_{u=0}^1 g_t^L(u,\gamma_t)dG(u) = \overline{g}_t K_{t-}. \tag{12}$$

Dividing both sides of the equation by K_{t-}, we obtain

$$\omega_{t-} \int_{u=0}^1 g_t^H(u,\gamma_t)dG(u) + (1-\omega_{t-}) \int_{u=0}^1 g_t^L(u,\gamma_t)dG(u) = \overline{g}_t. \tag{13}$$

In our model, all quantities are scaled by K_t at the aggregate level and k_t^j at the firm level. Because individual firms take γ_t as given, our analysis of an individual firm's problem will be based on γ_t instead of $\overline{g}_t$. However, as will be shown below, there exists a unique mapping from $\overline{g}_t$ to γ_t.

In this economy, crises have cleansing effects. A firm's financial constraint depends on the unit value of its capital, q_t^j, and the shock size u_t. For firms with a sufficiently low u_t, the collateral constraint may not bind, and therefore, the investment is set at the target level defined in (4). As will be discussed below, because of the productivity wedge (i.e., $A^H > A^L$), the unit value of type-H capital, q_t^H, is higher than that of type-L capital, q_t^L. Therefore, among the firms without a binding financial constraint, type-H firms invest more than type-L firms, which implies an upward jump in ω_t (i.e., the fraction of capital being type-H, defined in (2)). Such a cleansing effect is also active through firms' investment in normal times. Another cleansing effect works through the financial constraint. Because $q_t^L < q_t^H$ in equilibrium, among the firms that draw a sufficiently high u_t and face a binding financial constraints, type-L firms face a tighter financial constraint than type-H firms and therefore invest less. This also implies an upward jump in ω_t.

The government funding dampens the cleansing effects, resulting in a smaller upward jump in ω_t. Firms with a binding financial constraint seek funding from the government. In spite of their types, they face the same repayment (capital units paid to the government). Therefore, while the government funding successfully generates more capital (i.e., the total production capacity $K_t = K_t^H + K_t^L$), it biases downward the quality distribution represented by ω_t. Next, we characterize the equilibrium dynamics, with a focus on the difference between the laissez-faire economy and the one with government funding.

3 Benchmark Economy: The Cleansing Effects of Crises

To examine the impact of government funding, we establish the laissez-faire benchmark. First, we analyze a single firm's investment problem. As previously discussed, a firm invests at the targeted level given by (4) in normal times (i.e., when the idiosyncratic investment opportunities arrive). In a crisis, a type-j firm with shock u_t solves the following optimization problem:

$$\pi(u_t, q_t^j)k_{t-}^j \equiv \max_x q_t^j F\left(x\right) k_{t-}^j - x k_{t-}^j\,, \tag{14}$$

subject to the collateral constraint (8). We introduce $\pi(u_t, q_t^j)$ to denote the maximized profits per unit of capital in a crisis. The optimal choice, $x^j(u_t, q_t^j)$, depend on u_t through the collateral constraint (8) and capital value q_t^j. If u_t is sufficiently low such that the collateral value is greater than the targeted level of investment (i.e., $\bar{\iota}_t^j < \chi q_t^j(1-u_t)$), the firm attains the investment target: $x^j(u_t, q_t^j) = \bar{\iota}_t^j$. However, if $\bar{\iota}_t^j > \chi q_t^j(1-u_t)$, the collateral constraint (8) binds (i.e., $x^j(u_t, q_t^j) = \chi q_t^j(1-u_t)$). The following proposition summarizes the results.

Proposition 1 (Benchmark Economy: Investment and Financing in Crises) *If $u_t \le \hat{u}(q_t^j)$, where $\hat{u}(q_t^j)$ is defined by $\bar{\iota}_t^j = \chi q_t^j\left(1-\hat{u}(q_t^j)\right)$ and $\bar{\iota}_t^j$ is solved in (4), the collateral constraint does not bind, and the firm attains the investment target: $x^j(u_t, q_t^j) = \bar{\iota}_t^j$. If $u_t > \hat{u}(q_t^j)$, the collateral constraint binds, and the firm under-invests: $x^j(u_t, q_t^j) = \chi q_t^j(1-u_t) < \bar{\iota}_t^j$.*

Under Proposition 1, the law of motion of aggregate type-j capital is given by

$$\frac{dK_t^j}{K_{t-}^j} = \left(-\delta + \lambda_I F(\bar{\iota}_{t-}^j)\right)dt + \Delta_t^j dN_t, \tag{15}$$

where, as previously discussed, $\bar{\iota}^j_{t-}$ is given by (4), and the jump in a crisis is

$$\Delta^j_t = \int_0^{\hat{u}(q^j_t)} F(\bar{\iota}^j_t) dG(u) + \int_{\hat{u}(q^j_t)}^{\upsilon} F(\chi(1-u)q^j_t) dG(u) - U, \tag{16}$$

and the average size of capital-destruction shock is a constant $U \equiv \mathbb{E}[u_t]$.

The focus of our analysis is the aggregate output, which depends on both the firm quality distribution and the aggregate capital stock: per unit of time, the aggregate output is

$$Y_t = K^H_t A^H + K^L_t A^H = K_t \left(\omega_t A^H + (1-\omega_t) A^L\right). \tag{17}$$

Using the notations in (16), we solve the law of motion of the aggregate capital stock, K_t:

$$\frac{dK_t}{K_{t-}} = \underbrace{\left[-\delta + \lambda_I \left(\omega_{t-} F(\bar{\iota}^H_{t-}) + (1-\omega_{t-}) F(\bar{\iota}^L_{t-})\right)\right]}_{\equiv \mu^K_t(\omega_{t-})} dt + \underbrace{\left(\omega_{t-}\Delta^H_t + (1-\omega_{t-})\Delta^L_t\right)}_{\equiv \Delta^K_t(\omega_{t-})} dN_t\,. \tag{18}$$

Capital quality (i.e., the fraction of capital being of H type) has the following law of motion:

$$d\omega_t = \underbrace{\omega_{t-}\left(1-\omega_{t-}\right)\lambda_I \left(F(\bar{\iota}^H_{t-}) - F(\bar{\iota}^L_{t-})\right)}_{\equiv \mu^\omega_t(\omega_{t-})} + \Delta^\omega_t(\omega_{t-}) dN_t, \tag{19}$$

where

$$\Delta^\omega_t(\omega_{t-}) = \frac{\omega_{t-}(1+\Delta^H_t)}{\omega_{t-}(1+\Delta^H_t) + (1-\omega_{t-})(1+\Delta^L_t)} - \omega_{t-}. \tag{20}$$

The following proposition states that capital values and investment targets are constant.

Proposition 2 (Benchmark Equilibrium) *In the laissez-faire economy, capital value, q^j, and investment target (and normal-time investment, given by (4)), $\bar{\iota}^j$, are constant in equilibrium ($j \in H, L$). Capital value, q^j, is solved by*

$$r = \frac{A^j}{q^j} - \delta + \frac{\lambda_I \left(q^j F\left(\bar{\iota}^j\right) - \bar{\iota}^j\right)}{q^j} + \frac{\lambda \Pi(q^j)}{q^j} - \lambda U, \tag{21}$$

where $\Pi(q^j) \equiv \mathbb{E}[\pi(u_t, q^j)]$ is the u_t-average investment profits in a crisis.

Equation (21) states that the required return, r, is equal to the total return on capital that constitutes production, depreciation, expected investment profits in normal times, expected investment profits in crises, and expected capital destruction in crisis. Intuitively, the capital of type-H firms has a higher unit value as it has a higher productivity than the capital of type-L firms (i.e., $A^H > A^L$).

Corollary 1 (Benchmark Economy: Capital Value Wedge) *In equilibrium,* $q^H > q^L$.

Crises have cleansing effects through two channels. First, because $q^H > q^L$, the type-H firms have a higher investment target, (i.e., $\bar{\iota}^H > \bar{\iota}^L$ from (4)). Therefore, among the unconstrained firms, type-H firms invest more. Moreover, $q^H > q^L$ implies that type-H firms have more funding to invest as the right side of its collateral constraint (8) is higher than that of type-L firms.

Proposition 3 (Benchmark Economy: The Cleansing Effect) *In equilibrium,* $\Delta_t^{\omega}(\omega_{t-}) > 0$.

In sum, the aggregate dynamics of the laissez-faire economy are given by the laws of motion of K_t and ω_t, respectively, (18) and (19), and in equilibrium, the type-H capital share, ω_t, jumps upward, reflecting a positive cleansing effect of crisis on the overall capital quality.

4 Equilibrium under Government Intervention

In this section, we analyze the equilibrium where the government intermediates funding and is free from firms' limited commitment problem. We compare it with the benchmark equilibrium.

4.1 Investment and Financing

As previously discussed, we focus on the equilibrium where the firm attains its investment target when the idiosyncratic investment opportunities arrive (i.e., in normal times). In a crisis, a type-j firm with shock u_t solves the following optimization problem:

$$\pi(u_t, q_t^j, \gamma_t)k_{t-}^j \equiv \max_{x\geq 0,\, g\geq 0} q_t^j F\left(i\right) k_{t-}^j - xk_{t-}^j - q_t^j \gamma_t g k_{t-}^j \,, \tag{22}$$

subject to the constraint (8) and $i = x + g$. As in the benchmark economy, we use $\pi(u_t, q_t^j, \gamma_t)$ to denote the maximized investment profits per unit of capital. The optimal choices, $x^j(u_t, q_t^j, \gamma_t)$ and $g^j(u_t, q_t^j, \gamma_t)$, depend on u_t through the collateral constraint (8), capital value q_t^j, and the units of capital that are repaid to the government per unit of funding, γ_t.

If the government funding is cheaper than the fairly priced private funding (i.e., $q_t^j \gamma_t < 1$), the firm fully relies on the government funding (i.e., $x^j(u_t, q_t^j, \gamma_t) = 0$ and $i^j(u_t, q_t^j, \gamma_t) = g_t^j(u_t, q_t^j, \gamma_t)$). The first-order condition for g,

$$q_t^j F'\left(g_t^j(u_t, q_t^j, \gamma_t)\right) = q_t^j \gamma_t, \tag{23}$$

implies that $q_t^j F'\left(g_t^j(u_t, q_t^j, \gamma_t)\right) < 1$ so the firm over-invests, $i^j(u_t, q_t^j, \gamma_t) = g_t^j(u_t, q_t^j, \gamma_t) > \bar{\iota}_t^j$. In this case, the collateral constraint on private funding becomes irrelevant.

The collateral constraint is also irrelevant when government funding is fairly priced (i.e., $q_t^j \gamma_t = 1$). In this case, the firm invests at the targeted level, $i^j(u_t, q_t^j, \gamma_t) = \bar{\iota}_t^j$. How $i^j(u_t, q_t^j, \gamma_t)$ is allocated between $x^j(u_t, q_t^j, \gamma_t)$ and $g^j(u_t, q_t^j, \gamma_t)$ (i.e., the mixture of funding) is indeterminate.

Next, we analyze the case where government funding is more expensive than private funding (i.e., $q_t^j \gamma_t > 1$). If u_t is sufficiently low such that the collateral value is greater than the targeted level of investment (i.e., $\bar{\iota}_t^j < \chi q_t^j (1 - u_t)$), the firm fully replies on private funding to achieve the investment target: $i^j(u_t, q_t^j, \gamma_t) = x^j(u_t, q_t^j, \gamma_t) = \bar{\iota}_t^j$ and $g_t^j(u_t, q_t^j, \gamma_t) = 0$. However, if $\bar{\iota}_t^j > \chi q_t^j (1 - u_t)$, the collateral constraint (8) binds (i.e., $x^j(u_t, q_t^j, \gamma_t) = \chi q_t^j (1 - u_t)$). Whether the firm seeks funding from the government depends on the cost γ_t. At $g = 0$ and $i = x^j(u_t, q_t^j, \gamma_t) = \chi q_t^j (1 - u_t)$, the marginal benefit of government funding is $q_t^j F'\left(\chi q_t^j (1 - u_t)\right)$, and the marginal cost is $q_t^j \gamma_t$. Therefore, the optimal $g_t^j(u_t, q_t^j, \gamma_t) > 0$ if and only if $q_t^j F'\left(\chi q_t^j (1 - u_t)\right) > q_t^j \gamma_t$, i.e., the units of newly created capital are greater than the units of capital repaid to the government: $F'\left(\chi q_t^j (1 - u_t)\right) > \gamma_t$. The optimal amount of government funding is given by

$$F'\left(\chi q_t^j (1 - u_t) + g_t^j(u_t, q_t^j, \gamma_t)\right) = \gamma_t. \tag{24}$$

To summarize the optimal funding choices, we define the two thresholds, $\hat{u}(q_t^j)$ and $\tilde{u}(q_t^j, \gamma_t)$:

$$\bar{\iota}_t^j = \chi q_t^j \left(1 - \hat{u}(q_t^j)\right), \tag{25}$$

and

$$F'\left(\chi q_t^j (1 - \tilde{u}(q_t^j, \gamma_t))\right) = \gamma_t. \tag{26}$$

A firm with $u_t > \hat{u}(q_t^j)$ faces a binding financial constraint, and a firm with $u_t > \tilde{u}_t^j$ seeks over-priced funding from the government after it exhausts the fairly priced private funding. The following proposition summarizes the investment and financing decisions of the type-j firm with shock

u_t in a crisis. We provide the proof in the appendix.

Proposition 4 (Investment and Financing in Crises) *In a crisis, when government funding is underpriced (i.e., $q_t^j \gamma_t < 1$), the firm fully relies on government funding and over-invests, i.e., $x^j(u_t, q_t^j, \gamma_t) = 0$ and $i^j(u_t, q_t^j, \gamma_t) = g_t^j(u_t, q_t^j, \gamma_t) > \bar{\iota}_t^j$, where $g_t^j(u_t, q_t^j, \gamma_t)$ is given by (23).*

When government funding is fairly priced (i.e., $q_t^j \gamma_t = 1$), we have $i^j(u_t, q_t^j, \gamma_t) = \bar{\iota}_t^j$ and the allocation of $i^j(u_t, q_t^j, \gamma_t)$ between $x^j(u_t, q_t^j, \gamma_t)$ and $g^j(u_t, q_t^j, \gamma_t)$ is indeterminate.

When government funding is overpriced (i.e., $q_t^j \gamma_t > 1$), there are three scenarios:

(1) If $u_t \leq \hat{u}(q_t^j)$, the collateral constraint does not bind, and the firm attains the investment target by fully relying on private funding: $x^j(u_t, q_t^j, \gamma_t) = \bar{\iota}_t^j$ and $g_t^j(u_t, q_t^j, \gamma_t) = 0$.

(2) If $u_t \in \left(\hat{u}(q_t^j), \tilde{u}(q_t^j, \gamma_t)\right]$, the collateral constraint binds, and the firm fully relies on private funding and invests below the targeted level: $i^j(u_t, q_t^j, \gamma_t) = x^j(u_t, q_t^j, \gamma_t) = \chi q_t^j (1 - u_t) < \bar{\iota}_t^j$ and $g_t^j(u_t, q_t^j, \gamma_t) = 0$.

(3) If $u_t > \tilde{u}(q_t^j, \gamma_t)$, the firm exhausts its private-funding capacity, seeks government funding, and invests below the targeted level: $x^j(u_t, q_t^j, \gamma_t) = \chi q_t^j (1 - u_t)$, $g_t^j(u_t, q_t^j, \gamma_t)$ given by (24), and $i^j(u_t, q_t^j, \gamma_t) = x^j(u_t, q_t^j, \gamma_t) + g^j(u_t, q_t^j, \gamma_t) < \bar{\iota}_t^j$.

To sharpen the intuitions, we consider the following functional form of $F(\cdot)$:

$$F(i) = \phi \log(i/\underline{\iota}) \,, \tag{27}$$

where the parameters ϕ and $\underline{\iota}$ are positive constants. Through (4), the investment function implies

$$\bar{\iota}_t^j = q_t^j \phi. \tag{28}$$

The assumption that the collateral constraint (6) does not bind in normal times, i.e., $\bar{\iota}_t^j \leq \chi q_t^j$, is equivalent to the following parameter restriction that we maintain throughout the paper:

$$\phi \leq \chi. \tag{29}$$

Under the logarithm investment function, both types of firms have the same threshold (not dependent on q_t^j) for u_t that determines whether the collateral constraint binds in a crisis:

$$\hat{u}(q_t^H) = \hat{u}(q_t^L) = 1 - \frac{\phi}{\chi}. \tag{30}$$

Therefore, in a crisis, firms with $u_t > 1-\phi/\chi$ face a binding collateral constraint.[19] Next, we solve the threshold for u_t that determines whether the firm seeks funding from the government when the funding is overpriced (i.e., the equation (24) under $q_t^j \gamma_t > 1$):

$$\tilde{u}(q_t^j, \gamma_t) = 1 - \frac{\phi}{\chi q_t^j \gamma_t}. \tag{31}$$

Because $q_t^j \gamma_t > 1$, we have $\tilde{u}(q_t^j, \gamma_t) > \hat{u}(q_t^j)$. A type-$j$ firm with $u_t > \tilde{u}(q_t^j, \gamma_t) = 1 - \phi/\chi q_t^j \gamma_t$ faces a binding collateral constraint and seeks overpriced government funding.

Finally, when government funding is underpriced, (23) implies

$$g_t^j(u_t, q_t^j, \gamma_t) = \frac{\phi}{\gamma_t}. \tag{32}$$

When government funding is overpriced, (24) implies

$$g_t^j(u_t, q_t^j, \gamma_t) = \frac{\phi}{\gamma_t} - \chi q_t^j (1 - u_t). \tag{33}$$

Figure 1 illustrates a firm's investment and financing choices. In Panel A and C, we plot the investment and demand for government funding of a financially unconstrained firm (i.e., with $u_t < \hat{u}(q_t^j)$). The firm only seeks government funding when it is cheaper than private funding (i.e., when $\gamma < 1/q_t^j$), and in that case, the firm over-invests. Panel B and D illustrate the investment and demand for government funding of a financially constrained firm. When $\gamma < 1/q_t^j$, the firm fully relies on government funding as it is cheaper than private funding, and it over-invests. At $\gamma = 1/q_t^j$, the firm is indifferent between the two sources of funding and attains its investment target. Once γ passes $1/q_t^j$, the firm prioritizes the cheaper private funding, and its demand for government funding decreases gradually as it becomes increasingly expensive (i.e., γ_t increases). These results on firms' investment and financing decisions serve as the cornerstone of our equilibrium analysis.

4.2 The Distortionary Effects of Policy Intervention

We analyze a stationary equilibrium where γ_t is a constant. The stationary equilibrium under a constant γ presents the main mechanisms in a transparent fashion and allows a direct compari-

[19] To rule out the case where no firm faces a binding collateral constraint, we impose the following parameter restriction on the upper bound υ of the capital destruction shock u_t: $\upsilon > 1 - \phi/\chi$.

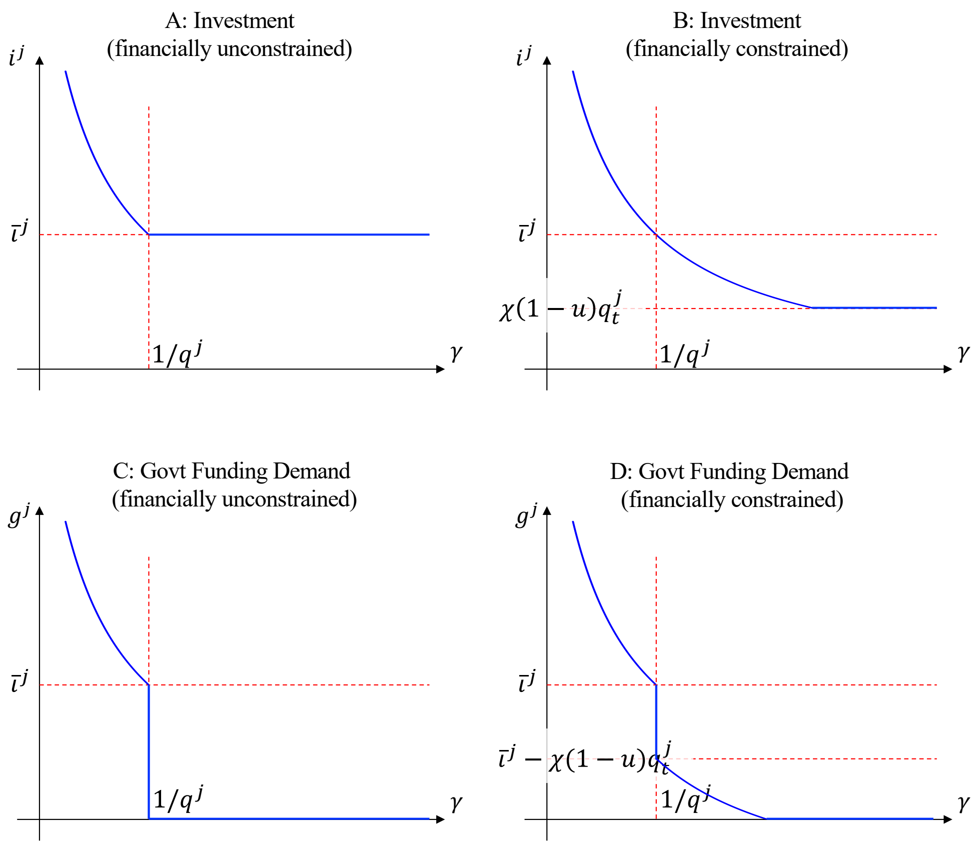

Figure 1: **Investment and Demand for Government Funding in a Crisis.**

son with the laissez-faire economy. Similar to Proposition 2, the next proposition significantly simplifies our analysis by showing that capital value and investment target are constant.

Proposition 5 (Stationary Equilibrium) *When γ_t is constant, capital value, q^j, and investment target (and normal-time investment, given by (4)), $\bar{\iota}^j$ are constant in equilibrium. Capital value, q^j, is solved by*

$$r = \frac{A^j}{q^j} - \delta + \frac{\lambda_I \left(q^j F\left(\bar{\iota}^j\right) - \bar{\iota}^j\right)}{q^j} + \frac{\lambda \Pi(q^j, \gamma)}{q^j} - \lambda U. \tag{34}$$

where $\Pi(q^j, \gamma) \equiv \mathbb{E}[\pi(u_t, q^j, \gamma)]$ is the u_t-average profits in a crisis. Moreover, $q^H > q^L$.

From Proposition 4, it is clear that the government should not set $\gamma > 1/q^L$ (i.e., the credit support is overpriced even from the type-L firms' perspective). If $\gamma > 1/q^L$, reducing γ moves both types of firms' investment closer to their targeted level. Moreover, setting $\gamma < 1/q^H$ (i.e., underpricing credit for both types) is also not optimal. By Proposition 4, $\gamma < 1/q^H$ induces over-investment of both types at the expense of consumption, reducing welfare. Therefore, we consider

$$\gamma \in [1/q^H,\, 1/q^L]\,. \tag{35}$$

According to Proposition 4, type-L firms fully rely on the government funding in crisis, while type-H firms face overpriced government funding when $\gamma > 1/q^H$.

Under government credit support, the laws of motion of K_t (capital quantity) and ω_t (capital quality) are still given by (18) and (19), respectively, but the jump sizes in a crisis now differ. Specifically, for the H type, $q^H\gamma \geq 1$, the jump in type-H capital is given by

$$\begin{aligned}\Delta^H = &\int_0^{\hat{u}(q^H)} F(\bar{\iota}^H)dG(u) + \int_{\hat{u}(q^H)}^{\tilde{u}(q^H,\gamma)} F(\chi(1-u)q^H)dG(u) \\ &+ \int_{\tilde{u}(q^H,\gamma)}^{\upsilon} F(\chi(1-u)q^H + g^H(u,q^H,\gamma))dG(u) - U,\end{aligned} \tag{36}$$

where the three integrals correspond to the three scenarios in Proposition 4 under overpriced government funding (i.e., $q^H\gamma > 1$). The thresholds, $\hat{u}(q^H)$ and $\tilde{u}(q^H,\gamma)$, are given by (30) and (31), respectively, and $g^H(u,q^H,\gamma)$ is given by (33). Note that by the continuity of integral operators, this expression also applies to the case where $q^H\gamma = 1$. For the L type, we have

$$\Delta^L = \int_{u=0}^{\upsilon} F\left(g^L(u,q^L,\gamma)\right) dG(u) - U, \tag{37}$$

where $g^H(u,q^H,\gamma)$ is given by (23). With Δ_t^H and Δ_t^L, the jumps of K_t is $\left(\omega_{t-}\Delta_t^H + (1-\omega_{t-})\Delta_t^L\right)$ and the jump size of ω_t is given by (20), as in the benchmark economy.

The next proposition states the trade-off that the government faces. More lenient credit pricing (i.e., reducing γ) results in more investment but biases the quality distribution downward.

Proposition 6 (Credit Policy and Aggregate Dynamics) *Given q^H and q^L, a more lenient government policy (i.e., a lower γ) reduces the decline of capital quantity but also weakens the cleansing effect, i.e.,*

$$\frac{\partial \Delta_t^K}{\partial \gamma} < 0\,, \quad \frac{\partial \Delta_t^\omega}{\partial \gamma} > 0$$

More lenient credit pricing stimulates firms' investment but dampens the positive cleansing

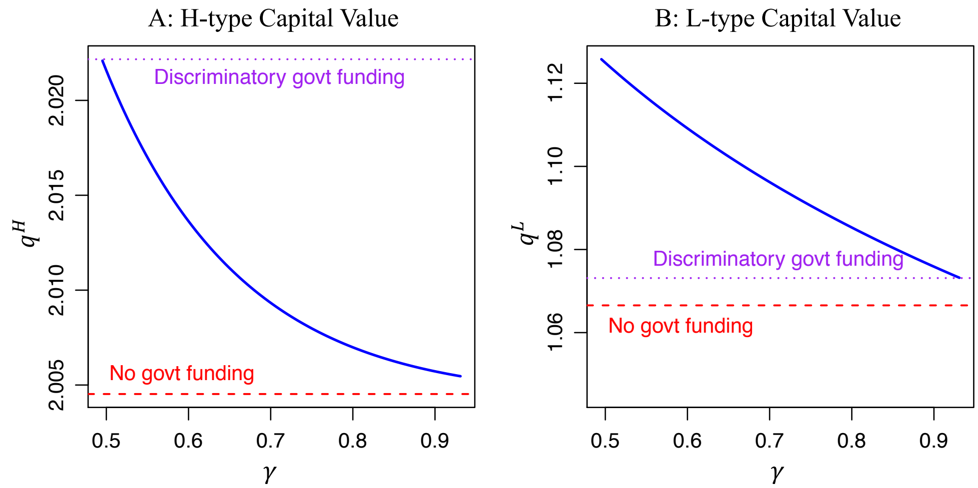

Figure 2: **Capital Value and Government Funding.**

effect of crises on the firm quality distribution. As previously discussed, the cleansing effect works through two channels. First, type-H firms want to invest more than type-L firms ($\bar{\iota}^H > \bar{\iota}^L$ from $q^H > q^L$ in Proposition 5). Second, type-H firms can invest more as their collateral value is higher. Government funding has a negative impact on both margins. First, as type-L firms fully rely on government funding, the collateral constraint becomes irrelevant for them. Second, government funding can cause an increase in q^L that is greater than an increase in q^H, resulting in a narrower wedge in capital value between the two types and, consequently, a narrower wedge between their targeted levels of investment. The mechanism works as follows. More lenient credit pricing implies more subsidy to type-L firms in all circumstances in crises, as they seek government funding at any value of u_t (see Proposition 4). However, type-H firms only benefit in the states where u_t is sufficiently high. Therefore, an reduction in γ can boost the expected investment profits for type-L firms more than for type-H firms, causing a percentage increase in q^L more than in q^H.

Figure 2 shows the positive impact of government funding on type-H capital value (Panel A) and type-L capital value (Panel B). In both panels, capital value increases when the cost of government funding, γ, decrease. We compare our economy with the benchmark economy without government intervention and with a hypothetical economy where the government differentiates firms of different types and set $\gamma^H = 1/q^H$ for type-H firms and $\gamma^L = 1/q^L$ for type-L firms. The discriminatory pricing of government funding essentially achieves the first-best scenario where financial constraints become irrelevant, because even though private funding requires collateral,

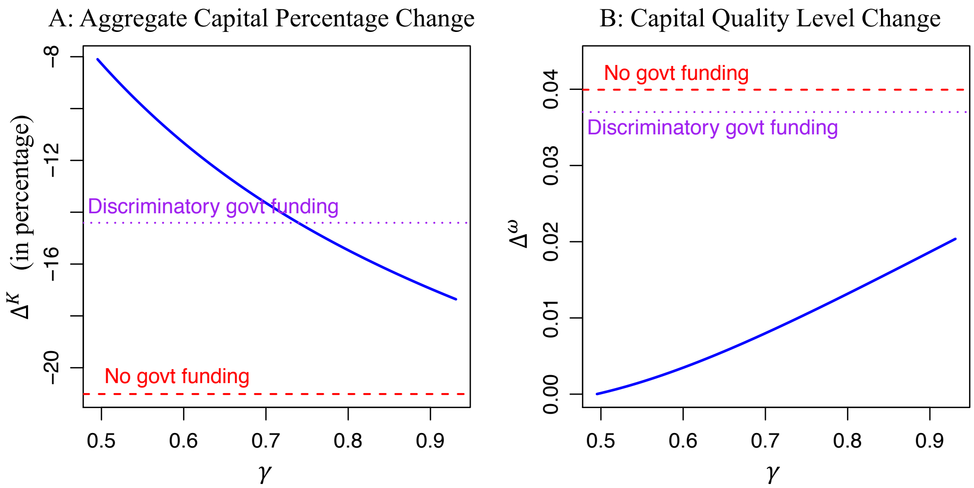

Figure 3: **Crisis Dynamics and Government Funding.**

government funding is free from firms' limited commitment and is also fairly priced.

Intuitively, q^H is the highest in the first-best scenario and lowest in the benchmark economy in Panel A of Figure 2. Government funding with $\gamma \in [1/q^H, 1/q^L]$ is either fairly or overpriced from a type-H firm's perspective, so lowering γ increases q^H by increasing the firm's profits in crises but q^H cannot exceed the first-best level. In contrast, q^L is higher than the first-best level in Panel B, because underpriced government funding offers a subsidy to type-L firms in crisis.

In sum, government funding reduces the cleansing effects of crises by both allowing type-L firms to invest more and raising type-L firms' investment targets more than that of type-H firms. Figure 3 illustrates the impact of government funding in a crisis. The solid line in Panel A plots the jump in K_t, which decreases in γ, and Panel B plots the jump in ω_t, which increases in γ. If the government wants to preserve the total production capacity, K_t, by offering cheaper credit (i.e., reducing γ), it dampens the cleansing effect of the crisis, biasing downward capital quality.

In both panels of Figure 3, we compare the jump size with that from the laissez-faire economy and with that under a fair and discriminatory credit policy (i.e., $q^H\gamma^H = 1$ and $q^L\gamma^L = 1$). In Panel A, the solid line is above the dashed line of the laissez-faire benchmark, showing that government funding successfully rescues the production capacity relative to the benchmark. It also shows that relative to the benchmark of fairly priced credit (dotted line), the rescue overshoots when γ is low and government funding is too lenient, encouraging type-L firms to over-invest.

The fairly priced government funding effectively eliminates the impact of financial constraints

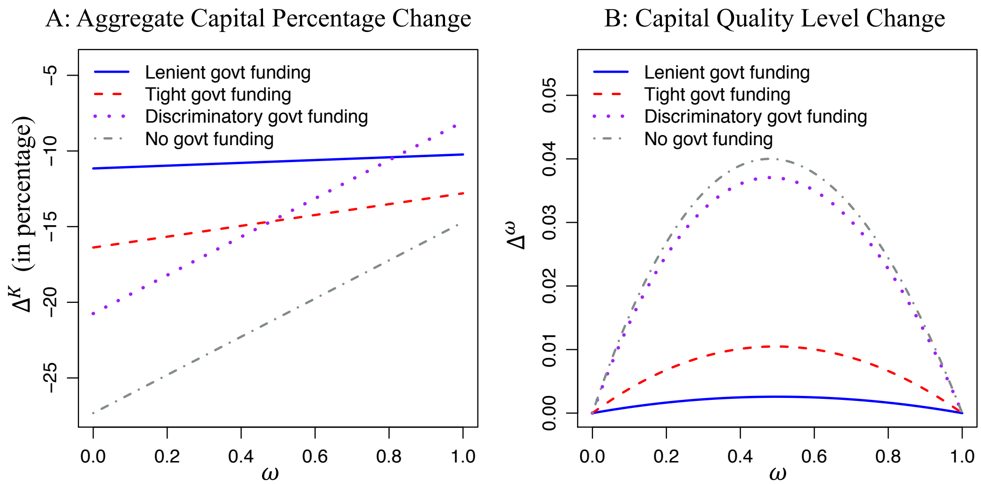

Figure 4: **Capital Quality and Crisis Dynamics.**

and allows both types to attain the targeted levels of investment. As previously discussed, the financial constraint contributes to the cleansing effect because, in a crisis, type-H firms have greater collateral values than type-L firms, and both types under-invest. In Panel B of Figure 3, we show that the fairly priced government funding (dotted line) reduces the cleansing effect by eliminating this inefficient channel. In comparison, the non-discriminatory pricing results in a greater (and inefficient) reduction of the cleansing effect, as shown by the solid line being below the dotted line. For any $\gamma \in (1/q^H, 1/q^L)$, government funding is underpriced for type-L firms and overpriced for type-H firms, so all type-L firms seek government funding and over-invest while a subset of type-H firms with sufficiently high u_t borrow and still under-invest (see Proposition 4).

4.3 The Dynamic Effects of Capital Quality Distortions

The distortions in a crisis brought by government funding have two important dynamic effects. As the government funding biases capital quality downward in a crisis, the economy enters into the next crisis with a lower capital quality. In Panel A of Figure 4, we plot the percentage drop of aggregate capital, K_t, against the pre-crisis capital quality, ω_{t-} in four scenarios: (1) lenient government funding (low γ, solid line), (2) tight government funding (high γ, dashed line), (3) discriminatory government funding ($\gamma^H = 1/q^H$ and $\gamma^L = 1/q^L$, dotted line), and (4) the laissez-faire economy (dash-dotted line). In all four scenarios, the percentage decline in K_t is deeper when

ω_{t-} is lower. Therefore, even though government funding can reduce the drop in K_t in the current crisis, the resultant downward bias in ω_t implies a greater drop in the next crisis.

The different slopes across the four scenarios in Panel A of Figure 4 reveals that policy intervention in a crisis should be conditioned on the firm quality distribution that the economy carries into the crisis. Near the right end of the curves where ω_{t-} is close to one, the economy enters into a crisis with a dominant share of high-quality firms. In this case, the outcome of a lenient pricing of government funding (i.e., low γ) is closest to the outcome of the first-best (discriminatory) credit pricing that effectively eradicates financial constraints. Intuitively, the cost of providing government funding is the overinvestment of type-L firms, which is not much of a concern when ω_{t-} is close to one. The government funding mainly serves to alleviate the underinvestment problem of type-H firms. In contrast, when ω_{t-} is close to zero, a tight supply of government funding (i.e., high γ) moves the economy closer to the first-best scenario, while the lenient pricing of government funding overshoots, causing severe overinvestment of type-L firms.

Panel B of Figure 4 again shows that the impact of policy intervention depends on the firm quality distribution that the economy carries into a crisis. If the economy enters into a crisis with ω_{t-} close to either zero (almost all firms are L type) or one (almost all firms are H type), the differences in the pricing of government funding do not result in large differences in the post-crisis value of ω_t. As shown in (20), this is a standard base effect – if one type constitutes a negligible share of the production sector, how it is treated differently from the other type does not affect the composition significantly. The pricing of government funding becomes a prominent issue when ω_{t-} is in the middle range, and as previously discussed, more lenient pricing results in a weaker cleansing effect (i.e., a small upward jump in ω_t) by inducing type-L firms to over-invest more.

In sum, the distortions in the firm quality distribution brought by government funding affect not only the economic outcome in the current crisis but also the aggregate dynamics in the next crisis. A downward bias in capital quality (ω_t), which is a necessary side-effect of rescuing the overall production capacity (K_t), exacerbates the drop in K_t in the next crisis. Moreover, it is important to condition policy intervention on the firm quality distribution. Specifically, in an economy with a balanced mix of high- and low-quality firms, different policy choices yield quite distinct outcomes.

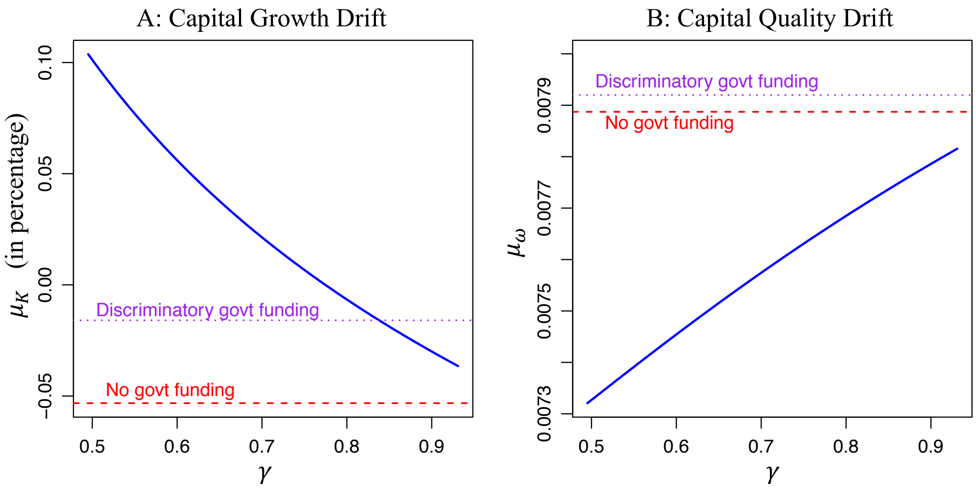

Figure 5: **Policy Intervention in Crises and Pre-Crisis Dynamics.**

4.4 The Expectation Effects of Policy Intervention

Next, we characterize another aspect of the dynamic effects of capital-quality distortions. When government funding is available in crises, firms rationally expect it and adjust their investment policies in normal times. Specifically, type-L firms expect to be subsidized and, as their capital value (or Tobin's q), q^L, increases. Their investment rate in normal times increases when the idiosyncratic investment opportunities arrive (see (4)), resulting in a reduction in the drift of ω_t (see (19)) and a downward bias in capital quality outside of crises. Admittedly, government funding also raises q^H and, as a result, H-type firms' investment rate also increases outside of crises. However, such impact is weaker than that on type-L firms, because as previously discussed, type-L firms benefit from government funding at all values of u_t, the capital destruction shock, while type-H firms only seek government funding when u_t is sufficiently large.

Capital value reflects firms' rational expectation of future crises and investment opportunities. Therefore, the distortions in capital value capture how credit policy in crises affects firms' expectations and investment decisions in normal times. As the normal-time investment wedge narrows, the drift of ω_t declines (see (19)), slowing down the gradual improvement of firm quality distribution. In Panel B of Figure 5, we show that the drift of ω_t (normal-time growth) increases as the government tightens its supply of funding in crises (i.e., γ increases). The drift of ω_t is higher when government funding is eliminated, suggesting that by dampening the cleansing effect, government funding negatively affects capital quality not only in crisis but also in normal times when firms' investment is guided by their expectations of policy interventions in future crises via capital values.

In Panel A of Figure 5, we show that the expected growth rate of K_t against γ. When γ is sufficiently low, K_t grows at a faster pace than what is implied by the first-best (discriminatory) pricing of government funding. The overinvestment problem of type-L firms in crises propagates into normal times as q^L increases, driving up the normal-time investment rate. Overinvestment comes at the cost of consumption, so in the next subsection, we provide a welfare analysis that comprehensively evaluates the impact of policy intervention on production and consumption.

4.5 Welfare

At time t, the social welfare is defined as the present value of household consumption streams

$$\mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)} \left(\begin{array}{l} (\omega_{s-}A^H + (1-\omega_{s-})A^L)K_{s-}ds - \lambda_I \left(\omega_{s-}\bar{\iota}_t^H + (1-\omega_{s-})\bar{\iota}_t^L\right) K_{s-}ds \\ - \left(\omega_{s-}\int_0^\upsilon i_t^H(u)dG(u) + (1-\omega_{s-})\int_0^\upsilon i_t^L(u)dG(u)\right) K_{s-}dN_s \end{array} \right)\right], \tag{38}$$

where, in the integral, we record the flow consumption in normal times net off the goods invested in normal times and in crises. To simplify the notation, we denote the aggregate investment in crises (scaled by the pre-crisis level of capital stock, K_{t-}) by I_t:

$$I_t \equiv \omega_{s-}\int_0^\upsilon i_t^H(u)dG(u) + (1-\omega_{s-})\int_0^\upsilon i_t^L(u)dG(u)\,. \tag{39}$$

The following proposition states the functional form of welfare (the planner's value function). In the appendix, we show how the welfare function is solved.

Proposition 7 (Social Welfare Function) *The social welfare at time t is a function of capital quantity, K_t, and capital quality, ω_t, in the form $W(\omega_t)K_t$, where the function $W(\omega)$ satisfies the following differential equation:*

$$\begin{aligned} rW(\omega) =& \omega A^H + (1-\omega)A^L - \lambda_I\left(\omega\bar{\iota}^H + (1-\omega)\bar{\iota}^L\right) + W(\omega)\mu_K(\omega) + W'(\omega)\mu_\omega(\omega) \\ & - \lambda I(\omega) + \lambda\left[W(\omega+\Delta^\omega(\omega))\left(1+\Delta^K(\omega)\right) - W(\omega)\right]\,. \end{aligned} \tag{40}$$

Panel A of Figure 6 shows the welfare improvement relative to the laissez-faire economy for three scenarios: (1) lenient government funding (low γ, solid line); (2) tight government funding

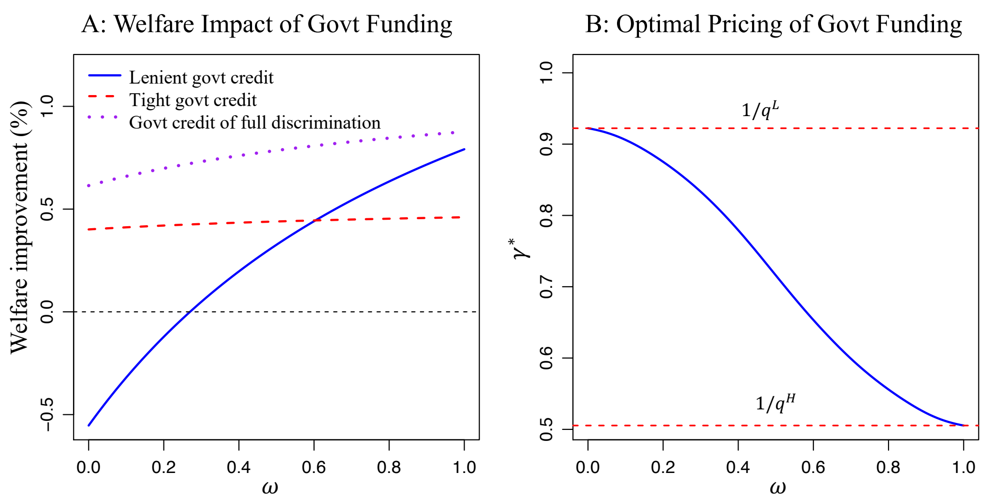

Figure 6: **Welfare and the Optimal Pricing of Government Funding.**

(high γ, dashed line); (3) discriminatory government funding ($\gamma^H = 1/q^H$ and $\gamma^L = 1/q^L$, dotted line). The last scenario shows the best possible improvement of welfare, as the fairly priced government funding effectively eliminates the impact of financial constraints.

A key message from Panel A of Figure 6 is that an ultra-lenient funding provision by the government destroys welfare when type-L firms dominate the economy (i.e., the left end of the solid curve). The overinvestment of type-L firms comes at the expense of aggregate consumption, so even though the total capital stock, K_t, grows faster, households' life-time consumption value declines. As L type's capital share shrinks (i.e., ω_t increases), such negative impact becomes smaller, so along the solid line, the welfare improvement increases in ω_t.

The curve of welfare improvement under a tight government funding supply stays above zero across different values of ω_t. When the government provides a relatively small amount of funding (or equivalent, prices funding at a relatively high γ), the marginal improvement of welfare due to type-H firms' efficient investment is large, while the resultant wasteful investment from type-L firms is still small. Therefore, a timid intervention almost guarantees a positive (but not necessarily great) outcome. This result favors gradualism in policy making, especially when the intervention cannot be discriminatory due to either the lack of information or political constraints.

Panel B of Figure 6 completes our analysis of welfare by plotting the optimal γ (which maximizes the time-0 welfare) against ω_0. Intuitively, on the left end where the economy is dominated with type-L firms, optimal intervention requires the government to price funding at the high end,

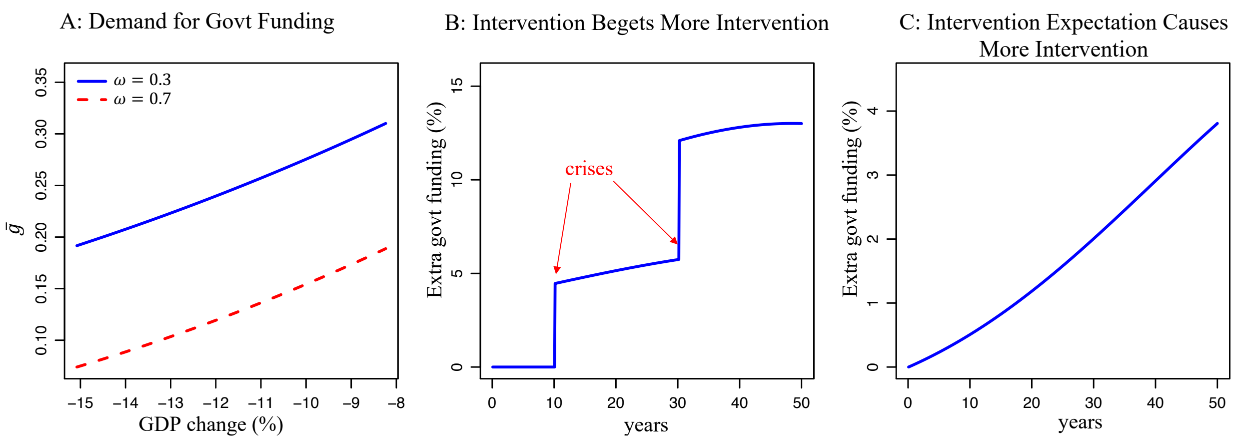

Figure 7: **The Slippery Slope of Government Funding.**

almost making type-L firms indifferent between government funding and private funding. This is motivated by the fact that a lenient pricing is more likely to result in type-L firms' wasteful investment than to boost type-H firms' efficient investment. In contrast, near the right end where type-H firms dominate, the optimal pricing leans towards being lenient, almost at the low end, to narrow the wedge between type-H firms' actual investment and their investment targets.

4.6 The Slippery Slope of Policy Intervention

The distortions in firm quality distribution brought by policy intervention imply a slippery slope of policy intervention. If the government aims to contain the output slump to a certain level in crises, intervention in one crisis begets interventions of greater scales in future crises. In Panel A of Figure 7, we plot the ratio of government funding to total capital stock against the percentage drop in output at two levels of ω_t. If we fix a value of output drop on the x-axis, the curves map out the minimal scale of government funding that can limit the decline of output up to that value. Naturally, the curves are upward-sloping: The smaller an output drop the government can accept, the larger scale of government funding it will have to provide. Across the levels of output drop, the necessary intervention is of a smaller scale when ω_t is higher (i.e., the economy has more high-quality firms). Therefore, when ω_t is biased downward by intervention in the current crisis, the economy enters into the next crisis with a lower ω_t (relative to the laissez-faire economy), which then implies a greater scale of intervention to contain the output drop to a certain level.

Panel B and C of Figure 7 show the two contributing factors behind the slippery slope of policy intervention: (1) the direct negative impact on ω_t in crises, (2) the expectation distortions. In Panel B, we calculate the amount of government funding that is needed to prevent an output drop from exceeding 10% *in the next crisis*, and compare it against the hypothetical amount of government funding needed should a policy intervention have never happened. The curve starts flat at zero because policy intervention has not yet happened. It jumps up to 5% when the first crisis and the first policy intervention happen after ten years, meaning that this intervention, by distorting ω_t, causes the government funding to be 5% higher in the next crisis than the amount needed in the absence of this intervention. When the next crisis hits in the thirtieth year, the second intervention induces more distortions in ω_t, thus further elevating the government funding needed in the next crisis relative to the amount needed without any of the two interventions. The drift upward between the two crises and after the second crisis is due to the fact that a distortion of ω_t by policy intervention has a persistent effect (see the law of motion given by (19)).

In Panel B of Figure 7, we simulate the paths using q^H and q^L from the laissez-faire economy to shut down the expectation effect (i.e., the distortion in ω_t due to firms' investments in normal times that incorporate expectations of future policy interventions). In Panel C of Figure 7, we focus on the expectation effect. In this graph, we calculate the amount of government funding that is needed to prevent an output drop from exceeding 10% *in the next crisis*, and compare it against the amount of funding needed in a hypothetical economy where intervention has never happened and firms do not expect interventions in crises. We do not input a crisis in the fifty years of simulation, but since firms expect government funding should a crisis occur, their investment decisions in normal times are distorted. In particular, ω_t is biased downward by type-L firms' overinvestment. Therefore, as time goes, the distortion in ω_t accumulates, implying that when the next crisis hits, the government has to provide more funding to prevent a 10% or higher drop of output than the amount of funding needed in the absence of firms' expectation of policy interventions.

In sum, policy intervention biases ω_t downwards in crises and in normal times (through firms' expectations of interventions). As a result, the economy enters into crises with a smaller share of firms being the high type than the laissez-faire benchmark, so the government funding needed to prevent a certain level of output drop is larger. Our model generates a slippery slope of policy intervention, a trap of policy makers' own making: Both the past interventions and agents' expectations of future interventions cause the government to spend more should a crisis occur. However, as previously discuss, this policy trap is a necessary evil because by relaxing firms' financial constraints in crises, especially those of the high-type firms, policy interventions can improve welfare.

4.7 Optimal Dynamic Intervention

So far, our analysis assumes a constant number of capital units, γ, charged by the government as repayments. Can the government avoid the slippery slope of intervention by dynamically adjusting the repayment rate? This is a important question, because in our model, the quality distribution, represented by ω_t, varies over time. When ω_t is low and there are many type-L firms, the government would prefer a high repayment rate and reduce its funding support, because the cost of type-L firms' overinvestment overweighs the benefit of relaxing type-H firms' financial constraints. In contrast, when ω_t is high, the government would prefer a lower repayment rate. Below we solve the optimal $\gamma(\omega_t)$ through the dynamic optimization of social welfare. We show that even though dynamically adjusted interventions improve welfare, it cannot help the government avoid the policy trap of interventions begetting more interventions.

When the cost of government funding depends on ω_t, both q_t^H and q_t^L become dependent on ω_t. For $j \in \{H,\ L\}$, the capital value has the following law of motion:

$$\frac{dq_t^j}{q_{t-}^j} = \mu_{q,t-}^j dt + \Delta_{q,t-}^j dN_t$$

With the capital value as a function of ω_t, (i.e., $q_t^j = q^j(\omega_t)$), we obtain the drift and jump size:

$$\mu_{q,t-}^j = \frac{dq^j(\omega_{t-})}{d\omega_{t-}} \mu^\omega(\omega_{t-}) dt,$$

and

$$\Delta_{q,t-}^j = \frac{q^j(\omega_{t-} + \Delta^\omega(\omega_{t-})) - q^j(\omega_{t-})}{q^j(\omega_{t-})},$$

where $\mu^\omega(\omega_{t-})$ and $\Delta^\omega(\omega_{t-})$ are functions of ω_{t-}, defined above in (19) and (20), respectively.[20]

The following equation of capital valuation is an ODE that solves $q^j(\omega_t)$:

$$r = \frac{A^j}{q_{t-}^j} + \mu_{q,t-}^j - \delta + \frac{\lambda_I \left(q_{t-}^j F(\overline{\iota}_{t-}^j) - \overline{\iota}_{t-}^j \right)}{q_{t-}^j} + \frac{\lambda \Pi_t^j}{q_{t-}^j} + \lambda \left((1 + \Delta_{q,t-}^j)(1-U) - 1 \right), \qquad (41)$$

where $\mu_{q,t-}^j$ depends on the first derivative of $q^j(\omega_t)$, $\overline{\iota}_{t-}^j$ depends on q_{t-}^j (see (4)), and the expected profits is the integral of investment profits in a crisis, $\pi(u_t, q^j(\omega_t), \gamma(\omega_t))$ in (22), over the c.d.f. of

[20] The calculation of $\Delta^\omega(\omega_{t-})$ requires $\Delta^H(\omega_{t-})$ and $\Delta^L(\omega_{t-})$ given by (36) and (37), respectively.

shock size, $G(u_t)$. Note that the investment profits in a crisis depends on the post-shock capital value, $q^j(\omega_t) = q^j(\omega_{t-} + \Delta^\omega(\omega_{t-}))$. Technically, equation (41) is an ODE with endogenous delay as it contains both the pre-shock and post-shock capital values. In comparison with the capital valuation equation (34) under a constant γ (and a constant q^j), the differences are in the additional drift term, $\mu^j_{q,t-}$, and the last term of return on capital in a crisis. Under a constant q^j, the return in a crisis is simply $-U$ (the fraction of capital being destroyed), but under a state-dependent q^j_t, we need to account for the capital reevaluation via $\Delta^j_{q,t-}$, so the total return is the product of reevaluation per unit of capital, $1 + \Delta^j_{q,t-}$, and the remaining fraction of capital, $1 - U$.

Solving the model requires jointly solving the two capital valuation ODEs above, for $j \in \{H, L\}$, and the following HJB equation of dynamic intervention and welfare optimization:

$$\begin{aligned} rW(\omega_{t-}) =& \omega_{t-}A^H + (1-\omega_{t-})A^L - \lambda_I \left(\omega_{t-}\bar{\iota}^H(\omega_{t-}) + (1-\omega_{t-})\bar{\iota}^L(\omega_{t-})\right) \\ &+ W(\omega_{t-})\mu^K(\omega_{t-}) + W'(\omega_{t-})\mu^\omega(\omega_{t-}) \\ &+ \lambda \max_\gamma \left[W(\omega_{t-} + \Delta^\omega(\omega_{t-},\gamma))\left(1 + \Delta^K(\omega_{t-},\gamma)\right) - W(\omega_{t-}) - I(\omega_{t-},\gamma)\right] . \end{aligned} \tag{42}$$

where $\bar{\iota}^j(\omega_{t-})$, $j \in \{H, L\}$, is defined in (4), $\mu^K(\omega_{t-})$ and $\Delta^K(\omega, \gamma)$ defined in (18), $\mu^\omega(\omega_{t-})$ and $\Delta^\omega(\omega_{t-}, \gamma)$ defined in (19), and the aggregate investment-to-pre-crisis capital ratio, $I(\omega_{t-}, \gamma)$ defined in (39). In comparison with the welfare HJB equation (40) under a constant γ, the last term on the right side of (42) reflects the optimization over γ given the firm quality distribution, represented by ω_{t-}, that the economy carries into a crisis.

The first-order condition for the optimal γ^* reveals the trade-off that the government faces:

$$\begin{aligned} &W'(\omega_{t-} + \Delta^\omega(\omega_{t-},\gamma^*))\frac{\partial \Delta^\omega(\omega_{t-},\gamma^*)}{\partial\gamma}\left(1 + \Delta^K(\omega_{t-},\gamma^*)\right) + \\ &W(\omega_{t-} + \Delta^\omega(\omega_{t-},\gamma^*))\frac{\partial \Delta^K(\omega_{t-},\gamma^*)}{\partial\gamma} - \frac{\partial I(\omega_{t-},\gamma)}{\partial\gamma} = 0\,. \end{aligned} \tag{43}$$

The first term shows the negative impact of reducing γ through the dampening of the cleansing effect, $\frac{\partial \Delta^\omega(\omega_{t-},\gamma^*)}{\partial\gamma}$, and its long-run effects are encoded in the marginal change of the present value of future consmptions (i.e., the forward-looking welfare measure) per unit of capital, $W'(\omega_{t-} + \Delta^\omega(\omega_{t-},\gamma^*))$. The second term shows the positive impact of reducing γ through the preservation of capital, $\frac{\partial \Delta^K(\omega_{t-},\gamma^*)}{\partial\gamma}$.Each unit of capital saved by the government funding raises welfare by $W(\omega_{t-} + \Delta^\omega(\omega_{t-},\gamma^*))$. The last term reflects the fact that stimulating investment through the

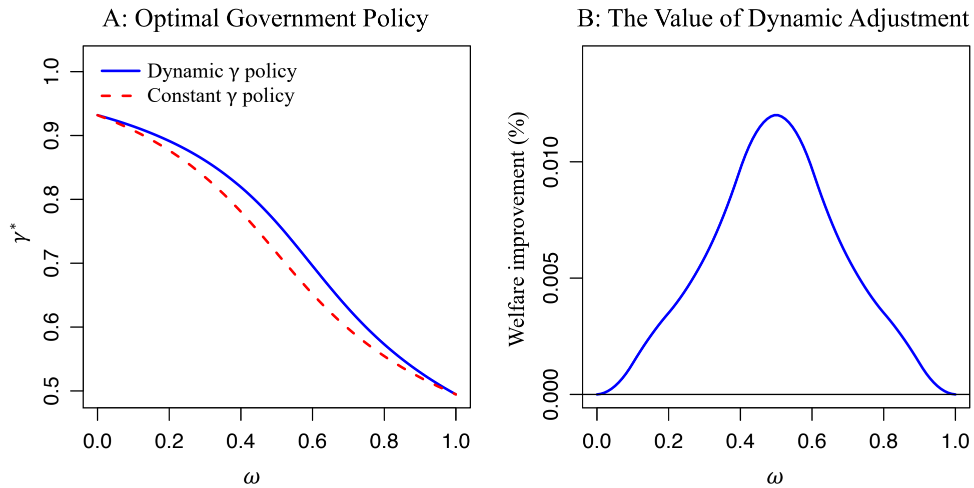

Figure 8: **Optimal Dynamic and Static γ and Welfare Difference** ($\chi = 0.25$)**.**

government funding directs goods towards creating capital instead of the current consumption.

Equation (43) implicitly defines the optimal γ^* as a function of ω. Once we solve the functions, $q^H(\omega)$, $q^L(\omega)$, $W(\omega)$, and $\gamma(\omega)$, we obtain the time-t values of the other endogenous variables (firms' investment and financing policies in crises in Proposition 4) as functions of ω.

Proposition 8 (Equilibrium under Dynamic Intervention) *In an equilibrium where the government dynamically adjusts the funding repayment based on the firm quality distribution, the capital value, q_t^j ($j \in \{H, L\}$), the welfare per unit of capital, W_t, and the optimal capital units repaid to the government per unit of funding support, γ_t, jointly satisfy the equations (41), (42), and (43), and firms' investment and financing decisions are given by Proposition (4).*

Panel A of Figure 8 compares the dynamically adjusted γ and the optimal constant γ set at $t = 0$. Overall, they are close numerically, and, at $\omega_0 = 0$ and 1, the two coincide because when the economy has only one type of firms, the government can simply offer the fairly priced funding and achieve the first-best outcome. In the interior region, the dynamic γ is higher, because given the upward trajectory of ω_t (i.e., type-H firms outgrowing type-L firms over time), the government can tighten its funding supply initially when the concern over type-L firms' overinvestment dominates, and later, at higher values of ω_t, loosen its finding supply to stimulate type-H firms' efficient investment. Such flexibility improves welfare as shown in Panel B of Figure 8 where we calculate the percentage increase in welfare under the dynamic γ (relative to the optimal constant γ).

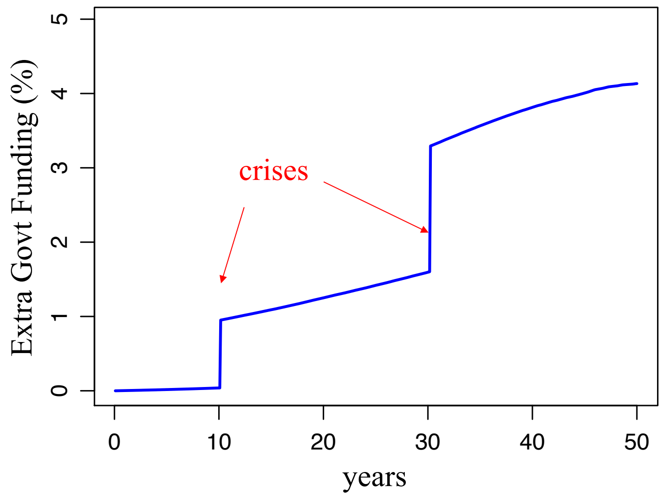

Figure 9: **The Slippery Slope of Dynamic Interventions.**

While being able to dynamically adjust the pricing of funding support improves welfare, the government still cannot avoid the slippery slope of intervention as shown in Figure 9. Specifically, we simulate a path of ω_t in the benchmark (laissez-faire) economy with two crises after ten and thirty years, respectively, and we calculate the optimal amount of funding support (implied by the optimal $\gamma(\omega_t)$) should the government intervene in the next crisis.[21] In this base case, the scale of funding support is not affected by past interventions or firms' expectations of interventions. Then we calculate the optimal amount of funding support in the next crisis under the dynamically adjusted $\gamma(\omega_t)$, i.e., the equilibrium in this subsection. Its percentage difference relative to the base case is plotted in Figure 9. The jumps in the two crises show that the distortions in ω_t brought by government interventions lead to larger interventions in the next crisis relative to the hypothetical economy without any intervention or intervention expectation. The upward drifts before the first crisis (visibly weak), between the two crises, and after the second crisis, are due to the distortions in ω_t from firms' normal-time investments that depend on their expectations of future interventions through the capital values. Overall, dynamically conditioning $\gamma(\omega_t)$ on the firm quality distribution cannot eliminate the slippery slope of interventions.

[21] The starting point of the simulation is $\omega_0 = 0.5$ for both cases.

5 Corporate Liquidity Management

In this section, we extend the model to incorporate firms' precautionary savings following the literature on theories of dynamic liquidity management (Froot, Scharfstein, and Stein, 1993; Riddick and Whited, 2009; Bolton, Chen, and Wang, 2011; Hugonnier, Malamud, and Morellec, 2015; He and Kondor, 2016; Li, 2018a; Nikolov, Schmid, and Steri, 2019). In normal times, firms may accumulate savings in anticipation of a potentially binding financial constraint in crises.

Let m_t^j denote the type-j firm's liquidity holdings per unit of capital, and r_m denote the interest rate on liquidity holdings. In normal times, firms may save their revenues or raise equity to buy liquid assets.[22] Given the cost of capital r (i.e., households' discount rate), firms hold an infinite amount of liquidity if $r_m > r$. In the following, we consider $r_m \leq r$, which is in line with the fact that money-market instruments (or "cash and cash equivalents") typically generate lower yields than corporate debts or equities. This also follows the models on dynamic liquidity management that typically assume a return on liquidity holdings below shareholders' discount rate (e.g., Bolton, Chen, and Wang, 2011). When $r_m < r$, holding liquidity incurs a carry cost.

Firms hold liquid assets to hedge the crisis risk. A firm has two types of savings, liquid assets and capital. While capital is subject to the destruction shock in crises, liquid assets do not. Therefore, a firm pays the (carry) cost of liquidity holdings for insurance against crises. In a crisis, the financial constraint (8) of a type-j firm with shock u is relaxed by its liquidity holdings:

$$x^j(u) \leq \chi q^j(1-u) + m^j. \tag{44}$$

We suppress the time subscripts and will show that the optimal m_t^j is constant in equilibrium.

The liquidity holdings change the u-thresholds in Proposition 4. Under the logarithm $F(\cdot)$, the financial constraint binds if $\overline{v}^j = q^j\phi \geq \chi q^j(1-u) + m^j$, so the new threshold is given by

$$\hat{u}(q^j, m^j) = 1 - \frac{\phi}{\chi} + \frac{m^j}{\chi q^j}. \tag{45}$$

Comparing (30) with (45), we can see that the thresholds now differ by firm types through the last term. Liquidity holdings also affect the threshold of whether to seek overpriced funding from the

[22] For simplicity, it is assumed that raising equity for investment in normal times or crisis is not possible, which can be motivated by asymmetric information (Myers and Majluf, 1984) or the disagreement between inside and outside shareholders (Dittmar and Thakor, 2007) on the quality and/or risk-return trade-off of the investment projects.

government. Following (26), the threshold is defined by the following condition

$$F'\left(\chi q^j(1-\tilde{u}(q^j,m^j,\gamma))+m^j\right)=\gamma\,, \tag{46}$$

so under the logarithm $F(\cdot)$, we solve the new threshold as follows:

$$\tilde{u}(q^j,m^j,\gamma)=1-\frac{\phi}{\chi q^j\gamma}+\frac{m^j}{\chi q^j}. \tag{47}$$

As discussed in Proposition 4, when the government funding is underpriced or fairly priced (i.e., $q^j\gamma\leq 1$), the financial constraint is irrelevant, so the firm does not hold liquidity. When the government funding is overpriced (i.e., $q^j\gamma>1$), we have $\hat{u}(q^j,m^j)<\tilde{u}(q^j,m^j,\gamma)$. By raising the u-thresholds, liquidity holdings reduce the likelihood of a binding financial constraint and, conditional on a binding financial constraint, liquidity holdings reduce the likelihood of seeking overpriced government funding. A firm faces a trade-off between the marginal (carry) cost of holding liquidity ($r-r_m$ per unit of time) and the marginal benefit which is characterized below.

Under $q^j\gamma>1$, the marginal benefit of liquidity holdings depends on different scenarios of u, the shock size in a crisis. A type-j firm draws $u\leq\hat{u}(q^j,m^j)$ with probability $G\left(\hat{u}(q^j,m^j)\right)$. As the financial constraint does not bind, it earns investment profits of $q^jF(\overline{\iota}^j)-\overline{\iota}^j$. The firm draws $u\in(\hat{u}(q^j,m^j),\,\tilde{u}(q^j,m^j,\gamma)]$ with probability $G\left(\tilde{u}(q^j,m^j,\gamma)\right)-G\left(\hat{u}(q^j,m^j)\right)$. The financial constraint binds, and the firm fully relies on private funding to finance investment, earning profits $q^jF\left(\chi q^j(1-u)+m^j\right)-\left(\chi q^j(1-u)+m^j\right)$. Finally, the firm draws $u>\tilde{u}(q^j,m^j,\gamma)$ with probability $1-G\left(\tilde{u}(q^j,m^j,\gamma)\right)$. The optimal amount of government funding, $g^j(u,q^j,m^j,\gamma)$ which now depends on internal liquidity, m^j, is given by

$$F'\left(\chi q^j(1-u)+m^j+g^j(u,q^j,m^j,\gamma)\right)=\gamma\,, \tag{48}$$

As in (24) of the baseline model, the amount of newly created capital at the margin is equal to the units of capital repaid to the government. Under the logarithm $F(\cdot)$, we have

$$g^j(u,q^j,m^j,\gamma)=\frac{\phi}{\gamma}-\chi q^j(1-u)-m^j\,, \tag{49}$$

i.e., the total investment is ϕ/γ. The profits, $q^jF\left(\phi/\chi\right)-\phi/\chi-\left(q^j\gamma-1\right)g^j(u,q^j,m^j,\gamma)$, adjusts for the premium $(q^j\gamma-1)$ of government funding in the last term.

In a crisis, the expected investment profits are given by

$$\begin{aligned}\pi\left(u, q^j, m^j, \gamma\right) \equiv & G\left(\hat{u}(q^j, m^j)\right)\left[q^j F(\bar{\iota}^j) - \bar{\iota}^j\right] \\ & + \int_{\hat{u}(q^j, m^j)}^{\tilde{u}(q^j, m^j, \gamma)} \left[q^j F\left(\chi q^j(1-u) + m^j\right) - \left(\chi q^j(1-u) + m^j\right)\right] dG(u) \\ & + \int_{\tilde{u}(q^j, m^j, \gamma)}^{\upsilon} \left[q^j F\left(\frac{\phi}{\chi}\right) - \frac{\phi}{\chi} - \left(q^j\gamma - 1\right) g^j(u, q^j, m^j, \gamma)\right] dG(u).\end{aligned} \tag{50}$$

Therefore, the firm sets its optimal liquidity holdings by equating the marginal cost and benefit:

$$r - r_m = \lambda \int_{u=0}^{\upsilon} \frac{\partial \pi\left(u, q^j, m^j, \gamma\right)}{\partial m^j} dG(u)\,. \tag{51}$$

Liquidity holdings also change the valuation of capital. Per unit of capital, m^j units of liquidity holdings incur a carry cost of $r - r_m$ per unit of time but boost investment profits in crises by relaxing the financial constraint. As in the baseline model, we integrate over u to define the expected profits in a crisis: $\Pi(q^j, m^j, \gamma) = \mathbb{E}\left[\pi\left(u, q^j, m^j, \gamma\right)\right]$. The following equation solves q^j:

$$r = \frac{A^j}{q^j} - \delta - \frac{(r - r_m)m^j}{q^j} + \frac{\lambda_I\left(q^j F\left(\bar{\iota}^j\right) - \bar{\iota}^j\right)}{q^j} + \frac{\lambda \Pi(q^j, m^j, \gamma)}{q^j} - \lambda U. \tag{52}$$

The following proposition summarizes the solutions of optimal liquidity holdings and capital value.

Proposition 9 (Optimal Liquidity Holdings and Capital Value) *For $j \in \{H, L\}$, the optimal liquidity holdings per unit of capital of a type-j firm, $m^j(q^j, \gamma, r_m)$, is solved in equation (51), and the unit value of type-j capital is solved in equation (52).*

When $\gamma \in [1/q^H,\ 1/q^L]$, type-$L$ firms will not hold any liquidity because government funding, which costs $q^L\gamma < 1$, is cheaper than its own liquidity holdings, which costs 1, as a source of funds for investment. Therefore, only type-H firms hold liquidity in equilibrium.

Corollary 2 (Liquidity Distribution) *In equilibrium, type-H firms hold liquidity m^H given by (51), while type-L firms do not hold liquidity (i.e., $m^L = 0$).*

In a general equilibrium setting, an increase in the supply of liquid assets, such as bank deposits and Treasury securities, will likely raise the yield on liquidity holdings, r_m, thus raising m^H. As long as the firm faces a binding financial constraint and/or seeks overpriced government funding in some states (u) in a crisis, a marginal increase of liquidity holdings always means more profits.

When the economy has an abundant supply of liquid assets and r_m rises up to r, the optimality condition (51) implies that $\int_{u=0}^{v} \frac{\partial \pi\left(u, q^H, m^H, \gamma\right)}{\partial m^H} dG(u) = 0$, i.e., the marginal value of liquidity is zero, which in turn suggests that type-H firms will no longer face a binding financial constraint in a crisis or in need of overpriced government funding. Therefore, when $r_m = r$, type-H firms' investment achieves the targeted levels both in and outside of crises, and their capital value, q^H, is equal to the value in a hypothetical first-best economy where the financial (collateral) constraint does not exist. Moreover, given that now only type-L firms seek government funding, the government can set a uniform yet fair price of funding (i.e., $\gamma = 1/q^L$), so that type-L firms no longer receive any subsidy and will not over-invest. As a result, type-L firms invest at the targeted levels, and their capital value is equal to the first-best value (without being inflated by the government funding subsidy). The following proposition summarizes the first-best scenario.

Corollary 3 (Abundant Liquidity and First-Best Allocation) *When $r_m = r$, the economy attains the first-best outcome (i.e., the equilibrium of the economy without financial constraints).*

Our analysis suggests a higher interest rate of cash instruments (i.e., nonfinancial corporations' cash and cash equivalents) can be beneficial as it facilitates firms' self-insurance through liquidity holdings and thereby reduces the distortionary effects of government funding. In contrast, a low-rate environment hurts firms, because firms may face a higher cost of self-insurance (i.e., liquidity carry cost). This mechanism is related to Quadrini (2020). Quadrini (2020) emphasizes that low interest rate suppresses producers' precautionary savings that are essential for buffering unhedgeable shocks in production process. The caution against low interest rate also echoes Brunnermeier and Koby (2018) who analyze the detrimental effects of low interest rate on banks.

Corollary 3 characterizes the extreme case of satiated firm liquidity demand under zero liquidity carry cost. In Figure 10, we compare two economies with different levels of liquidity carry costs. In Panel A, we plot the minimal amount of government funding (Y-axis) needed to contain the output drop in crisis to a given level (X-axis). In both economies, reducing the output drop requires more government funding, so the government funding-to-output ratio, $\bar{g}$, increases from the left to the right. In the economy with the low liquidity carry cost, type-H firms hold more liquidity to self-insure against crises, so less government funding is needed, which then implies a small degree of distortionary effects on the firm quality dynamics.

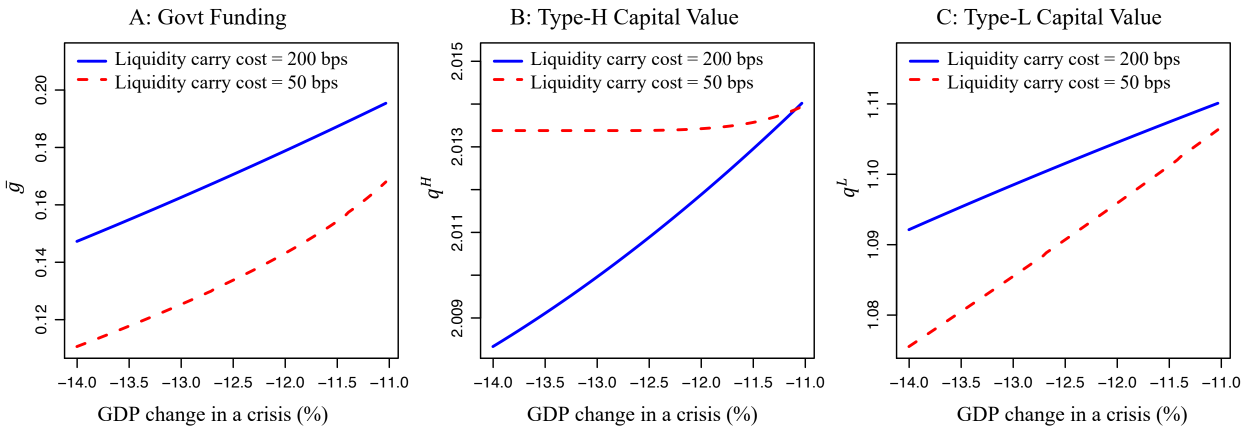

Figure 10: **Liquidity Carry Cost and the Distortionary Effects of Government Funding.**

In Panel B and C of Figure 10, we plot, respectively, the type-H and type-L capital values (Y-axis) under the minimal scales of government funding needed to contain the output drop to a certain level (X-axis). In all cases, capital values increase as the amount of government funding increases and the output drop becomes less severe. In Panel B, a lower liquidity carry cost can lead to a higher value of type-H capital, because self-insurance against crises is cheaper. A lower liquidity carry cost can also lead to a lower value of type-H capital, because as type-H firms hold more liquidity to self-insure, the government reduces its funding support given any targeted level of output drop. The former force tends to be the dominant force in Panel B, as shown by the dashed line (low liquidity carry cost) largely staying above the solid line (high liquidity carry cost).

In Panel C of Figure 10, type-L capital value, q^L, declines when liquidity carry cost decreases. This is a quite interesting result, because as previously discussed, type-L firms do not hold liquidity, thus not directly exposed to the variation in liquidity carry cost. Under a lower liquidity carry cost, type-H firms save more, so the required government funding in a crisis declines, which then implies that type-L firms now receive less underpriced government funding. Moreover, expecting less subsidy in crises, type-L firms invest less in normal times, guided by the now "deflated" q^L.

In Panel A of Figure 11, we plot the welfare-maximizing γ^* at different levels of liquidity carry cost. From the left to the right, as liquidity carry cost increases, the government optimally reduces γ.[23] A higher liquidity carry cost implies less liquidity holdings of type-H firms, so the self-insurance of the private sector weakens, and the government optimally becomes more lenient

[23]In Figures 11 and 12, we consider $\omega_0 = 30\%$ but the results are similar if we initiate the economy at a different value of ω_0. The results are available upon request.

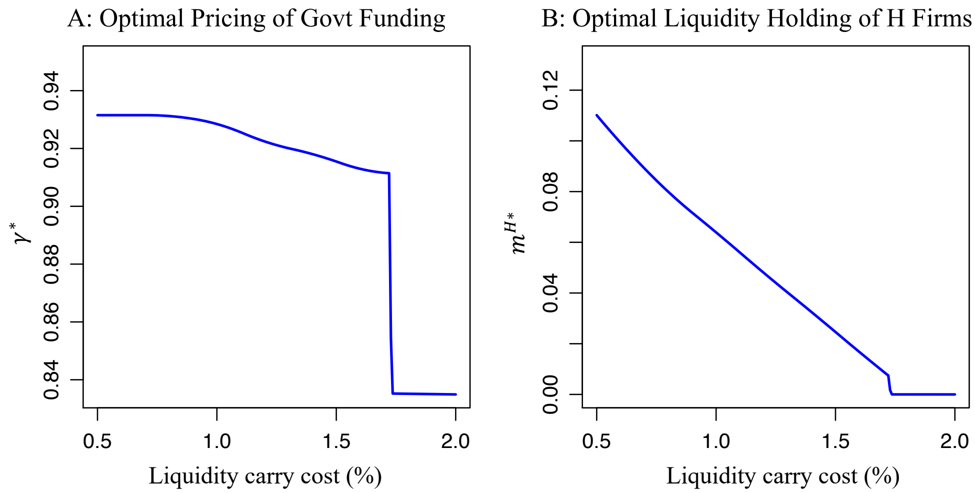

Figure 11: **Liquidity Carry Cost and the Distortionary Effects of Government Funding.**

in funding supply in crises. In Panel B of Figure 11, we show that the liquidity holdings per unit of capital of type-H firms decline as liquidity carry cost increases. When the liquidity carry cost is sufficiently hold, type-H firms simply give up on self-insurance.

Panel A of Figure 12 plots the optimal amount of government funding scaled by the total capital stock (implied by the optimal γ^* in Panel A of Figure 11) against different levels of liquidity carry cost. As liquidity carry cost increases and type-H firms' precautionary savings decline, the scale of funding support increases and eventually jumps up at the value of liquidity carry cost above which type-H firms no longer self-insure.

Panel B of Figure 12 shows that as liquid carry cost increases, welfare decreases with or without government funding in crises. Without government funding, both types of firms hold liquidity to self-insure against crises, so as the carry cost increases, they hedge less and the crises become more severe. Government funding improves welfare in spite of the distortions on the firm quality distribution. With government funding, only type-H firms may hold liquidity, and once the carry cost becomes sufficiently high, it becomes irrelevant because type-H firms no longer self-insure and fully rely government funding instead. Therefore, the solid line in Panel B of Figure 12 flattens out at the right end while the dotted line of welfare in the laissez-faire economy keeps declining.

In sum, our analysis points to the following conclusions. When it is costless to self-insure, firms will do so (by holding liquidity) and government intervention is unnecessary. However, under a positive liquidity carry cost, government intervention can improve welfare when carefully designed

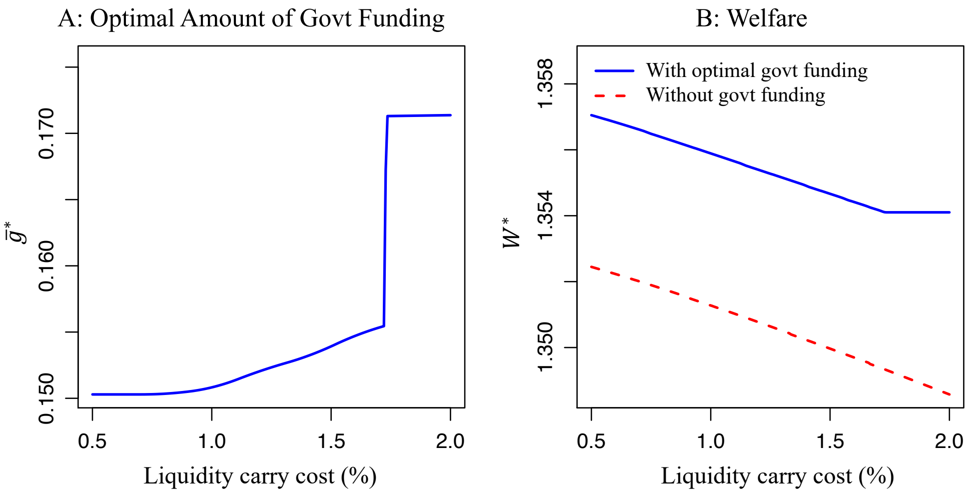

Figure 12: **Liquidity Carry Cost and the Distortionary Effects of Government Funding.**

to balance type-H firms' efficient investment and type-L firms' overinvestment. When liquidity carry cost declines, type-H firms rely more on self-insurance through internal liquidity management pre-crisis rather than seeking government support in crises. As a result, the government can reduce its funding support to minimize the inefficiency from type-L firms' overinvestment.

Our analysis takes as given a liquidity carry cost. In a general equilibrium setting, it will be endogenously determined by the firms' demand and the issuances of liquid instruments from both the government and the private sector (Woodford, 1990; Holmström and Tirole, 1998, 2001; Li, 2018a). Therefore, expanding the supply of liquid assets tends to reduce the equilibrium liquidity carry cost, which not only addresses type-H firms' underinvestment but also corrects type-L firms' overinvestment. The firm quality dynamics improve as a result. In a liquidity shortage, for example, due to a strong liquidity demand in foreign countries (Caballero, Farhi, and Gourinchas, 2008), the firm quality dynamics deteriorate as type-H firms reduce precautionary savings and type-L firms profit from distortionary yet necessary interventions in crises.

Under the distortionary effects of government funding support in crises, the government should play an active role in supplying liquid assets, for example, Treasury bills in the United States. This is also in line with Friedman's rule. The Friedman rule states that the private cost of holding money (liquidity), which is firms' liquidity carry cost in our model, should equal the social cost of supplying liquidity (Friedman, 1969). An expansion of public liquidity supply may have additional benefits (Stein, 2012; Krishnamurthy and Vissing-Jorgensen, 2015; Li, 2019) and costs (Li, 2018b) through the substitution between the government-issued and bank-issued liquid assets. We leave a comprehensive cost-benefit analysis of liquidity supply for future research.

6 The Bazooka Effect

On July 15, 2008, in his testimony before the Senate Banking Committee about government liquidity support during the global financial crisis, the U.S. Secretary of the Treasury Henry Paulson famously said: "*If you've got a bazooka, and people know you've got it, you may not have to take it out.*" An announcement of government support by itself can have a positive impact in crises, even before any funding is provided. We analyze the announcement effects and highlight that the success of a liquidity facility cannot be solely judged by its take-up rate. During the Covid-19 pandemic, the lower than expected utilization of certain liquidity facilities have drawn much attention (e.g., Hanson, Stein, Sunderman, and Zwick, 2020).[24] Our analysis directly speaks to this issue. Specifically, we show that an announcement of funding support enlarges firms' private funding capacities by boosting asset prices (Section 6.1) and makes the pricing of private funding more efficient by improving firms' bargaining power against their relationship banks.

6.1 The Announcement Effects

For simplicity, we analyze the announcement effects using the solution without firms' liquidity holdings (from Section 4).[25] Figure 13 show the announcement effects under $\chi = 0.2$ (Panel A) and $\chi = 0.6$ (Panel B). As a reminder, a lower value of χ means a tighter financial constraint (see (6)), which represents a more severe damage of the private funding market in crises.

In both panels, we plot the aggregate investment scaled by capital stock, i.e., I_t/K_t, against ω_t. A higher ω_t leads to more investments because, as previously discussed, type-H firms have higher investment targets and more valuable collaterals. Therefore, the slopes are positive in all cases. In both panels, the solid line shows the investment without government intervention (i.e., the benchmark case in Section 3). The dashed line shows the announcement effects. Specifically, to mimick a policy surprise, we increase q^H and q^L from the values in the laissez-faire economy to the

[24] See also "As Washington scrambles for more bailout money, the Fed sits on mountain of untapped funds" by Rachel Siegel and Jeff Stein, The Washington Post Octeber 19, 2020.

[25] Incorporating firms' liquidity holdings does not change the qualitative implications. The results are available upon request.

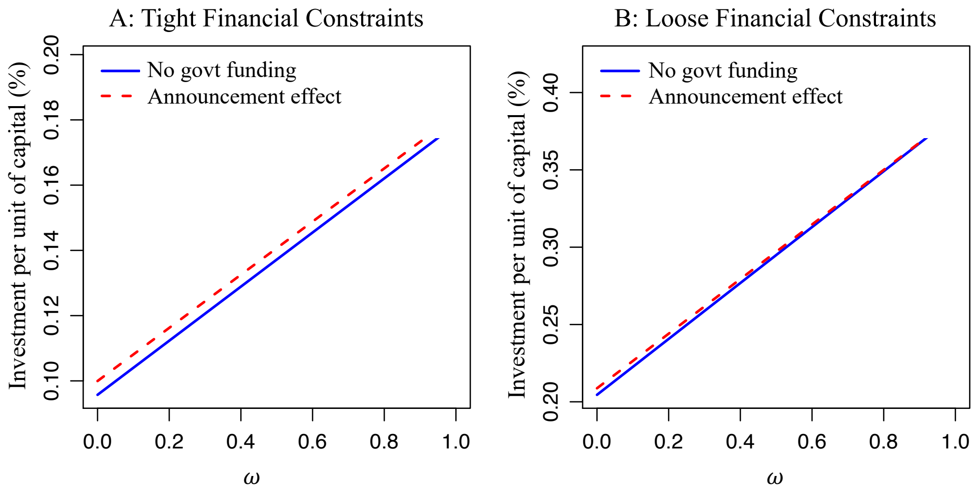

Figure 13: **The Announcement Effect.**

values in the economy with government intervention. Upon the announcement, the asset prices, q^H and q^L, increase immediately, reflecting firms' expectations of current and future funding support.

The announcement of government funding increases firms' private funding capacity by increasing the collateral values. The improvement from the solid line to the dashed line is fully attributed to the additional private funding. The additional investment is brought by the announcement of intervention not the actual funding provided by the government. The positive impact of policy announcement is stronger in Panel A under a tighter constraint on private funding, because the marginal impact of government funding on firms' investment profits is larger.

6.2 Government Funding as an Outside Option

In systematic crises, it is likely that financial intermediaries experience balance-sheet impairments and face financial constraints just as nonfinancial firms do. However, in the situations where intermediaries are well-capitalized, they may play an active role in relaxing the financial constraints on firms' investment, thus reducing the needs for government intervention.[26] We extend our model to incorporate banks and emphasize that unless the banking sector is fully competitive, government direct lending is still important as an outside option that increases firms' bargaining power. Our

[26]To the extent that technological constraints may bind, the entrants often bring in innovations and new capital (Erel and Liebersohn, 2020).

results are in line with the empirical findings of Jiménez, Peydró, Repullo, and Saurina (2017).

We assume that banks have perfect information on firms' types (Diamond, 1984; Ramakrishnan and Thakor, 1984; Heider and Inderst, 2012). When firms borrow from banks, they no longer face the collateral constraint, and unlike firms, banks do not face a collateral constraint when they raise funds from deep-pocket households. This setup is essentially an extreme case of Rampini and Viswanathan (2018) who model banks as collateralization specialists.[27] Here our purpose is to provide banks as much flexibility as possible, so that our evaluation of the benefit and necessity of government funding can be regarded as from a sufficiently conservative pointview.

Given banks' informational advantage and free access to financing, a competitive banking sector achieves the first-best outcome. Introducing competitive banks is equivalent to eliminating financial constraints for firms. However, inefficiency arises if banks have market power. In crises when alternative sources of financing are limited, firms typically rely on banks with long-term relationships (Santos and Winton, 2008).[28] We consider a unit mass of banks, each paired with one firm. The relationship bank is the firm's only source of private funding after the firm hits the collateral constraint. Banks distribute any profits to households. Within the collateral constraint, the firm can borrow from competitive households who break even. Therefore, banks are only relevant in crises and only for the subset of firms whose collateral constraint binds. As before, we conjecture and verify an equilibrium with constant capital value, q^j, and investment targets, $\bar{\iota}^j$.

A relationship bank extends a take-it-or-leave-it offer to the paired firm. Without government funding, the bank seizes the full surplus from the firm's investment beyond its collateral value. From a firm's perspective, any investment beyond its collateral value generates zero profits, so the capital valuation equation (21) from the laissez-faire benchmark still holds. Given the same q^H and q^L, firms invest in normal times at the same rates as they do in the laissez-faire economy, implying the same drifts of K_t and ω_t. The economy differs from the laissez-faire benchmark in crises, and in particular, the jumps in K_t and ω_t (i.e., Δ^K and Δ^ω, respectively). The financially unconstrained banks finance all profitable investments and seize all surplus, so Δ_t^j is $F(\bar{\iota}_t^j) - U$, where $\bar{\iota}_t^j$ is given by (4), $j \in \{H, L\}$. Given Δ_t^H and Δ_t^L, we can calculate Δ^K and Δ^ω using (18) and (20).

With the relationship banks, the economy achieves the targeted levels of investment for both types of capital in crises, but inefficiency still exists. Capital values, q^H and q^L, are still below the

[27] The collateral constraint can also be relaxed under relationship lending (Sharpe, 1990; Petersen and Rajan, 1994; Boot and Thakor, 2000; Detragiache, Garella, and Guiso, 2000; Degryse and Ongena, 2005; Bolton and Freixas, 2006; Parlour and Plantin, 2008; Repullo and Suarez, 2012; Bolton, Freixas, Gambacorta, and Mistrulli, 2016).

[28] Banks' credit market power has been well documented (e.g., Sunderam and Scharfstein, 2016; Cahn et al., 2017).

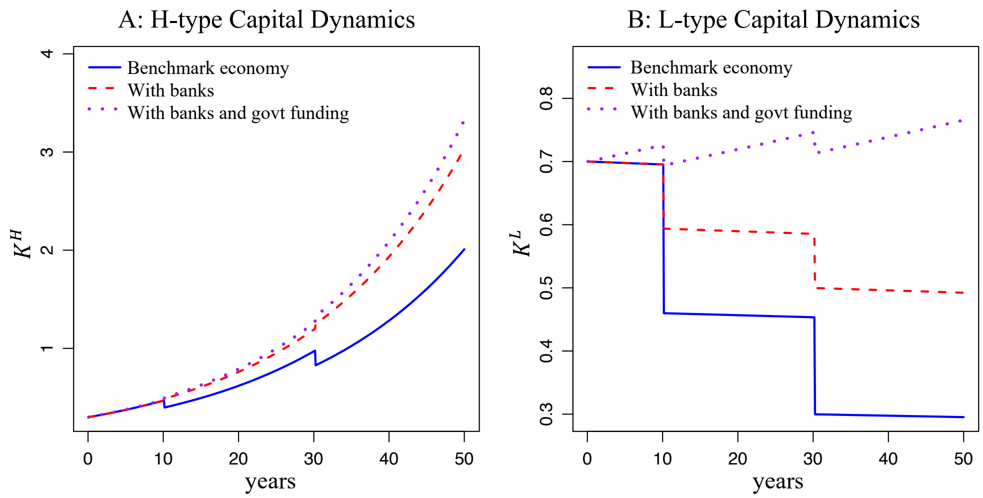

Figure 14: **Aggregate Dynamics with Relationship Banking and Government Intervention.**

first-best levels, as they reflect firms' expectations of losing investment profits to banks in crises. Such distortions to capital values lower the investment targets in both normal times and crises. Moreover, lower capital values imply tighter collateral constraints on private funding in crises.

Government funding serves as firms' outside option. As in our main model, we consider $\gamma \in [1/q^H,\ 1/q^L]$. Type-$L$ firms only rely on underpriced government funding, and q^L is solved by the capital valuation equation (34). Type-H firms only seek funding from their relationship banks when they have exhausted the competitive private funding within the collateral constraint and, if u is sufficiently low, seek government funding given by (24). A type-H firm keeps the profits from investment financed by the competitive private funding and government funding, so q^H is solved by the capital valuation equation (34) in the main model. With q^L and q^H, we solve firms' normal-time investments and the drifts of K_t and ω_t. In crises, banks finance all profitable investments, so the jumps in K_t and ω_t are the same as those in the previous case without government funding.

Government funding is a double-edged sword. On the bright side, it leaves more investment profits to type-H firms, thus boosting q^H (and type-H firms' investment targets both in and outside of crises). Raising q^H also relaxes type-H firms' collateral constraint on competitive (non-relationship) private funding in crises. The dark side of government funding is still the overinvestment of type-L firms both in and outside of crises, as previously discussed in the main model.

In Figure 14, we simulate the paths of K_t^H (Panel B) and K_t^L (Panel B) with two crises after ten and thirty years, respectively. Adding relationship banks to the laissez-faire benchmark lifts up

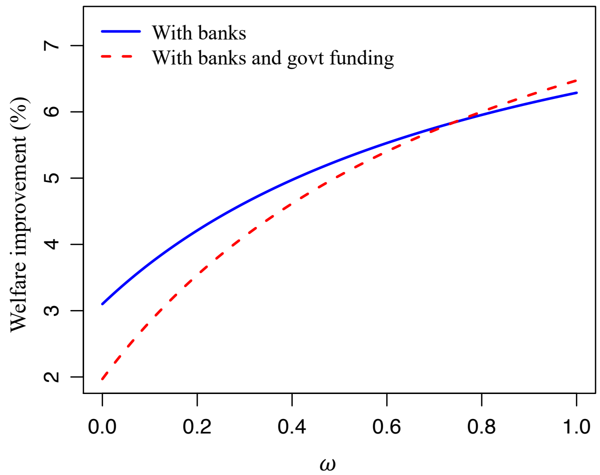

Figure 15: **Welfare Improvement from Relationship Banking and Policy Intervention.**

the growth trajectories, because bank financing allows the investments in both types of capital to reach the targeted levels in crises. However, as previously discussed, capital values are stuck at the values from the benchmark economy, because firms expect to lose all profits from investments beyond their collateral values to the relationship banks. Adding government funding allows type-H firms to seize back part of the lost profits. Therefore, in Panel A, q^H increases, driving up type-H firms' investment targets in both normal times and crises. In Panel B, adding government funding significantly distorts the dynamics of K_t^L, as type-L firms over-invest in crises, and their expectations of underpriced government funding translate into an inflated value of q^L, which elevate the targeted levels of investment in both crises and normal times.

In sum, government funding benefits the economy by allowing type-H firms to preserve more profits and thereby boosting q^H (which raises type-H firms' investment targets and relaxes their collateral constraints). However, the non-discriminatory pricing of government funding implies subsidy to type-L firms who conduct wasteful investment. In Figure 9, we first show the welfare improvement from adding banks to the laissez-faire benchmark (solid line), and then we add government funding. When the economy has many type-H firms (i.e., when ω_t is higher), the benefits of helping type-H firms dominate the costs of type-L firms' overinvestment, so adding government funding leads to a further improvement of welfare. However, when type-L firms constitute a dominant share, adding government funding reduces welfare, as the costs of type-L firms' overinvestment overwhelmes the benefits of helping type-H firms to seize profits from banks.

Our analysis does not change if we allow banks to match the government's offer to type-H firms. In fact, it is in a bank's interest to do so, because for every dollar lent, the bank reaps net profits of $q^H\gamma - 1 > 0$. The bank, who knows the firm's type, certainly recognizes such profits and improves its offer by an infinitesimal amount ϵ (i.e., charging a repayment of $q^H(\gamma - \epsilon)$) in order to win the deal. Now the firm borrows from the bank, but still keeps the profits that it would have earned by financing the investment with the government funding. In such a case, government funding is not utilized but still improves welfare by serving as an outside option.

7 Conclusion

Using a dynamic two-sector model, we analyze the impact of credit intervention on the long-run dynamics of firm quality. In a laissez-faire economy, crises have cleansing effects, because in crises, low-quality firms face tighter financial constraints and have lower Tobin's q than high-quality firms do. The economy emerges from crises with an improved firm quality distribution.

However, among both types of firms, a subset underinvest under binding financial constraints, and such inefficiency calls for government intervention. The government can step in as a lender with superior ability in contract enforcement, thereby relaxing firms' financial constraints, but due to either the lack of information on firm types or political constraints, the government offers the same repayment schedule to all firms. In equilibrium, credit intervention dampens the cleansing effects and distort the firm quality distribution over the long run by inducing overinvestment of the low-quality firms in both crises and, through the expectation effects, in normal times.

The model features a slippery slope of intervention. As the current intervention biases downward the firm quality distribution, the economy enters the next crisis with a lower total productivity, and an intervention of a greater scale becomes necessary. Larger interventions lead to stronger distortions, which in turn call for even larger interventions in the future. However, we show that when carefully designed, credit intervention improves welfare relative to the laissez-faire benchmark. Our analysis favors gradualism: A small intervention almost guarantees a positive outcome, while a large intervention may cause welfare loss due to the low-quality firms' overinvestment.

We extend our model to incorporate firms' dynamic liquidity management. In equilibrium, the low-quality firms benefit from mispriced government funding in crises, so they do not hold liquidity. The high-quality firms hold liquidity, as they expect relatively unfavorable terms when borrowing from the government in crises. When the supply of liquid assets increases, it helps the high-quality firms to save more and invest more out of their internal liquidity. As the overall needs for external financing decline in the economy, the government can scale back its credit support in crises, and thereby, reduce the distortionary effects on the firm quality distribution. Therefore, the government has two complementary tools at its disposal when addressing the breakdown of capital markets: Before a crisis hits, it can issue liquid securities (e.g., Treasury bills) held by firms as precautionary savings, and in a crisis, the government can offer credit support.

Finally, we incorporate relationship lending as an alternative way to relax firms' financial constraints in crises. Liquidity provision by the government improves welfare as an outside option for firms. It allows firms to seize back the some investment surplus from their relationship banks. The resultant boost in Tobin's q improves investment efficiency in both normal times and crises. Thus, the success of credit intervention cannot be judged by the lending volume.

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A Background

We review the responses of the Federal Reserve (Fed) and other central banks to the COVID-19 crisis. Traditionally, central banks provide liquidity through the banking system, relying on commercial banks to extend credit to the production sector. The standing facilities, for example the discount window of the Fed, effectively impose a ceiling rate in the interbank market to alleviate financial stress.[29] During the COVID-19 crisis, the Fed's initial response was a 150bp decrease in the primary credit (discount-window) rate. The "stigma effect" of borrowing from the lender of last resort limits the utilization of such facilities (Armantier, Ghysels, Sarkar, and Shrader, 2015).

New facilities were established during the Global Financial Crisis.[30] For example, Term Auction Facility (TAF) was introduced to avoid the stigma effect (Hu and Zhang, 2019). Many of these facilities, such as Primary Dealer Credit Facility, Money Market Mutual Fund Liquidity Facility, and Term Asset-Backed Securities Loan, are active during the ongoing COVID-19 crisis.

The Paycheck Protection Program Liquidity Facility (PPPLF), introduced in April 2020, is another example of liquidity provision through the banking system. In the Paycheck Protection Program (PPP), banks lend to employers at a uniform rate of 1% and the loans are guaranteed by the Small Business Administration (SBA). PPPLF allows banks to pledge PPP loans as collateral to borrow from the Fed at a rate of 0.35%. Similar liquidity facilities were set up by the Bank of England and Bank of Japan during the same period.[31]

On March 23, 2020, the Federal Reserve made a historic move by announcing two credit facilities that bypass the banking system and aim at directly easing the credit conditions for nonfinancial firms (Boyarchenko, Kovner, and Shachar, 2020). Primary Market Corporate Credit Facility (PMCCF) makes loans to and purchase bonds from large companies. Secondary Market Corporate Credit Facility (SMCCF) purchases corporate bonds in the secondary markets. For both programs, eligible companies must be investment-grade or were investment-grade as of March 22, 2020.[32] Such facilities extend the scope of quantitative easing (QE) that initially targets long-term govern-

[29]Other examples include the operational standing lending facility at the Bank of England, the marginal lending facility at the European Central Bank (ECB), and the complementary lending facility at the Bank of Japan (BOJ).

[30]Examples include the Primary Dealer Credit Facility (PDCF) of the Federal Reserve, Term Purchase and Resale Agreement (PRA) Facility of the Bank of Canada (BOC), and Long-Term Refinancing Operations (LTRO) of ECB. For more details, please refer to "*Timeline of Policy Response to the Global Financial Crises*"

[31]The Term Funding Scheme with additional incentives for SMEs (TFSME) at the Bank of England accepts SME loans as collateral with a haircut, but different from PPPLF, the loans do not necessarily have the same rate. A facility similar to PPPLF at the Bank of Japn allows banks to borrow at rate of -0.1% using SME loans as collateral.

[32]The lending is conducted through a special purpose vehicle and the U.S. Treasury provided the equity capital.

ment bonds and mortgage-related securities and was mainly introduced in response to the global financial crisis. Direct credit facilities for nonfinancial firms were also introduced in Europe, Japan, and other countries.[33] These facilities take advantage of the information production in the financial markets or by the rating agencies when it comes to the heterogeneity of firms' credit-worthiness.

During the COVID-19 crisis, small and medium enterprises (SMEs) experienced signficant disruptions (Gourinchas, Kalemli-Özcan, Penciakova, and Sander, 2020). To cover the liquidity needs of SMEs, the Main Street Lending Program (MSLP) was introduced on April 9, 2020. It is a collaboration between the Fed and U.S. Treasury. The Fed will buy up to \$600 billion in loans, with the U.S. Treasury contributing \$75 billion as risk-bearing capital. The program targets small and medium-sized businesses and non-profit employers that are impacted by the COVID-19 pandemic. In contrast to PMCCF and SMCCF, in which the Fed bypasses the banks and directly engage the corporate credit markets, the Fed works with banks on MSLP, again relying on banks' expertise in screening firms. Federal Reserve will buy 95% of new or existing loans to qualified employers, while the loan-issuing bank will keep 5% as skin in the game. Similar to the PPP loans, all borrowers receive *same* interest rate of LIBOR plus 3%.

B Proofs

B.1 Proof of Proposition 1

Proof provided in the main text.

B.2 Proof of Proposition 2 and Corollary 1

Since households are risk neutral, in equilibrium, the expected return of firm equity must be equal to the household discount rate r. For $q_j^j k_t^j$ amount of investment in firm equity, the expected return is given by

$$\frac{1}{q_t^j k_t^j}\mathbb{E}_t[\underbrace{A^j k_t^j dt}_{\text{dividend yield}} + \underbrace{d\left(q_t^j k_t^j\right)}_{\text{capital gain}} - \underbrace{\left(\bar{\iota}_t^j k_t^j dN_t^I + x_t^j k_t^j dN_t\right)}_{\text{investment cost (additional contribution)}}]$$

[33] Dell'Ariccia, Rabanal, and Sandri (2018) review the unconventional monetary policies in the Euro Area, Japan, and the U.K. QE applies to corporate equities in Japan (Charoenwong, Morck, and Wiwattanakantang, 2019).

We know that
$$\frac{dk_t^j}{k_t^j} = -\delta dt + F(\bar{\iota}_t^j)dN_t^I + \left(F(x_t^j) - u_t\right)dN_t$$
As a result, the household first order condition implies

$$r = \frac{A^j}{q_t^j} - \delta + \mathbb{E}_t\left[\frac{dq_t^j}{q_t^j}\right]/dt + \frac{\lambda_I\left(q_t^j F(\bar{\iota}_t^j) - \bar{\iota}_t^j\right)}{q_t^j} + \frac{\lambda\Pi(q_t^j)}{q_t^j} - \lambda U$$

where the investment is related to q_t^j through the q-relationship

$$F'(\bar{\iota}_t^j)q_t^j = 1$$

If we conjecture that q_t^j is a constant, then $dq_t^j = 0$, and the investment $\bar{\iota}_t^j$ is also a constant. The above first-order condition implies

$$r = \frac{A^j}{q^j} - \delta + \frac{\lambda_I\left(q^j F(\bar{\iota}^j) - \bar{\iota}^j\right)}{q^j} + \frac{\lambda\Pi(q^j)}{q^j} - \lambda U \tag{A-1}$$

which does not contain a time-varying component, and thus we confirm the conjecture that q_t^j is a constant.

Next, we prove the corollary that $q^H > q^L$. To prove this, we need to first show whether there is a solution and whether the solution is unique. To achieve this goal, we rewrite the first-order condition (A-1) as
$$r + \delta + \lambda U = \frac{A^j}{q^j} + \lambda_I\left(F(\bar{\iota}^j) - \frac{\bar{\iota}^j}{q^j}\right) + \frac{\lambda\Pi^j(q^j)}{q^j} \tag{A-2}$$
where the profit function is

$$\Pi^j(q^j) = E_u[\max_{x^j \le \chi(1-u)q^j}\left\{q^j F(x^j) - x^j\right\}]$$

To remove the q^j in the denominator of Π^j, we define

$$\tilde{\Pi}^j(q^j) \equiv \Pi^j/q^j = E_u[\max_{\tilde{x}^j \le \chi(1-u)}\left\{F(\tilde{x}^j q^j) - \tilde{x}^j\right\}]$$

Now it is clear that $\tilde{\Pi}^j(q^j)$ increases in q^j. However, the general functional form $F(\cdot)$ does not guarantee monotonicity. To proceed, we show the proof under the case of $F(i) = \phi \log(i/\underline{\iota})$, which implies

$$r+\delta+\lambda U = \frac{A^j}{q^j}+\lambda_I \left(F(\phi q^j) - \phi\right)+\lambda \left((F(\phi q^j) - \phi)G(\hat{u}) + \int_{\hat{u}}^{1} \left(F(\chi(1-u)q^j) - \chi(1-u)\right) dG(u) \right)$$

where $\hat{u} = 1 - \phi/\chi \in (0,1)$. Further simplifying the equation, we get

$$\begin{aligned} r + \delta + \lambda U = & \frac{A^j}{q^j} + (\lambda_I + \lambda (G(\hat{u}) + \phi (1 - G(\hat{u})))) \phi \log(\phi q^j/\underline{\iota}) \\ & - \lambda_I \phi - \lambda \phi G(\hat{u}) + \lambda \int_{\hat{u}}^{1} (\phi \log(\chi(1-u)/\phi) - \chi(1-u)) dG(u) \end{aligned} \tag{A-3}$$

Denote the right handside as $L(q^j)$. Then we get

$$L'(q) = -\frac{A^j}{q^2} + \phi (\lambda_I + \lambda (G(\hat{u}) + \phi (1 - G(\hat{u})))) \frac{1}{q}$$

which is below zero when

$$0 < q < \frac{A^j}{\phi (\lambda_I + \lambda (G(\hat{u}) + \phi (1 - G(\hat{u}))))} \tag{A-4}$$

and above zero when

$$q > \frac{A^j}{\phi (\lambda_I + \lambda (G(\hat{u}) + \phi (1 - G(\hat{u}))))} \tag{A-5}$$

Therefore, the function $L(q)$ decreases with q first, and then increases with q. Furthermore,

$$\lim_{q \to 0} L(q) \to \infty$$

Therefore, if (A-3) has a solution, then there must be a solution in the range of (A-4), where the $L(q)$ function decreases with q. If parameters also satisfy

$$\lim_{q \to \infty} L(q) > 0$$

then there may exist another solution in the range of (A-5). Since $L(q)$ increases with q in that range, a higher interest rate r (keeping everything else the same) will lead to a higher price of capital, which is counterintuitive. As a result, we restrict the solution in the range of (A-4).

Because $L(q)$ as a function uniformly increases with A^j, and $L(q)$ decreases with q for (A-4), the equilibrium capital price increases with productivity A^j. This leads to $q^H > q^L$.

B.3 Proof of Proposition 3

This result is a consequence of $q^H > q^L$ and collateral constraint being directly related to the value of capital. First, we note that $\Delta^\omega > 0$ is equivalent to $\Delta^H > \Delta^L$, so it suffices to prove $\Delta^H > \Delta^L$.

We rewrite Δ^j as

$$\Delta^j = \int_0^{\upsilon} \max\{F(\bar{\iota}^j), F(\chi(1-u)q^j)\}dG(u) - U$$

In the integrand, both $\bar{\iota}^j$ and $\chi(1-u)q^j$ are increasing functions of q^j, and the function $F(\cdot)$ is an increasing function. Therefore, a higher q^j leads to a larger Δ^j, which means that $\Delta^H > \Delta^L$.

B.4 Proof of Proposition 4

Let's first formally set up the optimization problem in a crisis. The firm objective function is

$$\max_{x_t^j, g_t^j} \left(q_t^j F\left(x_t^j + g_t^j\right) - x_t^j - q_t^j \gamma_t g_t^j\right) k_{t-}^j$$

s.t.

$$x_t^j \le \chi(1-u)q_t^j$$

$$x_t^j \ge 0$$

$$g_t^j \ge 0$$

The amount of new investment, x_t^j, and the borrowing from government, g_t^j, are both expressed as fractions over the pre-shock capital k_{t-}^j, while the post-shock capital is $k_t^j = (1-u)k_{t-}^j$. The benefits of having x_t^j and g_t^j are both in $q_t^j F'(x_t^j + g_t^j)$, but the costs are different.

Without financing frictions (removing constraint $x_t^j \le \chi(1-u)q_t^j$), the optimal total amount of financing is

$$\bar{\iota}_t^j = F'^{(-1)}(\frac{1}{q_t^j})$$

For simplicity, we denote

$$\Phi(x) = F'^{(-1)}(x) \tag{A-6}$$

which is a decreasing function that maps from the inverse of a price into investment. Therefore, we have $\bar{\iota}_t^j = \Phi(1/q_t^j)$, which implies that $\bar{\iota}_t^j$ increases with q_t^j.

Case 1: $q_t^j \gamma_t < 1$

When $q_t^j \gamma_t < 1$, the firm will prefer to raise everything through government financing. Since g_t^j has no upper limit, in this case, we will get $x_t^j = 0$, and

$$x_t^j + g_t^j = \Phi(\gamma_t) \tag{A-7}$$

We note that the total investment $\Phi(\gamma_t) > \Phi(1/q_t^j) = \bar{\iota}_t^j$, so that the total investment is above the first best.

Case 2: $q_t^j \gamma_t = 1$

Next, when $q_t^j \gamma_t = 1$, the firm is indifferent between the two. In this corner case, any choice that satisfies the collateral constraint and nonnegative constraints should be optimal. The relative amount of government financing versus self-financing will be determined by the scale of the government funding in equilibrium.

The total investment is given by

$$x_t^j + g_t^j = \Phi(1/q_t^j) = \bar{\iota}_t^j \tag{A-8}$$

so that the firm reaches the optimal level of investment.

Case 3: $q_t^j \gamma_t > 1$

Finally, when $q_t^j \gamma_t > 1$, private financing is preferred against government financing. However, there is an upper limit of how much private financing can be achieved.

If $\chi(1-u)q_t^j < \bar{\iota}_t^j$, then the firm is financially constrained, so that the firm will choose

$$x_t^j = \chi(1-u)q_t^j$$

$$g_t^j + x_t^j = \max\{\Phi(\gamma_t), \chi(1-u)q_t^j\}$$

As the government program becomes more lenient, γ_t is higher, and the total investment is closer to the efficient level. Clearly, under this scenario, total investment is below the efficient level,

$$i_t^j = g_t^j + x_t^j \leq \Phi(\gamma_t) < \Phi(\frac{1}{q_t^j}) = \bar{\iota}_t^j$$

If $\chi(1-u)q_t^j > \bar{\iota}_t^j$, then the firm is not financially constrained, so that the firm chooses

$$x_t^j = \bar{\iota}_t^j$$
$$g_t^j = 0$$

and the total investment reaches the optimal level.

To further discuss the scenarios, as in Proposition 4, we define the cutoffs,

$$\bar{\iota}_t^j = \chi q_t^j (1 - \hat{u}_t^j)$$

$$\Phi(\gamma_t) = \chi q_t^j (1 - \tilde{u}_t^j)$$

Clearly, the cutoffs satisfy $\hat{u}_t^j < \tilde{u}_t^j$. Since we have assumed that firms are unconstrained in normal times, i.e., $\chi q_t^j > \bar{\iota}_t^j$, we have $\hat{u}_t^j > 0$. In summary, the ranking is

$$0 < \hat{u}_t^j < \tilde{u}_t^j < 1$$

If we collect the above results using these cutoffs, then we arrive at Proposition 4.

B.5 Proof of Proposition 5

The derivation of the first-order condition is almost the same as Proposition 2. Then we proceed to prove that q_t^j is a constant if γ_t is a constant. First, we write the capital pricing equation as

$$r = \frac{A^j}{q_t^j} - \delta + \frac{\lambda_I \left(q_t^j F\left(\bar{\iota}_t^j\right) - \bar{\iota}_t^j\right)}{q_t^j} + \frac{\lambda \Pi(q_t^j, \gamma)}{q_t^j} - \lambda U. \tag{A-9}$$

where

$$\Pi(q_t^j;\gamma) = \int_0^{\hat{u}_t^j} \max\left\{q_t^j F(\bar{\iota}_t^j) - \bar{\iota}_t^j, q_t^j F(\Phi(\gamma)) - q_t^j\gamma\Phi(\gamma)\right\} dG(u)$$
$$+ \int_{\hat{u}_t^j}^1 \max\left\{\chi(1-u)q_t^j, q_t^j F(\Phi(\gamma)) - \chi(1-u)q_t^j - q_t^j\gamma(\Phi(\gamma) - \chi(1-u)q_t^j)\right\} dG(u) \tag{A-10}$$

We conjecture that q_t^j is constant. Then according to the investment FOC, the efficient level of investment, $\bar{\iota}_t^j$ is also a constant. From (A-10), we also know that once q_t^j and ι_t^j become constant, the threshold $\hat{u}_t^j$ is also a constant, so the whole integral becomes a constant. As a result, every component of (A-9) is a constant, confirming our conjecture.

Next, we will show the monotonicity of q^j. Similar to Section B.2, we provide our proof under the convenient specification of $F(i) = \phi \log(i/\underline{\iota})$.

In that case, (A-9) can be simplified as

$$r + \delta + \lambda U = \frac{A^j}{q^j} + \lambda_I \left(F(\phi q^j) - \phi\right) + \lambda \frac{\Pi(q^j;\gamma)}{q^j} \tag{A-11}$$

If $\gamma q^j > 1$ $\Pi(q^j;\gamma)/q^j$ is

$$\frac{\Pi^H(q^j;\gamma)}{q^j} = \int_0^{\hat{u}} \left(F(\phi q^j) - \phi\right) dG(u) + \int_{\hat{u}}^{\tilde{u}^j} \left(F(\chi(1-u)q^j) - \chi(1-u)\right) dG(u)$$
$$+ \int_{\tilde{u}^j}^1 \left(F(\Phi(\gamma)) - \chi(1-u) - \gamma(\Phi(\gamma) - \chi(1-u)q^j)\right) dG(u)$$

which increases with q^j. If $\gamma q^j \leq 1$,

$$\frac{\Pi^j(q^j;\gamma)}{q^j} = F(\Phi(\gamma)) - \gamma\Phi(\gamma)$$

which is not affected by q^j. Denote the right hand side of (A-11) as $h(q)$. Then

$$\frac{\partial h^L}{\partial q} = -\frac{A^j}{q^2} + \lambda_I \frac{\phi}{q}$$

for $q \leq 1/\gamma$, and

B.6 Proof of Proposition 6

Government Funding and Change in Units of Capital

First, Δ^K can be expressed as

$$\Delta^K = \omega\Delta^H + (1-\omega)\Delta^L$$

where

$$\begin{aligned}\Delta^H = &\int_0^{\hat{u}(q^H)} F(\bar{\iota}^H)dG(u) + \int_{\hat{u}(q^H)}^{\tilde{u}(q^H,\gamma)} F(\chi(1-u)q^H)dG(u) \\ &+ \int_{\tilde{u}(q^H,\gamma)}^{\upsilon} F(\chi(1-u)q^H + g^H(u,q^H,\gamma))dG(u) - U\end{aligned}$$

$$\Delta^L = \int_{u=0}^{\upsilon} F\left(g^L(u,q^L,\gamma)\right)dG(u) - U$$

From the individual firm optimization problem, we know that $g^j(u,q^j,\gamma)$ decreases with γ. As a result, Δ^L decreases with γ.

To prove the monotonicity of Δ^H, we note that

$$\tilde{u}(q^H,\gamma) = 1 - \Phi(\gamma)\frac{1}{\chi q^H}\,,$$

and the derivative of Δ^H over γ is

$$\frac{\partial\Delta^H}{\partial\gamma} = \int_{\tilde{u}(q^H,\gamma)}^{\upsilon}\left(F'\left(\chi(1-u)q^H + g^H(u,q^H,\gamma)\right)\frac{\partial g^H}{\partial\gamma}\right)dG(u)\,,$$

where the differentiation over $\tilde{u}(q^H,\gamma)$ is zero because the terms of the two integration limits cancel out. Since $F'(\cdot) > 0$ and g^H decreases in γ, we get $\partial\Delta^H/\partial\gamma < 0$.

Taking the results of Δ^H and Δ^L together, we get

$$\frac{\partial\Delta^K}{\partial\gamma} = \omega\frac{\partial\Delta^H}{\partial\gamma} + (1-\omega)\frac{\partial\Delta^L}{\partial\gamma} < 0$$

Government Funding and Change of Capital Quality

Rewriting (20), the cleansing effect is

$$\Delta_\omega = \frac{\omega}{\omega + (1-\omega)\frac{1+\Delta_L}{1+\Delta_H}}$$

As a result, as long as we can prove that

$$\frac{1+\Delta_H}{1+\Delta_L}$$

increases with γ_t, then we can get that the cleansing effect increases with tighter government lending programs. Expending that expression, we get

$$\frac{1 + \int_0^{\hat{u}^H} F(\bar{\iota}^H)dG(u) + \int_{\hat{u}^H}^{\tilde{u}^H} F(\chi(1-u)q^H)dG(u) + \int_{\tilde{u}^H}^1 F(\Phi(\gamma))dG(u) - U}{1 + F(\Phi(\gamma)) - U}$$

To facilitate discussions, we denote $y = \Phi(\gamma)$, and define

$$h(y) = \frac{a + h_1(y)}{a + h_2(y)}$$

with

$$h_1(y) = \int_0^{\hat{u}^H} F(\bar{\iota}^H)dG(u) + \int_{\hat{u}^H}^{\tilde{u}^H} F(\chi(1-u)q^H)dG(u) + \int_{\tilde{u}^H}^1 F(y)dG(u)$$

$$h_2(y) = F(y)$$

$$a = 1 - U > 0$$

It is sufficient to prove that $h(y)$ decreases with y. To achieve this goal, we note that

$$h_1(y) > h_2(y) > 0$$

Furthermore,

$$0 < {h_1}'(y) = \int_{\tilde{u}^H}^1 F'(y)dG(u) < h_2'(y) = F'(y)$$

As a result,

$$h_1{}'(y) - h_2{}'(y) < 0$$

$$h_1{}'(y)h_2(y) - h_2{}'(y)h_1(y) < 0$$

Therefore,

$$\begin{aligned} h'(y) &= \frac{1}{(a+h_2(y))^2}\left(h_1{}'(y)\left(a+h_2(y)\right) - h_2{}'(y)\left(a+h_1(y)\right)\right) \\ &= \frac{1}{(a+h_2(y))^2}\left(a\left(h_1{}'(y)-h_2{}'(y)\right) + h_1{}'(y)h_2(y) - h_2{}'(y)h_1(y)\right) < 0 \end{aligned}$$

Since $y = \Phi(\gamma)$ decreases with γ, $h'(y) < 0$ implies that $h(y)$ increases with γ. Combining this result with the fact that Δ_ω increases with $h(y)$, we get Δ_ω increasing in γ.

B.7 Aggregate Investment and Demand for Government Funding

Aggregate Firm Investment

Next, we consider how the aggregate firm investment is affected by government credit support. As discussed in the main text, we only consider the more general case that not all firms are financially constrained, i.e., $\chi q_t^j \geq \bar{\iota}_t^j$ for $j \in \{L, H\}$.

Then the total amount of investment is

$$\begin{aligned} I_t = \omega_t &\left(\int_0^{\bar{u}_t^H} \max\{\Phi(\gamma_t), \bar{\iota}_t^H\} dG(u) + \int_{\bar{u}_t^H}^1 \max\{\Phi(\gamma_t), \chi(1-u)q_t^H\} dG(u)\right) \\ + (1-\omega_t) &\left(\int_0^{\bar{u}_t^L} \max\{\Phi(\gamma_t), \bar{\iota}_t^L\} dG(u) + \int_{\bar{u}_t^L}^1 \max\{\Phi(\gamma_t), \chi(1-u)q_t^L\} dG(u)\right) \end{aligned} \tag{A-12}$$

Case 1: When $\Phi(\gamma_t) \in [\bar{\iota}_t^L, \bar{\iota}_t^H]$, we get

$$\chi q_t^H > \bar{\iota}^H > \Phi(\gamma_t) > \bar{\iota}^L$$

so that

$$\max\{\Phi(\gamma_t), \bar{\iota}^H\} = \bar{\iota}^H; \quad \max\{\Phi(\gamma_t), \bar{\iota}^L\} = \Phi(\gamma_t)$$

For any $u \geq \hat{u}_t^L$, we have

$$\chi(1-u)q_t^L < \bar{\iota}_t^L < \Phi(\gamma_t)$$

so that

$$\max\{\Phi(\gamma_t), \chi(1-u)q_t^L\} = \Phi(\gamma_t)$$

However, there exists a solution $\tilde{u}_t^H \in (\hat{u}_t^H, 1)$, such that

$$\Phi(\gamma_t) = \chi(1-\tilde{u}_t^H)q_t^H$$

indicating

$$\max\{\Phi(\gamma_t), \chi(1-u)q_t^H\} = \begin{cases} \Phi(\gamma_t), & \text{if} \quad u \geq \tilde{u}_t^H \\ \chi(1-u)q_t^H, & \text{if} \quad u < \tilde{u}_t^H \end{cases}$$

Finally, we can get

$$I_t = \omega_t \left(\bar{\iota}^H G(\hat{u}_t^H) + \int_{\hat{u}_t^H}^{\tilde{u}_t^H} \chi(1-u)q_t^H dG(u) + \int_{\tilde{u}_t^H}^{1} \Phi(\gamma_t) dG(u) \right) + (1-\omega_t)\Phi(\gamma_t)$$

Clearly, γ_t only affects the highly-constrained H-type firms, but all the L-type firms. As γ_t becomes smaller, the government lending program is more lenient, reducing the "cleansing effect" for $u < \tilde{u}_t^H$ between H and L.

Note that there is a **pecking order** of financing by the H type firms. When the liquidity shock size $u < \hat{u}_t^H$, the firm utilizes its own financing from the market and invest at the efficient level. When $u \in (\hat{u}_t^H, \tilde{u}_t^H)$, the firm reaches its binding collateral constraint but doesn't participate in the government lending program. When $u > \tilde{u}_t^H$, the bank participates in the government lending program and the amount of new investment is determined by the leniency of the government credit program.

Case 2: When $1/\gamma_t > q_t^H$, we get

$$\Phi(\gamma_t) > \bar{\iota}_t^H > \bar{\iota}_t^L$$

Thus both H and L type firms will always participate in the government credit program, leading to a total investment of

$$I_t = \Phi(\gamma_t)$$

Case 3: When $1/\gamma_t < q_t^L$, we get

$$I_t = \omega_t \left(\int_0^{\hat{u}_t^H} \bar{\iota}_t^H dG(u) + \int_{\hat{u}_t^H}^{\tilde{u}_t^H} \chi(1-u) q_t^H dG(u) + \int_{\tilde{u}_t^H}^1 \Phi(\gamma_t) dG(u) \right)$$

$$+(1-\omega_t) \left(\int_0^{\hat{u}_t^L} \bar{\iota}_t^L dG(u) + \int_{\hat{u}_t^L}^{\tilde{u}_t^L} \chi(1-u) q_t^L dG(u) + \int_{\tilde{u}_t^L}^1 \Phi(\gamma_t) dG(u) \right)$$

which implies that γ_t only affects the investment of highly constrained H and L type firms.

Discussions: the efficient intervention will be two separate lending terms for H and L, $\gamma_t^H = 1/q_t^H$, and $\gamma_t^L = 1/q_t^L$ (intuitively this should be true, but we should prove it in the future). If the government picks a single $\gamma_t > 1/q_t^L > 1/q_t^H$, by reducing γ_t, the government can make the intervention for H and L type firm both more efficient. If the government picks a single $\gamma_t < 1/q_t^H < 1/q_t^L$, by increasing γ_t, the government can also make the intervention for both types more efficient. Consequently, the optimal intervention should be $\gamma_t \in [1/q_t^H, 1/q_t^L]$.

Summary: efficient government intervention should satisfy $\gamma_t \in [1/q_t^H, 1/q_t^L]$. Therefore,

$$I_t = \omega_t \left(\bar{\iota}_t^H G(\hat{u}_t^H) + \int_{\hat{u}_t^H}^{\tilde{u}_t^H} \chi(1-u) q_t^H dG(u) + \int_{\tilde{u}_t^H}^1 \Phi(\gamma_t) dG(u) \right) + (1-\omega_t)\Phi(\gamma_t)$$

The total new capital (as a multiplier over K_t) in a crisis is

$$\omega_t \left(\int_0^{\hat{u}_t^H} F(\bar{\iota}_t^H) dG(u) + \int_{\hat{u}_t^H}^{\tilde{u}_t^H} F(\chi(1-u) q_t^H) dG(u) + \int_{\tilde{u}_t^H}^1 F(\Phi(\gamma_t)) dG(u) \right) + (1-\omega_t) F(\Phi(\gamma_t))$$

Aggregate Demand for Government Financing

For financially unconstrained firms, i.e., $\bar{\iota}_t^j \leq \chi(1-u) q_t^j$, the demand for government financing is

$$g_t^j = \begin{cases} \Phi(\gamma_t) & \text{if} \quad q_t^j \gamma_t < 1 \\ [0, \bar{\iota}_t^j] & \text{if} \quad q_t^j \gamma_t = 1 \\ 0 & \text{if} \quad q_t^j \gamma_t > 1 \end{cases}$$

For financially constrained firms, i.e., $\bar{\iota}_t^j > \chi(1-u)q_t^j$, the demand for government financing is

$$g_t^j = \begin{cases} \Phi(\gamma_t) & \text{if} \quad q_t^j\gamma_t < 1 \\ [\bar{\iota}_t^j - \chi(1-u)q_t^j, \bar{\iota}_t^j] & \text{if} \quad q_t^j\gamma_t = 1 \\ \max\{\Phi(\gamma_t) - \chi(1-u)q_t^j, 0\} & \text{if} \quad q_t^j\gamma_t > 1 \end{cases}$$

For aggregate demand of government financing, we have the following cases:
If $\gamma_t < 1/q_t^H$, then the total demand for government financing is

$$g_t = \Phi(\gamma_t)$$

If $\gamma_t = 1/q_t^H$, then the total demand for government financing is

$$g_t = \omega_t \cdot \underbrace{[\int_0^1 \left(\bar{\iota}_t^H - \chi(1-u)q_t^H\right)^+ dG(u), \bar{\iota}_t^H]}_{\text{range}} + (1-\omega_t)\Phi(\gamma_t)$$

If $\gamma_t \in (1/q_t^H, 1/q_t^L)$, then the total demand for government financing is

$$g_t = \omega_t \int_0^1 \left(\Phi(\gamma_t) - \chi(1-u)q_t^j\right)^+ dG(u) + (1-\omega_t)\Phi(\gamma_t)$$

If $\gamma_t = 1/q_t^L$, then the total demand for government financing is

$$g_t = \omega_t \int_0^1 \left(\Phi(\gamma_t) - \chi(1-u)q_t^j\right)^+ dG(u) + (1-\omega_t) \underbrace{[\int_0^1 \left(\bar{\iota}_t^L - \chi(1-u)q_t^j\right)^+ dG(u), \bar{\iota}_t^L]}_{\text{range}}$$

If $\gamma_t > 1/q_t^L$, then the total demand for government financing is

$$g_t = \int_0^1 \left(\Phi(\gamma_t) - \chi(1-u)q_t^j\right)^+ dG(u)$$

An illustration of the total demand for government financing is shown in Figure 16.

Remark 1 *An important conclusion from this subsection is that there is a monotonic mapping between the total government lending $\bar{g}_t$ and the government lending tightness γ_t. At the two*

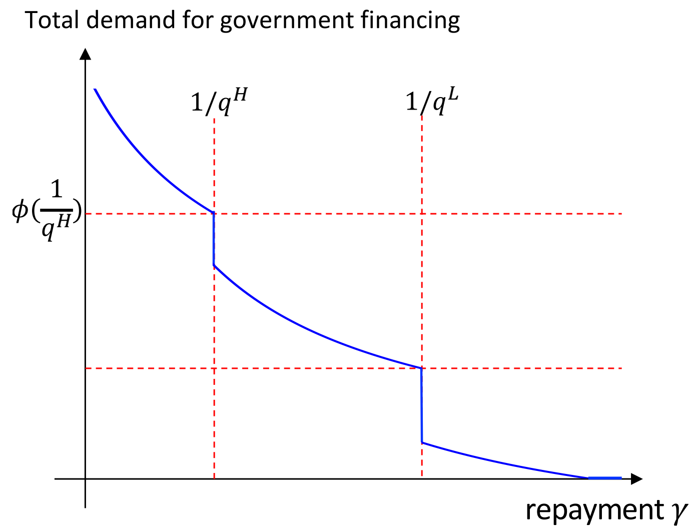

Figure 16: **Aggregate Demand for Government Funding in Crises**.

boundaries $\gamma_t = 1/q_t^H$ and $\gamma_t = 1/q_t^L$, one γ_t can be mapped to multiple $\bar{g}_t$, since the allocation between individual financing versus government financing is not fully determined. However, in terms of investment, consumption, and welfare, it is sufficient to only know γ_t. As a result, in what follows, we only consider γ_t as a control for the government, with the caveat in mind that if $\gamma_t \in \{1/q_t^L, 1/q_t^H\}$, the total scale of the government credit program is undetermined and the government has discretion of a range that yields the same equilibrium allocations in the economy.

B.8 Proof of Proposition 7

The social welfare is

$$E_t[\int_t^\infty e^{-r(s-t)} \begin{pmatrix} (\omega_s A^H + (1-\omega_s)A^L)K_{s-}ds - \lambda_I \left(\omega_s \bar{\iota}_t^H + (1-\omega_s)\bar{\iota}_t^L\right) K_{s-}ds \\ -\left(\omega_s \int_0^1 i_t^H(u)dG(u) + (1-\omega_s)\int_0^1 i_t^L(u)dG(u)\right) K_{s-}dN_s \end{pmatrix}]$$

Denote

$$d\omega_t \equiv \mu_\omega(\omega_{t-})dt + \Delta_\omega(\omega_{t-})dN_t$$

$$\frac{dK_t}{K_{t-}} \equiv \mu_K(\omega_{t-})dt + \Delta_K(\omega_{t-})dN_t$$

$$C(\omega) = \omega A^H + (1-\omega)A^L - \lambda_I \left(\omega \bar{\iota}^H + (1-\omega)\bar{\iota}^L\right)$$

Given government strategy γ_t, the welfare is

$$W(\omega_0)K_0 = E[\int_0^\infty e^{-rt} \left(C(\omega_t)K_t dt - I_t K_{t-} dN_t\right)]$$

with

$$I_t = \omega_{t-} \int_0^1 i_t^H(u) dG(u) + (1-\omega_{t-}) \int_0^1 i_t^L(u) dG(u)$$

If we conjecture a welfare function of

$$W(\omega_t)K_t$$

then the HJB equation for welfare is

$$rW(\omega) = C(\omega) - \lambda I(\omega) + W(\omega)\mu_K(\omega) + W'(\omega)\mu_\omega(\omega) + \lambda \left[W(\omega + \Delta_\omega(\omega)) \left(1 + \Delta_K(\omega)\right) - W(\omega)\right]$$

where depending on assumptions, we can either assume a fixed γ, or a dynamically optimized $\gamma(\omega)$ that has already been solved and plugged into the above formula.

There are two obsorbing states, $\omega_t = 0$ and $\omega_t = 1$, with $d\omega_t|_{\omega_t \in \{0,1\}} = 0$. As a result, at the two boundaries, $W(\omega)$ satisfies

$$rW(\omega) = C(\omega) - \lambda I(\omega) + W(\omega)\mu_K(\omega) + \lambda W(\omega)\Delta_K(\omega)$$

which is a simple algebra equation that leads to

$$W(\omega) = \frac{C(\omega) - \lambda I(\omega)}{r - \mu_K(\omega) - \lambda \Delta_K(\omega)}$$

for $\omega \in \{0, 1\}$.

B.9 Optimal Welfare

We discuss three possible cases of optimal welfare, depending on the action space of the government.

Optimal Constant γ

Suppose that the government commits to a constant γ. Then for each γ, we can solve for the welfare function $W(\omega_0;\gamma)$. The optimal γ is chosen as

$$\gamma^*(\omega_0) = \max_{\gamma} W(\omega_0;\gamma)$$

To interpret the trade off in the optimization, we can expand the welfare as

$$E[\int_0^\tau e^{-rt}C(\omega_t)dt] + e^{-r\tau}\left(-\underbrace{I(\omega_{\tau-};\gamma)}_{\text{consumption costs}} + \underbrace{W\left(\omega_{\tau-}+\Delta_\omega(\omega_{\tau-});\gamma\right)\left(1+\Delta_K(\omega_{\tau-})\right)K_{\tau-}}_{\text{continuation welfare}}\right)$$

where τ is the arrival time for the first crisis shock dN_t starting from time 0. This makes the trade off clear. For a smaller γ, the government is more lenient, so the consumption cost at time τ is higher. Furthermore, another cost is that the cleansing effect, $\Delta_{H,t}(\omega_{\tau-})$, will be smaller. The benefit is coming from a higher after-intervention capital, $1+\Delta_K$.

Optimal $\gamma(\omega)$ without long-term commitment

Suppose that we allow the government to commit a $\gamma(\omega_{t-})$ at $t-$, right before a possible crisis shock. However, the government cannot commit longer. Then the problem is essentially optimize over γ under each ω_{t-}, and assure dynamic optimality. The HJB equation becomes

$$rW(\omega) = \max_{\gamma}\left\{\begin{array}{c} C(\omega) - \lambda I(\omega;\gamma) + W(\omega)\mu_K + W'(\omega)\mu_\omega \\ +\lambda\left(W(\omega+\Delta_\omega(\omega;\gamma))\cdot(1+\Delta_K(\omega;\gamma)) - W(\omega)\right)\end{array}\right\} \tag{A-13}$$

where crisis period investment, $I(\omega;\gamma)$, jump in state variables, Δ_ω, Δ_K, are directly influenced by the choice of current γ, while other terms are taken as given as standard in HJB equations.

Optimal $\gamma(\omega)$ without commitment at all

Suppose that the government cannot commit to γ outside a crisis, but can only choose a γ during a crisis that optimizes the welfare. Then the problem becomes

$$W(\omega_{t-})K_{t-} = (1-\lambda dt)C(\omega_{t-})K_{t-}dt + (1-\lambda dt)(1-rdt)W(\omega_{t+dt})K_{t+dt}$$

$$+\lambda dt \cdot \max_{\gamma_t}\left\{-\left(\int_0^1 \big(\omega_{t-}i^H(u)+(1-\omega_{t-})i^L(u)\big)dG(u)\right)K_{t-}+W(\omega_{t-}+\Delta\omega_t)K_{t-}(1+\Delta_{K,t})\right\}$$

Then we can simplify the above HJB into

$$\begin{aligned} rW(\omega) &= C(\omega)+(W'(\omega)\mu_\omega+W(\omega)\mu_K) \\ &\quad +\lambda\max_{\gamma}\left\{W(\omega+\Delta_\omega(\omega;\gamma))\cdot(1+\Delta_{K,t}(\omega;\gamma))-W(\omega)\right\} \end{aligned} \tag{A-14}$$

We can rewrite this equation the same as (A-13).

In summary, the optimal $\gamma(\omega)$ without long-term commitment is the same as the optimal $\gamma(\omega)$ when the government has no commitment. But they are different from the case where the government can commit to a constant γ.

C Details on Numeric Solutions

C.1 Numerical Solutions

Equilibrium and Welfare with Constant γ

For any given constant γ, the prices of capital are constant. Therefore, the key of solving the model is to solve the equations for q^H and q^L. We use the standard non-linear equation solver for that problem and restrict the solution to the (unique) one that q^j decreases with discount rate r. Once q^H and q^L are solved, the dynamic evolutions of state variables are available for model simulations.

To solve for welfare given constant γ, we use the "false time derivative" method, which starts with $W(\omega, T)$ for a large T. And then iterate back, using

$$\begin{aligned} &{W_t}'(\omega,t) = C(\omega)-\lambda I(\omega)+W(\omega,t)\mu_K(\omega)+{W_\omega}'(\omega,t)\mu_\omega(\omega) \\ &+\lambda\left[W(\omega+\Delta_\omega(\omega),t)\left(1+\Delta_K(\omega)\right)-W(\omega,t)\right]-rW(\omega,t) \end{aligned} \tag{A-15}$$

$$W(\omega,t-dt)=W(\omega,t)-{W_t}'(\omega,t)dt$$

We continue the iteration until $W_t'(\omega,t)\approx 0$, in which case the HJB equation is well satisfied. Note that this method applies also to a dynamic $\gamma(\omega)$ policy, as long as we update the functions $C(\omega)$, $I(\omega)$, $\mu_K(\omega)$, $\mu_\omega(\omega)$, accordingly

Optimal Policy with Constant γ (depending on initial state ω_0)

To solve for the optimal static government policy $\gamma^*(\omega_0)$ as a function of the initial state ω_0, we proceed in the following steps:

- First, we solve for the q^H and q^L under perfect discrimination. Then we discretize a grid of γ over the range $[1/q^H, 1/q^L]$. Denote this discrete set as Γ
- Second, for each γ in the grid, we solve for the welfare function, $W(\cdot;\gamma)$.
- Third, at each $\omega_0 \in [0, 1]$, we pick

  $$\gamma^*(\omega_0) = \max_{\gamma\in\Gamma} \{W(\omega_0; \gamma)\}$$

  as the optimal solution.

By choosing a fine grid over ω and γ, we are able to get a smooth $\gamma^*(\omega_0)$ as a function of ω_0.

Equilibrium and Welfare with Dynamic $\gamma(\omega)$

To solve for the equilibrium under a dynamic government policy $\gamma(\cdot)$, we again use the false-time derivative method, by assuming that $q_t^j = q^j(\omega_t, t)$. Then we arrive at the following equation, with additional dt terms compared to the underlying equation (**??**):

$$\frac{dq^j(\omega_{t-}, t)/dt}{q^j(\omega_{t-}, t)} = r - \left(\frac{A^j}{q_{t-}^j} + \mu_{q,t}^j - \delta + \frac{\lambda_I \left(q_{t-}^j F(\bar{\iota}_{t-}^j) - \bar{\iota}_{t-}^j \right)}{q_{t-}^j} + \frac{\lambda \Pi_t^j}{q_{t-}^j} + \lambda \left((1 + \Delta_{q,t}^j)(1 - U) - 1 \right) \right) \tag{A-16}$$

Then we apply the iteration

$$q^j(\omega_{t-}, t - \Delta t) = q^j(\omega_{t-}, t) - \frac{dq^j(\omega_{t-}, t)}{dt} \cdot \Delta t \tag{A-17}$$

until $dq^j(\omega_{t-}, t)/dt \approx 0$ for all ω_{t-}.

We initialize the algorithm by first suppressing the $\mu_{q,t}^j$ and $\Delta_{q,t}^j$ components in (A-16), and solving for q^j at a constant $\gamma = \gamma(\omega)$. Then we vary the reference point ω to fit a function q^j over ω as the initial value for the iteration.

Once we solve for the equilibrium under dynamic $\gamma(\omega)$, then we can update the normal time consumption function $C(\omega)$, crisis investment function $I(\omega)$, state variable drifts and jumps, $\mu_K(\omega)$, $\mu_\omega(\omega)$, $\Delta_K(\omega)$, and $\Delta_\omega(\omega)$. With these updated functions, we use the same false time derivative method as in equation (A-15) to solve for the welfare function.

Optimal Policy with Dynamic $\gamma(\omega)$

Finally, we solve for the optimal dynamic government policy function $\gamma^*(\omega)$ as a function of the current state ω. The HJB equation is in (A-13). To solve for this equation, we start with a initial policy function that is the optimal static policy $\gamma^*(\omega_0)$. Then we apply a "double-iteration false time derivative method" as follows:

- For each round $\gamma^{(n)}(\cdot)$, solve for the associated equilibrium and the welfare function $W^{(n)}(\cdot)$.
- After solving for the n-round welfare function $W^{(n)}(\cdot)$, we solve for the optimal government policy problem indicated by (A-14), where terms not directly affected by γ are removed:

  $$\gamma^{(n+1)}(\omega) = \max_{\gamma} \left\{ -\lambda I^{(n)}(\omega;\gamma) + \lambda \left(W^{(n)}(\omega + \Delta_\omega^{(n)}(\omega;\gamma)) \cdot (1 + \Delta_K^{(n)}(\omega;\gamma)) - W^{(n)}(\omega) \right) \right\}$$

  where superscript (n) denote the functions corresponding to round-n government strategy, $\gamma^{(n)}(\cdot)$.
- Iterate over the policy function until two consecutive rounds are close enough, i.e.,

  $$\int_0^1 |\gamma^{(n+1)}(\omega) - \gamma^{(n)}(\omega)| d\omega < \varepsilon$$

  for some small $\varepsilon > 0$.

C.2 Parameters

In the baseline of the model, we set the parameters as follows.

- Discount rate $r = 10\%$.
- Depreciation rate $\delta = 10\%$.

- Frequency of investment opportunities, $\lambda_I = 0.25$, i.e., every four years.
- Frequency of liquidity crisis, $\lambda = 0.05$, i.e., every twenty years.
- Investment function,
$$F(i) = 0.2 \log(i/0.04)$$
which is setting $\phi = 0.2$ and $\underline{\iota} = 0.04$.
- The distribution of liquidity shocks in a crisis is set to a truncated normal distribution with mean of 0.6, standard deviation of 0.2, and upper bound $\upsilon = 0/8$.
- Productivity of H type capital is 0.3, and L type capital is 0.2.
- The financial constraint multiplier, χ, is set to 0.35, so that $\chi > \phi$ and firms are not financially constrained in normal times.
- The default γ for constant-γ government policy is set to be
$$\gamma_{\text{default}} = \frac{0.5}{q^H} + \frac{0.5}{q^L}$$
where q^H and q^L are the capital values consistent with such a default γ (i.e., we solve for a fixed-point problem).
- The "lenient government funding" case sets
$$\gamma_{\text{lenient}} = \frac{0.8}{q^H} + \frac{0.2}{q^L}$$
and the "tight government funding" case sets
$$\gamma_{\text{tight}} = \frac{0.2}{q^H} + \frac{0.8}{q^L}$$

These parameters are only for the purpose of illustrating the model mechanism and are not targeted at data moments. For those that require a starting point of simulation, we use a default value of $\omega_0 = 0.3$.