Fragile New Economy: Intangible Capital, Corporate Savings Glut, and Financial Instability

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Abstract

Intangible-intensive firms in the U.S. hold an enormous amount of liquid assets that are in fact short-term debts issued by financial intermediaries. This paper builds a macro-finance model that captures this structure. A self-perpetuating savings glut emerges in equilibrium. As intangibles become increasingly important for production, firms hoard more liquidity to finance investments in intangibles with limited pledgeability. The resulting low interest rates induce intermediaries to increase leverage and bid up asset prices, which in turn encourages firms to invest more and hoard even more liquidity to fund expansion. Along these secular trends, endogenous risk accumulates in the financial system.

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1 Introduction

U.S. nonfinancial corporations have accumulated a substantial amount of cash, turning from net borrowers in aggregate to net lenders (Quadrini, 2017). Their liquid assets increased from 5% of GDP in the 1980s to 9.6% of GDP by 2019. The cash-to-assets ratio of the average public firm more than doubled (Bates, Kahle, and Stulz, 2009; Gao, Whited, and Zhang, 2018). This paper provides the first theory of the macroeconomic causes and consequences of corporate savings gluts.

The first ingredient is firms’ trade-off between investing in tangible capital, which can be externally financed, and investing in unpledgeable intangibles, which needs liquidity holdings. Empirically, intangible investment has surpassed tangible investment (Corrado and Hulten, 2010), and corporate savings are concentrated in intangible-intensive sectors. In the model, intangible-investment efficiency rises exogenously, tilting investment towards intangibles. The emergence of a corporate savings glut and its macroeconomic effects depend on the general equilibrium forces.

The second ingredient is a distinguishing feature of the model. Firms hold liquidity in the form of financial intermediaries’ debts. Empirically, corporate savings consist of deposits and money-market securities issued by intermediaries. In the model, firms hold intermediary debts that are in turn backed by intermediaries’ claims on firms’ pledgeable cash flows from tangible capital. Thus, firms’ liquidity holdings and investments depend on intermediaries’ balance-sheet capacity.

A unique financial accelerator generates a self-perpetuating corporate savings glut and amplifies the economy’s responses to both the structural change of intangible-investment efficiency and business-cycle shocks. Firms’ savings push down interest rates, so intermediaries can borrow cheaply and grow, driving up the market value of tangible capital. A higher value of pledgeable capital implies that firms’ liquidity holdings can be levered up to larger investments, so firms are more eager to save and interest rates decline further, which encourages intermediaries to take on more risks and keep bidding up the value of tangible capital. In the model, firms optimally choose

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1 Liquid assets include currency, deposits, open market papers, repurchase agreements, and Treasury securities held directly or indirectly via money-market or mutual funds (source: Financial Accounts of the United States).

investment composition. Investment is financially constrained and liquidity holdings are necessary due to the intangible component, while the tangible component triggers feedback effects.

This paper is the first to jointly analyze the corporate savings glut and other secular trends, such as the decline of interest rates, the rise of asset valuations, and the growth of financial intermediation. It is also the first to analyze endogenous financial risks along these trends, showing that a transition towards an intangible-intensive economy is accompanied by an increasingly fragile financial system. Following Caballero, Farhi, and Gourinchas (2008), the model highlights the role of savings glut in explaining low interest rates under asset shortages. It differs by connecting the asset supply to intermediaries’ balance-sheet capacity, which is key to the endogenous origination of financial risk. It also differs by modelling corporate savings in a closed-economy instead of foreign savings in a world economy. Driven by investment needs, corporate savings comove with asset prices. The resulting feedback mechanism is key to the amplification of financial risk.

The continuous-time economy has entrepreneurs, bankers, and households. Their roles are discussed sequentially. First, I focus on entrepreneurs and, in particular, their liquidity demand.

A unit mass of infinitely-lived entrepreneurs manage tangible and intangible capital to produce non-durable generic goods. Capital represents efficiency units and its output is normalized to one unit of goods per unit of time. Capital depreciates stochastically, loading on an aggregate Brownian shock. A negative shock reduces capital stocks, i.e., the production capacity in the economy. In spite of these common features, tangible and intangible capital differ in liquidity.

As in Holmström and Tirole (1998), entrepreneurs face idiosyncratic Poisson shocks that entail a restart of business – their existing capital is destroyed, but they may create new capital. They choose the amount of goods to invest (scale) and their intangible share of investment (composition). To finance the investment, entrepreneurs can sell the ownership of tangible capital at a competitive-market price, and commit to dutifully managing the capital on behalf of buyers, delivering goods it produces. In other words, tangible capital is liquid (tradable and pledgeable). In contrast, intangible capital is not tradable or pledgeable, representing technological, human, and organizational capital that are inalienable or difficult for creditors to repossess.

The output of tangible capital is capitalizable, while intangible capital contributes the non-
capitalizable share of output. The dichotomy follows Caballero, Farhi, and Gourinchas (2008) who study global savings gluts and the shortages of financial assets. Later, the model is extended to incorporate a third type of capital, tradable intangibles (Akcigit, Celik, and Greenwood, 2016).

The illiquidity of intangible capital gives rise to a funding constraint on entrepreneurs’ investment. Even though investing in tangible capital relaxes the constraint, intangible investment creates sufficiently many units of intangible capital such that entrepreneurs optimally choose a positive intangible share. Importantly, the productivity of intangible investment increases over time. This captures exogenous technological changes. Because capital is essentially a stream of future consumption units, the fact that intangible investment creates increasingly more capital also captures the shift of consumers’ preference towards products and services generated by intangibles.

The funding constraint implies that entrepreneurs want to hold liquidity and finance investment with a combination of internal funds and external funds (raised against tangible capital). A potential solution of liquidity provision, in the spirit of Holmström and Tirole (1998), is to pool all entrepreneurs’ tangible capital – the source of capitalizable output – into a mutual fund whose shares are distributed to entrepreneurs. The fund diversifies away the idiosyncratic Poisson shocks, so when the shock hits an individual entrepreneur, her holdings of fund shares are still valuable and can be used to buy goods as investment inputs, even though her own capital is destroyed.

However, such diversification services require expertise. In reality, firms mainly hold money-market instruments issued by financial intermediaries in their portfolios of “cash and cash equivalents”. A unit mass of infinitely-lived bankers are introduced to intermediate the liquidity supply.

Bankers buy tangible capital with their own wealth (equity) and by issuing short-term safe debts (“deposits”) that entrepreneurs hold as liquidity buffers. Bankers create value not as lenders (their typical roles in macro-finance models) but instead as the issuers of liquid assets. The model highlights bankers’ role in addressing asset shortages (Caballero, 2006; Caballero, Farhi, and Gourinchas, 2017b). Entrepreneurs assign a liquidity premium to deposits, which is equal to the marginal value of liquidity due to the Poisson-arriving investment needs (Holmström and Tirole, 2001). This liquidity premium lowers the deposit rate, encouraging bankers to expand their balance sheets. However, acquiring tangible capital and issuing safe deposits involve risk-taking,
so bankers’ capacity to intermediate the liquidity supply depends on their wealth as the risk buffer.

Finally, households are introduced, competing with entrepreneurs to hold deposits. Following the literature, households’ demand is from deposit-in-utility, motivated by the roles of deposits as means of payment. Households can also trade tangible capital. This is the first model featuring both firms’ and households’ liquidity demand. Their relative importance in driving interest rates, asset prices, and endogenous financial risk will be analyzed via counterfactual analysis.

The Markov equilibrium has four state variables that have a hierarchical structure. The highest level is time and it triggers secular trends via the productivity of intangible investment. At the next level, the ratio of bankers’ wealth to tangible capital stock measures the size of intermediation capacity relative to the amount of assets available for intermediation. Together with time, it determines all prices, such as the deposit rate and the value of tangible capital. At the third and fourth levels, respectively, are tangible and intangible capital stocks, driving the aggregate quantities. The law of motion of a state variable depends on itself and the state variables at higher levels. The problem of solving the full equilibrium dynamics is converted to solving a system of partial differential equations. Next, I discuss how the economy responds to structural changes in the investment technology and to the business-cycle shocks (Brownian shock to the capital stocks).

The first year in the model is calibrated to represent the U.S. economy in the years around 1990, matching the intangible share of investment, the level and composition (firms’ holdings vs. households’ holdings) of intermediary debts, deposit rates, and (tangible) capital valuation. Then I input a linear trend of intangible-investment productivity and examine the model’s performances.

The model generates a trend in the intangible share of investment that matches data with an error below 0.5% (average in 1990–2010). A larger intangible share tightens the funding constraint on investment, so entrepreneurs hold more deposits and assign a larger liquidity premium, pushing down the deposit rate. A lower debt cost encourages bankers to expand their balance sheets, bidding up the market value of tangible capital. The resulting trend in capital valuation matches data with an error below 1.4% (average in 1990–2010). The rise of firms’ holdings of intermediary debts also replicates data, increasing from 9.8% of households’ holdings in 1990 to 14% in 2010.

Tangible capital plays two roles, production and liquidity provision (via the bankers’ balance
The deposit liquidity premium from entrepreneurs is transmitted by bankers to tangible capital, so the equilibrium value of tangible capital is beyond the present value of goods it produces. Therefore, the rise of entrepreneurs’ liquidity demand translates into the rising capital valuation.

A feedback mechanism reinforces the trends through a self-perpetuating savings glut. As the deposit rate declines and bankers bid up the value of tangible capital, the financing capacity of entrepreneurs’ investments becomes larger, allowing deposit holdings to be levered up to larger investments. Moreover, investments are more profitable as the tangible capital is more valuable. Despite a greater financing capacity, the cornerstone holdings of internal funds are still necessary due to the intangible part of investment. Therefore, the marginal value of liquidity increases, resulting in an even stronger deposit demand of entrepreneurs and an even lower deposit rate.

The feedback mechanism also amplifies the economy’s response to the aggregate shock that hits capital stocks. Endogenous financial risk accumulates after positive shocks, and materializes into a downward spiral when negative shocks trigger the decline of intermediated liquidity supply.

Consider a positive shock. Given bankers’ levered positions in tangible capital, the ratio of bankers’ wealth to tangible capital stock, a key state variable, increases. The liquidity premium on deposits makes bankers’ marginal costs of financing (and discount rates) lower than those of the rest of the economy. Therefore, when they become richer relative to the supply of tangible capital, their demand drives up the market value of tangible capital, which in turn leads to a higher leverage on entrepreneurs’ deposits and higher investment profits. So, entrepreneurs assign a larger liquidity premium on deposits, driving down the bankers’ discount rate. A sequence of positive shocks widen the discount-rate gap between bankers and the rest of the economy, making the value of tangible capital increasingly sensitive to shocks that trigger reallocation between the two groups.

The accumulation of endogenous financial risk is biased towards the downside. Positive shocks trigger the reallocation of tangible capital to bankers with low discount rates but eventually cause bankers to consume their wealth. However, negative shocks cause a continuing reallocation of tangible capital to the rest of the economy with high discount rates. Such asymmetry sheds light on the recent findings that longer booms precede more severe crises.3

3Please refer to Baron and Xiong (2017), Jordà, Schularick, and Taylor (2013), Krishnamurthy and Muir (2016), and López-Salido, Stein, and Zakrajšek (2017) among others. The mechanism is consistent with banks’ procyclical
The accumulation of financial risk in the form of asset-price volatility affects the real economy. Given a high volatility, the value of tangible capital falls significantly after negative shocks, reducing entrepreneurs’ leverage on deposit holdings and investments. By reducing bankers’ wealth, the decline of tangible capital value also causes bankers to shrink balance sheets, so entrepreneurs hold fewer deposits and investments decline further. The composition of capital stock, which determines the fraction of output that is capitalizable, is also affected as a result. Depending on entrepreneurs’ choices of their intangible share of investment and the investment technology, the decline of investments affect the two types of capital stocks disproportionately.

As the economy becomes more intangible-intensive, new markets emerge for the exchange of intangibles. Akcigit, Celik, and Greenwood (2016) document that 16% of the U.S. registered patents between 1976 and 2006 are traded. For this reason, the model departs from the dichotomy of liquid tangible capital and illiquid intangible capital by incorporating a third type, tradable intangibles, which are traded among entrepreneurs and households. Note that as long as some intangibles are illiquid, the cornerstone liquidity holdings are still necessary for entrepreneurs’ investments. Therefore, by enlarging entrepreneurs’ external financing capacity, tradable intangibles increase the leverage on deposit holdings and lead to a larger liquidity premium. The mechanism is amplified. The linear trend of intangible-investment productivity triggers an increasing and convex trend in the intangible share of investment, in contrast to the linear trend in the baseline model. As a result, corporate savings rise sharply, resulting in a lower deposit rate and a higher tangible capital value. The endogenous volatility of tangible capital value is 50% higher.

Finally, I conduct a counterfactual analysis to evaluate the quantitative importance of entrepreneurs’ deposit demand, relative to households’, in driving interest rates, asset prices, and endogenous financial risk. Despite being less than 15% of households’ holdings (both in the model and data), entrepreneurs’ deposit holdings have a significant impact due to the unique feedback mechanism that links the endogenous deposit rate to the value of tangible capital, and then, to the leverage on entrepreneurs’ deposit holdings. Eliminating entrepreneurs’ liquidity demand adds around 2.5% to the equilibrium deposit rate and causes the valuation of tangible capital to fall by payout in data (Baron, 2014; Adrian, Boyarchenko, and Shin, 2016).
Moreover, the endogenous volatility of tangible capital value is reduced by 30%.

**Literature.** The model makes two contributions to the literature on savings gluts, asset shortages, and low interest rates: (1) asset demand arises from endogenous corporate savings; (2) asset supply depends on financial intermediaries’ balance-sheet capacity. These two ingredients generate a feedback mechanism that amplifies both secular trends and cycles. On asset demand, the literature focuses on foreign savings (Bernanke, 2005; Caballero and Krishnamurthy, 2006; Caballero, Farhi, and Gourinchas, 2008; Caballero and Krishnamurthy, 2009; Gourinchas and Rey, 2016; Maggiore, 2017; Bolton, Santos, and Scheinkman, 2018).

On asset supply, the model is related to Giglio and Severo (2012) and Miao and Wang (2018) who emphasize the shortage of collateral capital, but financial intermediation is absent in their models and absent in the broad literature on asset shortages due to limited pledgeability (Holmström and Tirole, 2001; Kiyotaki and Moore, 2001; Farhi and Tirole, 2012; Martin and Ventura, 2012; Hirano and Yanagawa, 2017).

The disconnect between the macro-finance literature and the literature on corporate liquidity management is quite surprising given that corporate savings have become a major component of national savings (Pozsar, 2011; Carlson et al., 2016; Greenwood et al., 2016; Chen et al., 2017). The downward trend in interest rates has drawn enormous attention and has been studied jointly with other secular trends (Caballero, Farhi, and Gourinchas, 2017a; Eggertsson, Robbins, and Wold, 2018; Farhi and Gourio, 2018; Marx, Molon, and Velde, 2018; Corhay, Kung, and Schmid, 2019). Liquidity demand has been proposed as a key contributing factor (Del Negro, Giannone, Giannoni, and Tambalotti, 2017). However, corporate savings have been ignored. Moreover, the financial instability implications of such trends have not been studied. This paper bridges the gap.

The disconnect between the two literatures is also apparent in the partial-equilibrium approach taken by models of corporate liquidity management (e.g., Froot, Scharfstein, and Stein, 1993; Riddick and Whited, 2009; Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, 2008).

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4 U.S. nonfinancial corporations’ holdings of liquid intermediary debts are comparable in magnitude to foreigners’ holdings. The ratio of the former to the latter is stable since the 1990s, around 75%. Liquid intermediary debts include currency and deposits, open market papers, and repurchase agreements held directly or indirectly via money-market or mutual funds (source: Financial Accounts of the United States).

5 The first money market fund was set up to meet the demand of nonfinancial corporations (Hershey, 1973).
except Holmström and Tirole (1998), the models assume perfect storage technology, leaving out the question of who issues the liquid securities.

Recent studies in the macro-finance literature highlight the value of bank liabilities in incomplete markets (Brunnermeier and Sannikov, 2016; Quadrini, 2017) and as liquid assets for households (Kiyotaki and Moore, 2000; Moreira and Savov, 2017; Krishnamurthy and Vissing-Jorgensen, 2015; Piazzesi and Schneider, 2016; Egan, Lewellen, and Sunderam, 2018; Van den Heuvel, 2018; Begnaud, 2019; Begnaud and Landvoigt, 2018). This paper is the first to model both households’ and firms’ liquidity demand, and the model is calibrated so their relative contributions to intermediaries’ funding match data. This allows for a counterfactual analysis to show the relative importance of firms’ liquidity demand in affecting interest rates and financial instability.

Section 2 and 6 provide evidence on the distinct responses of households’ and firms’ liquidity demand to asset-price variations that are consistent with the model’s predictions. In the run-up to the Great Recession, the financial sector grew significantly (Greenwood and Scharfstein, 2013; Schularick and Taylor, 2012), feeding on cheap funds from major cash pools (Adrian and Shin, 2010; Pozsar, 2014). This paper provides the first analysis of the important role of corporate savings.

In comparison with households’ savings and foreign savings, corporate savings have two distinct empirical features: corporate savings comove with asset prices and concentrate in intangible-intensive sectors (Section 2). The first feature is essential for the feedback mechanism. The second feature brings the rise of intangibles into the analysis of interest rates and financial instability.

On the feedback mechanism, this paper develops a new financial accelerator based on firms’ savings. It amplifies both trends and cycles. The previous literature focuses on firms’ borrowing and business cycles (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999; Gertler and Kiyotaki, 2010). The feedback mechanism exhibits fire-sale dynamics. The liquidity premium on deposits creates a procyclical wedge in the discount rate between bankers, “natural buyers”

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6See also the banking theory literature (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990; Goldstein and Pauzner, 2005; Dang, Gorton, Holmström, and Ordonez, 2014; Hart and Zingales, 2014; DeAngelo and Stulz, 2015).

7Related, using a structural model, Eisfeldt (2007) show that the liquidity premium of Treasury bills cannot be explained by the liquidity demand from consumption smoothing under standard preferences. Eisfeldt and Rampini (2009) document empirically that corporate liquidity needs are correlated with measures of liquidity premium.

8Except Eisfeldt and Muir (2016), the empirical literature on corporate cash holdings focuses on trends but not cycles. Another contribution of this paper is the finding of comovement between corporate savings and asset prices.
(Shleifer and Vishny, 2011), and the rest of the economy. Thus, the longer a boom lasts, the sharper asset price falls when negative shocks reduce natural buyers’ wealth. This procyclical discount-rate wedge is distinct from the constant cash-flow wedge between intermediaries and households in Brunnermeier and Sannikov (2014) due to agents’ differences in asset-management skills.

By connecting intangible investment and corporate savings, this paper contributes to the literature on the macroeconomic implications of intangibles. Motivated by the empirical studies on

the massive cash holdings in intangible-intensive firms (Pinkowitz, Stulz, and Williamson, 2015; Graham and Leary, 2018; Falato, Kadyrzhanovaz, Sim, and Steri, 2018; Begenau and Palazzo, 2019), exogenous variation of intangible-investment efficiency is introduced to trigger firms’ liquidity demand. Through corporate savings, intangible investment has an impact on interest rates and financial instability. Moreover, in the model, the fraction of output attributed to intangible capital evolves endogenously in response to firms’ investment, which in turn depends on intermediaries’ liquidity supply. Therefore, this paper establishes a new connection between industrial structure and financial development (Levine, 1997; Rajan and Zingales, 1998).

2 Corporate Liquidity Demand

This section establishes a robust empirical link between intangible investment and firms’ demand for liquid securities. The model has two main features regarding the relationship between firms’ characteristics and their liquidity holdings (“cash”): (1) firms hold more cash when invest more in intangibles; (2) an increase in the value of tangible capital (capital with capitalizable output) causes firms to hold more cash due to the intangible (illiquid) share of investment.

The cross-sectional analysis is based on Compustat annual data from 1980 to 2019. Previous studies has shown that the rise of intangible capital is important for explaining the secular trends in corporate profits and investment (McGrattan and Prescott, 2010b; Crouzet and Eberly, 2018; Gutiérrez and Philippon, 2017; Peters and Taylor, 2017). Dell’Ariccia, Kadyrzhanova, Minoiu, and Ratnovski (2018) and Döttling and Perotti (2017) emphasize the decline of firms’ borrowings from banks as a result of less collateral assets. In contrast, this paper focuses on the liability side of banks’ balance sheets, i.e., firms holding banks’ liabilities as liquidity buffer. Previous studies also explores broad implications of intangible capital on productivity (Atkeson and Kehoe, 2005; McGrattan, 2016), current account (McGrattan and Prescott, 2010a), stock valuation (Hansen, Heaton, and Li, 2005; Ai, Croce, and Li, 2013; Eisfeldt and Papanikolaou, 2014), and investment (Daniel, Naveen, and Yu, 2018).

10This includes Compustat firm-year observations with non-missing data for total assets and sales. All firms incor-

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key variable of interest is nonfinancial firms’ liquidity holdings, i.e., “cash and cash equivalents”. A firms’ intangible intensity is measured by the ratio of intangible investment to total assets (averaged over time). Following the literature, intangible investment includes R&D and organizational-capital investment that is 30% of selling, general and administrative expenses. Firms are then sorted into deciles by intangible intensity to form the ranking variable “Intan./Assets”. Two aggregate measures of capital valuations are constructed. Each year, I calculate the market capitalization-weighted average ratio of enterprise value (EV) to earnings before interest, tax, depreciation, and amortization (EBITDA). EV is the market-based measure of a firm’s capitalizable output, and EBITDA is the annual output. For a more restrictive measure of tangible capital valuation, I calculate “Tangible EV/EBITDA” by only using firms in the lowest quintile of Intan./Assets.

Figure 1 reports scatter charts (with regression lines) of the cash-to-asset ratio against capital valuation for each Intan./Assets quintile. Each point is given by the quintile’s market capitalization-weighted average cash-to-asset ratio in a year and the average EV/EBITDA in that year. More
### Panel A: Capital Intangibility & Corporate Cash Holdings

<table>
<thead>
<tr>
<th>Cash Assets</th>
<th>Intangibility = Intan./Assets</th>
<th>Intangibility = – PPE/Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Intangibility</td>
<td>3.400***</td>
<td>3.346***</td>
</tr>
<tr>
<td></td>
<td>(0.227)</td>
<td>(0.235)</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Year FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>152,549</td>
<td>152,549</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.1761</td>
<td>0.1993</td>
</tr>
</tbody>
</table>

### Panel B: Capital Valuation & Intangible-Driven Corporate Cash Holdings

<table>
<thead>
<tr>
<th>Cash Assets</th>
<th>Valuation = Ave. EV/EBITDA</th>
<th>Valuation = Tangible EV/EBITDA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Intan./Assets</td>
<td>-1.259*</td>
<td>-1.422**</td>
</tr>
<tr>
<td></td>
<td>(0.633)</td>
<td>(0.600)</td>
</tr>
<tr>
<td>Valuation</td>
<td>-0.969***</td>
<td>-0.948***</td>
</tr>
<tr>
<td>Intan./Assets×Valuation</td>
<td>0.438***</td>
<td>0.454***</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>Controls</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Year FE</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>152,549</td>
<td>152,549</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.2110</td>
<td>0.2231</td>
</tr>
</tbody>
</table>

Firm-year clustered standard errors in parentheses

* $p < 0.1$  ** $p < 0.05$  *** $p < 0.01$

Intangible firms hold more cash and show a sharper correlation between cash and capital valuation. Figures C.1 and C.2 in Appendix C show the same pattern with tangible EV/EBITDA and Tobin’s Q as alternative measures of capital valuation.\(^\text{12}\)

Table 1 reports the results of a regression analysis that allows the presentation of conditional correlations by controlling for firm characteristics and time fixed effects.\(^\text{13}\)

\(^\text{12}\)Tangible EV/EBITDA is the average EV/EBITDA of the firms in the lowest Intan./Assets quintile. Two versions of Tobin’s Q are calculated, the whole market’s average Tobin’s Q and tangible Tobin’s Q that is the average Tobin’s Q of firms in the lowest Intan./Assets quintile. All averages are market capitalization-weighted.

\(^\text{13}\)The control variables are selected and winsorized following Opler et al. (1999) and Bates et al. (2009). They
Figure 2: **Decomposing Nonfinancial Firms’ Holdings of Liquid Securities.**

shows that more intangible-intensive firms hold more cash.\(^{14}\) The most conservative estimate in Column (4) implies a difference of 13.7% in the cash-to-asset ratio between the top and bottom deciles of Intan./Assets. Columns (5) to (8) show that less intangible firms (ranked in higher deciles of property, plant, and equipment, “PPE”) hold less cash. These findings are consistent with previous studies.\(^{15}\) Panel B reports a positive coefficient of the interaction between asset valuation and firms’ intangibility that is robust across specifications. Intangible firms’ cash holdings are more sensitive to capital valuation, consistent with the increasingly steep slope in Figure 1. Table C.5 in Appendix C reports the same pattern with Tobin’s Q measuring capital valuation.\(^{16}\)

Figure 2 shows nonfinancial firms’ liquidity holdings are mainly issued by financial interme-include (Compustat codes in parenthesis): acquisition activity (aqc/at), capex (capx/at), cash flow ([oibdp – xint – dvc – txt]/at), net working capital ([wcap – chel]/at), payout dummy (equal to 1 if dvc is positive), leverage ([dlc – dltl]/at), market to book ratio ([at + prcc_f*csho – ceq]/at), R&D to sales ratio (xrd/sale), size (log of at in 2005 dollars), Tobin’s Q ([at + prcc_f*csho – ceq – txdb]/[0.1*( at + prcc_f*csho – ceq – txdb) + 0.9*at]), and industry sigma, which is the 10-year mean of the cross-sectional standard deviations of firms’ cash flow/assets in a two-digit SIC industry.

\(^{14}\)This is consistent with previous studies (Begenau and Palazzo, 2019; Pinkowitz et al., 2015; Falato et al., 2018).

\(^{15}\)Investment need is a key determinant of firms’ cash holdings (Denis and Sibilkov, 2010; Duchin, 2010). Firms with less collateral also tend to hold more cash (Almeida and Campello, 2007; Li, Whited, and Wu, 2016).

\(^{16}\)Table C.4 reports the same pattern when firms are sorted by asset tangibility (i.e., PPE-to-asset ratio). Less tangible firms exhibit stronger correlation between cash and capital valuation (measured by both EV/EBITDA and Tobin’s Q).
diaries (source: Financial Accounts of the U.S.). Mutual fund and money market fund holdings are attributed to the underlying assets based on sector level tables. Nonfinancial firms are among the major cash pools that feed leverage to intermediaries (Carlson et al., 2016; Pozsar, 2014). Their liquidity holdings scaled by GDP almost doubled by 2019. The trend was disrupted in the financial crisis, falling to the 1980s level, and firms flighted to Treasuries. This loss of funding for intermediaries is recognized by regulators. Retail deposits are assigned 90% to 95% stable funding factor while corporate deposits are assigned 50% (Basel Committee on Banking Supervision, 2014).

The rise of firms’ liquidity holdings and its procyclicality can be explained by the findings in Figure 1 and Table 1. Figure 3 shows the rise of the intangible share of investment (Panel A),

\[ \theta_i \equiv \frac{\text{Intangible Investment}}{\text{Intangible Investment} + \text{Capital Expenditure}}, \]

and EV/EBITDA (Panel B) that contribute to the rise of firms’ liquidity demand. Panel C reports the ratio of nonfinancial firms’ holdings of intermediary debts to households’ holdings (source: Financial Accounts of the U.S.). Recession years are marked by shaded areas. The ratio trends

Figure 3: The Rise of Intangible Investment, Capital Valuation, and Corporate Savings.
upward with cyclical drops in recessions, suggesting that, as a source of funding for intermediaries, corporate liquidity holdings are more procyclical than households’. This is likely due to the fact that corporate liquidity holdings increase in the procyclical capital valuation. In contrast, households’ holdings decrease in various measures of capital valuation (Section 6).

Next, a model is built to generate these trends in the intangible share of investment, capital valuation, corporate liquidity holdings. It also generates the trends in interest rates and the size of intermediation sector, which have been documented extensively by previous studies.

3 Model

Consider a continuous-time, infinite-horizon economy. The production sector is set up first with a focus on its liquidity demand that is driven by intangible investment. Later, bankers and households are introduced. The model fixes a probability space and an information filtration that satisfy the usual regularity conditions (Protter, 1990). Competitive agents act under rational expectations.

3.1 The Production Sector and Liquidity Demand

Preferences. There is a unit mass of entrepreneurs. Let $\mathbb{E} = [0, 1]$ denote the set of entrepreneurs. Let $c_t^E$ denote a representative entrepreneur’s cumulative consumption up to time $t$. Throughout this paper, subscripts denote time, and whenever necessary, superscripts are used to denote agents’ type, with “$E$” for entrepreneurs (and later, “$B$” for bankers and “$H$” for households). An entrepreneur maximizes the life-time, risk-neutral expected utility with discount rate $\rho$:

$$
\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-\rho t} dc_t^E \right].
$$

(1)

Appendix B discusses the implications of incorporating risk-averse preferences.

Capital and production. Each entrepreneur manages a firm that has tangible and intangible capital. Capital represents efficiency units and is counted by its output. One unit of capital produces one unit of non-durable generic goods per unit of time. In aggregate, the economy has $K_t^T$ and
$K_t^I$ units of tangible and intangible capital, respectively, at time $t$ that generate a flow of output, $(K_t^T + K_t^I) dt$. A fraction $\delta dt - \sigma dZ_t$ of capital are destroyed over $dt$. The standard Brownian motion $Z_t$ captures aggregate shocks to production capacity.\(^1\)

The two types of capital differ in liquidity. Tangible capital is perfectly liquid. Entrepreneurs may sell the ownership of their firms’ tangible capital, and after the sale, dutifully manage the capital on behalf of the buyers and deliver the goods produced. Therefore, tangible capital is free from frictions that compromise the cash-flow pledgeability or secondary-market liquidity. We may think of tangible capital as inventory, equipment, plant, and property. While in reality, certain physical assets are not actively traded, but the securities backed by their cash flows are traded.

In contrast, intangible capital is illiquid. Its ownership cannot be traded, and the goods it produces cannot be pledged for external funds. Later, an extension incorporates tradable intangibles. Intangible capital represents entrepreneurs’ human capital, organizational capital, proprietary technologies, and brand names that are difficult to repossess by creditors.

**Investment and liquidity demand.** Entrepreneurs face idiosyncratic shocks of investment needs. The Poisson arrival of such shocks is independent across entrepreneurs with intensity $\lambda$. When hit by the shock, an entrepreneur’s firm loses all capital but is endowed with a technology to transform goods into new capital instantaneously.\(^2\) She chooses $i_t$, the amount of goods invested, and $\theta_t$, the intangible share, to create $\kappa_t^I \theta_i i_t$ units of intangible capital and $\kappa_t^T (1 - \theta_t) i_t$ units of tangible capital, where $\kappa_t^I$ and $\kappa_t^T$ are the productivities of intangible and tangible investments, respectively.

The productivity of intangible investment is an increasing function of time, $\kappa_t^I = \kappa^I (t)$. Because capital represents production units that correspond to a stream of goods, an increase of $\kappa_t^I$ means that intangible investment generates more production capacity.

Let $q_t^I$ denote the value of intangible capital (denominated in goods). The entrepreneur is indifferent in consumption timing, so she values the goods from non-tradeable intangible capital

\(^1\)For parsimony, the stochastic depreciation rates are set to be the same for both types of capital. Introducing different depreciation rates for intangible and intangible capital will not change the mechanism and the solution method.

\(^2\)This specification reflects the lumpiness of investment at micro levels (e.g. Doms and Dunne, 1998). Due to the idiosyncratic nature of investment opportunities, the aggregate investment is smooth, in line with Thomas (2002).
by Gordon growth formula, accounting for normal-time depreciation and Poisson destruction

\[ q^I_t = \frac{1}{\rho + \delta + \lambda}. \]  

(2)

Henceforth, the time subscript is dropped for \( q^I \). In contrast, the value of tangible capital, denoted by \( q^T_t \), depends on agents’ trading, so it varies over time and loads on the aggregate shock,

\[ dq^T_t = q^T_t \mu^T_t \, dt + q^T_t \sigma^T_t \, dZ_t. \]  

(3)

where the drift and diffusion terms, \( \mu^T_t \) and \( \sigma^T_t \), will be endogenously determined in equilibrium.

Given \( q^I \) and \( q^T_t \), an investing entrepreneur maximizes the investment profits:

\[
\max_{(i_t, \theta_t)} \left\{ q^I_t \kappa^I_t \theta_t + q^T_t \kappa^T_t (1 - \theta_t) - F(\theta_t) \right\} i_t - i_t,
\]

(4)

where a convex \( F(\theta_t) \) is introduced to avoid counterfactual corner solutions (i.e., \( \theta_t \in \{0, 1\} \)).

Due to the illiquidity of intangible capital, the scale of investment is constrained by tangible value:

\[ i_t \leq q^T_t \kappa^T_t i_t (1 - \theta_t). \]  

(5)

Self-financing, \( 1 \leq q^T_t \kappa^T_t (1 - \theta_t) \), is ruled out by the following condition (Appendix A).

**Assumption 1** *Investment projects are not self-financed: \( 1 > \kappa^T_t \left( \frac{1}{\rho + \delta + \lambda} \right) \).***

Under Assumption 1, entrepreneurs would prefer to hold liquidity, i.e., assets other than their own capital, which are immune to the Poisson shocks and can be traded for goods as investment inputs when the Poisson shocks hit.\(^{19}\) Holmström and Tirole (1998) point out a theoretical solution that is to pool all liquid assets (tangible capital) in mutual funds where idiosyncratic shocks are diversified away. Then entrepreneurs hold the mutual-fund shares and use them as liquidity for

\(^{19}\)It has been well documented that intangible investments rely heavily on firms’ internal liquidity (for example, R&D investments in Hall (1992), Himmelberg and Petersen (1994), and Hall and Lerner (2009)).
investment. Let \( m_t^E \) denote an entrepreneur’s liquidity holdings, so the constraint (5) becomes

\[
i_t \leq m_t^E + q_t^T \kappa^T i_t (1 - \theta_t) .
\]  

(6)

### 3.2 Intermediated Liquidity Supply

In reality, firms rarely hold direct claims on other firms but instead hold debt securities largely issued by financial intermediaries as illustrated by Figure 2. There are several reasons why firms hold intermediated liquidity. The service of diversifying away idiosyncratic shocks is likely to require expertise.

Agency frictions limit the issuance of equity (e.g., He and Krishnamurthy, 2013), so firms hold intermediaries’ debt instead of equity. Intermediated liquidity supply is also motivated by the literature on banks as inside money creators (e.g., Kiyotaki and Moore, 2000).

Bankers are introduced to intermediate the supply of liquidity. Entrepreneurs are assumed to hold liquidity in the form of short-term bank debts (referred to as “deposits”) that are in turn backed by banks’ holdings of tangible capital. With a slight abuse of notation, \( m_t^E \) now represents entrepreneurs’ deposit holdings that mature in \( dt \) with interests \( r_t dt \). I characterize a Markov equilibrium where banks never default, so bank debt is safe and \( r_t dt \) is the realized return. Entrepreneurs use deposits to buy goods as investment inputs when hit by the Poisson shocks.

In contrast to the existing macroeconomic models with financial intermediation that emphasize bankers’ expertise on lending, this model emphasizes the liability side of bank balance sheets – banks add value to the economy because their debts are held by entrepreneurs as liquidity buffers.

**Preferences.** There is a unit mass of bankers. Let \( \mathbb{B} = [0, 1] \) denote the set of bankers. A representative banker maximizes the life-time, risk-neutral expected utility with discount rate \( \rho \):

\[
\mathbb{E} \left[ \int_0^\infty e^{-\rho t} dc_t^B \right] ,
\]  

(7)

---

20 Introducing intermediaries can also be motivated by their expertise in monitoring (Diamond, 1984), restructuring (Bolton and Freixas, 2000), or enforcing collateralized claims (Rampini and Viswanathan, 2019).

21 Macro-finance models that are built upon diffusion processes typically do not feature bank default (e.g., Brunnermeier and Sannikov, 2014). Default may be introduced through an aggregate Poisson shock that destroys capital.
where $c_t^B$ denotes a banker’s cumulative consumption up to time $t$.

**Balance sheet.** A banker incurs interest expenses $r_t dt$ on debt liabilities and earns risky return $dr_t^T$ on her holdings of tangible capital, where $r_t^T$ denotes the cumulative return that loads on shocks. To characterize $dr_t^T$, let $k_t^{TB}$ denote a banker’s holdings of tangible capital, with “T” and “B” indicating “tangible” and “banker” respectively. Capital stock depreciates stochastically, so

$$dk_t^{TB} = -k_t^{TB} (\delta dt - \sigma dZ_t) - k_t^{TB} \lambda dt. \tag{8}$$

The last term is from the $\lambda dt$ firms that lose capital at the Poisson shocks. Through diversification, the banker faces a constant rate of capital destruction instead of idiosyncratic Poisson shocks.

By Itô’s lemma, equations (3) and (8) imply the following tangible capital return:

$$dr_t^T = \frac{k_t^{TB} dt}{q_t^T k_t^{TB}} + \frac{d \left( q_t^T k_t^{TB} \right)}{q_t^T k_t^{TB}} = \left( \frac{1}{q_t^T} + \mu_t^T - \delta - \lambda + \sigma_t^T \sigma \right) dt + (\sigma_t^T + \sigma) dZ_t \tag{9}$$

The first term, $1dt/q_t^T$, is dividend yield – the production flow, $1dt$, divided by the unit value, $q_t^T$. The second to fourth terms, $(\mu_t^T - \delta - \lambda) dt$, account for the expected unit value change, the quantity depreciation, and the measure of firms hit by the Poisson shocks. The fifth term, $\sigma_t^T \sigma$, is Itô’s quadratic covariation. The return loads on the aggregate shock via $\sigma_t^T$, the endogenous return volatility of $q_t^T$ (price risk), and $\sigma$, the exposure to exogenous depreciation shock (quantity risk).

Let $n_t^B$ denote a representative banker’s wealth that has the following law of motion,

$$dn_t^B = x_t^B n_t^B dr_t^T - (x_t^B - 1)n_t^B r_t dt - dc_t^B, \tag{10}$$

where $x_t^B \equiv q_t^T k_t^{TB} / n_t^B$ is the asset-to-wealth ratio and debt value is $(x_t^B - 1)n_t^B = q_t^T k_t^{TB} - n_t^B$.

As shown by (10), intermediation involves risk-taking. Bankers issue safe deposits while holding risky tangible capital. Wealth (equity) buffers risk, and an undercapitalized banking sector cannot fulfill its role as liquidity supplier. To capture this idea, I assume that banks cannot issue outside equity to replenish wealth, i.e., $dc_t^B \geq 0$ as in Brunnermeier and Sannikov (2014).\(^{22}\)

\(^{22}\)By inspecting equation (9), we can see that negative consumption is equivalent to issuing equity to replenish net
can be motivated by agency frictions. As a result, bankers’ wealth drives the \textit{intermediation capacity}. In this model, entrepreneurs’ liquidity demand from Holmström and Tirole (1998) meets banks’ limited balance-sheet capacity from Holmström and Tirole (1997).

### 3.3 Households

The main purpose of the banking sector is to hold tangible capital and issue debts to entrepreneurs who need liquidity buffers. In reality, households also hold financial intermediaries’ debts, for example, in the U.S., their holdings were on average 8.5 times those of nonfinancial firms’ from 1980 to 2019.\textsuperscript{23} While households’ demand for deposits is not essential for the main mechanism, it is important to incorporate it in order to calibrate model and to perform the counterfactual analysis, in which I will compare the economies with and without entrepreneurs’ liquidity demand.

The literature on households’ demand for intermediaries’ debts takes a money-in-utility approach, motivated by the role of intermediaries’ debts (e.g., deposits) as means of payment (Sidrauski, 1967; Stein, 2012; Van den Heuvel, 2018).\textsuperscript{,} Households’ utility from deposits is specified as a power utility function separable from the consumption utility (Poterba and Rotemberg, 1986; Nagel, 2016; Begena and Landvoigt, 2018) and deposits are normalized by measures of income (Begena, 2019; Krishnamurthy and Vissing-Jorgensen, 2015). Consider a unit mass of households, $\mathbb{H} = [0, 1]$. Following the literature, a representative household’s utility is given by

$$
\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-\rho t} \left( dc_t^H + \beta_t \left( \frac{m_t^H}{w_t^H} \right)^{1-\xi} \frac{w_t^H}{1-\xi} \right) \right],
$$

(11)

where $c_t^H$ is the cumulative consumption process, $m_t^H$ is the holdings of deposits, and $w_t^H$ is labor income per unit of time. The scaling parameter is a function of time, $\beta_t = \beta(t)$, and will be calibrated to generate a ratio of firms’ to households’ holdings of bank debts that match data.

\textsuperscript{23}Intermediary debts are those listed in Figure 2 (source: Financial Accounts of the United States).
It is assumed that the aggregate labor income is proportional to the output of tangible capital,

\[ \int_{i \in H} w^H_t (i) \, di = \alpha K_t. \]  

(12)

This follows the models of output attribution to labor, physical capital, and intangibles (Lucas, 1978b; Atkeson and Kehoe, 2005). The parameter \( \alpha \) is redundant in aggregate analysis. To see that, consider the optimality condition for \( m^H_t \), which equates the marginal utility of holding deposits and the marginal cost, i.e., the spread between the required return, \( \rho \), and the deposit rate \( r_t \):

\[ \beta_t \left( \frac{m^H_t}{w^H_t} \right)^{-\xi} = \rho - r_t. \]  

(13)

Rearranging the equation and aggregating across households, we obtain

\[ M^H_t = \int_{i \in H} m^H_t (i) \, di = \left( \int_{i \in H} w^H_t (i) \, di \right) \left( \frac{\rho - r_t}{\beta_t} \right)^{-\xi} = \alpha K^T_t \left( \frac{\rho - r_t}{\beta_t} \right)^{-\xi}, \]  

(14)

where the last step utilizes (12). The parameter \( \alpha \) simply scales the aggregate deposit demand, but the same can be achieved by scaling \( \beta_t \). Therefore, only the calibration of \( \beta_t = \beta (t) \) is necessary.

### 3.4 Aggregation and Markov Equilibrium

**Overview.** Figure 4 summarizes the model. As in Caballero, Farhi, and Gourinchas (2008), only a fraction of output is capitalizable – the output of tangible capital – and the key inefficiency is a liquidity shortage. Depending on the bankers’ risk-taking capacity (wealth), a fraction of tangible capital is owned by bankers who in turn issue deposits to households and entrepreneurs. Entrepreneurs’ deposits relax the liquidity constraint (6) on investment. Therefore, economic growth depends on the intermediated liquidity supply. An increase of \( \kappa^I_t \) means that more production capacity is generated by intangible investment and the liquidity shortage is exacerbated.

The economy has three markets to clear (goods, the ownership of tangible capital, and deposits). The aggregate output is \( (K^I_t + K^T_t + \alpha K^T_t) \, dt \), generated by intangible capital, tangible capital, and labor. The \( \lambda dt \) entrepreneurs who are hit by the Poisson shocks acquire goods to
create new capital, and the remaining goods are consumed by the rest of the economy.\textsuperscript{24} Entrepreneurs, bankers, and households can all trade the ownership of tangible capital at competitive price $q^T_t$ given the stock $K^T_t$. In the deposit market, bankers’ supply is equal to the demand from entrepreneurs, $M^E_t = \int_{i \in E} m^E_t (i) \, di$, and the demand from households, $M^H_t = \int_{i \in H} m^H_t (i) \, di$.

The real-financial linkage. Rearranging the binding constraint (6), we obtain investment

$$i_t = \left( \frac{1}{1 - q^T_t \kappa^T (1 - \theta_t)} \right) m^E_t .$$

(15)

One unit of liquidity is leveraged up to $1 / \left[ 1 - q^T_t \kappa^T (1 - \theta_t) \right]$ units of goods invested because tangible capital is pledgeable. Let $M^E_t$ denote the entrepreneurs’ aggregate deposit holdings. In $dt$, the aggregate investment, $I_t$, from the $\lambda dt$ entrepreneurs (hit by the Poisson shocks) is

$$\left( \frac{1}{1 - q^T_t \kappa^T (1 - \theta_t)} \right) M^E_t \lambda dt .$$

(16)

The deposit-market clearing condition links the entrepreneurs’ liquidity to bankers’ wealth:

$$M^E_t = (x^B_t - 1) N^B_t - M^H_t .$$

(17)

\textsuperscript{24}Under agents’ risk-neutral utility on consumption flows, their demand for consumption goods is perfectly elastic.
where the right side is the deposits issued by bankers minus the households’ deposit holdings.

Given the aggregate investment, the law of motion of aggregate intangible capital is

$$dK^I_t = \left(\frac{1}{1 - q^T_t \kappa^T_t (1 - \theta_t)}\right) \left[(x^B_t - 1) N^B_t - M^H_t\right] \theta_t \kappa^I_t \lambda dt - (\delta dt - \sigma dZ_t) K^I_t, \quad (18)$$

and the law of motion of aggregate tangible capital is

$$dK^T_t = \left(\frac{1}{1 - q^T_t \kappa^T_t (1 - \theta_t)}\right) \left[(x^B_t - 1) N^B_t - M^H_t\right] (1 - \theta_t) \kappa^T_t \lambda dt - (\delta dt - \sigma dZ_t) K^T_t. \quad (19)$$

Total investment in (16) is split into the tangible and intangible parts by entrepreneurs’ choice of intangible share, $\theta_t$. Then investments are multiplied by the productivities, $\kappa^I_t$ and $\kappa^T_t$.25

Equations (18) and (19) highlight the link between intermediation capacity and economic growth. When bankers are well-capitalized, more deposits are issued. Intermediation creates liquidity that can be leveraged up to finance the growth of productive capital in the economy.

Equations (18) and (19) also show how the financial conditions drive economic fluctuations. Entrepreneurs’ leverage on liquidity increases in the value of tangible capital, $q^T_t$. Therefore, the endogenous asset-price volatility, i.e., $\sigma^T_t$ in (3), feeds into investment fluctuation, and thus, has a direct impact on the real economy. The ratio of total risk to exogenous risk, $(\sigma^T_t + \sigma) / \sigma$, measures the strength of financial amplification and shall be the focus of financial instability analysis.

In response to liquidity creation by bankers and entrepreneurs’ choices of investment scale and composition, tangible and intangible capital stocks evolve continuously as documented by the empirical literature (Peters and Taylor, 2017; Falato et al., 2018; Begenau and Palazzo, 2019).

**State variables.** I solve a Markov equilibrium where endogenous variables are functions of state variables. The economy has three stock variables, $N^B_t \equiv \int_{i \in B} n^B_{i,t} di$, the bankers’ aggregate wealth, $K^I_t$, and $K^T_t$ that are predetermined before agents’ decision making. They are naturally the state

25The lost capital of entrepreneurs hit by the Poisson shock is evenly endowed to the rest of entrepreneurs. Therefore, the $\lambda dt$ measure of lost capital lost is not in (18) and (19), and the idiosyncratic Poisson shocks do not affect aggregate production capacity. Accordingly, one interpretation of the Poisson shock is that the $\lambda dt$ entrepreneurs’ customer base is seized by the other entrepreneurs through creative destruction (Aghion, Akcigit, and Howitt, 2014)
variables. Moreover, due to the time trends in the productivity of intangible investment, \( \kappa^I_t \), and the scaling parameter of households’ deposit utility, \( \beta_t \), time is also a state variable.

**State Variables**: \( t, N^B_t, K^I_t \), and \( K^T_t \).

These four state variables have a convenient *hierarchical* property. First, apparently, time progresses linearly and has an autonomous law of motion. Second, \( (N^B_t, K^I_t, K^T_t) \) can be equivalently represented by \( (\eta_t, K^I_t, K^T_t) \), where \( \eta_t \), the intermediation intensity, is defined by

\[
\text{Intermediation Intensity} : \quad \eta_t = \frac{N^B_t}{K^T_t}.
\]

(20)

It is a ratio of intermediation capacity to the total amount of assets to be intermediated. The next proposition states that its evolution only depends on itself and time, and that the market prices, such as \( q^T_t \) and \( r_t \), and the \( K^T_t \)-scaled aggregates quantities are functions of \( \eta_t \) and time only. Appendix A provides the proof. Therefore, to solve the Markov equilibrium, I first focus on the sub-system where \( \eta_t \) and time are the two state variables and solve the market prices and the \( K^T_t \)-scaled aggregate quantities, which require solving a system of partial differential equations (PDEs). The solutions of these variables are then fed into the laws of motion of \( K^I_t \) and \( K^T_t \) (see (18) and (19)) for a complete characterization of equilibrium dynamics.

**Proposition 1 (Financial System)** The equilibrium law of motion of intermediation intensity is

\[
\frac{d\eta_t}{\eta_t} = \mu^\eta(\eta_t, t) \, dt + \sigma^\eta(\eta_t, t) \, dZ_t,
\]

(21)

for \( \eta_t \in (0, \bar{\eta}(t)) \), where \( \bar{\eta}(t) \) is an upper reflecting boundary that varies over time. Appendix A defines \( \mu^\eta(\eta_t, t) \), \( \sigma^\eta(\eta_t, t) \), and \( \bar{\eta}(t) \). The following market price variables and \( K^T_t \)-scaled aggregate quantities (denoted by “\( \sim \)”, for example, \( \tilde{M}^E_t \equiv M^E_t/K^T_t \)) are functions of \( \eta_t \) and time only: (1) the value of tangible capital, \( q^T_t = q^T(\eta_t, t) \); (2) the deposit rate, \( r_t = r(\eta_t, t) \); (3) the \( K^T_t \)-scaled households’ deposits, \( \tilde{M}^H_t = \tilde{M}^H(\eta_t, t) \); (4) the \( K^T_t \)-scaled entrepreneurs’ deposits, \( \tilde{M}^E_t = \tilde{M}^E(\eta_t, t) \); (5) the optimal intangible share of investment, \( \theta_t = \theta(\eta_t, t) \); (6) bankers’ asset-to-wealth ratio, \( x^B_t = x^B(\eta_t, t) \); (7) bankers consume only when \( \eta_t = \bar{\eta}(t) \).
Time $t$ drives entrepreneurs’ and households’ deposit demands via $\kappa_t^I$ and $\beta_t$ respectively. The status of financial system is given by $\eta_t$ that drives the value of tangible capital and the deposit rate. As the leverage on liquidity and investment profits vary over time, entrepreneurs respond in their choices of deposit holdings and investment composition. Capital stocks, $K_t^T$ and $K_t^I$, evolve, determining the relative contributions of tangible and intangible capital to output.$^{26}$

Proposition 1 simplifies the presentation of equilibrium dynamics. Instead of presenting the endogenous variables in a four-dimensional state space, I plot each variable against $\eta_t$ to show how the financial condition drives the price variables and $K_t^T$-scaled quantities, given the values of $\kappa_t^I$ and $\beta_t$. Then, averaging over $\eta_t$, I compare these variables under different values of $\kappa_t^I$ and $\beta_t$.

4 Equilibrium Dynamics

This section starts with calibration, and then presents the results on trends and cycles. The variations of $\kappa_t^I$ and $\beta_t$ over time trigger trends in intangible investment, liquidity demand from the production sector, interest rates, capital valuations, and the size of banking sector. Along the trends, the financial condition, i.e., $\eta_t$, drives economic fluctuations through endogenous asset-price volatility, $\sigma_t^T$ in (3), that amplifies the impact of exogenous shocks on investment and growth.

4.1 Mapping the Model to Data

The model is calibrated and solved numerically. I use annual data on U.S. firms, households, and financial markets from CRSP/Compustat, Federal Reserve Economic Data (FRED), and Financial Accounts of the United States (Flow of Funds). As in Section 2, the sample period is 1980–2019.

Trends in the model and data. One unit of time is one year in the model. Time drives trends via $\kappa^I (t)$ and $\beta (t)$, and $\eta_t$ drives economic fluctuations along the trends. To examine the model’s predictions on trends, I obtain the simulated paths of prices and $K_t^T$-scaled quantities (functions of $t$

$^{26}$The intangible share of capital stock increases over time, consistent with empirical findings (Begenau and Palazzo, 2019; Peters and Taylor, 2017). Capital composition is a key state variable in Eberly and Wang (2008) who study agents’ trade-off between diversification benefits and reallocation costs when two sectors are available for investment.
and $\eta_t$), and for every $t$, average across the simulated paths (over $\eta_t$) to obtain the $\eta$-averages. For example, $\mathbb{E}^\eta[\theta(\eta,1)]$ is the average intangible share of investment at the end of first year. This calculation can be done at the end of each year to obtain annual time series of $\eta$-averages that reveal the trends. For example, the deposit-rate trend over twenty years is given by $\{\mathbb{E}^\eta[r(\eta, t)]\}_{t=0}^{20}$.

The trend in data is extracted by moving averages. Specifically, I calculate the twenty-year rolling averages of variables, so, given that the sample begins in 1980, the first rolling average centers at 1990, which maps to $t = 0$ in the model. These twenty-year rolling averages extract low-frequency variations in data. Given the sample from 1980 to 2019, I obtain 21 rolling-averages for each variable that centers at 1990, 1991, ..., 2010 respectively, which in turn maps to $t = 0, 1, \ldots, 20$ in the model. The trends are robust to the choice of rolling window from ten to twenty years.

**Calibrating $\kappa^I(t)$ and $\beta(t)$**. I calibrate $\kappa^I(t)$ to generate a trend in intangible share of investment from the data ($\theta_t$ in the model), and calibrate $\beta(t)$ to generate a trend in the ratio of firms’ to households’ holdings of intermediary debt ($M_t^E/M_t^H$ in the model). Note that to obtain the simulated paths of $M_t^E/M_t^H$, I do not need to simulate all four state variables, because $M_t^E/M_t^H = \tilde{M}_t^E/\tilde{M}_t^H$, where “$\tilde{}$” denotes $K_t^T$-scaled variables, $\tilde{M}_t^E/\tilde{M}_t^H$ only depends on $\eta_t$ and $t$ (Proposition 1).

By using the ratio $M_t^E/M_t^H$ as a calibration target, I avoid imposing any trends in either the level of aggregate deposits (the size of the banking sector) or the level of firms’ deposit holdings (corporate savings). Therefore, the trends in the total deposits and firms’ deposits are the model’s outputs, not inputs. Moreover, using this ratio as a calibration target makes sure that in the model, firms’ and households’ relative contributions to bank funding are realistic, which is important for the comparison of economies with and without firms’ liquidity demand in Section 6.

The function $\kappa^I(t)$ is specified as a linear function of time,

$$\kappa_t^I = \kappa_0^I + \kappa_1^I t. \quad (22)$$

The value of $\kappa_0$ is chosen for $\mathbb{E}^\eta[\theta(\eta,0)]$ to match the rolling average of data centering at 1990.

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27 Simulation starts from the stationary distribution of $\eta_t$ when time is fixed at $t = 0$.

28 Instead of averaging over the simulated path, the $\eta$-averages can be calculated using the $t$-conditional stationary distribution of $\eta_t$ (implied by (21)) and the solved functions of endogenous variables, for example, $q_t^T = q_T^T(\eta_t, t)$. Appendix A solves the $t$-conditional stationary distribution of $\eta_t$. 

25
### Table 2: Parameter Calibration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
<th>Moment Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Intangible investment productivity: Intercept</td>
<td>$\kappa_0^I$</td>
<td>1.076</td>
<td>$\mathbb{E}^{\eta} [\theta (\eta, t)], t = 0$</td>
<td>55.2% 54.4%</td>
</tr>
<tr>
<td>(2) Intangible investment productivity: Time coefficient</td>
<td>$\kappa_1^I$</td>
<td>0.018</td>
<td>Average annual change of $\mathbb{E}^{\eta} [\theta (\eta, t)]$</td>
<td>0.9% 0.9%</td>
</tr>
<tr>
<td>(3) Poisson shock intensity</td>
<td>$\lambda$</td>
<td>0.050</td>
<td>Average annual change of $\mathbb{E}^{\eta} \left[ \theta (\eta, t) \tilde{I} (\eta, t) \right]$</td>
<td>1.6% 1.4%</td>
</tr>
<tr>
<td>(4) Tangible investment productivity</td>
<td>$\kappa_T$</td>
<td>0.011</td>
<td>Average annual change of $\mathbb{E} \left[ (1 - \theta (\eta, t)) \tilde{I} (\eta, t) \right]$</td>
<td>0.0% -0.1%</td>
</tr>
<tr>
<td>(5) Investment cost $F(\theta) = \phi \theta^2 I_t^2$</td>
<td>$\phi$</td>
<td>0.954</td>
<td>$\frac{\text{Vol.} [\theta I_t]}{\text{Vol.} [1 - \theta I_t]}$</td>
<td>1.84 2.06</td>
</tr>
<tr>
<td>(6) Agents’ discount rate</td>
<td>$\rho$</td>
<td>0.062</td>
<td>$\mathbb{E}^{\eta} [r (\eta, t)], t = 0$</td>
<td>3.3% 3.1%</td>
</tr>
<tr>
<td>(7) Capital depreciation rate: Mean</td>
<td>$\delta$</td>
<td>0.088</td>
<td>$\mathbb{E}^{\eta} \left[ q^T (\eta, t) \right], t = 0$</td>
<td>6.6 6.8</td>
</tr>
<tr>
<td>(8) Capital depreciation rate: Vol.</td>
<td>$\sigma$</td>
<td>0.020</td>
<td>Vol. of bank asset return</td>
<td>2.9% 2.6%</td>
</tr>
<tr>
<td>(9) Household deposit demand elasticity to deposit rate</td>
<td>$\xi$</td>
<td>1.100</td>
<td>$\mathbb{E}^{\eta} \left[ \frac{\tilde{M}^E (\eta, t) + \tilde{M}^H (\eta, t)}{q^T (\eta, t)} \right], t = 0$</td>
<td>84.3% 73.3%</td>
</tr>
<tr>
<td>(10) Household deposit utility scale: Intercept</td>
<td>$\beta_0$</td>
<td>0.791</td>
<td>$\mathbb{E}^{\eta} \left[ \frac{\tilde{M}^E (\eta, t)}{M^H (\eta, t)} \right], t = 0$</td>
<td>9.8% 9.6%</td>
</tr>
<tr>
<td>(11) Household deposit utility scale: Time coefficient ($\leq 1992$)</td>
<td>$\beta_1$</td>
<td>1.825</td>
<td>Average annual change of $\mathbb{E}^{\eta} \left[ \frac{\tilde{M}^E (\eta, t)}{M^H (\eta, t)} \right], t = 0, 1, 2$</td>
<td>1.3% 1.2%</td>
</tr>
<tr>
<td>(12) Household deposit utility scale: Time coefficient ($&gt; 1992$)</td>
<td>$\beta_2$</td>
<td>2.139</td>
<td>Average annual change of $\mathbb{E}^{\eta} \left[ \frac{\tilde{M}^E (\eta, t)}{M^H (\eta, t)} \right], t = 2, \ldots, 20$</td>
<td>3.0% 3.2%</td>
</tr>
</tbody>
</table>

The value of $\kappa_1^I$ is chosen so the average annual change of $\{\mathbb{E}^{\eta} [\theta (\eta, t)]\}_{t=0}^{20}$ matches the empirical counterpart. The first two rows of Table 2 report the calibration results. Because the moments are sufficiently sensitive to the corresponding parameters, the parameters are calibrated sequentially.

The function $\beta (t)$ is specified as a linear function of time with a structural break,

$$\beta_t = \beta_0 + (\beta_1 \mathbb{1}_{t \leq 2} + \beta_1 \mathbb{1}_{t > 2}) t. \quad (23)$$

The logarithm of households’ holdings of intermediary debts has a structural break in its time trend at 1992 ($t = 2$ in the model), detected by both the supremum Wald and LR tests with p-
values below 0.0001 (Andrews, 1993; Perron, 2006). I take logarithm because households’ deposit holdings in the model grow exponentially along with capital stock (see (14)). In data, households’ holdings of intermediary debts also show exponential growth. I also use supremum Wald and LR tests on the ratio of households’ holdings of intermediary debts to households’ total asset and detect a structural break in the level at 1992. Figure C.4 in Appendix C reports the raw data.

The value of \( \beta_0 \) is chosen so that \( \mathbb{E}_\eta \left[ \frac{\tilde{M}_E(\eta,0)}{M_H(\eta,0)} \right] \), i.e., the \( \eta \)-average ratio of entrepreneurs’ to households’ holdings of deposits at \( t = 0 \) matches the rolling average of data centering at 1990.\(^{29}\) The value of \( \beta_1 \) is chosen so the average annual change of \( \left\{ \mathbb{E}_\eta \left[ \frac{\tilde{M}_E(\eta,t)}{M_H(\eta,t)} \right] \right\}_{n=0}^{20} \) matches its empirical counterpart, and \( \beta_2 \) is set so the average annual change of \( \left\{ \mathbb{E}_\eta \left[ \frac{\tilde{M}_E(\eta,t)}{M_H(\eta,t)} \right] \right\}_{n=3}^{20} \) matches its empirical counterpart. The last three rows of Table 2 report the calibration results.

**Calibrating parameters at** \( t = 0 \). The model generates trends in: (1) the intangible share of investment; (2) firms’ deposit holdings; (3) the size of the banking sector; (4) the deposit rate; and (5) tangible capital value. The initial value of and the trend in (1) (i.e., average annual change) are used to calibrate \( \kappa_{0}^{I} \) and \( \kappa_{1}^{I} \), but I avoid using the trends in (2) to (5) to calibrate parameters so these four trends can be viewed as the model’s outputs rather than inputs. To let these trends take off from realistic starting points, I match their \( \eta \)-averages at \( t = 0 \) to the empirical counterparts.

Since the value of \( \beta_0 \) is chosen to match the *initial composition* of aggregate deposit holdings (the ratio of firms’ to households’ holdings), the value of \( \xi \), households’ deposit demand elasticity, is set for the *initial level* of aggregate deposit holdings to match its empirical counterpart. Once the initial composition and level of deposits are pinned down, I have obtained the starting points of trend (2) and (3). I use total deposits outstanding as a measure of the size of the banking sector.

I scale the aggregate deposit holdings by aggregate value of tangible capital,

\[
\frac{M_E^t + M_H^t}{q_t^T K_t^T} = \frac{\tilde{M}_E(\eta,t) + \tilde{M}_H(\eta,t)}{q_t^T(\eta,t)}. \tag{24}
\]

which measures the fraction of capitalizable assets that has been transformed into liquidity by bankers. The data counterpart is the value of nonfinancial corporations’ and households’ holdings

\(^{29}\)Data is plotted in Panel C of Figure 3. Figure 2 list the debt instruments that map to deposits in the model.
of intermediary debts (listed in Figure 2) scaled by the value of fixed assets in the nonfinancial corporate sector from the Bureau of Economic Analysis (current-cost net stock). I calculate the 20-year average centering at 1990, and use it as a target for \[ \mathbb{E}_\eta \left[ \frac{\hat{M}^E(\eta,0) + \hat{M}^H(\eta,0)}{q^T(\eta,0)} \right] \] when calibrating \( \xi \). The value of \( \xi \), 1.1, is close to households’ deposit-demand elasticity in other banking models (e.g., 1.4 from Begenau, 2019). Row 9 in Table 2 reports the calibration result. Note that further increasing \( \xi \) improves the calibration result by reducing households’ deposit holdings, but this will drive the ratio of firms’ to households’ deposits away from its target. Because I compare models with and without firms’ liquidity demand in Section 6, I choose to prioritize matching deposit composition so firms’ and households’ relative contributions to bank funding are realistic.

The discount factor, \( \rho \), is chosen so \[ \mathbb{E}_\eta [r(\eta,0)] \] matches the average rate of intermediary debts in 1990. The expected capital depreciation rate, \( \delta \), is chosen so \[ \mathbb{E}_\eta [q^T(\eta,0)] \] matches the average EV/EBITDA ratio in 1990. Capital generates one unit of goods per year, so \( q^T = q_T / 1 \) is the ratio of capital value to its annual output. Tangible capital generates all capitalizable output, so its value maps to firms’ aggregate enterprise value (EV), which is the present value of future cash flows reflected in debt and equity markets. The calibration of \( \rho \) and \( \delta \) give the starting points of trend (4) and (5). The calibration results are reported in Row 6 and 7 of Table 2.

**Other parameters.** The parameter \( \lambda \) governs the frequency of Poisson-arriving investment needs. Because \( \theta_t \) trends upward over time, \( \lambda \) governs how much the rise of the intangible investment share translates into the rise of the intangible investment level. As a reminder, the \( K^T_t \)-scaled aggregate investment is denoted by \( \tilde{I}_t \), so the \( K^T_t \)-scaled intangible investment is \( \theta_t \tilde{I}_t \), which is a function \( \eta_t \) and \( t \). Given any \( t \), I calculate the \( \eta \)-average of \( \theta_t \tilde{I}_t \). The twenty-year rolling averages of the ratio of intangible investment to firms’ physical capital exhibit a linear trend. Therefore,

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30 I subtract the value of intellectual properties from the aggregate fixed assets to obtain the value of tangible asset.

31 The short-term interest rates are the real rates with the consumer price index as deflator. The debt instruments correspond to the list in Figure 2, which include: (1) jumbo (\( \geq \$100,000 \)) and non-jumbo checking deposits, savings deposits, certificate of deposits, and money market; (2) 1-, 2-, and 3-month AA-rated financial commercial papers; (3) 3- and 6-month bankers acceptance; (4) 1-, 2-, and 3-month AA-rated asset-backed commercial papers; (5) GCF repo rate with Treasury securities, mortgage-backed securities, and agency- and GSE-backed securities as collateral; (6) Fed fund. Data is from FRED, except the repo rates from the Federal Reserve Bank of New York.

32 The average is taken over median EV/EBITDA ratios of 11 Fama-French nonfinancial industries (Compustat).

33 Each year, I calculate cross-section total asset-weighted average of ratio of intangible investment to tangible capital (PPE) (Panel A of Figure C.5 in Appendix C), and calculate the twenty-year rolling averages.
the value of $\lambda$ is chosen so the average annual changes of $E^n \left[ \theta (\eta, t) \tilde{I} (\eta, t) \right]$, $n = 0, 1, \ldots, 20$, matches the average annual changes of the data rolling averages. The productivity of tangible investment, $\kappa^T$ is chosen so the $\eta$-average of the ratio of tangible investment to physical capital, $E^n \left[ (1 - \theta (\eta, t)) \tilde{I} (\eta, t) \right]$, $n = 0, 1, \ldots, 20$, has an average annual change that matches its empirical counterpart.\footnote{Each year, I calculate cross-section total asset-weighted average of ratio of tangible investment (capital expenditure) to tangible capital (PPE) (Panel B of Figure C.5 in Appendix C), and calculate the twenty-year rolling averages.} The calibration results of $\lambda$ and $\kappa^T$ are reported in Rows 3 and 4 of Table 2.

The convex investment cost $F(\theta)$ is specified as $F(\theta_t) = \frac{\phi}{2} \theta_t^2$. When $\phi$ is high, entrepreneurs adjust $\theta_t$ slowly; when $\phi$ is low, $\theta_t$ varies more. Therefore, $\phi$ governs the relative volatilities of intangible and tangible investment. Given time $t$, the conditional distribution of $\eta_t$ (implied by (21)) can be used to calculate the volatility ratio of intangible to tangible investment (both scaled by $K^T_t$). Then the volatility ratios are averaged over time, denoted by $\frac{\text{Vol.}[\theta]}{\text{Vol.}[1 - \theta \tilde{I}]}$. The value of $\phi$ is chosen so that this average volatility ratio matches the volatility ratio of detrended intangible to tangible investment.\footnote{Each year, I take the ratio of intangible investment (scaled by PPE) and tangible investment (scaled by PPE) (see footnote 33 and 34). The resulting time series exhibits a linear trend.} Finally, a larger $\sigma$ translates into a greater volatility in the bankers’ returns on assets. Therefore, the value of $\sigma$ is chosen so that the volatility of bankers’ return on asset matches its empirical counterpart (Gornall and Strebulaev, 2018). Rows 5 and 8 in Table 2 report the calibration results. Note that the parameters are calibrated sequentially, so $\phi$ and $\sigma$ are calibrated last and the calibration results are not as close as previous parameters.

### 4.2 Corporate Savings Glut and Long-Run Trends

The results are in two categories: (1) the economy’s response to the increase of $\kappa^I_t$ overt time (trends) and (2) the economy’s response to the aggregate shock, $dZ_t$ (cycles). This subsection focuses on the former. The next subsection analyzes economic fluctuations and financial instability.

For a coherent characterization of the mechanism, the analytical results are presented alongside the numerical solutions. First, I characterize entrepreneurs’ intangible-driven liquidity demand. When hit by the Poisson shock, an entrepreneur maximizes investment profits given by (4) facing the liquidity constraint (6). Let $\pi_t$ denote the marginal value of liquidity, i.e., the Lagrange
multiplier of constraint (6). The Lagrange function summarizes the entrepreneur’s problem:

$$\mathcal{L} = \max_{\{i_t, \theta_t\}} \left[ q^I I^t \theta_t + q^T i^T (1 - \theta_t) - F(\theta_t) \right] i_t - i_t + \pi_t \left[ m^E_i + q^T i^T (1 - \theta_t) \right].$$

As the liquidity constraint binds, one unit of liquidity $m^E_i$ leads to $1/ \left[ 1 - q^T i^T (1 - \theta_t) \right]$ more units of goods invested, as previously shown in (15). Because external funds are raised against tangible capital at a fair price, the entrepreneur captures the full surplus per unit of investment, i.e., $[q^I I^t \theta_t + q^T i^T (1 - \theta_t) - F(\theta_t)] - 1$. Therefore, the marginal value of liquidity, $\pi_t$, is the product of marginal profit of investment and the leverage on liquidity holdings:

$$\pi_t = \left\{ \left[ q^I I^t \theta_t + q^T i^T (1 - \theta_t) - F(\theta_t) \right] - 1 \right\} \frac{1}{1 - q^T i^T (1 - \theta_t)}$$

Applying Appendix A states the parameter condition under which the constraint (6) always binds and $\pi_t > 0$.

The entrepreneur’s choice of $\theta_t$ is characterized by the first-order condition that equates the marginal values of intangible and tangible investments:

$$q^I I^t - F'(\theta_t) = (1 + \pi_t) q^T i^T.$$

Note that on the right side of (27), the marginal value of tangible capital, $q^T i^T$, is amplified by $\pi_t$, because investing more in tangible capital not only creates more production units but also relaxes the funding constraint (6). The next proposition summarizes the entrepreneur’s liquidity-holding and investment decisions with a focus on the value of liquidity. Appendix A provides the proof.

**Proposition 2 (Liquidity Premium)** Entrepreneurs’ investment has the following properties:

1. The optimal intangible share of investment, $\theta_t$, in (27) is increasing in $\kappa^I_t$;
2. The marginal value of liquidity, $\pi_t$, given by (26), is increasing in $\kappa^I_t$ and $q^T_t$.

Given the arrival intensity of investment needs, $\lambda$, entrepreneurs accept a deposit rate below $\rho$:

$$r_t = \rho - \lambda \pi_t.$$
Table 3: Trends in Intangible Investment, Corporate Liquidity, Interest Rate, and Capital Valuation

<table>
<thead>
<tr>
<th>Time</th>
<th>Intangible Inv. Share</th>
<th>Firm Deposits</th>
<th>Firm Deposits</th>
<th>Interest Rate</th>
<th>Capital Valuation</th>
<th>Total Deposits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \mathbb{E}^\eta [\theta(\eta, t)] )</td>
<td>( \mathbb{E}^\eta \left[ \frac{\bar{M}^E(\eta, t)}{M^E(\eta, t)} \right] )</td>
<td>( \mathbb{E}^\eta \left[ \frac{\bar{M}^H(\eta, t)}{q^E(\eta, t)} \right] )</td>
<td>( \mathbb{E}^\eta [r(\eta, t)] )</td>
<td>( \mathbb{E}^\eta \left[ q^T(\eta, t) \right] )</td>
<td>( \mathbb{E}^\eta \left[ \frac{\bar{M}^E + \bar{M}^H}{q^T(\eta, t)} \right] )</td>
</tr>
<tr>
<td>( t = 0 )</td>
<td>55.2%</td>
<td>9.8%</td>
<td>7.7%</td>
<td>3.27%</td>
<td>6.6</td>
<td>84.3%</td>
</tr>
<tr>
<td>Data '90</td>
<td>54.4%</td>
<td>9.6%</td>
<td>6.3%</td>
<td>3.09%</td>
<td>6.8</td>
<td>73.3%</td>
</tr>
<tr>
<td>( t = 4 )</td>
<td>58.7%</td>
<td>11.1%</td>
<td>8.6%</td>
<td>2.10%</td>
<td>6.9</td>
<td>83.5%</td>
</tr>
<tr>
<td>Data '94</td>
<td>58.2%</td>
<td>10.8%</td>
<td>7.1%</td>
<td>2.46%</td>
<td>6.9</td>
<td>74.6%</td>
</tr>
<tr>
<td>( t = 8 )</td>
<td>62.2%</td>
<td>10.7%</td>
<td>8.3%</td>
<td>0.95%</td>
<td>7.3</td>
<td>83.7%</td>
</tr>
<tr>
<td>Data '98</td>
<td>61.9%</td>
<td>12.2%</td>
<td>7.9%</td>
<td>1.72%</td>
<td>7.3</td>
<td>73.5%</td>
</tr>
<tr>
<td>( t = 12 )</td>
<td>65.7%</td>
<td>13.0%</td>
<td>9.3%</td>
<td>-0.07%</td>
<td>7.7</td>
<td>84.7%</td>
</tr>
<tr>
<td>Data '02</td>
<td>66.0%</td>
<td>13.1%</td>
<td>8.4%</td>
<td>0.92%</td>
<td>7.5</td>
<td>72.4%</td>
</tr>
<tr>
<td>( t = 16 )</td>
<td>69.1%</td>
<td>13.7%</td>
<td>10.3%</td>
<td>-1.50%</td>
<td>7.8</td>
<td>83.8%</td>
</tr>
<tr>
<td>Data '06</td>
<td>69.1%</td>
<td>13.8%</td>
<td>9.1%</td>
<td>0.42%</td>
<td>7.7</td>
<td>75.1%</td>
</tr>
<tr>
<td>( t = 20 )</td>
<td>72.6%</td>
<td>14.1%</td>
<td>10.6%</td>
<td>-2.88%</td>
<td>7.9</td>
<td>83.5%</td>
</tr>
<tr>
<td>Data '10</td>
<td>72.6%</td>
<td>14.0%</td>
<td>9.7%</td>
<td>-0.38%</td>
<td>8.0</td>
<td>79.3%</td>
</tr>
</tbody>
</table>

As \( \kappa_t \) increases over time, intangible investment creates increasingly more production capacity than tangible investment, so the entrepreneurs optimally choose to tilt investment towards intangibles, i.e., to increase \( \theta_t \). Table 3 reports the model-generated trend in \( \theta_t \) (i.e., the \( \eta \)-averages introduced in Section 4.1) and their empirical counterpart from 1990 to 2010 (mapping to \( t = 0 \) to 20 in the model). The model generates \( \mathbb{E}^\eta [\theta(\eta, t)] \) very close to the twenty-year rolling averages.

As the intangible share of investment increases, the entrepreneurs face a tighter liquidity constraint, so the marginal value of liquidity, \( \pi_t \), increases and the entrepreneurs hold more liquidity in the form of bank debts. Table 3 reports the ratio of entrepreneurs’ (firms’) holdings of bank debts to households’ holdings, which increases from below 10% to 14% in both the model and data. The table also reports an upward trend in the cash-to-asset ratio (i.e., deposits scaled by tangible capital value, \( \mathbb{E}^\eta \left[ \frac{\bar{M}^E}{q^T K^T_t} \right] \)), which directly maps to the finding of a rising cash-to-asset ratio in the two decades leading up to the financial crisis (e.g., Bates, Kahle, and Stulz, 2009).

The increasing value of liquidity, \( \pi_t \), drives down the equilibrium deposit rate \( r_t \). As stated in
Proposition 2, the entrepreneurs accept a deposit rate below $\rho$, and the wedge, $\lambda \pi_t$, is the product of the probability of investment needs and the marginal value of liquidity in the investment project. The model predicts a lower and more negative real rate in the 2000s than found in the data, which is likely due to the zero lower bound on nominal rates that binds in reality (Eggertsson and Woodford, 2003; Fischer, 2016). This suggests that when the rise of intangible investment increases the liquidity demand from the production sector, the zero lower bound and liquidity trap become more acute problems (Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson and Krugman, 2012; Caballero and Farhi, 2017; Guerrieri and Lorenzoni, 2017). Moreover, a liquidity trap triggered by the rise of intangibles in one country can spread globally (Caballero, Farhi, and Gourinchas, 2015).

Table 3 also reports an upward trend in capital valuation that closely matches the data. Tangible capital represents the capitalizable production capacity. The ratio of $q^T_t$ to one unit of goods produced per unit of time (one year) maps to EV-to-EBITDA ratio, since, by definition, enterprise value (EV) is the present value of the capitalizable output of a firm, reflected in the debt and equity markets. The valuation of capital is purely driven by the discount-rate variation as cash flow is fixed in the model. When entrepreneurs or households own tangible capital, the discount rate is $\rho$. However, when bankers are sufficiently wealthy, the economy achieves full intermediation – all tangible capital is owned by bankers who issue deposits to meet entrepreneurs’ and households’ demand. Then the discount rate for tangible capital can fall below $\rho$, driven by a low deposit rate.

The last column of Table 3 shows that in the model, the ratio of total deposits to tangible capital value is relatively stable over time. Therefore, as the value of tangible capital increases, the total amount of deposits issued by banks increases, which implies an upward trend in the size of the banking sector. In the last few decades, the financial intermediation sector grew significantly (Greenwood and Scharfstein, 2013; Schularick and Taylor, 2012), feeding on cheap funds from major cash pools (Adrian and Shin, 2010; Pozsar, 2014). This paper is the first to formally establish the link between the rise of corporate savings and the secular growth of financial intermediation.

Next, I explain the mechanism of capital valuation and the associated endogenous financial risk. Consider the bankers’ discount rate, i.e., their required expected return on tangible capital holdings, $r_t + \text{bankers’ risk premium}$, where the deposit rate, $r_t$, is the marginal cost of financing.
To analyze the risk-premium component, we need the bankers’ value function. The homogeneity property of the bankers’ problem implies a linear value function \( q_t^B n_t^B \), where, the marginal value of wealth, \( q_t^B \), has an equilibrium law of motion:

\[
\frac{dq_t^B}{q_t^B} = \mu_t^B \, dt - \gamma_t^B \, dZ_t. \tag{29}
\]

Appendix A provides the proof. In equilibrium, \( dZ_t < 0 \) reduces bankers’ wealth and increases their marginal value of wealth, so \( \gamma_t^B > 0 \). A negative shock also leads to lower realized returns on tangible capital via faster depreciation of holdings and the decline of \( q_t^T \) in equilibrium (see (9)). Therefore, to hold tangible capital, bankers require a risk premium of \( \gamma_t^B (\sigma_t^T + \sigma) \, dt \). This is a standard asset-pricing result – the risk premium is equal to the covariance between the growth rate of the marginal value of wealth and the asset’s return. Here \( \gamma_t^B \) is the price of risk and \( (\sigma_t^T + \sigma) \) is the quantity of risk, a sum of exogenous risk, \( \sigma \), and endogenous price risk, \( \sigma_t^T \) (see (3)).

**Proposition 3 (Asset Pricing)** The expected return on tangible capital, in equilibrium, is the sum of \( r_t \) and the compensation for bankers’ risk exposure:

\[
E_t [dr_t^T] = r_t + \gamma_t^B (\sigma_t^T + \sigma) . \tag{30}
\]

Bankers’ price of risk \( \gamma_t^B = 0 \) when \( \eta_t = \bar{\eta}(t) \), where \( \bar{\eta}(t) \) is the time-\( t \) upper bound of \( \eta_t \) that is defined in Proposition 1. Given the expression of \( dr_t^T \) in (9) and \( r_t = \rho - \lambda \pi_t \) from Proposition 2, the equilibrium value of tangible capital satisfies the following equation

\[
q_t^T = \frac{1}{[\rho - \lambda \pi_t + \gamma_t^B (\sigma_t^T + \sigma)] - [\mu_t^T + \sigma_t^T \sigma - \delta - \lambda]} . \tag{31}
\]

\(^{36}\)Like Tobin’s Q, \( q_t^B \), is a forward looking measure of profits per unit of equity. This offers an alternative to understand why \( \gamma_t^B > 0 \). Due to the negative shocks and their persistent effects under the equity issuance constraint, the whole banking sector becomes undercapitalized and shrinks for a sustained period of time. To clear the markets of tangible capital and deposits, the spread between the expected return on tangible capital and deposit rate will have to widen so that banks would hold tangible capital and issue deposits. As the expected future profits rise, \( q_t^B \) increases.

\(^{37}\)Note that \( q_t^B \in [1, +\infty) \) because if \( q_t^B < 1 \), bankers prefer consuming than retaining wealth. When \( \eta_t = \bar{\eta}(t) \), \( q_t^B = 1 \) and bankers consume (Proposition 1). Consumption reduces \( N_t^B \), but once \( q_t^B \) is above one, consumption stops (retaining wealth is worth \( q_t^B \) but consuming is worth \( q \)). Thus, \( \bar{\eta}(t) \) is an upper reflecting boundary of \( \eta_t \).
Equation (31) resembles the Gordon growth formula. The numerator is cash flow (production). In the denominator, the first component is the discount rate and the second component is the expected growth rate.\textsuperscript{38} As $\kappa^I_t$ drives up $\theta_t$, the intangible share of investment, and $\pi_t$, the marginal value of liquidity, entrepreneurs accept an increasingly low deposit rate $r_t$, which drives down the discount rate in (31) and pushes up $q^T_t$, as shown in the last column of Table 3. Appendix A shows that (31) implies a partial differential equation for the function $q^T_t = q^T (\eta_t, t)$.

Tangible capital has two sources of value. It produces goods, and it provides liquidity indirectly through the bankers’ balance sheet by backing the deposits. In this economy, entrepreneurs assign a liquidity premium on deposit holdings, which, through the reduction of the bankers’ discount rate, is partially transmitted to the value of tangible capital.\textsuperscript{39} Comparing (31) and the valuation of illiquid intangible capital (2), we can see that the source of variation in $q^T_t$ is the liquidity value of tangible capital instead of the production value. Without the liquidity demand, the value of tangible capital would have been the discounted sum of goods produced, $q^T_t = \frac{1}{\rho_+\delta+\lambda} $.

The transmission of the liquidity premium is incomplete due to the risk-premium component of the bankers’ discount rate, except when bankers are sufficiently wealthy (i.e., $\eta_t = \underline{\eta} (t)$), $\gamma^B_t = 0$, and their discount rate is precisely $r_t$. The risk premium can be shut down and the transmission of liquidity premium is complete, if bankers are allowed to freely raise outside equity and replenish their net worth. This is because when retaining wealth by forgoing consumption does not add value, the marginal value of wealth, $q^B_t$, is pinned to one, and thus, $\gamma^B_t = 0$. Under the equity issuance constraint, $q^B_t$ becomes responsive to shocks, generating procyclical intermediation capacity.

Along the trend of rising intangible investment, a feedback mechanism arises that strengthens the downward trend in interest rates and the upward trend in tangible capital valuation, creating an endogenous corporate savings glut. As shown by (26) in Proposition 2, the marginal value of liquidity, $\pi_t$, is increasing in $q^T_t$. A higher value of tangible capital enlarges the external-financing capacity of investment projects, allowing liquidity holdings to be leveraged to larger investments. Moreover, a higher value of $q^T_t$ means investments are more profitable. Therefore, the decline of $r_t$ leads to the increase of $q^T_t$, which in turn leads to a higher value of liquidity, $\pi_t$, a stronger demand

\textsuperscript{38}Investment creates new capital instead of grows the existing capital, so it does not add to the growth rate.

\textsuperscript{39}Related, Giglio and Severo (2012) analyze the liquidity value of tangible capital without financial intermediation.
Table 4: Trend in Endogenous Financial Risk

<table>
<thead>
<tr>
<th>Model Time $t$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Financial Risk Multiplier</td>
<td>2.7</td>
<td>3.2</td>
<td>3.6</td>
<td>4.0</td>
<td>4.4</td>
<td>4.7</td>
</tr>
</tbody>
</table>

for liquidity from the entrepreneurs, and an even lower interest rate, $r_t$. The corporate savings glut arises endogenously in a closed-economy, distinct from an exogenous savings glut that has been shown to affect interest rates and asset prices (e.g., Caballero, Farhi, and Gourinchas, 2008).

As $\kappa^I_t$ increases and the economy becomes more intangible-intensive, it also becomes increasingly fragile through the endogenous financial risk. The total value of capitalizable output is the value of tangible capital, $q^T_t K^T_t$. By Itô’s lemma, its growth rate is

$$
\frac{d \left( q^T_t K^T_t \right)}{q^T_t K^T_t} = \left( \mu^T_t - \delta - \lambda + \sigma^T_t \sigma \right) dt + \left( \sigma^T_t + \sigma \right) dZ_t,
$$

Therefore, a natural measure of endogenous financial risk is the ratio of total shock exposure of $q^T_t K^T_t$ to the exogenous shock exposure from stochastic capital depreciation:

$$
\textit{Financial Risk Multiplier} : \frac{\sigma^T_t + \sigma}{\sigma}.
$$

The endogenous variation of $q^T_t$ amplifies the impact of shocks. This ratio is a function of $t$ and $\eta_t$. It exhibits a trend driven by time $t$ and, along the trend, fluctuates with $\eta_t$. Table 4 reports the maximum (over $\eta_t$) of this ratio with time fixed at $t = 0, 4, \ldots, 20$. It also reports the corresponding years in the data. Table 4 shows that endogenous financial risk rises alongside the trends in intangible investment, corporate savings, interest rates, and asset prices. Next, I analyze the financial cycle and the unique amplification mechanism from an endogenous corporate savings glut.

**Remark.** Agents have risk-neutral preferences so the elasticity of intertemporal substitution (EIS) is infinite. If EIS were finite, an increase in $\kappa^I_t$ may put upward pressure on $r_t$: as the economy grows faster, consumption grows faster, so the risk-free rate rises, which implies a higher deposit
rate. However, the downward trend in $r_t$ is robust because, in order for this counteracting force to be active, $r_t$ must decline in the first place. A higher growth rate is due to larger investments, which in turn rely on more deposits issued by banks (held by entrepreneurs) and/or higher leverage on liquidity. To induce banks to issue more deposits, $r_t$ must decrease. And, to have a higher leverage on liquidity, $q_t^T$ must increase, which in turn requires a lower $r_t$. Bankers’ discount rate has two components, $r_t$ and the risk premium. Due to the upward trend in $\sigma_t^T$, the risk premium increases over time, so a decrease in $r_t$ is necessary for higher $q_t^T$ and higher leverage on liquidity.

4.3 Endogenous Financial Risk and Economic Fluctuation

This subsection focuses on economic fluctuations along the trend, driven by the intermediation intensity, $\eta_t$. Figure 5 plots six endogenous variables against $\eta_t$. The plots end at $\eta(t)$, the upper boundary of $\eta_t$, and is conditional on time $t = 20$ (i.e., 2010 in data). To understand the economy’s
response to shocks, first consider positive shocks that move $\eta_t$ to the right. Panel A of Figure 5 plots the bankers’ price of risk (or required Sharpe ratio) for holding tangible capital:

$$
\gamma^B_t = \frac{\mathbb{E}_t [dr^T_t] - r_t}{\sigma^T_t + \sigma}, \tag{34}
$$

which declines as $\eta_t$ increases, and eventually reaches zero at the upper boundary of $\eta_t$ where, according to Proposition 1, bankers consume. This implies a procyclical intermediation capacity.

In Panel B, the discount rate for tangible capital, i.e., the expected return $\mathbb{E} [dr^T_t]$, is initially at $\rho$ to attract entrepreneurs’ and households’ demand. However, as $\eta_t$ increases, bankers eventually hold all tangible capital and the discount rate falls below $\rho$. Recall that the cash flow of tangible capital is constant, so what drives the variation of $q^T_t$ is the discount rate. Therefore, as the discount rate declines following positive shocks, the value of tangible capital, $q^T_t$, increases as shown in Panel C. Note that the increase of $q^T_t$ in $\eta_t$ is smooth even though the decrease of discount rate in $\eta_t$ is not. Under rational expectation, $q^T_t$ is forward-looking, so any increase of $\eta_t$ raises the probability of low discount-rate regions, and therefore, increases $q^T_t$ in every state of the world.

As $q^T_t$ increases, a feedback mechanism emerges. Investment becomes more profitable, and the leverage on liquidity becomes higher. Therefore, entrepreneurs accept a lower $r_t$ (Proposition 2), holding more deposits as shown in Panel D of Figure 5. A lower $r_t$ further reduces the bankers’ discount rate, leading to an even higher $q^T_t$. In the process, entrepreneurs hold more liquidity and invest more as shown in Panels E and F. Note that when scaled by $K^T_t$, the run-up of entrepreneurs’ deposits and investments stops when the growth of the bankers’ wealth outpaces that of the tangible capital value (bank asset value). When this occurs, bank equity crowds out debt on the balance sheet, resulting in a reduction of $K^T_t$-scaled deposits.

While $\kappa^T_t$ generates the trends in the economy, the endogenous variation of $\gamma^B_t$ generates the cycle. Positive shocks enlarge bankers’ intermediation capacity, i.e., reducing $\gamma^B_t$, so bankers bid up $q^T_t$, which in turn leads to a larger value of liquidity for entrepreneurs and a lower deposit rate $r_t$. As $r_t$ declines, bankers expand balance sheets further, pushing up $q^T_t$ in an upward spiral.

---

40 When $\eta_t < 0.28$ (1.7% probability), $M^F_t = 0$ and $r_t$ is below what entrepreneurs accept (i.e., $r_t < \rho - \lambda \pi_t$ and (28) no longer holds). $r_t$ is solved by equating households’ demand and bankers’ supply. See Appendix A.2
The amplification mechanism requires bankers to face a financial constraint, because, if bankers could raise equity freely to replenish net worth, their marginal value of wealth would be pinned to one with $\gamma_t^B$ pinned to zero. As a result, entrepreneurs’ liquidity demand lowers the deposit rate and raises the value of tangible capital but does not cause economic fluctuations.\footnote{Allowing limited equity issuance (e.g., He and Krishnamurthy, 2013) may change the quantitative performances and cause the calibration to deliver different parameter values, but will not change the mechanism.}

The upward spiral triggered by positive shocks, $dZ_t > 0$, seems benign, featuring a boom of liquidity creation and investment. However, endogenous risk accumulates along the process, so when negative shocks hit, i.e., $dZ_t < 0$, the risk materializes into a downward spiral.

To understand the accumulation of endogenous financial risk in booms, consider a value of $\eta_t$ near zero in Panel A of Figure 6 (reproducing Panel B of Figure 5). The discount rate stays at $\rho$ with a large probability. However, as we move to the right, $\eta_t$ approaches the cutoff point where the discount rate falls below $\rho$. As a result, even small shocks can cause a large discount-rate change.
and a large variation of $q_t^T$. Therefore, the asset price, $q_t^T$, becomes more sensitive to shocks (i.e., higher $\sigma_t^T$) as $\eta_t$ moves to the right. This explains why in Panel B, the ratio of total return volatility, $\sigma_t^T + \sigma$, to exogenous volatility, $\sigma$, is increasing in $\eta_t$. The shock amplification mechanism becomes stronger as booms prolong. The amplification subdues eventually as the sensitivity of the discount rate to $\eta_t$ becomes increasingly smaller. The financial risk multiplier peaks 4.73.

The endogenous asset-price volatility has impact on the real economy. As shown in (18) and (19), the economic growth is directly tied to $q_t^T$ through the scale of investment because an increase of $q_t^T$ enlarges entrepreneurs’ financing capacity, i.e., the leverage on liquidity holdings. Therefore, the asset-price volatility translates into the volatility of the economic growth rate.

The accumulation of endogenous risk is also asymmetric. Panels C and D plot respectively the probabilities of a $2\sigma$ decrease and increase of $q_t^T$ in one quarter.\(^{42}\) Note that at sufficiently low (high) values of $\eta_t$, a further decrease (increase) by $2\sigma$ is impossible because it goes beyond the equilibrium range of $q_t^T$. The probability of a drop in $q_t^T$ increases as $\eta_t$ increases. It eventually declines as the shock amplification weakens (Panel B). The probability of an increase in $q_t^T$ also increases but declines earlier, suggesting that the risk accumulation is asymmetric, biased towards the downside. Such asymmetry sheds light on the findings that periods of banking expansion often precede severe crises (e.g., Jordà, Schularick, and Taylor, 2013; Baron and Xiong, 2017).

When negative shocks hit, the feedback mechanism turns into a vicious cycle. As $\gamma_t^B$ (and bankers’ discount rate) increases, $q_t^T$ declines, which in turn discourages entrepreneurs from saving for investments and, thereby, causes an increase of $r_t$, bankers’ cost of debt financing. Bankers’ discount rate increases further, causing $q_t^T$ to continue falling. The decline of $q_t^T$ erodes bankers’ wealth, further increasing $\gamma_t^B$, and the impact is amplified by leverage. As the economy moves leftward in Figure 6, the downside risk in $q_t^T$ rises in Panel C, while the upside risk is relatively insensitive in Panel D. This prediction speaks directly to the findings of Adrian, Boyarchenko, and Giannone (2019) – downside risks increase relative upside risks as financial conditions deteriorate.

**Remark.** In crises, there is an economic force that counteracts the decline of firms’ deposits and exerts downward pressure on $r_t$. When $\eta_t = N_t^B / K_t^T$ is low, deposit supply scaled by $K_t^T$ is low.

\(^{42}\)Given the model solution, these probabilities can be calculated using the Feynman–Kac PDEs.
Households tend to hold less deposits. Dividing (14) by \( K_t^r \), we see \( \frac{M_H^T}{K_T^r} = \alpha \left( \frac{\rho - r_t}{\beta} \right) ^{-\frac{1}{\xi}} \). When the left side is low, \( r_t \) tends to be low. This captures the spirit of flight to safety in crises (Caballero and Krishnamurthy, 2008). The actual value of \( r_t \) still depends on firms’ liquidity demand.

5 The Illiquidity of Intangible Capital

A key ingredient of the model is the illiquidity of intangible capital. It gives rise to a funding constraint (6) that leads to entrepreneurs’ liquidity demand and bankers’ roles as liquidity suppliers.

The panel regression reported in Table 5 shows that more intangible firms borrow less, which indicates tighter credit constraints. The sample is from Section 2. Intangibility measures include Intan./Asset decile rank and the inverse PPE/Asset decile rank (Section 2). Control variables are included following Lian and Ma (2019) who also share their loan categorization data.\(^{43}\) Time fixed effects are added to eliminate common variations.\(^{44}\) All specifications show a negative correlation between intangibility and leverage. In Column (1), a one-decile difference in intangibility is associated with a difference of 1.244% in the level of leverage. Column (4) reports an estimate of

\[^{43}\] Control variables: size (log total assets in 2005 dollars); market-to-book ratio; cash-to-asset ratio; EBITDA-to-asset ratio ([sale – cogs – xsga]/at); net cash receipts-to-asset ratio ([oancf + xint]/at); inventory-to-asset ratio (invt/at).

\[^{44}\] Examples of common variations include tax changes, banking regulation changes, bankruptcy law change etc.
similar magnitude. Credit relies on creditors’ claims on collateral or contractual rights to future cash flows (Lian and Ma, 2019). With different measures of intangibility, Columns (2) and (5) and Columns (3) and (6) show that intangible firms are disadvantaged on both fronts.

The illiquidity of intangible capital motivates a dichotomy in the main model: as in Caballero, Farhi, and Gourinchas (2008), the output is split into a capitalizable component, produced by “tangible” trees, and a non-capitalizable component, produced by “intangible” trees.

As the U.S. economy becomes more intangible-intensive, new markets emerge for the exchange of intangibles. Akcigit, Celik, and Greenwood (2016) document that, between 1976 to 2006, 16% of the U.S. registered patents were traded. Therefore, I extend the model to incorporate tradable intangibles. Just like non-tradable intangibles and tangible capital, tradable intangibles represent efficiency units, so its output is normalized to one unit of goods per unit of time. To focus on the implications of its liquidity, the stochastic deprecation and Poisson-arriving destruction of tradable intangibles are the same as those of capital in the main model.

Entrepreneurs and households can buy and sell the tradable intangibles. It is assumed that bankers do not own tradable intangibles. Otherwise, tradable intangibles and tangible capital are indistinguishable. Given that tradable intangibles are traded among entrepreneurs and households, the discount rate is \( \rho \), and, given the same production flow and depreciation/destruction dynamics as those of non-tradable intangibles, the equilibrium unit value of tradable intangibles is \( q^I \) in (2). Tradable intangibles differ from non-tradable intangibles in that they relax the funding constraint:

\[
\begin{align*}
    i_t & \leq m^E_t + q^T_t \kappa^T (1 - \theta) i_t + \chi (q^I \kappa^I \theta i_t), \\
\end{align*}
\]

where \( q^I \kappa^I \theta i_t \) is the value of all intangibles, and the parameter \( \chi \) is the fraction that is tradable.\(^45\)

According to Corrado et al. (2016), intellectual properties accounted for 37.7% of intangible investment in the U.S. from 1995 to 2016.\(^46\) Therefore, given that 16% of patents were traded (Akcigit, Celik, and Greenwood, 2016), \( \chi \) is calibrated to be 6.0% = 37.7% × 16%.\(^47\)

---

\(^45\)External financing from intangibles can be related to venture capital (VC). Akcigit, Dinlersoz, Greenwood, and Penciakova (2019) examine both empirically and theoretically the role of VC in creating endogenous growth.

\(^46\)The other categories of intangibles include brand, database, design, organizational capital, softwares, training etc.

\(^47\)This value is in the same magnitude as the value implied by the finding of Mann (2018). He finds that 38% of
Table 6: Tradable Intangibles and the Reinforcing Trends

<table>
<thead>
<tr>
<th>Time</th>
<th>Intangible Share</th>
<th>Firm Deposits Value Rate</th>
<th>Interest Rate</th>
<th>Capital Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model $t=0$</td>
<td>55.2%</td>
<td>7.7%</td>
<td>3.27%</td>
<td>6.6%</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>57.3%</td>
<td>27.8%</td>
<td>2.60%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Model $t=4$</td>
<td>58.7%</td>
<td>8.6%</td>
<td>2.10%</td>
<td>6.9%</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>61.8%</td>
<td>30.7%</td>
<td>1.06%</td>
<td>8.5%</td>
</tr>
<tr>
<td>Model $t=8$</td>
<td>62.2%</td>
<td>8.3%</td>
<td>0.95%</td>
<td>7.3%</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>66.6%</td>
<td>33.6%</td>
<td>-0.64%</td>
<td>9.2%</td>
</tr>
<tr>
<td>Model $t=12$</td>
<td>65.7%</td>
<td>9.3%</td>
<td>-0.07%</td>
<td>7.7%</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>71.6%</td>
<td>36.7%</td>
<td>-2.59%</td>
<td>9.9%</td>
</tr>
<tr>
<td>Model $t=16$</td>
<td>69.1%</td>
<td>10.3%</td>
<td>-1.50%</td>
<td>7.8%</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>76.9%</td>
<td>39.4%</td>
<td>-4.81%</td>
<td>10.4%</td>
</tr>
<tr>
<td>Model $t=20$</td>
<td>72.6%</td>
<td>10.6%</td>
<td>-2.88%</td>
<td>7.9%</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>82.7%</td>
<td>42.6%</td>
<td>-7.32%</td>
<td>10.7%</td>
</tr>
</tbody>
</table>

The extended model has three types of capital in a liquidity hierarchy: illiquid intangibles, intangibles traded among entrepreneurs and households, and tangible capital that are traded by all agents. The assumption that tradable intangibles are less liquid than tangible capital can be motivated by the search friction in patent trading (Akcigit, Celik, and Greenwood, 2016).\footnote{US patenting firms had previously pledged patents as collateral for financing, and these firms performed 20% of R&D expense and patenting in Compustat, implying $\chi = 38\% \times 20\% = 7.6\%$.}

Table 6 shows that tradable intangibles significantly amplify the mechanism. The intangible share of investment is higher, and its increase over time becomes convex. In contrast, the main model produces a linear trend, yielding an increase of 3.5% every four years driven by the linear increase of $\kappa_t^I$. The difference widens from 2.1% at $t = 0$ to 10.1% by $t = 20$. The non-linearity in trend arises because now the increase of $\kappa_t^I$ allows entrepreneurs to create more tradable intangibles.\footnote{The authors also point out (1) the market is specialized (often involving lawyers as middlemen); (2) the sensitivity of intellectual property makes potential participants reluctant to reveal information; (3) buyers face lemons problems.}
Table 7: Tradable Intangibles and the Financial Risk Multiplier

<table>
<thead>
<tr>
<th>Time $t$</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Model:</td>
<td>2.7</td>
<td>3.2</td>
<td>3.6</td>
<td>4.0</td>
<td>4.4</td>
<td>4.7</td>
</tr>
<tr>
<td>Tradable Intan.:</td>
<td>3.6</td>
<td>4.4</td>
<td>5.2</td>
<td>6.0</td>
<td>6.8</td>
<td>7.6</td>
</tr>
</tbody>
</table>

that bring external funds to create both intangibles (with constant unit value $q^I$) and tangible capital whose value, $q^T_t$, also trends up over time. Tradable intangibles also increase the overall leverage on (and the marginal value of) liquidity holdings. Therefore, the feedback mechanism is strengthened, resulting in a much higher level and faster growth of entrepreneurs’ liquidity holdings, a sharper decline of the deposit rate, and a stronger upward trend in the value of tangible capital.

By strengthening the feedback mechanism, tradable intangibles also amplify the economy’s response to shocks. Table 7 shows that the financial risk multiplier is higher than that of the main model. A stronger amplification mechanism makes the value of tangible capital more volatile, which in turn translates into more volatile leverage on entrepreneurs’ liquidity and investment. Moreover, the concave upward trend in the financial risk multiplier of the main model becomes a linear trend once tradable intangibles are added. A lower level of the deposit rate widens the discount-rate wedge between bankers and the rest of the economy, making the value of tangible capital more sensitive to shocks that trigger reallocation between the two groups.

6 Economy without Corporate Liquidity Demand

In this section, I conduct a counterfactual analysis to evaluate the quantitative importance of corporate savings in driving interest rates, asset prices, and endogenous financial risk. The model features two sources of demand for bankers’ debts. The upward trend of entrepreneurs’ demand is driven by $\kappa^I (t)$, while households’ demand exhibits a weaker upward trend driven by $\beta (t)$.

The trends in both sources of demand contribute to the decline in interest rates and the increase in asset prices, but the endogenous trend in $q^T_t$ pushes up entrepreneurs’ demand further by increasing the leverage on liquidity holdings. In contrast, as $r_t$ declines, the upward trend in
Table 8: The Impact of Corporate Liquidity Demand

<table>
<thead>
<tr>
<th>Time</th>
<th>Interest Rate $E^n [r(\eta, t)]$</th>
<th>Capital Valuation $E^n [q^T (\eta, t)]$</th>
<th>Financial Risk Multiplier $\max \left{ \frac{\sigma(T(\eta, t) + \sigma)}{\sigma} \mid t = n \right}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model $t = 0$</td>
<td>3.27%</td>
<td>6.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>5.78%</td>
<td>5.7</td>
<td>1.4</td>
</tr>
<tr>
<td>Model $t = 4$</td>
<td>2.10%</td>
<td>6.9</td>
<td>3.2</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>4.73%</td>
<td>6.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Model $t = 8$</td>
<td>0.95%</td>
<td>7.3</td>
<td>3.6</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>3.42%</td>
<td>6.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Model $t = 12$</td>
<td>-0.07%</td>
<td>7.7</td>
<td>4.0</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>2.30%</td>
<td>6.5</td>
<td>2.8</td>
</tr>
<tr>
<td>Model $t = 16$</td>
<td>-1.50%</td>
<td>7.8</td>
<td>4.4</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>0.92%</td>
<td>6.6</td>
<td>3.1</td>
</tr>
<tr>
<td>Model $t = 20$</td>
<td>-2.88%</td>
<td>7.9</td>
<td>4.7</td>
</tr>
<tr>
<td>Counterfactual</td>
<td>-0.31%</td>
<td>6.7</td>
<td>3.4</td>
</tr>
</tbody>
</table>

households’ demand is dampened (see (14)). The two sources of demand also exhibit distinct cyclical properties. Panel C of Figure 3 shows that, empirically, firms’ holdings of intermediary debt are more procyclical than households’. The model produces this pattern. Positive shocks reduce the bankers’ risk price, $\gamma_t^B$, so the value of tangible capital increases, which in turn stimulates entrepreneurs’ deposit demand by increasing the leverage on liquidity, resulting in a lower $r_t$ (Proposition 2). Meanwhile, households’ demand is dampened by the decrease of $r_t$.

Therefore, despite being less than 1/7 of households’ holdings of intermediary debts, entrepreneurs’ liquidity demand can have a large impact under the feedback mechanism. In Table 8, I compare the model’s performances with and without entrepreneurs’ liquidity demand. In the counterfactual model, the Poisson-arriving liquidity needs are eliminated, so, in equilibrium, entrepreneurs do not hold deposits while households hold all deposits. Appendix A.2 provides solution details. In the counterfactual model, $r_t$ is around 2.5% above $r_t$ in the main model, and the tangible capital value falls by 14%. Moreover, the financial risk multiplier declines by 30%.
Table 9: Asset Valuations and Household Holdings of Financial Intermediaries’ Debts

### Panel A: Regression Analysis of Aggregate Data

<table>
<thead>
<tr>
<th>LHS: HH Holdings of Intermediary Debts scaled by GDP</th>
<th>Financial-Market Valuation</th>
<th>Housing-Market Valuation (Price/Rent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHS: Financial-Market Valuation Metrics =</td>
<td>Tangible EV/EBITDA</td>
<td>Average EV/EBITDA</td>
</tr>
<tr>
<td></td>
<td>Tangible Tobin’s Q</td>
<td>Average Tobin’s Q</td>
</tr>
<tr>
<td></td>
<td>Financial-Market Valuation</td>
<td>Housing-Market Valuation (Price/Rent)</td>
</tr>
<tr>
<td></td>
<td>-0.017***</td>
<td>-0.142***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.024)</td>
</tr>
<tr>
<td></td>
<td>-0.010***</td>
<td>-0.060**</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.024)</td>
</tr>
<tr>
<td></td>
<td>-0.189***</td>
<td>-0.142***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.024)</td>
</tr>
<tr>
<td></td>
<td>-0.095***</td>
<td>-0.060**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>Observations</td>
<td>160</td>
<td>160</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.2971</td>
<td>0.1943</td>
</tr>
</tbody>
</table>

Heteroscedasticity-consistent standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

### Panel B: Regression Analysis of Micro Data

<table>
<thead>
<tr>
<th>HH Cash Holdings scaled by Income</th>
<th>$\Delta \ln$ (Housing Price Index)</th>
<th>Controls</th>
<th>Household FE</th>
<th>State FE</th>
<th>Year FE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-0.059</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.119***</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.114***</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.046</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.086***</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(0.031)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.081**</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>70,442</td>
<td>70,032</td>
<td>70,032</td>
<td>65,280</td>
<td>65,215</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.0001</td>
<td>0.2389</td>
<td>0.2495</td>
<td>0.1370</td>
<td>0.3438</td>
</tr>
</tbody>
</table>

State-time clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

The key difference between entrepreneurs’ demand for intermediary debts and households’ demand is that they respond differently when asset prices rise. Section 2 provides evidence that entrepreneurs’ demand is increasing in capital valuation. Next, I show, empirically, that households’ demand is decreasing in various measures of asset valuation.

In Panel A of Table 9, time-series regressions show that households’ holdings of intermediary debts depend negatively on asset valuation ratios. The dependent variable is the quarterly household holdings of intermediary debts scaled by GDP from 1980 to 2019 (source: Financial Accounts of the U.S.). Intermediary debts are listed in Figure 2 and indirect holdings via money-
market funds and mutual funds are attributed to underlying securities. The explanatory variables are measures of capital valuation (see Section 2) and the national housing price-to-rent ratio. Appendix C provides summary statistics. Column (6) shows that financial-market and housing-market valuations together explain 31% of variation.\footnote{This is the ratio of two time series in FRED: (1) “All-Transactions House Price Index for the United States”; (2) “Consumer Price Index for All Urban Consumers: Rent of Primary Residence in U.S. City Average”.

} The analysis of aggregate data has a small sample size and does not utilize cross-sectional variations. Next, I use household-level micro data. The financial-market valuation metrics do not have regional variation, so they are not included.

The Panel Study of Income Dynamics (PSID) reports biannual information on households’ financials from 1999 to 2017.\footnote{Liquidity holdings include checking/savings deposits, money market funds, certificates of deposit, Treasury securities (not including I.R.A.). A breakdown into instruments issued by intermediaries and the government is unavailable, but as shown in Figure C.3 in Appendix C, Treasury securities account for less than 15%. Related, to analyze households’ mortgage refinancing behavior, Chen, Michaux, and Roussanov (2020) use data from Financial Accounts of the U.S. for time-series analysis and PSID (including households’ liquidity holdings) for panel-data analysis. The regression samples start in 2001 because the calculation of log difference requires housing price.

} Guided by the model, the dependent variable is the ratio of liquidity holdings to household income. The explanatory variable of interest is the log difference of state-level home price index from the Federal Housing Finance Agency (FHFA). Rent data are unavailable so the log difference is taken to address apparent non-stationarities in these house prices.

Panel B of Table 9 reports a statistically significant negative response of households’ liquidity holdings to an increase in house prices, conditional on different combinations of control variables and fixed effects (FE).\footnote{Following studies on household consumption-savings decisions and portfolio allocation (Bergstresser and Poterba, 2004; Campbell and Cocco, 2007; Bogan, 2015; Chetty, Sándor, and Sziedl, 2017; Stroebel and Vavra, 2019), I construct the following control variables using PSID data: the log difference of total household income, the log difference of total household wealth, the number of people in a household, the age of household head, the education level of household head, a homeowner dummy, and a couple dummy (equal to one if the household head lives with a partner). I consider household, state, and year fixed effects. Note that the number of observations decline after household FE is added because 65 households only appear once in the panel. Appendix C provides summary statistics.

} Including control variables and fixed effects increases the adjusted $R^2$ to above 34% (in Columns (5) and (6)) by reducing noise, allowing the correlation to emerge between households’ liquidity holdings and housing price variation. The magnitudes of the estimates are consistent across specifications. The evidence suggests that households’ liquidity holdings respond negatively to asset-price increase, opposite to firms’ liquidity holdings.
7 Conclusion

This paper studies the macroeconomic causes and consequences of corporate savings gluts. In response to the structural changes in investment technology, a self-perpetuating savings glut can arise endogenously in the production sector as firms optimally choose to invest more in intangibles while financial intermediaries issue an increasing amount of liquid securities, taking advantage of the resulting low interest rates and bidding up asset prices. The economy becomes increasingly fragile along the trends because financial intermediaries’ funding cost advantage becomes increasingly large. As a result, asset prices, which drive firms’ external financing capacity and investments, become increasingly sensitive to shocks that trigger reallocation between agents with low (bankers) and high discount rates (the rest of the economy). The counterfactual analysis shows that corporate savings have a significant impact on interest rates, asset prices, and endogenous financial risk. Recent developments in making intangibles more tradable amplify the mechanism.

An interesting direction for future research is to incorporate nominal frictions into the model. The corporate savings glut contributes to a negative real rate in the model that, combined with low inflation, implies a binding lower bound on the nominal rates (Eggertsson and Woodford, 2003; Fischer, 2016). This suggests that the rise of intangible investment can trigger a more severe liquidity trap (Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson and Krugman, 2012; Caballero and Farhi, 2017; Guerrieri and Lorenzoni, 2017). Moreover, a liquidity trap triggered by one country’s transition towards an intangible-intensive economy can spread to the rest of the world through current accounts and policy responses (Caballero, Farhi, and Gourinchas, 2015).
References


A Proofs and Solution Algorithm

A.1 Proofs

Ruling out self-financing. If entrepreneurs’ investment projects can be self-financed, entrepreneurs do not need to hold liquidity for investment and the liquidity premium is zero. The equilibrium value of tangible capital is the production value, i.e., $1 / (\rho + \delta + \lambda)$. If Assumption 1 holds, then even if entrepreneurs set the intangible share of investment, $\theta_t$, to zero, the external financing capacity, $\kappa^T q^T_t = \kappa^T (\frac{1}{\rho + \delta + \lambda})$ is still below 1, which is the cost of investment. This contradicts that investment is self-financed. Therefore, under Assumption 1, the investment project cannot be self-financed.

Proof of Proposition 1. First, I show that there exists an upper bound $\bar{\eta}(t)$ such that $\eta_t \leq \bar{\eta}(t)$. Note that $q^B_t \geq 1$ in equilibrium because if $q^B_t < 1$, bankers are better off consuming (worth 1) than retaining wealth (worth $q^B_t$). As will be shown later, $q^B_t$ is a bivariate function, $q^B_t = q^B(\eta_t, t)$. Fixing $t$, let $\bar{\eta}(t)$ denote that lowest value of $\eta_t$ where bankers consume. Therefore, $q^B(\bar{\eta}(t), t) = 1$ and $q^B_t > 1$ at $\eta_t < \bar{\eta}(t)$. Suppose there exists $\eta' > \bar{\eta}(t)$ such that $\eta_t$ reaches $\eta'$. This leads to a contradiction – it is no longer optimal for bankers to consume at $\bar{\eta}(t)$ because their marginal value of wealth will surely increase: at $\bar{\eta}(t)$, if $\eta_t$ increases, $q^B_t$ will not decline because $q^B_t \geq 1$, and if $\eta_t$ decreases, $q^B_t$ will surely increase because, by definition of $\bar{\eta}(t)$, $q^B_t > 1$ for $\eta_t < \bar{\eta}(t)$. Therefore, $\eta_t$ cannot increase beyond $\bar{\eta}(t)$, the upper boundary given by bankers’ consumption optimality.

Next, I derive the law of motion of $\eta_t$ in $(0, \bar{\eta}(t))$. According to (10), bankers’ wealth satisfies the following law of motion in the region where bankers’ consumption is zero, i.e., $\eta_t \in (0, \bar{\eta}(t))$:

$$\frac{dN^B_t}{N^B_t} = \mu^N_t dt + \sigma^N_t dZ_t, \quad (A.1)$$

where

$$\mu^N_t = r_t + x^B_t (\mathbb{E}_t[dr^T_t] - r_t), \quad (A.2)$$

and

$$\sigma^N_t = x^B_t (\sigma^T_t + \sigma). \quad (A.3)$$

The expression of expected return of tangible capital holdings, $\mathbb{E}_t[dr^T_t]$, can be obtained from (9).
By Itô’s lemma, the law of motion of $\eta_t$ is given by
\[
\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ ,
\] (A.4)
where
\[
\mu_t^\eta = \mu_t^N - \mu_t^{KT} - \sigma_t^N \sigma + \sigma^2 ,
\] (A.5)
(\text{where} $\mu_t^{KT}$ \text{is the expected instantaneous growth rate of} $K_t^T$) \text{and}
\[
\sigma_t^\eta = x^B_t \left( \sigma_t^T + \sigma \right) - \sigma .
\] (A.6)
According to (19), the expected instantaneous growth rate of $K_t^T$ is given by
\[
\mu_t^{KT} = \left( \frac{1}{1 - q_t^T \kappa^T (1 - \theta_t)} \right) \left[ (x_t^B - 1) N_t^B - M_t^H \right] (1 - \theta_t) \kappa^T \lambda - \delta
\] (A.7)
where the second equation uses the definition of $\eta_t$ and households’ aggregate deposit demand given by (14). In A.2, $q_t^T, r_t, x_t^B, \theta_t, E_t [dr_t^T], \sigma_t^T, \text{and the rest of variables listed in Proposition 1}$ are shown to be bivariate functions of $\eta_t$ and $t$.

**Proof of Proposition 2.** First, I solve the investment problem of entrepreneurs who are hit by the Poisson shocks, and then embed the solution to the entrepreneurs’ dynamic optimization. An investing entrepreneur solves the problem summarized by the Lagrange function (25):
\[
\mathcal{L} = \max_{\{i_t, \theta_t\}} [q_t^I \kappa_t^I \theta_t + q_t^T \kappa^T (1 - \theta_t) - F'(\theta_t)] i_t - i_t + \pi_t [m_t^E + q_t^T \kappa^T i_t (1 - \theta_t) - i_t] .
\] (A.8)
Given $\kappa_t, q_t^I, q_t^T, \text{and} \kappa^T$, the entrepreneur chooses $\theta_t$ and $i_t$. The first-order condition (F.O.C.) for $\theta_t$ is
\[
q_t^I \kappa_t^I - q_t^T \kappa^T (1 + \pi_t) - F'(\theta_t) = 0 ,
\] (A.9)
and the F.O.C. for $i_t$ is (i.e., (26) in the main text)

$$
\pi_t = \left\{ \left[ q^I \kappa^I_t \theta_t + q^T \kappa^T (1 - \theta_t) - F(\theta_t) \right] - 1 \right\} \left( \frac{1}{1 - q^T \kappa^T (1 - \theta_t)} \right)
$$

(A.10)

The F.O.C. for $\theta_t$ equates the marginal value of investing in intangibles and the marginal value of investing in tangibles (which includes both the value of tangible capital and the shadow value from relaxing the liquidity constraint). The F.O.C. for $i_t$ solves the marginal value of liquidity as equal to the net profits of investment multiplied by the leverage on liquidity holdings. The liquidity constraint binds so the total investment is given by

$$
i_t = \frac{m^E_t}{1 - (1 - \theta_t) \kappa^T q^T}.
$$

(A.11)

Next, I prove that $\theta_t$ is increasing in $\kappa^I_t$. First, note that, from (A.10),

$$
\frac{\partial \pi_t}{\partial \theta_t} = \left[ q^I \kappa^I_t - q^T \kappa^T - F'(\theta_t) \right] \left( \frac{1}{1 - q^T \kappa^T (1 - \theta_t)} \right) - \left\{ \left[ q^I \kappa^I_t \theta_t + q^T \kappa^T (1 - \theta_t) - F(\theta_t) \right] - 1 \right\} \frac{q^T \kappa^T}{[1 - q^T \kappa^T (1 - \theta_t)]^2} = 0,
$$

(A.12)

where the second equation follows from (A.9) and (A.10). Differentiating (A.9) with respect to (w.r.t.) $\kappa^I_t$, I obtain

$$
q^I - q^T \kappa^T \frac{\partial \pi_t}{\partial \kappa^I_t} \frac{\partial \theta_t}{\partial \kappa^I_t} - q^T \kappa^T \frac{\partial \pi_t}{\partial \kappa^I_t} - F''(\theta_t) \frac{\partial \theta_t}{\partial \kappa^I_t} = 0.
$$

(A.13)

Rearranging the equation and using (A.12), I solve

$$
\frac{\partial \theta_t}{\partial \kappa^I_t} = \frac{q^I - q^T \kappa^T \frac{\partial \pi_t}{\partial \kappa^I_t}}{F''(\theta_t)}.
$$

(A.14)

According to (A.10), the partial derivative of $\pi_t$ w.r.t. $\kappa^I_t$ is

$$
\frac{\partial \pi_t}{\partial \kappa^I_t} = \frac{q^I \theta_t}{1 - q^T \kappa^T (1 - \theta_t)}.
$$

(A.15)
Using this equation to substitute out \( \frac{\partial \pi_t}{\partial k_{T}'} \) in (A.14), I obtain

\[
\frac{\partial \theta_t}{\partial k_{T}'} = \frac{1}{F'(\theta_t)} \left[ q'I - q_t k'T \frac{q'I \theta_t}{1 - q_t k'T (1 - \theta_t)} \right] = \frac{q'I (1 - q_t k'T)}{F'(\theta_t) [1 - q_t k'T (1 - \theta_t)]}.
\]  (A.16)

In equilibrium, \( q_t k'T \) must be smaller than 1, because otherwise the entrepreneur sets \( \theta_t = 0 \) (i.e., investing all in tangible capital) and self-finances the project to achieve infinite profits. Therefore, the right side of (A.16) is positive, i.e., \( \theta_t \) is increasing in \( k_{T}' \).

The right side of (A.15) is positive, so \( \pi_t \) is increasing in \( k_{T}' \). Finally, I prove that \( \pi_t \) is increasing in \( q_t k'T \). Differentiating (A.10) w.r.t. \( q_t k'T \), I obtain

\[
\frac{\partial \pi_t}{\partial q_t k'T} = k'T (1 - \theta_t) - \left\{ [q'I k_{T}' \theta_t + q_t k'T (1 - \theta_t) - F'(\theta_t)] - 1 \right\} \frac{-k'T (1 - \theta_t)}{[1 - q_t k'T (1 - \theta_t)]^2} = \frac{k'T (1 - \theta_t)}{1 - q_t k'T (1 - \theta_t)} (1 + \pi_t) > 0. \]  (A.17)

Next, I solve (28), i.e., the optimality condition for entrepreneurs’ optimal liquidity holdings. Entrepreneurs maximize the life-time utility, \( \mathbb{E} \left[ \int_{t=0}^{+\infty} e^{-\rho t} dc_t' \right] \) given the following law of motion of wealth:

\[
dw_t = -dc_t + \mu_t w_t dt + \sigma_t w_t dZ_t + (\hat{w}_t - w_t) dN_t,
\]

where \( \mu_t w_t \) and \( \sigma_t w_t \) are the drift and diffusion terms that depend on choices of tangible capital and deposit holdings and will be elaborated later. \( dN_t \) is the increment of the idiosyncratic counting (Poisson) process. At the Poisson time, an entrepreneur’s wealth jumps by the total profits from investment minus the value of lost tangible capital holdings (denoted by \( k'_{TE} \))

\[
\hat{w}_t - w_t = \left\{ [q'I k_{T}' \theta_t + q_t k'T (1 - \theta_t) - F'(\theta_t)] - 1 \right\} \left( \frac{1}{1 - q_t k'T (1 - \theta_t)} \right) m_t - q_t k'T
\]

Note that \( w_t \) does not contain the existing stock of intangible capital, because when analyzing entrepreneurs’ decisions, the production flows from intangible capital can be treated as goods that are directly consumed, given entrepreneurs’ indifference in the timing of consumption.

I conjecture that the value function is linear in wealth \( v_t \): \( v_t = \zeta_t w_t + v'I \), where \( \zeta_t \) is the marginal value of liquid wealth (i.e., without counting the value of intangible capital), and \( v'I \) is
the present value of consumption from intangible capital. Consider a generic equilibrium diffusion process for \( \zeta_t^E \):
\[
d\zeta_t^E = \zeta_t^E \mu_t^E dt + \zeta_t^E \sigma_t^E dZ_t,
\]
where \( \zeta_t^E \mu_t^E \) and \( \zeta_t^E \sigma_t^E \) are the drift and diffusion terms, respectively. Entrepreneurs’ marginal value of wealth, \( \zeta_t^E \), is a summary statistic of their investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual is hit by the Poisson shocks.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is
\[
\rho \zeta_t^E w_t^E dt = \max_{dc_t^E, k_t^TE, m_t^E} dc_t^E - \zeta_t^E dc_t^E + \left\{ w_t^E \zeta_t^E \mu_t^E + w_t^E \zeta_t^E \mu_t^w + w_t^E \zeta_t^E \sigma_t^E \sigma_t^w + \lambda \zeta_t^E (\hat{w}_t - w_t) \right\} dt.
\]
Note that the consumption flow from intangible capital and \( \rho v^I dt \) cancel each other out, because, by definition, \( v^I \) is the \( \rho \)-discounted present value of consumption flow.

Entrepreneurs can choose any \( dc_t^E \in \mathbb{R} \), so \( \zeta_t^E \) must be equal to one, and thus, I have also confirmed the value function conjecture. Since \( \zeta_t^E \) is a constant equal to one, \( \mu_t^E \) and \( \sigma_t^E \) are both zero. The HJB equation can be simplified:
\[
\rho \zeta_t^E w_t^E dt = \max_{k_t^TE \geq 0, m_t^E \geq 0} \mu_t^w w_t^E dt + \lambda dt \left( \pi_t m_t^E - q_t^T k_t^TE \right).
\] (A.19)

Wealth drift includes production, the value change of tangible capital holdings, and the deposit return:
\[
\mu_t^w w_t^E dt = k_t^TE dt + \mathbb{E}_{t} \left( q_{t+dt}^T - q_t^T k_t^TE \right) + r_t m_t^E dt.
\]
Let \( d\psi_t^E \) denote the Lagrange multiplier of the budget constraint, \( q_t^T k_t^TE + m_t^E \leq w_t^E \). The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: \( m_t^E \geq 0 \), and
\[
m_t^E \left( r_t dt + \pi_t \lambda dt - d\psi_t^E \right) = 0.
\]
The F.O.C. for optimal tangible capital holdings is: \( k_t^TE \geq 0 \), and
\[
k_t^TE \left( -\mathbb{E}_{t} \left[ dt_t^T \right] + d\psi_t^E \right) = 0.
\]
Substituting these optimality conditions into the HJB equation, we have

\[ \rho v_t^E dt = w_t^E d\psi_t^E. \]

Because \( \zeta_t^E = 1 \), \( v_t^E = w_t^E \), and \( d\psi_t^E = \rho dt \). Substituting \( d\psi_t^E = \rho dt \) into the F.O.C. for \( m_t^E \), we have

\[ \rho - r_t = \lambda \pi_t. \]

Substituting \( d\psi_t = \rho dt \) into the F.O.C. for \( k_t^{TE} \) and rearranging the equation, we have

\[ \mathbb{E}_t [dr_t^T] = \rho dt, \]

that is, when entrepreneurs hold tangible capital, they require an expected return of \( \rho \).

**Binding liquidity constraint.** Consider the following inequalities:

\[
\max_{\theta_t} \left\{ q_t^I \kappa_t^I \theta_t + q_t^T \kappa_t^T (1 - \theta_t) - F (\theta_t) \right\} \geq q_t^I \kappa_t^I - F (1) \geq q_t^I \kappa_0^I - F (1),
\]

where the first step follows \( q_t^T \geq q_t^I \) (due to the additional liquidity value of tangible capital) and the optimality of \( \theta_t \), and the second step follows from \( \kappa_t^I \geq \kappa_0^I \). Therefore, as long as

\[ q_t^I \kappa_0^I - F (1) > 1, \tag{A.20} \]

we have

\[
\pi_t = \max_{\theta_t} \left\{ q_t^I \kappa_t^I \theta_t + q_t^T \kappa_t^T (1 - \theta_t) - F (\theta_t) \right\} - 1 \left( \frac{1}{1 - q_t^T \kappa_t^T (1 - \theta_t)} \right) \geq q_t^I \kappa_0^I - F (1) - 1 \left( \frac{1}{1 - q_t^I \kappa_t^I (1 - \theta_t)} \right) > 0
\]

and the liquidity constraint binds. Note that \( \left( \frac{1}{1 - q_t^I \kappa_t^I (1 - \theta_t)} \right) > 0 \) from Assumption 1. The calibrated parameter values satisfy the condition given by (A.20).

**Proof of Proposition 3.** Conjecture that the bank’s value function takes the linear form: \( v_t^B = \)
Consider the following generic equilibrium diffusion process for $q_t^B$,

$$dq_t^B = q_t^B \mu_t^B dt - q_t^B \gamma_t^B dZ_t.$$  

Define $dy_t^B = dc_t^B / n_t^B$, the consumption-to-wealth ratio of bankers. Under the conjectured functional form, the HJB equation is

$$\rho v_t^B dt = \max_{dy_t^B} \left\{ \left(1 - q_t^B \right) \mathbb{I}_{\{dy_t^B > 0\}} n_t^B dy_t^B \right\} + \mu_t^B q_t^B n_t^B + \max_{x_t^B} \left\{ r_t + x_t^B \left( \mathbb{E}_t \left[ dr_t^T \right] - r_t \right) - x_t^B \gamma_t^B \left( \sigma_t^T + \sigma \right) \right\} q_t^B n_t^B,$$

Dividing both sides by $q_t^B n_t^B$, $n_t^B$ is eliminated, which confirms the homogeneity property,

$$\rho = \max_{dy_t^B} \left\{ \left(1 - q_t^B \right) \mathbb{I}_{\{dy_t^B > 0\}} dY_t^B \right\} + \mu_t^B + \max_{x_t^B} \left\{ r_t + x_t^B \left( \mathbb{E}_t \left[ dr_t^T \right] - r_t \right) - x_t^B \gamma_t^B \left( \sigma_t^T + \sigma \right) \right\} ,$$

and the conjecture of linear value function. The indifference condition for $x_t^B$ is

$$\mathbb{E}_t \left[ dr_t^T \right] = r_t + \gamma_t^B \left( \sigma_t^T + \sigma \right).$$

Substituting the expression of $\mathbb{E}_t \left[ dr_t^T \right]$ given by (9) and using (28), I obtain (31).

Substituting the optimality conditions into the HJB equation, I obtain

$$\mu_t^B = \rho - r_t.$$  

The result that $\gamma_t^B = 0$ when bankers consume is given by the smooth-pasting condition, $\partial q_t^B (\eta_t, t) / \partial \eta_t = 0$ (so by Itô’s lemma, $\gamma_t^B = 0$), which is discussed in more details in A.2. The upper boundary $\eta_t(t)$ is given by the value-matching condition of bankers’ consumption, $q_t^B (\eta_t(t), t) = 1$, and is jointly determined with the function $q_t^B = q_t^B (\eta_t, t)$ in the solution of PDEs of $q_t^B (\eta_t, t)$ and $q_T^B (\eta_T, t)$ in A.2.

**Conditional stationary distribution of $\eta_t$.** Following Brunnermeier and Sannikov (2014), I derive the conditional stationary probability density of $\eta_t$. Fixing $\kappa_t^I$ and $\beta_t$, the probability density of $\eta_t$
at time $t$, $p(\eta, t)$, satisfies the Kolmogorov forward equation

$$\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta} (\eta \mu^\eta(\eta) p(\eta, t)) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} \left( \eta^2 \sigma^\eta(\eta)^2 p(\eta, t) \right).$$

Note that, fixing $\kappa^I_t$ and $\beta_t$, $\mu^\eta_t$ and $\sigma^\eta_t$ are functions of $\eta_t$ as shown in A.2. A stationary density is a solution to the forward equation that does not vary with time (i.e. $\frac{\partial}{\partial t} p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over $\eta$, $p(\eta)$ solves the following first-order ordinary differential equation within the reflecting boundary:

$$0 = C - \eta \mu^\eta(\eta) p(\eta) + \frac{1}{2} \frac{d}{d\eta} \left( \eta^2 \sigma^\eta(\eta)^2 p(\eta) \right), \quad \eta \in [0, \eta_\text{f}].$$

The integration constant $C$ is zero because of the reflecting boundary. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_0^{\eta_\text{f}} p(\eta) \, d\eta = 1$).

### A.2 Solution Algorithm

The full solution of the model consists of two parts: first, the laws of motion of state variables, and, second, the endogenous variables as functions of state variables, for example, $q^T_t = q^T(\eta_t, t)$. The Markov equilibrium has four state variable: time, $\eta_t$, $K^I_t$, and $K^T_t$. As shown in the main text, the last three variables’ laws of motions depend on the endogenous variables that are functions of these state variables. To simplify the notation, I suppress the time subscripts in the following.

First, I construct a mapping from $\eta, t, q^B(\eta, t), q^T(\eta, t), \partial q^B(\eta, t) / \partial \eta, \partial q^T(\eta, t) / \partial \eta, \partial q^B(\eta, t) / \partial t$ and $\partial q^T(\eta, t) / \partial t$ to the second-order derivatives with respect to $\eta$, $\partial^2 q^B(\eta, t) / \partial \eta^2$ and $\partial^2 q^T(\eta, t) / \partial \eta^2$, i.e., a system of second-order partial differential equations for $q^B(\eta, t)$ and $q^T(\eta, t)$. Once I solved these two functions, the rest of the price variables and $K^T$-scaled aggregate quantities can be solved as they will be shown to depend only on $\eta, t$, the levels and derivatives of $q^B(\eta, t)$ and $q^T(\eta, t)$. This confirms the statement in Proposition 1 that these variables are bivariate functions of $\eta$ and $t$. After solving the price variables and $K^T$-scaled aggregate quantities, the laws of motion of $\eta_t, K^I_t$, and the $K^T_t$ are given by (21), (18) and (19).

**Constructing PDEs for $q^B(\eta, t)$ and $q^T(\eta, t)$**. Inputs are $\eta, t$ (and thus, $\kappa^I(t)$ and $\beta(t)$), the levels and first derivatives of $q^B(\eta, t)$ and $q^T(\eta, t)$. Outputs are $\partial^2 q^B(\eta, t) / \partial \eta^2$ and $\partial^2 q^T(\eta, t) / \partial \eta^2$. It
is convenient to define the following notations of elasticities:

$$
\epsilon^T \equiv \frac{\partial q^T}{\partial \eta} / q^T \quad \text{and} \quad \epsilon^B \equiv \frac{\partial q^B}{\partial \eta} / q^B .
$$

**Step 1: Calculate** $\sigma^\eta$, $\sigma^T$, $\gamma^B$, $x^B$, and $r$.

Proposition 2 solves the optimal intangible share of investment, $\theta$, and the marginal value of liquidity, $\pi$, that entrepreneurs assign to deposits, as functions of $q^I$ (constant, see (2), $q^T$, and $\kappa^I (t)$ and the parameters. Given $F(\theta_t) = \frac{1}{2} \theta^2$, (A.9) implies a quadratic equation for $\theta$ when $\pi$ is substituted out using (A.10). Once $\theta$ is solved, (A.10) solves $\pi$. In the following, I will discuss different cases, but the values of these variables will not change.

First, consider the case where entrepreneurs do not hold any deposits. With $M^E = 0$, the deposit-market clearing condition (17) is

$$
(x^B - 1) \eta = M^H / K^T = \alpha \left( \frac{\rho - r}{\beta (t)} \right)^{-\frac{1}{\xi}} ,
$$

where the second equation is obtained from households’ aggregate deposit demand (14). Within this case, there are two scenarios. First, bankers hold all tangible capital, so $q^T K^T = x^B N^B$, i.e.,

$$
x^B = q^T / \eta ,
$$

and then from (A.49), $r$ is calculated. If $r > \rho - \lambda \pi$, then entrepreneurs prefer to hold deposits, and I switch a different case where entrepreneurs hold deposits (to be discussed shortly). If $r \leq \rho - \lambda \pi$, I proceed to calculate $\sigma^\eta$, $\sigma^T$, and $\gamma^B$. Using $\sigma^\eta = x^B (\sigma^T + \sigma) - \sigma$ from (A.6), and, by Itô’s lemma, $\sigma^T = \epsilon^T \sigma^\eta$, I obtain $\sigma^\eta$. Using Itô’s lemma again, I obtain $\sigma^T = \epsilon^T \sigma^\eta$ and $\gamma^B = -\epsilon^B \sigma^\eta$. Now I calculate bankers’ discount rate, $r + \gamma^B (\sigma^T + \sigma)$. If $\rho < r + \gamma^B (\sigma^T + \sigma)$, then the rest of the economy has a lower discount rate than bankers, so bankers cannot hold all tangible capital, and I switch to the scenario where entrepreneurs do not hold deposits and bankers do not hold all tangible capital. If $\rho > r + \gamma^B (\sigma^T + \sigma)$, this scenario is valid and proceed to Step 2.

Now consider the scenario where entrepreneurs do not hold deposits and bankers do not hold all tangible capital. In this scenario, $x^B$ is calculated as follows. Given that the rest of the economy
holds tangible capital, the expected return on tangible capital is $\rho$, and from Proposition 3,

$$\rho = r + \gamma^B \left( \sigma^T + \sigma \right). \quad (A.26)$$

By Itô’s lemma,

$$\sigma^T = \epsilon^T \sigma^\eta \text{ and } \gamma^B = -\epsilon^B \sigma^\eta. \quad (A.27)$$

I substitute these expressions of $\sigma^T$ and $\gamma^B$ into (A.51) to obtain a quadratic equation of $\sigma^\eta$, and the roots are

$$\sigma^\eta = \frac{-\epsilon^B \sigma \pm \sqrt{(\epsilon^B \sigma)^2 - 4 \epsilon^B \epsilon^T (\rho - r)}}{2 \epsilon^B \epsilon^T}. \quad (A.28)$$

I study a Markov equilibrium where $\epsilon^B \leq 0$ (i.e., bankers’ marginal value of wealth declines in $\eta_t$), $\epsilon^T \geq 0$ (i.e., the value of tangible capital increases in $\eta_t$), and $\rho - r \geq 0$, so the only positive root is

$$\sigma^\eta = \frac{-\epsilon^B \sigma - \sqrt{(\epsilon^B \sigma)^2 - 4 \epsilon^B \epsilon^T (\rho - r)}}{2 \epsilon^B \epsilon^T}. \quad (A.28)$$

A positive root is selected because bankers have levered positions in tangible capital, so the shock impact is greater on $N^B$ than on $K^T$, and thus, $\eta$ responds positively to the Brownian shock. Using $\sigma^\eta = x^B \left( \sigma^T + \sigma \right) - \sigma$ from (A.6), I obtain

$$x^B = \sigma^\eta + \sigma \sigma^\eta \epsilon^T + \sigma. \quad (A.29)$$

Using (A.54) to substitute out $x^B$ in (A.49), I obtain

$$\left( \frac{\sigma^\eta + \sigma}{\sigma^\eta \epsilon^T + \sigma} - 1 \right) \eta = \alpha \left( \frac{\rho - r}{\beta(t)} \right)^{-\frac{1}{2}}. \quad (A.30)$$

Using (A.53) to substitute out $\sigma^\eta$ on the left side of (A.55), I obtain an equation for $r$. Once $r$ is solved, I use (A.53) to solve $\sigma^\eta$, use (A.54) to solve $x^B$, and use (A.52) to solve $\sigma^T$ and $\gamma^B$. Proceed to Step 2.

Finally, consider the case where entrepreneurs hold deposits. From Proposition 2, the equilibrium deposit rate is given by

$$r = \rho - \lambda \pi. \quad (A.31)$$
Given this deposit rate, I can calculate the deposit demand of households (scaled by $K^T$) using (14), and obtain the aggregate deposit demand, $(M^E + M^H) / K^T$. Next, consider the scenario where bankers hold all tangible capital, i.e., $x^B = q^T / \eta$. From (A.54), I solve $\sigma^\eta$, and from (A.52), I solve $\sigma^T$ and $\gamma^B$. Now I calculate bankers’ discount rate, $r + \gamma^B \left( \sigma^T + \sigma \right)$. If $\rho < r + \gamma^B \left( \sigma^T + \sigma \right)$, then the rest of economy have lower discount rate than bankers, so bankers cannot hold all tangible capital and I switch to the scenario where entrepreneurs hold deposits and bankers do not hold all tangible capital. If $\rho > r + \gamma^B \left( \sigma^T + \sigma \right)$, this scenario is valid and proceed to Step 2.

Now consider the scenario where entrepreneurs hold deposits and bankers do not hold all tangible capital. The expected return on tangible capital is $\rho$, so from Proposition 3,

$$\rho = r + \gamma^B \left( \sigma^T + \sigma \right).$$  \hspace{1cm} (A.32)

Using (A.31) to substitute $r$ with $\rho - \lambda \pi$, I obtain

$$\lambda \pi = \gamma^B \left( \sigma^T + \sigma \right).$$  \hspace{1cm} (A.33)

Using Itô’s lemma, i.e., (A.52), I substitute $\sigma^T$ and $\gamma^B$ out with $\epsilon^T \sigma^\eta$ and $- \epsilon^B \sigma^\eta$ respectively to obtain a quadratic equation of $\sigma^\eta$, and the roots are

$$\sigma^\eta = \frac{- \epsilon^B \sigma \pm \sqrt{\left( \epsilon^B \sigma \right)^2 - 4 \epsilon^B \epsilon^T \lambda \pi}}{2 \epsilon^B \epsilon^T}.$$  

I study a Markov equilibrium where $\epsilon^B \leq 0$ (i.e., bankers’ marginal value of wealth declines in $\eta$), $\epsilon^T \geq 0$ (i.e., the value of tangible capital increases in $\eta$), and, as the shadow price of funding constraint on investment, $\pi \geq 0$, so the only positive root is

$$\sigma^\eta = \frac{- \epsilon^B \sigma - \sqrt{\left( \epsilon^B \sigma \right)^2 - 4 \epsilon^B \epsilon^T \lambda \pi}}{2 \epsilon^B \epsilon^T}. $$  \hspace{1cm} (A.34)

Using Itô’s lemma again, i.e., (A.52), I solve $\sigma^T$ and $\gamma^B$. Using $\sigma^\eta = x^B \left( \sigma^T + \sigma \right) - \sigma$ from (A.6), I solve $x^B$. Proceed to Step 2.

**Step 2: Calculating the Second-Order Derivatives**

The drift and diffusion of $\eta$ are given in the proof of Proposition 1. Given $q^T$, $\pi$, $\gamma^B$, and $\sigma^T$,
(31) solves \( \mu^T \). The following equation, obtained by Itô’s lemma, solves \( \frac{\partial^2 q^T}{\partial \eta^2} \):

\[
\mu^T q^T = \frac{\partial q^T}{\partial t} + \frac{\partial q^T}{\partial \eta} \mu^\eta \eta + \frac{1}{2} \frac{\partial^2 q^T}{\partial \eta^2} (\sigma^\eta \eta)^2 .
\] (A.35)

According to (A.23), \( \mu^B_t = \rho - r_t \), so the following equation, obtained by Itô’s lemma, solves \( \frac{\partial^2 q^B}{\partial \eta^2} \):

\[
\mu^B q^B = \frac{\partial q^B}{\partial t} + \frac{\partial q^B}{\partial \eta} \mu^\eta \eta + \frac{1}{2} \frac{\partial^2 q^B}{\partial \eta^2} (\sigma^\eta \eta)^2 .
\] (A.36)

**Boundary conditions for PDEs for** \( q^B (\eta, t) \) **and** \( q^T (\eta, t) \). Tangible capital has constant cash flow, one unit of goods per unit of time, so what causes its price to vary is the discount-rate changes. Close to \( \eta = 0 \), an absorbing state, the banking sector is extremely small, so the discount rate (expected return) is fixed at \( \rho \) to induce the rest of economy to own tangible capital and clear the market. Thus, \( q^T \) should not vary as \( \eta \) approaches zero:

\[
\lim_{\eta \to 0} \frac{\partial q^T(\eta, t)}{\partial \eta} = 0 .
\] (A.37)

Moreover, when bankers are extremely undercapitalized, their marginal value of wealth approaches infinity,

\[
\lim_{\eta \to 0} q^B (\eta, t) = +\infty ,
\] (A.38)

because \( q^B \) is the present value of one unit of equity, and it increases when the banking sector shrinks, widening the return spread between holding tangible capital and issuing deposits.

The upper boundary of \( \eta, \bar{\eta} \), where bankers consume, is a reflecting boundary, so to rule out arbitrage (i.e., perfectly predictable variation of asset price),

\[
\frac{\partial q^T(\bar{\eta}, t)}{\partial \eta} = 0 .
\] (A.39)

For consumption to be optimal at \( \bar{\eta} \), bankers’ marginal value of wealth, \( q^B \), satisfies the value-matching condition,

\[
q^B (\bar{\eta}, t) = 1 .
\] (A.40)
and the smooth-pasting condition
\[
\frac{\partial q^B (\eta, t)}{\partial \eta} = 0. \quad (A.41)
\]

Finally, it is assumed that the linear trends of \( \kappa^I \) and \( \beta \) end at \( t = \bar{t} \). When solving the model, I map \( \bar{t} \) to 2010 in the data. When \( t \) reaches \( \bar{t} \) and \( \kappa^I \) and \( \beta \) no longer vary, the economy converges to a time-homogeneous Markov equilibrium where the price variables and \( K^T \)-scaled quantities are functions of \( \eta_t \) only. Therefore, the boundary condition on the time dimension for \( q^B (\eta, t) \) and \( q^T (\eta, t) \) is the convergence to \( \bar{q}^B (\eta) \) and \( \bar{q}^T (\eta) \) of the time-homogeneous Markov equilibrium.

The functions, \( \bar{q}^B (\eta) \) and \( \bar{q}^T (\eta) \), of the time-homogeneous Markov equilibrium at \( \bar{t} \) can be solved by a system of ordinary differential equations (ODEs) that are constructed following the same aforementioned procedure, except that at the very last step, by Itô’s lemma, the second-order derivatives are solved by
\[
\mu^B q^B = \frac{d q^B}{d \eta} \mu^B + \frac{1}{2} \frac{d^2 q^B}{d \eta^2} (\sigma^B)^2. \quad (A.42)
\]
and
\[
\mu^T q^T = \frac{d q^T}{d \eta} \mu^T + \frac{1}{2} \frac{d^2 q^T}{d \eta^2} (\sigma^T)^2. \quad (A.43)
\]
The ODEs have the following conditions in analogy to (A.37) to (A.41):

As \( \eta \) approaches zero: (1) \( \lim_{\eta \to 0} \frac{d q^T (\eta)}{d \eta} = 0 \); (2) \( \lim_{\eta \to 0} \bar{q}^B (\eta) = +\infty \).

At the upper reflecting boundary, \( \bar{\eta} \): (3) \( \frac{d q^T (\bar{\eta})}{d \eta} = 0 \); (4) \( \bar{q}^B (\bar{\eta}) = 1 \); (5) \( \frac{d \bar{q}^B (\eta)}{d \eta} = 0 \).

**Prices and \( K^T \)-scaled quantities in Proposition 1.** The solution procedure has solved \( q^T_t \), \( r_t \), \( x^B_t \), \( \theta_t \) as bivariate functions of \( \eta_t \) and \( t \) because they only depend on \( \eta_t \), \( t \), \( q^T_t \), \( \varepsilon^T \), and \( \varepsilon^B \). From (14), households’ aggregate deposit holdings, \( M^H_t / K^T_t \), is \( \alpha \left( \frac{\rho - r_t}{\beta (t)} \right)^{-\frac{1}{\gamma}} \). Entrepreneurs’ aggregate deposit holdings (scaled by \( K^T_t \)), \( M^E_t / K^T_t \), is given by
\[
\frac{M^E_t}{K^T_t} = \frac{(x^B_t - 1) N^B_t - M^H_t}{K^T_t} = (x^B_t - 1) \eta_t - \alpha \left( \frac{\rho - r_t}{\beta (t)} \right)^{-\frac{1}{\gamma}}. \quad (A.44)
\]
The aggregate intangible investment (scaled by \( K^T_t \)) is \( \theta_t M^E_t / K^T_t \) and the aggregate tangible investment (scaled by \( K^T_t \)) is \( (1 - \theta_t) M^E_t / K^T_t \). Now it has been proven that the price variables and \( K^T \)-scaled aggregate quantities listed in Proposition 1 are bivariate functions of \( \eta_t \) and \( t \).
The hierarchy of state variables. Time has its autonomous law of motion. The law of motion of $\eta_t$ in the proof of Proposition 1 only depends on $\eta_t$ and time $t$. The law of motion of $K_t^T$ (i.e., (19) in the main text) only depends on $\eta_t$, time $t$, and $K_t^T$: using (A.7), I obtain

$$
\frac{dK_t^T}{K_t^T} = \left[ \left( \frac{x_t^B - 1}{1 - q_t^I K_t^T (1 - \theta_t)} \right) \lambda (1 - \theta_t) \right] \eta_t - \alpha \left( \frac{q_t}{\beta(t)} \right)^{-\frac{1}{2}} \kappa^T (1 - \theta_t) \lambda - \delta \right] dt + \sigma dZ_t , \tag{A.45}
$$

where the drift is solved in the proof of Proposition 1 and the endogenous variables on the right side are bivariate functions of $\eta_t$ and $t$. Finally, rewriting (18) from the main text, I obtain the law of motion of $K_t^I$, which depends on all four state variables,

$$
\frac{dK_t^I}{K_t^I} = \frac{K_t^T}{K_t^I} \left[ \left( \frac{x_t^B - 1}{1 - q_t^I K_t^T (1 - \theta_t)} \right) \lambda (1 - \theta_t) \right] \theta_t \kappa^I (t) \lambda dt - (\delta dt - \sigma dZ_t) , \tag{A.46}
$$

Solving the model with tradable intangibles. Allowing $\chi$ fraction of intangible capital to be tradable among entrepreneurs and households only change the optimality conditions for $\theta$ and $i$. The rest of solution algorithm is the same as that of the main model. The F.O.C. for $\theta_t$ is

$$
q_t^I \kappa^I (1 + \chi \pi) - q_t^T \kappa^T (1 + \pi) - F' (\theta) = 0 . \tag{A.47}
$$

In contrast to (A.9), the marginal benefit of creating intangible capital has an additional component $q_t^I \kappa^I \chi \pi$ from relaxing the financial constraint. The F.O.C. for $i$ is

$$
\pi = \left\{ \left[ q_t^I \kappa^I \theta + q_t^T \kappa^T (1 - \theta) - F (\theta) \right] - 1 \right\} \left( \frac{1}{1 - q_t^I \kappa^I (1 - \theta) - \chi q_t^I \kappa^I \theta} \right) , \tag{A.48}
$$

Given that $F (\theta) = \frac{\varphi}{2} \theta^2$, (A.47) implies a quadratic equation for $\theta$ when $\pi$ is substituted out by (A.48). Once $\theta$ is solved, (A.48) solves $\pi$. 

72
Solving the model without entrepreneurs’ liquidity demand. The model without entrepreneurs’ liquidity demand can be solved with a similar procedure. A system of PDEs are constructed for \( q^B(\eta, t) \) and \( q^T(\eta, t) \), and the construction process reveals that once \( q^B(\eta, t) \) and \( q^T(\eta, t) \) are known, the rest of the endogenous variables can be solved. The boundary conditions for the PDEs are the same as the main model.

Next, a system of PDEs are constructed. Inputs are \( \eta, t \) (and thus, \( \kappa^I(t) \) and \( \beta(t) \)), the levels and first derivatives of \( q^B(\eta, t) \) and \( q^T(\eta, t) \). Outputs are \( \partial^2 q^B(\eta, t) / \partial \eta^2 \) and \( \partial^2 q^T(\eta, t) / \partial \eta^2 \).

**Step 1: Calculate \( \sigma^\eta, \sigma^T, \gamma^B, x^B, \) and \( r \).**

The deposit-market clearing condition (17) is

\[
(x^B - 1) \eta = M^H / K^T = \alpha \left( \frac{\rho - r}{\beta(t)} \right)^{-\frac{1}{\xi}}, \tag{A.49}
\]

where the second equation is obtained from households’ aggregate deposit demand (14). There are two scenarios. First, bankers hold all tangible capital, so \( q^T K^T = x^B N^B \), i.e.,

\[
x^B = q^T / \eta, \tag{A.50}
\]

and, then, from (A.49), \( r \) is calculated. If \( r > \rho - \lambda \pi \), then entrepreneurs prefer to hold deposits and I switch a different case where entrepreneurs hold deposits (to be discussed shortly). If \( r \leq \rho - \lambda \pi \), I proceed to calculate \( \sigma^\eta, \sigma^T, \) and \( \gamma^B \). Using \( \sigma^\eta = x^B (\sigma^T + \sigma) - \sigma \) from (A.6), and, by Itô’s lemma, \( \sigma^T = \epsilon^T \sigma^\eta \), I obtain \( \sigma^\eta \). Using Itô’s lemma again, I obtain \( \sigma^T = \epsilon^T \sigma^\eta \) and \( \gamma^B = -\epsilon^T \sigma^\eta \).

Now I calculate bankers’ discount rate, \( r + \gamma^B (\sigma^T + \sigma) \). If \( \rho < r + \gamma^B (\sigma^T + \sigma) \), then the rest of the economy has lower discount rate than bankers, so bankers cannot hold all tangible capital and I switch to the scenario where bankers do not hold all tangible capital. If \( \rho > r + \gamma^B (\sigma^T + \sigma) \), this scenario is valid so I can proceed to Step 2.

Now consider the scenario where bankers do not hold all the tangible capital. In this scenario, \( x^B \) is calculated as follows. Given that the rest of the economy holds tangible capital, the expected return on tangible capital is \( \rho \), and from Proposition 3,

\[
\rho = r + \gamma^B (\sigma^T + \sigma). \tag{A.51}
\]
By Itô’s lemma,
\[ \sigma^T = \epsilon^T \sigma^n \text{ and } \gamma^B = -\epsilon^B \sigma^n. \]  
(A.52)

I substitute these expressions of \( \sigma^T \) and \( \gamma^B \) into (A.51) to obtain a quadratic equation of \( \sigma^n \), and the roots are
\[ \sigma^n = \frac{-\epsilon^B \sigma \pm \sqrt{(\epsilon^B \sigma)^2 - 4\epsilon^B \epsilon^T (\rho - r)}}{2 \epsilon^B \epsilon^T}. \]

I study a Markov equilibrium where \( \epsilon^B \leq 0 \) (i.e., bankers’ marginal value of wealth declines in \( \eta_t \)), \( \epsilon^T \geq 0 \) (i.e., the value of tangible capital increases in \( \eta_t \)), and \( \rho - r \geq 0 \), so the only positive root is
\[ \sigma^n = \frac{-\epsilon^B \sigma - \sqrt{(\epsilon^B \sigma)^2 - 4\epsilon^B \epsilon^T (\rho - r)}}{2 \epsilon^B \epsilon^T}. \]  
(A.53)

A positive root is selected because bankers have levered positions in tangible capital, so the shock impact is greater on \( N^B \) than on \( K^T \), and thus, \( \eta \) responds positively to the Brownian shock. Using \( \sigma^n = x^B (\sigma^T + \sigma) - \sigma \) from (A.6), I obtain
\[ x^B = \frac{\sigma^n + \sigma}{\sigma^n \epsilon^T + \sigma}. \]  
(A.54)

Using (A.54) to substitute out \( x^B \) in (A.49), I obtain
\[ \left( \frac{\sigma^n + \sigma}{\sigma^n \epsilon^T + \sigma} - 1 \right) \eta = \alpha \left( \frac{\rho - r}{\beta (t)} \right)^{-1}. \]  
(A.55)

Using (A.53) to substitute out \( \sigma^n \) on the left side of (A.55), I obtain an equation for \( r \). Once \( r \) is solved, I use (A.53) to solve \( \sigma^n \), use (A.54) to solve \( x^B \), and use (A.52) to solve \( \sigma^T \) and \( \gamma^B \).

Proceed to Step 2.

**Step 2: Calculating the Second-Order Derivatives**

The drift and diffusion of \( \eta \) are given in the proof of Proposition 1. Given \( q^T, \pi, \gamma^B, \) and \( \sigma^T, \) (31) solves \( \mu^T \). The following equation, obtained by Itô’s lemma, solves \( \frac{\partial^2 q^T}{\partial \eta^2} \):
\[ \mu^T q^T = \frac{\partial q^T}{\partial t} + \frac{\partial q^T}{\partial \eta} \mu^T \eta + \frac{1}{2} \frac{\partial^2 q^T}{\partial \eta^2} (\sigma^n \eta)^2. \]  
(A.56)
According to (A.23), $\mu^B_t = \rho - r_t$, so the following equation, obtained by Itô’s lemma, solves $\frac{\partial^2 q^B}{\partial \eta^2}$:

$$
\mu^B q^B = \frac{\partial q^B}{\partial t} + \frac{\partial q^B}{\partial \eta} \mu^\eta \eta + \frac{1}{2} \frac{\partial^2 q^B}{\partial \eta^2} (\sigma^\eta \eta)^2.
$$

(A.57)
B Risk Aversion and Finite EIS

In this section, I extend the model to incorporate risk-averse preferences and finite EIS (elasticity of intertemporal substitution) showing that the solution of the extended model can be achieved by making two modifications to the solution of the model in the main text. First, the functions of endogenous variables, such as $q^T(\eta_t, t)$, can be derived by the procedure in A.2 with the time discount rate $\rho$ replaced by a function $\rho(\eta_t, t)$. The functional form of $\rho(\eta_t, t)$ depends on the risk-averse utility functions in the extended model.

Second, the laws of motion of state variables in the solution of the main model become the laws of motion under the risk-neutral measure in the extended model with risk aversion. To characterize the dynamics under the physical measure (probability measure of data generating process), a change of measure shall be performed using Girsanov’s Theorem. The Markov equilibrium has four state variable: time, $\eta_t$, $K^t_1$, and $K^T_1$. A change of measure affects the laws of motion of the last three by adjusting their drifts. The adjustments depend on (1) the state variables’ loadings of the aggregate shock (i.e., their diffusions) and (2) the consumers’ price of risk implied by the risk-averse utility functions and the equilibrium process of aggregate consumption. This method of extending risk-neutral models to incorporate risk-averse preferences can be applied to other macrofinance models with risk-neutral preferences, for example, Brunnermeier and Sannikov (2014).

After establishing these results, I characterize the conditions under which the equilibrium of the main (risk-neutral) model serves as an adequate approximation to the equilibrium of the extended model. The model solution has two parts, first, the endogenous variables as functions of state variables, for example, $q^T_t = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. I show that the first part of the solution is an adequate approximation if the expected growth rate of consumption is stable, which holds in the model and is consistent with the theories and evidence on long-run consumption risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008; Schorfheide, Song, and Yaron, 2018). I also show that ignoring risk aversion has little impact on the laws of motion of state variables under the standard risk aversion parameter in the asset pricing literature.

Incorporating preferences with risk aversion and finite EIS. Next, I introduce risk-averse preferences to the household sector. Entrepreneurs and bankers are reinterpreted as firms that maximize the present value of payouts to household shareholders, so households are the ultimate consumers in this economy. It is assumed that households face a complete market. For households, the relevant shock is the aggregate Brownian shock $dZ_t$. The market is complete if households can trade
tangible capital and risk-free assets.\footnote{For risk-free assets, it is assumed that households can lend to and borrow from each other (in equilibrium, at the risk-free rate $\rho_t$), i.e., the negative drift of SDF in (B.1), and unlike deposits, such risk-free assets do not bring deposit-in-utility for households or relax entrepreneurs’ liquidity constraints. They may represent personal IOUs.} No arbitrage and complete markets imply the existence of a unique stochastic discount factor (SDF), denoted by $\Lambda_t$, which, in equilibrium, is determined by the households’ marginal value of wealth (Duffie, 2001). The following analysis takes the equilibrium process of $\Lambda_t$ as given,

$$\frac{d\Lambda_t}{\Lambda_t} = -\rho_t dt - \gamma_t^H d\tilde{Z}_t, \quad \text{(B.1)}$$

where $\rho_t$ is the households’ time discount rate in equilibrium and $\gamma_t^H$ is the households’ price of risk. The endogenous discount rate $\rho_t$ replaces the parameter $\rho$ in the main text. After analyzing the entrepreneurs’ and banks’ problems, I specify the households’ preferences and solve $\Lambda_t$. The stochastic process $\tilde{Z}_t$ is the cumulative aggregate shock under the physical measure.

By Girsanov’s Theorem, we know the following connection between the aggregate shock to capital stock, $dZ_t$, under the risk-neutral measure and $d\tilde{Z}_t$, the shock under the physical measure,

$$dZ_t = d\tilde{Z}_t + \gamma_t^H dt. \quad \text{(B.2)}$$

The idiosyncratic Poisson shocks do not affect the change of measure because they are not priced in the SDF. Entrepreneurs’ information filtration under the physical measure is generated by $\tilde{Z}_t$ and the idiosyncratic Poisson shocks that trigger investment needs. Their information filtration under the risk-neutral measure is generated by $Z_t$ and the same idiosyncratic Poisson shocks. For bankers, idiosyncratic risks are diversified away, so the relevant information filtration is generated by $Z_t$ under the risk-neutral measure and $\tilde{Z}_t$ under the physical measure.

Girsanov’s Theorem implies a connection between objective functions under the physical and risk-neutral measures: a representative entrepreneur $i$ maximize

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho_t t} dc_{i,t}^E \right] = \hat{\mathbb{E}}\left[\int_{0}^{\infty} \frac{\Lambda_t}{\Lambda_0} dc_{i,t}^E \right], \quad \text{(B.3)}$$

and

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho_t t} dc_{i,t}^B \right] = \hat{\mathbb{E}}\left[\int_{0}^{\infty} \frac{\Lambda_t}{\Lambda_0} dc_{i,t}^B \right], \quad \text{(B.4)}$$

where $\hat{\mathbb{E}}[\cdot]$ is the rational-expectation operator under the physical measure, distinguished from
\( \mathbb{E} [\cdot] \), the rational-expectation operator under the risk-neutral measure, and, following the notations in Appendix A, \( c^E_{i,t} \) and \( c^B_{j,t} \) denotes the cumulative payout of non-financial firms and banks.

The full solution of the model consists of two parts: first, the endogenous variables as functions of state variables, for example, \( q^T_t = q^T(\eta_t, t) \), and, second, the laws of motion of state variables. The next proposition states the connection between an extended model with power utility and the model in the main text. The proof is at the end of this section. This method of incorporating risk-averse preferences into an originally risk-neutral model applies to any utility function. I use power (CRRA) utility as an example.

**Proposition B.1** Households have power utility over consumption and deposit-in-utility introduced in (11) and maximize

\[
\mathbb{E} \left[ \int_{t=0}^{\infty} e^{-\delta_H t} \left( \left( \frac{c^H_i}{1 - \gamma_H} \right)^{1 - \gamma_H} + \beta_t \left( \frac{m^H_t / w^H_t}{1 - \xi} \right)^{1 - \xi} \right) dt \right],
\]

where \( \delta_H \) and \( \gamma_H \) are the parameters for discount factor and relative risk aversion, respectively, and \( c^H_i \) denote the rate of consumption (instead of cumulative consumption).

The solutions of endogenous variables as functions of state variables can be obtained by the procedure in A.2 with the parameter \( \rho \) replaced by the following function:

\[
\rho (\eta_t, t) = \delta_H + \gamma_H H (\eta_t, t) + \gamma_H \tilde{C}_1 (\eta_t, t) \left[ \mu^\eta (\eta_t, t) + \frac{1}{2} \tilde{C}_2^1 (\eta_t, t) \sigma^\eta (\eta_t, t)^2 + \sigma^\eta (\eta_t, t) \sigma \right] + \frac{1}{2} \left( \gamma^2_H - \gamma_H \right) \left[ \tilde{C}_1 (\eta_t, t) \sigma^\eta (\eta_t, t) + \sigma \right]^2,
\]

where \( \mu^\eta (\eta_t, t), \sigma^\eta (\eta_t, t), \) and \( \mu^{KT} (\eta_t, t) \) are given by (A.5), (A.6), and (A.7) respectively in A.1, and \( \tilde{C}_1 (\eta_t, t) \) is the elasticity of \( K^T \)-scaled aggregate consumption, \( \tilde{C}_i^H \equiv C^H_i / K^T_i \), to \( \eta_t \),

\[
\tilde{C}_1 (\eta_t, t) \equiv \frac{\partial C^H_i (\eta_t, t)}{\partial \eta_t} \frac{\eta_t}{\tilde{C}^H (\eta_t, t)},
\]

and \( \tilde{C}_2 (\eta_t, t) \) is the elasticity of \( \frac{\partial C^H_i (\eta_t, t)}{\partial \eta_t} \) to \( \eta_t \),

\[
\tilde{C}_2 (\eta_t, t) \equiv \frac{\partial^2 C^H_i (\eta_t, t)}{\partial \eta_t^2} \left( \frac{\eta_t}{\partial C^H_i (\eta_t, t)} \right). \]

78
By Girsanov’s Theorem, the laws of motion of $\eta_t$, $K_t^T$, and $K_t^I$ are given by (A.4), (A.45), and (A.46) respectively with $dZ_t$, the Brownian shock under the risk-neutral measure, replaced by

$$d\hat{Z}_t + \gamma^H(\eta_t, t)\, dt$$  \hspace{1cm} (B.9)

where $d\hat{Z}_t$ is the Brownian shock under the physical measure, and $\gamma^H(\eta_t, t)$ is given by

$$\gamma^H(\eta_t, t) = \gamma^H[\epsilon^{\tilde{C}1}(\eta_t, t)\sigma^\eta(\eta_t, t) + \sigma],$$  \hspace{1cm} (B.10)

which is the households’ price of risk in equilibrium.

Comparing risk-neutral and risk-averse models. In the comparison between the risk-neutral and risk-averse models, a key object is $\epsilon^{\tilde{C}1}(\eta_t, t)$, the the elasticity of $K_t^T$-scaled aggregate consumption, $\tilde{C}_t^H \equiv C_t^H / K_t^T$, to $\eta_t$. Given $C_t^H = \tilde{C}_t^H K_t^T$, by Itô’s lemma, the volatility of consumption growth, $\sigma^C_t$ is given by

$$\sigma^C_t = \epsilon^{\tilde{C}1}(\eta_t, t)\sigma^\eta(\eta_t, t) + \sigma.$$  \hspace{1cm} (B.11)

The constant return-to-scale technology implies that the volatility of capital growth, $\sigma$, is the volatility of output growth. Empirically, consumption growth is less volatile in data than output growth (e.g., Blanchard and Simon, 2001). Therefore, if the preference parameters are calibrated to match consumption volatility (as typically done in the asset-pricing literature (Cochrane, 2005a)), we have

$$\epsilon^{\tilde{C}1}(\eta_t, t) < 0.$$  \hspace{1cm} (B.12)

The model solution has two parts: first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. Therefore, according to Proposition B.1, a potential misspecification from ignoring risk aversion has two consequences. First, in the algorithm that solves the functions of endogenous variables in A.2, $\rho$ should be replaced by $\rho(\eta_t, t)$. Second, the laws of motions of state variables are in fact risk-neutral dynamics. The dynamics under the physical measure require an adjustment of drifts by replacing $dZ_t$ with $d\hat{Z}_t + \gamma^H(\eta_t, t)\, dt$ (see B.2).

To analyze the impact of ignoring risk aversion on the functions of endogenous variables, I examine whether $\rho(\eta_t, t)$ can be approximated by a constant. The expression of $\rho(\eta_t, t)$ in (B.6)
can be simplified with the consumption growth volatility in (B.11):

\[
\rho(\eta_t, t) = \delta_H + \gamma_H \mu^{KT}(\eta_t, t) + \gamma_H \epsilon^{C1}(\eta_t, t) \left[ \mu^n(\eta_t, t) + \frac{1}{2} \epsilon^{C2}(\eta_t, t) \sigma^n(\eta_t, t)^2 + \sigma^n(\eta_t, t) \sigma \right] + \frac{1}{2} \left( \gamma_H^2 - \gamma_H^H \right) (\sigma'_C)^2 .
\]  

(B.13)

Let \( O(\sigma^2) \) denote the terms that involve the squared volatilities of growth rates (which all contain \( \sigma^2 \)). Because volatilities and expectations of growth rates are of similar magnitudes in this model where aggregate quantities are driven by geometric Brownian motions, these volatility-squared terms tend to be small. Therefore, I use the following expression

\[
\rho(\eta_t, t) = \delta_H + \gamma_H \mu^{KT}(\eta_t, t) + \gamma_H \epsilon^{C1}(\eta_t, t) \mu^n(\eta_t, t) + O(\sigma^2) .
\]  

(B.14)

The first and second terms are standard in consumption-based asset pricing models. Given the constant return-to-scale technology, the capital growth rate, \( \mu^{KT}(\eta_t, t) \), is the growth rate of aggregate output. In an endowment economy, the aggregate output (agents’ endowments) is equal to the aggregate consumption in equilibrium, so, in these consumption-based models, the equilibrium risk-free rate only contains the first two terms on the right side of (B.14) (e.g., Lucas, 1978a).

The third term is unique to this model. The drift of \( \eta_t, \mu^n(\eta_t, t) \), is the expected growth rate of the ratio of bankers’ wealth to tangible capital value. Because bankers hold a leveraged position in tangible capital, and the expected return on tangible capital is positive, bankers’ wealth grows faster than tangible capital in expectation, and \( \mu^n(\eta_t, t) \) is positive. Given that \( \epsilon^{C1}(\eta_t, t) < 0 \) (see (B.12)), the third term on the right side of (B.14) is negative.

The economy becomes more intangible-intensive over time, and firms hold more cash, which leads to an upward trend in investment and output growth. The counteracting force is also getting stronger. As the economy becomes more intangible-intensive, the liquidity premium on deposits, \( \rho_t - r_t \), becomes larger, which increases bankers’ return on wealth, and thus, pushes up \( \mu^n(\eta_t, t) \).

Assuming \( \rho(\eta_t, t) \) is a constant in the main model is equivalent to assuming that these two forces, \( \gamma_H \mu^{KT}(\eta_t, t) > 0 \) and \( \gamma_H \epsilon^{C1}(\eta_t, t) \mu^n(\eta_t, t) < 0 \), cancel each other out. The first force is from output growth. The second is from the fact that consumption is less volatile than output growth and, due to leverage, bankers’ expected return on wealth is greater than tangible capital. Empirically, this assumption means that the expected consumption growth rate is stable. A stable consumption growth rate is consistent with the findings of highly persistent expected consump-
tion growth in the literature on long-run risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008; Schorfheide, Song, and Yaron, 2018). approximating \( \rho(\eta_t, t) \) by a constant does not cause significant misspecification. When this approximation is adequate, the functions of endogenous variables, for example, \( q^T_t = q^T(\eta_t, t) \), that are solved in A.2 and presented in Section 4.2, are adequate approximations to the functions from the risk-averse model.

Next, I examine the impact of ignoring risk-aversion on the laws of motion of state variables. According to Proposition B.1, the dynamics of capital stocks given by (A.45) and (A.46) should be adjusted by replacing the Brownian shock under the risk-neutral measure, \( dZ_t \), by the Brownian shock under the physical measure (i.e., the real shock that drives the data generating processes), \( d\hat{Z}_t \), plus a drift adjustment \( \gamma_H(\eta_t, t) dt \):

\[
\frac{dK^T_t}{K^T_t} = \left[ \left( \frac{x^B - 1}{1 - q^T_t \kappa^T (1 - \theta_t)} \right) \kappa^T (1 - \theta_t) \lambda - \delta \right] dt + \sigma \gamma_H(\eta_t, t) dt + \sigma d\hat{Z}_t, \tag{B.15}
\]

and

\[
\frac{dK^I_t}{K^I_t} = \left[ \kappa^I (t) \theta_t \lambda - \delta \right] dt + \sigma \gamma_H(\eta_t, t) dt + \sigma d\hat{Z}_t. \tag{B.16}
\]

Consider a relative risk aversion \( \gamma_H = 5 \), which is a common value in the asset pricing literature (Cochrane, 2005a). Given that \( C^1(\eta_t, t) < 0 \) and \( \sigma^\eta(\eta_t, t) > 0 \) (see (A.6)), I obtain the following upper bound on the households’ price of risk: from (B.34),

\[
\gamma_H(\eta_t, t) = \gamma_H \left[ e^{C^1(\eta_t, t)} \sigma^\eta(\eta_t, t) + \sigma \right] \leq \gamma_H \sigma = 0.1, \tag{B.17}
\]

where the last equation substitutes in the value of \( \gamma_H \) and \( \sigma \). Given that \( \sigma = 0.02 \) and \( \gamma_H(\eta_t, t) < 0.1 \), the risk adjustment term is bounded above by 0.002. Therefore, ignoring risk aversion understates the expected growth rate of capital (and output), and the wedge is bounded above by 0.2%.

The physical-measure dynamics feature higher growth rates than those of the risk-neutral dynamics because, when changing from the physical measure to the risk-neutral measure, probability mass shifts towards the relatively worse states of the world, i.e., the risk-averse attitude is reflected by
probability re-weighting. A similar calculation can be applied to the law of motion of \( \eta_t \). The risk adjustment increases the drift of \( \eta_t \), and, averaging over time \( t \) and \( \eta_t \) on the simulated paths, such an increase is less than 6% of the drift, (i.e., \( < 0.06 \times \mu_\eta \)).

In the main text, I report the model’s solutions in two ways: (1) the values of endogenous variables at different points in time, averaged over \( \eta_t \) (e.g., Section 4.2) and (2) endogenous variables as functions of \( \eta_t \) (e.g., Section 4.3). The impact of ignoring risk aversion on the laws of motion of state variable only affects (1), and (2) depends on whether replacing \( \rho(\eta_t, t) \) with a constant \( \rho \) is an adequate approximation, as previously discussed.

This concludes the discussion on the consequences of model misspecification from ignoring risk aversion. Next, I derive the equations in Proposition B.1.

**Proof of Proposition B.1.** First, I solve the entrepreneurs’ problem and the bankers’ problem under the risk-neutral measure, taking as given the stochastic discount factor (SDF). After specifying the households’/consumers’ risk-averse utility function, I solve the SDF and perform the change of measure to obtain the physical-measure dynamics of the extended model. This method of solving models under the risk-neutral measure and then analyzing the physical-measure dynamics by applying Girsanov’s Theorem is often used in settings of complete markets (Duffie, 2001).

The entrepreneurs’ investment problem stays intact as it is a static problem happening only at idiosyncratic Poisson times. Therefore, the Lagrange function defined by (25) still summarizes the investment problem, and the marginal value of liquidity for the investment projects, \( \pi_t \), is given by (26). Because the time discount rate changes from \( \rho \) to \( \rho_t \), (28) in Proposition 2 is now

\[
 r_t = \rho_t - \lambda \pi_t .
\]  

(B.18)

The rest of Proposition 2 hold.

Given the homogeneity property of the bankers’ problem, their value function is still \( q_t^B \eta_t^B \), where the marginal value of equity, \( q_t^B = q^B(\eta_t, t) \), has the law of motion (29) under the risk-neutral measure. Proposition 3 can still be used to characterize the valuation of tangible capital and the bankers’ required expected return on tangible capital holdings under the risk-neutral measure. Note that if bankers can also access complete markets as households can, their marginal value of wealth, \( q_t^B \), will be pinned to one, and their price of risk, \( \gamma_t^B \), to zero. Being able to freely trade the aggregate shock with households is equivalent to being able to freely raise equity from households.
Bankers’ required expected return under the risk-neutral measure is (30) in Proposition 3. Under the physical measure, by Girsanov’s Theorem, it becomes

\[ \hat{E}_t [dR^T_t] = r_t + \gamma_B T (\sigma_t^T + \sigma) + \gamma_H T (\sigma_t^T + \sigma). \] (B.19)

Under the physical measure, banks require risk compensations not only due to the equity issuance constraint, \( \gamma_B T (\sigma_t^T + \sigma) \), but also on behalf of the household shareholders, \( \gamma_H T (\sigma_t^T + \sigma) \).

The valuation equation (31) for tangible capital in Proposition 3 still holds. The derivation in Appendix A applies under the risk-neutral measure. Equation (31) can also be derived under the physical measure but the law of motion of \( q_t^T \) and the stochastic depreciation of capital holdings have to be adjusted by the change of measure. Under the risk-neutral measure:

\[ \frac{dq_t^T}{q_t^T} = \mu_t^T dt + \sigma_t^T dZ_t, \] (B.20)

and, given (B.2), under the physical measure

\[ \frac{dq_t^T}{q_t^T} = (\mu_t^T + \gamma_H T \sigma_t^T) dt + \sigma_t^T d\hat{Z}_t, \] (B.21)

where the price of risk \( \gamma_H T \) is multiplied by the quantity of risk \( \sigma_t^T \). When moving from (B.21) to (B.20), the drift is adjusted downward, reflecting a risk adjustment via the shift of probability mass towards relatively worse states of the model. Risk aversion is reflected in the adjustment of the probability mass. The stochastic depreciation rate of capital under the physical measure is

\[ (\delta - \gamma_H T \sigma) + \sigma d\hat{Z}_t. \] (B.22)

The expected depreciation rate is adjusted upward when moving from the physical measure, \( \delta - \gamma_H T \sigma \)

---

Imperfect hedging can be easily incorporated. For example, bankers can only hedge a fraction \( \chi^B \) of aggregate risk due to agency friction and the need to keep “skin in the game” (He and Krishnamurthy, 2013). Note that given that hedging is free and bankers are effectively risk averse, bankers will hedge as much as they can. Then bankers’ risk exposure for one dollar of holdings of tangible capital is \( (1 - \chi^B) (\sigma_t^T + \sigma) \) in equilibrium, i.e., scaled down by \( \chi^B \) fraction. Bankers’ required expected return under the risk-neutral measure becomes \( r_t + \gamma_B (1 - \chi^B) (\sigma_t^T + \sigma) \). After the scaling, the same solution procedure still applies. Because the scaling reduces bankers’ discount rate and increase the value of tangible capital and entrepreneurs’ leverage on liquidity holdings, it amplifies the feedback mechanism.
\( \gamma_t^H \sigma \), to the risk-neutral measure, \( \delta \), as the probability mass is shifted towards relatively worse states of the world to reflect risk aversion encoded in the SDF. The expected return of tangible capital holdings consists of the dividend yield, \( 1/q_t^T \), the expected price appreciation, \( \mu_t^T + \gamma_t^H \sigma_t^T \), the expected capital depreciation, \( (\delta - \gamma_t^H \sigma) + \lambda \) (counting both the normal-time depreciation and idiosyncratic Poisson destruction), and the quadratic covariation \( \sigma_t^T \sigma \) from Itô’s calculus, which does not change due to the volatility-invariance property of change of measure Duffie (2001). In equilibrium, the expected return is equal to bankers’ required expected return in (B.19):

\[
\begin{align*}
    r_t + \gamma_t^B (\sigma_t^T + \sigma) + \gamma_t^H (\sigma_t^T + \sigma) &= \frac{1}{q_t^T} + (\mu_t^T + \gamma_t^H \sigma_t^T) - [(\delta - \gamma_t^H \sigma) + \lambda] + \sigma_t^T \sigma.
\end{align*}
\]

Note that \( \gamma_t^H (\sigma_t^T + \sigma) \) shows up on both sides. Rearranging the equation, we obtain (31).

For any stochastic process, its dynamics under the risk-neutral measure can be adjusted to obtain the dynamics under the physical measure. For instance, under the risk-neutral measure, the law of motion of \( q_t^B \) under the physical measure is given by

\[
\begin{align*}
    \frac{dq_t^B}{q_t^B} &= \mu_t^B + \sigma_t^B dZ_t, \tag{B.23}
\end{align*}
\]

so, (B.2) implies that the law of motion of \( q_t^B \) under the physical measure is given by

\[
\begin{align*}
    \frac{dq_t^B}{q_t^B} &= (\mu_t^B - \gamma_t^B \gamma_t^H) dt - \gamma_t^B d\widehat{Z}_t, \tag{B.24}
\end{align*}
\]

where the diffusion stays the same (i.e., the standard diffusion-invariance result) and the drift of \( q_t^B \) is “risk-adjusted”. Note that \( q_t^B \) is high in the relatively worse states of the world where banks are undercapitalized. The expected appreciation of \( q_t^B \) is adjusted upward when moving from (B.24) to (B.23) because, when changing from the physical measure to risk-neutral measure, more probability mass is shifted towards the relatively worse states of the world.

Given the function \( \rho(\eta_t, t) \), the procedure in A.2 can be used to solve all the variables listed in Proposition 1, and then, the laws of motion of \( \eta_t, K_t^T \), and \( K_t^I \) can be derived. These laws of motion are under the risk-neutral measure, so a change of measure needs to be performed to obtain the physical-measure laws of motion. As I have shown for \( q_t^T \) and \( q_t^B \), change of measure simply entails substituting out the Brownian shock under the risk-neutral measure, \( dZ_t \), using (B.2).

Using the procedure in A.2 to solve the model’s dynamics under the risk-neutral measure only requires the function \( \rho(\eta_t, t) \). It does not require the households’ utility function. To perform
the change of measure, I need to have the price of risk, \( \gamma_H^{-1} \), as a function of the state variables.

Next, I solve the SDF, linking \( \rho_t \) and \( \gamma_H^{-1} \) to households’ consumption (and wealth) dynamics. Specifically, I confirm that \( \rho_t \) only depends on \( \eta_t \) and time \( t \), i.e., \( \rho_t = \rho(\eta_t, t) \), and solve the functional form. I also solve the households’ price of risk, \( \gamma_H^{-1} \), as a function of these state variables.

In the following, I consider the standard time-separable power utility as an example. In this case, the stochastic discount factor is the time-discounted marginal utility of consumption (Cochrane, 2005b):

\[
\Lambda_t = e^{-\delta_H t} \left(c_t^H\right)^{-\gamma_H} .
\] (B.25)

In equilibrium, given that there is a unit mass of households, individual consumption is equal to the aggregate consumption, \( C_t^H \). Denote the equilibrium law of motion of aggregate consumption under the physical measure by

\[
\frac{dC_t^H}{C_t^H} = \mu^C_t \, dt + \sigma^C_t \, d\tilde{Z}_t .
\] (B.26)

By Itô’s lemma, the law of motion of the SDF, \( \Lambda_t \), is given by

\[
\frac{d\Lambda_t}{\Lambda_t} = -\left[\delta_H + \gamma_H \mu^C_t - \frac{1}{2} \gamma_H \left(\gamma_H + 1\right) \left(\sigma^C_t\right)^2\right] dt - \gamma_H \sigma^C_t \, d\tilde{Z}_t .
\] (B.27)

To solve \( \mu^C_t \) and \( \sigma^C_t \), consider the goods market-clearing condition:

\[
C_t^H \, dt + \frac{M_t^E}{1 - q_t^F \kappa^T (1 - \theta_t)} \lambda \, dt = (1 + \alpha) \, K_t^T \, dt .
\] (B.28)

The left side is the sum of households’ consumption and the goods invested by the \( \lambda \, dt \) measure of entrepreneurs who are hit by the Poisson shock. The right side is the goods produced by tangible capital and labor. For simplicity, the goods produced by intangibles are assumed to be consumed directly by the entrepreneurs, who run the firms, as compensation for their human capital (Hart and Moore, 1994; Bolton, Wang, and Yang, 2019). Adding intangibles’ output to (B.29) expands the dimension of state variables in \( \rho_t \) from two (i.e., \( \eta_t \) and \( t \)) to four, because both \( K_t^T \) and \( K_t^I \) show up in (B.29), and, given their distinct laws of motion, they have to be tracked separately. Dividing both sides of (B.29) by \( K_t^T \, dt \) and rearranging it, we have

\[
\tilde{C}_t^H = 1 + \alpha - \frac{\lambda M_t^E}{1 - q_t^F \kappa^T (1 - \theta_t)} ,
\] (B.29)
where, following the notations in the main text, I denote $K_t^T$-scaled values by “~”.

The procedure in A.2 solves the endogenous variables on the right side of (B.29) as functions of $\eta_t$ and time $t$. Therefore, $K_t^T$-scaled aggregate consumption,

$$\tilde{C}_t^H = \tilde{C}^H (\eta_t, t),$$  \hspace{1cm} (B.30)

is a known function of $\eta_t$ and $t$. so I can obtain $\epsilon \tilde{C}_1 (\eta_t, t)$ and $\epsilon \tilde{C}_2 (\eta_t, t)$. Note that under the risk-neutral measure, by Itô’s lemma, I obtain

$$\frac{dC_t^H}{C_t^H} = \frac{d\tilde{C}^H (\eta_t, t)}{\tilde{C}^H (\eta_t, t)} + \frac{dK_t^T}{K_t^T} + \epsilon \tilde{C} (\eta_t, t) \sigma^\eta (\eta_t, t) \sigma dt, \hspace{1cm} (B.31)$$

$$= \left\{ \epsilon \tilde{C}_1 (\eta_t, t) \left[ \mu^\eta (\eta_t, t) + \frac{1}{2} \epsilon \tilde{C}_2 (\eta_t, t) \sigma^\eta (\eta_t, t)^2 + \sigma^\eta (\eta_t, t) \sigma \right] + \mu^{KT} (\eta_t, t) \right\} dt$$

$$+ \left[ \epsilon \tilde{C}_1 (\eta_t, t) \sigma^\eta (\eta_t, t) + \sigma \right] dZ_t \hspace{1cm} (B.32)$$

where the risk-neutral measure dynamics, $\mu^\eta (\eta_t, t)$, $\sigma^\eta (\eta_t, t)$, and $\mu^{KT} (\eta_t, t)$ are given by (A.5), (A.6), and (A.7) respectively in A.1. To change the measure, using (B.2) to substitute $dZ_t$ with $d\hat{Z}_t + \gamma_t^H dt$, I obtain

$$\frac{dC_t^H}{C_t^H} = \left\{ \epsilon \tilde{C}_1 (\eta_t, t) \left[ \mu^\eta (\eta_t, t) + \frac{1}{2} \epsilon \tilde{C}_2 (\eta_t, t) \sigma^\eta (\eta_t, t)^2 + \sigma^\eta (\eta_t, t) \sigma \right] + \mu^{KT} (\eta_t, t) \right\} dt$$

$$+ \left[ \epsilon \tilde{C}_1 (\eta_t, t) \sigma^\eta (\eta_t, t) + \sigma \right] \gamma_t^H dt + \left[ \epsilon \tilde{C}_1 (\eta_t, t) \sigma^\eta (\eta_t, t) + \sigma \right] d\hat{Z}_t \hspace{1cm} (B.33)$$

According the law of motion of $\Lambda_t$ given by (B.27), the price of risk is

$$\gamma_t^H = \gamma_H \sigma_t^C = \gamma_H \left[ \epsilon \tilde{C}_1 (\eta_t, t) \sigma^\eta (\eta_t, t) + \sigma \right]. \hspace{1cm} (B.34)$$

I substitute out $\gamma_t^H$ in the drift term of (B.33) with the solution (B.34) and obtain

$$\mu_t^C = \mu^C (\eta_t, t)$$

$$= \epsilon \tilde{C}_1 (\eta_t, t) \left[ \mu^\eta (\eta_t, t) + \frac{1}{2} \epsilon \tilde{C}_2 (\eta_t, t) \sigma^\eta (\eta_t, t)^2 + \sigma^\eta (\eta_t, t) \sigma \right] + \mu^{KT} (\eta_t, t)$$

$$+ \gamma_H \left[ \epsilon \tilde{C}_1 (\eta_t, t) \sigma^\eta (\eta_t, t) + \sigma \right]^2 \hspace{1cm} (B.35)$$
Substituting the solutions of $\mu^C_t$ and $\sigma^C_t$ into the drift term of (B.27), I obtain

$$\rho_t = \rho(\eta_t, t)$$

(B.36)

$$= \delta_H + \gamma_H \mu^{KT}(\eta_t, t) + \gamma_H C_1(\eta_t, t) \left[ \mu^n(\eta_t, t) + \frac{1}{2} \epsilon C_2(\eta_t, t) \sigma^n(\eta_t, t)^2 + \sigma^n(\eta_t, t) \sigma \right]$$

$$+ \frac{1}{2} \left( \gamma_H^2 - \gamma_H \right) \left[ \epsilon C_1(\eta_t, t) \sigma^n(\eta_t, t) + \sigma \right]^2.$$
C Additional Tables and Figures

C.1 Summary Statistics

Table C.1: Summary Statistics for Nonfinancial Firm Cash and Leverage Regressions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Below Median Intan./Asset</th>
<th>Above Median Intan./Asset</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash/Assets (%)</td>
<td>12.385</td>
<td>24.514</td>
</tr>
<tr>
<td>Leverage (%)</td>
<td>29.388</td>
<td>18.056</td>
</tr>
<tr>
<td>Asset-backed Loans/Total Assets (%)</td>
<td>10.775</td>
<td>7.952</td>
</tr>
<tr>
<td>Cashflow-backed Loans/Total Assets (%)</td>
<td>20.282</td>
<td>12.634</td>
</tr>
<tr>
<td>Intangible Investment/Total Assets</td>
<td>0.043</td>
<td>0.239</td>
</tr>
<tr>
<td>PPE/Total Assets</td>
<td>0.364</td>
<td>0.194</td>
</tr>
<tr>
<td>Acquisitions/Total Assets</td>
<td>0.027</td>
<td>0.015</td>
</tr>
<tr>
<td>Capex/Total Assets</td>
<td>0.077</td>
<td>0.054</td>
</tr>
<tr>
<td>Cashflow/Total Assets</td>
<td>0.049</td>
<td>-0.055</td>
</tr>
<tr>
<td>Dividend Dummy</td>
<td>0.397</td>
<td>0.227</td>
</tr>
<tr>
<td>EBITDA/Total Assets</td>
<td>0.103</td>
<td>-0.034</td>
</tr>
<tr>
<td>Industry Sigma</td>
<td>0.075</td>
<td>0.092</td>
</tr>
<tr>
<td>Inventory/Total Assets</td>
<td>0.120</td>
<td>0.177</td>
</tr>
<tr>
<td>Net Cash Receipts/Total Assets</td>
<td>0.091</td>
<td>0.186</td>
</tr>
<tr>
<td>Net Working Capital/Total Assets</td>
<td>0.071</td>
<td>0.107</td>
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<tr>
<td>Real Size</td>
<td>5.754</td>
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<tr>
<td>Tobin’s Q</td>
<td>1.451</td>
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Table C.2: Summary Statistics for Household Time Series Regression

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<th>p40</th>
<th>p60</th>
<th>p80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Holdings/GDP</td>
<td>0.505</td>
<td>0.066</td>
<td>0.436</td>
<td>0.496</td>
<td>0.542</td>
<td>0.571</td>
</tr>
<tr>
<td>Average Tobin’s Q</td>
<td>1.823</td>
<td>0.342</td>
<td>1.527</td>
<td>1.699</td>
<td>1.905</td>
<td>2.085</td>
</tr>
<tr>
<td>Tangible Tobin’s Q</td>
<td>1.669</td>
<td>0.201</td>
<td>1.506</td>
<td>1.626</td>
<td>1.747</td>
<td>1.871</td>
</tr>
<tr>
<td>Price/Rent Ratio</td>
<td>1.270</td>
<td>0.129</td>
<td>1.177</td>
<td>1.201</td>
<td>1.249</td>
<td>1.335</td>
</tr>
<tr>
<td>Observations</td>
<td>160</td>
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Table C.3: Summary Statistics for Household Panel Data Regression

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<th>Variable</th>
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<th>p40</th>
<th>p60</th>
<th>p80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash/Income</td>
<td>0.205</td>
<td>0.584</td>
<td>0</td>
<td>0.011</td>
<td>0.052</td>
<td>0.180</td>
</tr>
<tr>
<td>Δ ln (Housing Price Index)</td>
<td>0.064</td>
<td>0.128</td>
<td>-0.038</td>
<td>0.058</td>
<td>0.098</td>
<td>0.139</td>
</tr>
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<td>Age</td>
<td>45.296</td>
<td>16.390</td>
<td>30</td>
<td>38</td>
<td>48</td>
<td>59</td>
</tr>
<tr>
<td>Couple Status</td>
<td>0.693</td>
<td>0.791</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Education Level</td>
<td>13.124</td>
<td>2.657</td>
<td>12</td>
<td>12</td>
<td>14</td>
<td>16</td>
</tr>
<tr>
<td>Home ownership Status</td>
<td>0.551</td>
<td>0.497</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Household Size</td>
<td>2.641</td>
<td>1.483</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<tr>
<td>Δ ln (Household Income)</td>
<td>0.036</td>
<td>1.390</td>
<td>-1.073</td>
<td>-0.286</td>
<td>0.364</td>
<td>1.144</td>
</tr>
<tr>
<td>Δ ln (Wealth excluding Home Equity)</td>
<td>-0.034</td>
<td>6.808</td>
<td>-4.879</td>
<td>-0.870</td>
<td>0.862</td>
<td>4.623</td>
</tr>
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<td>Observations</td>
<td>76,834</td>
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<td></td>
</tr>
</tbody>
</table>
### C.2 Corporate Liquidity Demand

Table C.4: Asset Tangibility, Capital Valuation, and Corporate Cash Holdings

#### Panel A: EV/EBITDA & Intangible-Driven Corporate Cash Holdings

<table>
<thead>
<tr>
<th>Cash Assets</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPE/Assets</td>
<td>-0.495</td>
<td>-0.225</td>
<td>-1.258***</td>
<td>-1.350***</td>
<td>-0.829*</td>
<td>-0.387</td>
<td>-1.416***</td>
<td>-1.491***</td>
</tr>
<tr>
<td>Ave. EV/EBITDA</td>
<td>1.849***</td>
<td>0.139</td>
<td>(0.274)</td>
<td>(0.164)</td>
<td>(0.243)</td>
<td>(0.413)</td>
<td>(0.397)</td>
<td>(0.260)</td>
</tr>
<tr>
<td>PPE/Assets × Ave. EV/EBITDA</td>
<td>-0.259***</td>
<td>-0.280***</td>
<td>-0.069***</td>
<td>-0.058**</td>
<td>(0.034)</td>
<td>(0.032)</td>
<td>(0.024)</td>
<td>(0.022)</td>
</tr>
<tr>
<td>Tan. EV/EBITDA</td>
<td>1.842***</td>
<td>-0.047</td>
<td>(0.332)</td>
<td>(0.172)</td>
<td>(0.243)</td>
<td>(0.413)</td>
<td>(0.397)</td>
<td>(0.260)</td>
</tr>
<tr>
<td>PPE/Assets × Tan. EV/EBITDA</td>
<td>-0.269***</td>
<td>-0.309***</td>
<td>-0.062**</td>
<td>-0.052*</td>
<td>(0.040)</td>
<td>(0.041)</td>
<td>(0.028)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Controls</td>
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<td>No</td>
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<td>Yes</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Year FE</td>
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<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Observations</td>
<td>152,524</td>
<td>152,524</td>
<td>130,204</td>
<td>130,204</td>
<td>152,524</td>
<td>152,524</td>
<td>130,204</td>
<td>130,204</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.1797</td>
<td>0.1861</td>
<td>0.4458</td>
<td>0.4488</td>
<td>0.1748</td>
<td>0.1840</td>
<td>0.4461</td>
<td>0.4487</td>
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</table>

Firm-year clustered standard errors in parentheses
* $p < 0.1$  ** $p < 0.05$  *** $p < 0.01$

#### Panel B: Tobin’s Q & Intangible-Driven Corporate Cash Holdings

<table>
<thead>
<tr>
<th>Cash Assets</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPE/Assets</td>
<td>-0.527</td>
<td>0.023</td>
<td>-1.254***</td>
<td>-1.434***</td>
<td>1.016</td>
<td>1.381*</td>
<td>-1.082**</td>
<td>-1.364***</td>
</tr>
<tr>
<td>Ave. Tobin’s Q</td>
<td>9.934***</td>
<td>0.176</td>
<td>(2.598)</td>
<td>(1.045)</td>
<td>(0.324)</td>
<td>(0.829)</td>
<td>(0.732)</td>
<td>(0.432)</td>
</tr>
<tr>
<td>PPE/Assets × Ave. Tobin’s Q</td>
<td>-1.487***</td>
<td>-1.732***</td>
<td>-0.396**</td>
<td>-0.286*</td>
<td>(0.328)</td>
<td>(0.333)</td>
<td>(0.185)</td>
<td>(0.169)</td>
</tr>
<tr>
<td>Tan. Tobin’s Q</td>
<td>20.708***</td>
<td>0.466</td>
<td>(4.530)</td>
<td>(1.954)</td>
<td>(0.324)</td>
<td>(0.829)</td>
<td>(0.732)</td>
<td>(0.432)</td>
</tr>
<tr>
<td>PPE/Assets × Tan. Tobin’s Q</td>
<td>-3.003***</td>
<td>-3.200***</td>
<td>-0.632**</td>
<td>-0.420</td>
<td>(0.572)</td>
<td>(0.511)</td>
<td>(0.297)</td>
<td>(0.286)</td>
</tr>
<tr>
<td>Controls</td>
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<td>Yes</td>
<td>No</td>
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<td>Yes</td>
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<tr>
<td>Year FE</td>
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<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
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</tr>
<tr>
<td>Observations</td>
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<td>152,524</td>
<td>130,204</td>
<td>130,204</td>
<td>152,524</td>
<td>152,524</td>
<td>130,204</td>
<td>130,204</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.1737</td>
<td>0.1826</td>
<td>0.4454</td>
<td>0.4486</td>
<td>0.1737</td>
<td>0.1826</td>
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### Table C.5: Intangible Investment, Tobin’s Q, and Corporate Cash Holdings

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<tr>
<th>Cash Assets</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intan./Assets</td>
<td>-1.405</td>
<td>-1.737</td>
<td>-0.566</td>
<td>-0.508</td>
<td>-4.425***</td>
<td>-4.700***</td>
<td>-1.798**</td>
<td>-1.698**</td>
</tr>
<tr>
<td>Ave. Tobin’s Q</td>
<td>-6.492***</td>
<td>(1.333)</td>
<td>-5.924***</td>
<td>(1.466)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intan./Assets × Ave. Tobin’s Q</td>
<td>2.572***</td>
<td>(0.640)</td>
<td>2.742***</td>
<td>(0.651)</td>
<td>1.070***</td>
<td>(0.351)</td>
<td>1.032***</td>
<td>(0.351)</td>
</tr>
<tr>
<td>Intan./Assets × Tan. Tobin’s Q</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.459***</td>
<td>(0.980)</td>
<td>5.643***</td>
<td>(0.946)</td>
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<td>Yes</td>
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<tr>
<td>Observations</td>
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<td>152,549</td>
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<tr>
<td>Adjusted $R^2$</td>
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<td>0.1988</td>
<td>0.2162</td>
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Firm-year clustered standard errors in parentheses
* $p < 0.1$  ** $p < 0.05$  *** $p < 0.01$

---

**Figure C.1:** Intangible Intensity, Tangible Capital Valuation, and Corporate Cash Holdings.

---

91
Figure C.2: **Intangible Intensity, Tobin’s Q, and Corporate Cash Holdings.**
C.3 Household Liquidity Demand

Figure C.3: Decomposing Households’ Holdings of Liquid Securities.

Figure C.4: Households’ Holdings of Intermediary Debts.
C.4 Additional Figures from Calibration

Figure C.5: PPE-Scaled Corporate Investments.