

Firm Quality Dynamics and the Slippery Slope of Credit Intervention *

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Abstract

A salient trend in crisis intervention has emerged in recent decades: Government and central banks offered funding directly to nonfinancial firms, bypassing banks and other credit intermediaries. We analyze the long-term consequences of such policies by focusing on firm quality dynamics. In a laissez-faire economy, firms with high productivity are more likely to survive crises than those with low productivity. The government provides funding support to save more firms but cannot customize it based on firm productivity, dampening the cleansing effect of crises. The policy distortion is self-perpetuating: A downward bias in firm quality distribution necessitates interventions of greater scale in future crises. Our mechanism is quantitatively important: We show that if policy makers ignore the distortionary effects on firm quality dynamics, the resultant credit intervention would almost double the optimal amount.

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1 Introduction

Since the Global Financial Crisis (GFC), credit intervention has grown in size and become more direct. The programs during the GFC injected liquidity through the banking sector, with only a few exceptions of direct funding for nonfinancial firms, such as the bond purchase programs at the Bank of Japan and European Central Bank. During the Covid-19 crisis, not only the GFC-era programs were promptly reinstated, but central banks and governments across the world initiated new programs that directly provided liquidity support to nonfinancial firms, such as the Primary and Secondary Market Corporate Credit Facilities (PMCCF and SMCCF), Main Street Lending Program (MSLP), and Paycheck Protection Program (PPP) in the U.S.

This observation underscores a trend where credit intervention has been ambitiously designed under the mantra of doing “whatever it takes” to salvage the economy. In light of this trend, it’s important to consider: What could be the long-term effects of credit intervention, and in particular, what causes credit intervention to grow in scale from one crisis to the next?

We answer this question from the perspective of firm quality dynamics. In our model, firms accumulate capital and produce, but they differ in productivity. High-quality firms have larger financing capacity because, relative to low-quality firms, they are less likely to hold up creditors through strategic default and prefer continuing operations. Therefore, high-quality firms have better access to liquidity and thus are more likely to survive in crises.¹ Crises exhibit a cleansing effect: The economy emerges from crises with a higher fraction of firms being high-quality.

Funding support from the government helps firms survive crises but dampens the cleansing effect because, unlike private-sector creditors, the government is unable to differentiate firms by their productivity. Therefore, credit intervention alleviates the decline of aggregate output but reduces the average firm productivity. As a result, a slippery slope of intervention emerges: The economy enters into the next crisis with lower productivity and thus requires a greater scale of intervention to sustain the output. In our calibrated model, increasing the scale of intervention in the current crisis by one dollar per unit of firm capital leads to an increase of 4 cents per unit of capital in the next intervention should another crisis happen in ten years. Given that capital stock grows over time, the inter-crisis pass-through rate in the total dollar amount is even larger.

A key feature of our model is the trade-off between quantity and quality. Optimal intervention

¹A larger financing capacity leads to a higher survival probability but does not imply that high-quality firms are less financially constrained, because the tightness of financial constraint is measured by the gap between funding and the targeted level of investment. High-quality firms may also have a higher investment target as we shall explain later.

in crises strikes a balance between distorting firm quality dynamics and preserving overall production capacity. It is important for policymakers to recognize our mechanism when deciding on the intervention scale. We show that ignoring the distortionary effects on firm quality dynamics results in a more aggressive intervention that almost doubles the welfare-maximizing amount.

In the following, we summarize the key modeling ingredients and provide more details on our main results. We follow the continuous-time formulation of the multi-sector models (Eberly and Wang, 2008) but simplify households' preferences to be risk-neutral. There are two types of firms that produce generic goods for consumption and investment using their productive capital with constant return-to-scale technology. Type-*H* firms have a higher productivity than type-*L* firms. In normal times, firms are not financially constrained and invest to grow their capital. The forward-looking valuation of capital (i.e., Tobin's q) is the present value of production flows and drives the optimal investment (Hayashi, 1982; Abel and Eberly, 1994; Brunnermeier and Sannikov, 2014).

The arrival of a crisis follows a Poisson process (Gourio, 2012; Wachter, 2013). We model a liquidity crisis in the spirit of Holmström and Tirole (1998). Firms differ in their crisis exposure, modeled as different baseline levels of survival probability, and firms can raise debt financing to increase their survival probability. Type-*H* firms have a higher targeted level of investment in survival as their capital is more valuable than that of type-*L* firms. Here, the first form of cleansing effect emerges: Type-*H* firms want to invest more, which may translate into a higher survival probability. Our emphasis shall be on the second form of cleansing effect: Type-*H* firms can borrow more than type-*L* firms.² It is the interference with this force that causes government intervention to distort firm quality dynamics and exhibit intertemporal dependence as a result.

Firms face a debt limit imposed by private-sector creditors because they may hold up creditors through strategic default (e.g., Bolton and Scharfstein, 1990; Hart and Moore, 1998). In strategic default, firm owners lose capital to the creditors but can extract value from the creditors through renegotiation.³ Therefore, type-*L* firms have a stronger incentive to strategically default than type-*H* firms because, due to lower productivity, their capital is less valuable. Consequently, type-*L* firms face a tighter debt limit than type-*H* firms, i.e., the second form of cleansing effect.

²The cleansing effects of crises in our model are distinct from those via creative destruction or cyclical reallocation (e.g., Eisfeldt and Rampini, 2006; Kogan et al., 2017; Acemoglu et al., 2018) and the efficiency gain is from creditors' disciplining firms against over-investment rather than weeding out unproductive firms that crowd out productive peers in product or input markets (e.g., Caballero and Hammour, 1994; Acharya et al., 2021).

³Firm owners' bargaining power can be understood as from moral hazard: Firm owners may threaten to divert resources, engage in wasteful spending, or withhold human capital (Aghion and Bolton, 1992; Hart and Moore, 1994).

This cleansing effect arises from the link between firm type and financing capacity. This link is quite general and can be modeled differently, for example, through a collateral constraint, in which case type-*H* firms pledge their more valuable capital as collateral and can raise more funds.⁴ Our modeling choice is motivated by the empirical literature on credit intervention.⁵

The government extends credit to firms in order to help more firms survive the crisis, effectively acting as a financial intermediary (Lucas, 2016). It finances lending with taxes on households and transfers the repayments in lump sums, in line with the models of credit policy and unconventional monetary policy (e.g., Gertler and Karadi, 2011a; Gertler et al., 2012; Araújo et al., 2015; Del Negro et al., 2017). When setting interest rates, the government follows private-sector creditors as in practice.⁶ However, when setting the size of credit support, the government imposes a uniform limit to firms of both types: A firm can borrow up to a multiple of its capital stock. This reflects the policy design in reality: Credit limit is set proportional to accounting measures of operation scale, which map to the capital units in our model, such as the programs during the Covid-19 crisis.⁷

Firms can choose any amount of borrowing from the credit facilities within the limit set by the government. The impact of credit intervention cannot be solely judged by the take-up. In our model, credit intervention enlarges firms' financing capacity both directly through the liquidity available and indirectly by crowding in private-sector funding. By improving the survival probability, credit intervention also affects the pricing of credit by private-sector creditors. Any change in the scale of credit intervention results in a new equilibrium of financing capacity and costs.

Credit intervention benefits type-*L* firms more than type-*H* firms, dampening the cleansing effect of crises. Intuitively, since type-*L* firms obtain less funds from private-sector creditors, the marginal impact of more funds from the government is greater for type-*L* firms. While private-sector investors properly account for type-*H* and *L* firms' difference in incentive to hold up creditors, the government does not, and therefore, its funding support generates distortionary effects.

⁴Debt capacity depends on collateral value under limited commitment (Kehoe and Levine, 1993; Kiyotaki and Moore, 1997; Geanakoplos, 2010; Rampini and Viswanathan, 2010; Li, Whited, and Wu, 2016).

⁵The recent evidence documented in the literature suggests a significant conflict of interests between borrowers and lenders in crises (Hanson et al., 2020; Lynch, 2021; Griffin et al., 2023).

⁶For publicly traded bonds, the government can rely on market prices, e.g., PMCCF and SMCCF in the U.S., and if a firm's debt is not publicly traded, the government can lend alongside informed banks and rely on banks to screen out and exclude firms that are riskier for a given level of interest rate, e.g., MSLP during the Covid-19 pandemic.

⁷MSLP sets a limit to six times the borrower's EBITDA, a measure of operating income rather than profitability per unit of resources deployed. The limit in PPP is tied to payroll rather than labor productivity. PMCCF and SMCCF impose limits tied to an issuer's existing debts (liability size rather than productivity). In addition to the uniform credit limit, if the government forgoes differentiation on interest rates, our mechanism is amplified (Appendix B.3).

There has been enormous attention on the lack of firm differentiation in credit facilities (English and Liang, 2020). There are several explanations. The government lacks information on firm productivity.⁸ Political considerations may go against treating firms differently in crises. And, customizing credit support for individual firms is infeasible when speedy implementation is required.

In summary, the productivity difference between type-*H* and *L* firms translates into a difference in private-sector financing capacity. A cleansing effect of crises emerges from the fact that type-*H* firms have a greater financing capacity and thus a higher survival probability. Credit intervention dampens such cleansing effect. Since both types of firms can obtain funding from the government, our analysis focuses on policy distortions among firms that can access the facility but have different levels of productivity. In practice, credit facilities may exclude certain firms, but the criterion is rarely productivity-based and thus unlikely to help avoid distortions on firm quality distribution.⁹

In our model, the government faces a trade-off between quantity and quality in line with the empirical evidence.¹⁰ Consider increasing credit support from zero to 30% of GDP.¹¹ In our calibrated model, the policy saves 8% of production capacity (i.e., capital units) but reduces the quality improvement (i.e., the increase in the fraction of firms being type-*H*) from about 9% to 4%. Optimal intervention strikes a balance between allowing more firms to survive and distorting the quality distribution. Reducing funding support strengthens the cleansing effect, so the economy has higher productivity post-crisis but has to climb out of a deeper decline in total output. If the government supplies more funding, the output drop is contained at the expense of lowering productivity.

Accordingly, the welfare is a bivariate function of the state variables, the total number of firms and fraction of firms being type-*H*. When solving the constrained-efficient scale of intervention, we first consider a policy maker that ignores the distortionary impact on quality dynamics and is only aware of the positive impact on capital quantity. Here, the trade-off is standard—investing

⁸Firms differ in both type (productivity) and crisis exposure (baseline survival probability), so the government cannot infer the type from credit pricing in the market and thus cannot properly account for firms' incentive to hold up creditors. Credit risk and productivity are correlated but not perfectly aligned in our model and data (Appendix C).

⁹For example, PMCCF and SMCCF introduced in the U.S. during the Covid-19 pandemic excluded firms with high credit risk. In Appendix C, we show that credit risk and productivity are far from being perfectly correlated.

¹⁰Intervention was effective in preserving production capacity during the Covid-19 crisis (Bartik et al., 2020; Bartlett and Morse, 2020; Hubbard and Strain, 2020; Denes et al., 2021; Kawaguchi et al., 2021). On the cleansing effect of the Covid-19 crisis, Muzi et al. (2023) find less productive firms were more likely to cease operations, and Bruhn et al. (2023) find economic activity was reallocated toward more productive firms beyond what is implied by cyclical variation. These two papers document that intervention dampens the cleansing effect. Moreover, Dörr et al. (2022) find that credit support disproportionately benefited firms that were financially vulnerable pre-Covid 19.

¹¹This is close in magnitude to the size of credit support in the U.S. during the Covid-19 crisis, including various credit programs such as PPP and MSLP. See Section 3 for more discussions on the magnitude of crisis interventions.

goods in firm survival vs. consuming goods—and the resultant intervention is almost twice the size of optimal intervention that properly accounts for the impact on both state variables, quantity and quality. Therefore, ignoring the impact on quality dynamics leads to excessive intervention. This exercise shows that recognizing the mechanism in this paper is quantitatively important.

Another perspective on welfare analysis is under- vs. over-investment under credit intervention. For both type-*H* and *L* firms, credit intervention enlarges the financing capacity of firms that under-invest due to a large crisis exposure and induces over-investment among firms that are relatively less liquidity-constrained and over-borrow. Over-investing happens because the option to hold up creditors through strategic default is in-the-money only if the firm survives. Therefore, under a sufficiently large scale of intervention, over-investment is guaranteed to happen, and it can happen among both type-*H* and *L* firms, though more prominent among type-*L* firms.

The distortionary effects of credit intervention on firm quality distribution persist over time. In our calibrated model. Increasing credit support from zero to 30% of GDP reduces the fraction of firms being type-*H* by 2 percentage points even ten years after the crisis. When a subsequent crisis arrives, the economy requires an even greater scale of intervention because, a lower average firm productivity translates into a smaller private-sector financing capacity in aggregate. Therefore, a slippery slope of intervention emerges in our model: Credit intervention in the current crisis begets interventions of greater scales in future crises. This slippery slope is robust even when we shut down agents' expectations of ever-growing intervention. The main takeaway from our paper is that credit intervention faces a quantity-quality trade-off, and any one-time deviation towards more aggressive intervention generates a ripple effect that permeates indefinitely into the future.

Literature. The role of governments, especially, the central banks, as lenders of last resort constantly evolves throughout the history in response to crises, political struggles, and technological innovations (Goodhart, 1998; Calomiris, Flandreau, and Laeven, 2016). Direct lending to nonfinancial firms is a meaningful addition to the policy toolbox. During a credit market freeze (Stiglitz and Weiss, 1981; De Meza and Webb, 1987), the government can step in, effectively functioning as a financial intermediary (Bebchuk and Goldstein, 2011; Lucas, 2016).¹² The Covid-19 crisis normalized the use of direct liquidity support to nonfinancial firms and will have a long-lasting effect on firms' expectations and their investment and financing decisions (Elenev et al., 2020).

The models of unconventional monetary policy assume an exogenous dead-weight loss of di-

¹²Bassetto and Cui (2020) analyze tax/subsidy as an alternative to credit policy in addressing financial frictions.

rect lending (Gertler and Kiyotaki, 2010; Cúrdia and Woodford, 2011; Gertler and Karadi, 2011a; Gertler, Kiyotaki, and Queralto, 2012; Araújo, Schommer, and Woodford, 2015; Del Negro, Eggertsson, Ferrero, and Kiyotaki, 2017). We unpack the black box of costs of government lending to nonfinancial firms, or asset purchases in general, and emphasize the endogenous evolution of firm quality distribution and a novel dynamic mechanism that leads to a slippery slope of intervention.¹³

Broadly, our paper contributes to the literature on the costs of crisis intervention, such as risk cost (Lucas, 2012), tax distortions as a form of financing costs (Hanson, Scharfstein, and Sunderam, 2018), feedback loop between sovereign and private-sector risk (Acharya et al., 2014; Brunnermeier et al., 2016), distortions on bank capital allocation (Antill and Clayton, 2021), and debt overhang and bankruptcy costs (Balloch et al., 2020; Brunnermeier and Krishnamurthy, 2020; Crouzet and Tourre, 2020; Greenwood et al., 2020; Wang et al., 2020).¹⁴

In our model, crises exhibit cleansing effects that emerge from type-*H* and *L* firms' differences in debt capacity, which is, in turn, due to creditors imposing discipline against over-investment. By dampening the cleansing effect, government intervention weakens such discipline. The efficiency gain from the crisis cleansing effect in our model is different from that in the existing models of cleansing effects that emphasize unproductive firms crowding out productive firms in output (product) or input (factor) markets and weeding out unproductive firms improves efficiency (e.g., Caballero and Hammour, 1994). The mechanism in our model is solely built on the financial aspects of crises.¹⁵ Incorporating type-*L* firms crowding out type-*H* firms in product or factor markets amplifies the inefficiency of credit intervention, dampening the cleansing effect of crises.

In our model, firms that over-invest can be viewed as engaging in a negative NPV transaction, but these firms are not zombies. In crises, a firm has a temporary liquidity need; once the firm survives the crisis, it becomes a normal firm that produces and invests optimally at the level dictated by its Tobin's *q*. In contrast, zombies are firms that are permanently impaired and continue operating only by relying on external financial resources (e.g., Caballero, Hoshi, and Kashyap, 2008;

¹³Beyond our emphasis of endogenous quality, our paper focuses on the intensive margin of policy intervention—among firms that receive credit support, the costs of capital of firms with different productivities are homogenized—while other studies emphasize the distortions from the extensive margin, i.e., a subset of firms receive a disproportionately large amount of credit support (Kurtzman and Zeke, 2020; Papoutsis, Piazzesi, and Schneider, 2021).

¹⁴The recent contributions on the benefits of credit-market intervention focus on the positive externalities that cannot be internalized by private lenders (e.g., Bebchuk and Goldstein, 2011; Philippon and Schnabl, 2013; Liu, 2016; Giannetti and Saidi, 2019; Hanson, Stein, Sunderman, and Zwick, 2020).

¹⁵The cleansing effects of crises in our model are also distinct from those via creative destruction or cyclical reallocation (e.g., Eisfeldt and Rampini, 2006; Kogan et al., 2017; Acemoglu et al., 2018).

Acharya, Eisert, Eufinger, and Hirsch, 2019; Acharya, Lenzu, and Wang, 2021). In spite of the temporary nature of firms' liquidity problem in our model, the policy distortions persist over time.

Firm quality distribution evolves endogenously in our model under a fixed information structure. Our paper focuses on the interaction between policy intervention and firm quality and differs from studies on endogenous asset quality that emphasize an evolving information structure and agents' incentive to improve asset quality, motivated by the GFC (Eisfeldt, 2004; Chari, Shourideh, and Zetlin-Jones, 2014; Chemla and Hennessy, 2014; Kurlat, 2013; Gorton and Ordoñez, 2014; Bigio, 2015; Zryumov, 2015; Bolton, Santos, and Scheinkman, 2016; Moreira and Savov, 2017; Caramp, 2017; Hu, 2017; Vanasco, 2017; Fukui, 2018; Neuhann, 2018; Asriyan, Fuchs, and Green, 2019; Daley, Green, and Vanasco, 2020; Lee and Neuhann, 2021; Farboodi and Kondor, 2021).¹⁶

Our model features resource misallocation. Unlike the literature that studies factor allocation (Ramey and Shapiro, 1998; Eisfeldt and Rampini, 2006, 2008; Jovanovic and Rousseau, 2008), our focus is on liquidity allocation between firms and households and between high- and low-quality firms. Intervention causes misallocation among firms but improves efficiency by channeling liquidity from households to firms. We analyze the trade-off, contributing to the literature on financial frictions and misallocation (Banerjee and Moll, 2010; Gilchrist, Sim, and Zakrajšek, 2013; Midrigan and Xu, 2014; Moll, 2014; Fuchs, Green, and Papanikolaou, 2016; Dou, Ji, Tian, and Wang, 2020; David and Zeke, 2021). As is David, Schmid, and Zeke (2018), there is a connection between firm productivity and risk exposure (type-*H* firms are less affected by crises). By changing firms' crisis exposure, credit intervention distorts the firm quality dynamics.

2 The Model

2.1 Preferences and Technology

Consider a continuous-time economy with a unit measure of representative agents ("households") and a government. Households have risk-neutral utility with time discount rate r :

$$\mathbb{E} \left[\int_{t=0}^{\infty} e^{-rt} dc_t \right], \quad (1)$$

¹⁶Policy makers may actively alter the information structure, which in turn affects the optimal intervention (Goldstein and Sapra, 2014; Bouvard, Chaigneau, and Motta, 2015; Shapiro and Skeie, 2015; Williams, 2015; Faria-e-Castro, Martinez, and Philippon, 2016; Goldstein and Leitner, 2018).

where c_t is the cumulative consumption. Households can own and trade equity and debt of firms. Firms maximize shareholders' value by managing capital to produce non-durable numeraire goods.

There are two types of firms. Type- H firms' capital produces A^H units of goods per unit of time. The productivity of type- L capital is A^L ($A^H > A^L$). Capital depreciates at rate, δ . We use superscripts for type and subscripts for time. Given the aggregate capital stocks, K_t^H and K_t^L , the total output of numeraire goods over dt is

$$Y_t \equiv A^H K_t^H + A^L K_t^L = (A^H \omega_t + A^L (1 - \omega_t)) K_t, \quad (2)$$

where ω_t , the fraction of type- H capital, represents the firm quality distribution

$$\omega_t \equiv \frac{K_t^H}{K_t^H + K_t^L}, \quad (3)$$

and K_t is the total units of capital

$$K_t \equiv K_t^H + K_t^L. \quad (4)$$

The output in the economy depends on both the capital quality, ω_t , and capital quantity, K_t .

Firms can grow capital through investment. Let q_t^j , $j \in \{H, L\}$, denote the endogenous value of capital that will be solved in equilibrium. It plays an important role in our analysis, as it incorporates the expectation of future growth path and disruptions in crises. Given the time- t value of capital, q_t^j , a type- j firm chooses the investment rate (or capital growth rate), ι_t^j :

$$\max_{\iota_t^j} q_t^j \iota_t^j k_t - \Phi(\iota_t^j, k_t), \quad (5)$$

where we adopt the cost function from the literature on Q-theory of investment (Hayashi, 1982):

$$\Phi(\iota_t^j, k_t) = \underbrace{\left(\iota_t^j + \frac{\theta}{2} \iota_t^{j2} \right)}_{\phi(\iota_t^j)} k_t \quad (6)$$

Under this functional form, we obtain the classic Q-theory formula of optimal investment:

$$\iota_t^j = \frac{q_t^j - 1}{\theta}. \quad (7)$$

The arrival of crises follows a Poisson process, N_t , with intensity λ (e.g., Gourio, 2012; Wachter, 2013). We model a liquidity crisis in the spirit of Holmström and Tirole (1998), where firms need to inject additional resources to preserve their production capacity. In a crisis (i.e., $dN_t = 1$), firms raise money for survival. Let $x_t \geq 0$ denote the amount of financing per unit of capital a firm obtains and spends on surviving the crisis. The firm then faces a survival probability $F(x_t + \zeta)$, where ζ is drawn from a common cumulative distribution function $H(\zeta)$ independently across firms and is realized before the firm makes its financing decision x_t . We assume that $F(\cdot)$ is strictly increasing and concave with $F(0) = 0$, so a higher ζ means a higher chance of survival (since $F'(\cdot) > 0$) and a lower marginal value of liquidity (since $F''(\cdot) < 0$). The random variable ζ captures different factors that affect the severity of a firm's liquidity crisis. For example, a low ζ reflects a severe mismatch between cash inflows and outflows, such as customers delaying payment and suppliers suspending trade credit. A firm's decision to raise external funds depends on its type $j \in \{H, L\}$ (through the capital value q_t^j) and the severity of its liquidity crisis given by ζ , so we denote the optimal amount of financing per unit of capital as $x_t^j(\zeta)$, $j \in \{H, L\}$.

Incorporating both normal-time growth through investment and crisis-time exit with probability $1 - F(x_t^j(\zeta) + \zeta)$, aggregate type- j capital stock has the following law of motion:

$$dK_t^H = K_t^H(\iota_t^H - \delta)dt - K_t^H \left[\int_{\zeta} (1 - F(x_t^H(\zeta) + \zeta)) dH(\zeta) \right] dN_t, \quad (8)$$

$$dK_t^L = K_t^L(\iota_t^L - \delta)dt - K_t^L \left[\int_{\zeta} (1 - F(x_t^L(\zeta) + \zeta)) dH(\zeta) \right] dN_t + \eta K_t dt. \quad (9)$$

We will show that in equilibrium, $q_t^H > q_t^L$, so type- H firms grow faster in normal times ($\iota_t^H > \iota_t^L$). Moreover, we will show that type- H firms have higher survival rates in crises. To maintain the stationarity of quality distribution, we introduce exogenous birth $\eta K_t dt$ for L-type firms.

2.2 Financial Frictions and Government Intervention in Crises

We model debt financing in crises as our focus is on credit intervention. The creditors charge an interest rate $r_t^j(\zeta, x)$ on a type- j firm with realized ζ and debt level x per unit of capital. The firm chooses the amount of debt financing to maximize the expected value:

$$x_t^j(\zeta) = \arg \max_{0 \leq x \leq d+\bar{g}} F(x + \zeta) [q_t^j - (1 + r_t^j(\zeta, x))x]. \quad (10)$$

Given a financing amount x , the firm survives the crisis with probability $F(x + \zeta)$. After paying the principal and interest to creditors, the firm owners' value is $q_t^j - (1 + r_t^j(\zeta, x))x$. The private-sector credit supply is constrained at \bar{d} . Government intervention, \bar{g} , relaxes the constraint. Note that what distinguishes crises and normal times is the limited private-sector credit in crises. In normal times, firms' investment is unconstrained, given by (7). Therefore, the tightening of credit conditions in crises reminisces the concept a financial shock (Jermann and Quadrini, 2012). Credit freeze can be attributed to lenders' balance-sheet impairment, elevated uncertainty, or informational frictions.¹⁷

The interest rate, $r_t^j(\zeta, x)$, is set by the creditors' break-even condition:

$$F(x + \zeta)(1 + r_t^j(\zeta, x))x = x. \quad (11)$$

When lending to firms, the government also charges this interest rate, taking advantage of the market signals. There are different types of policy design that allow the government to rely on market signals. Examples include PMCCF and SMCCF in the U.S. during the Covid-19 pandemic.¹⁸

Using (11), we substitute out $r_t^j(\zeta, x)$ in the firm value in (10) and obtain a simplified objective function $F(x + \zeta)q_t^j - x$. Without considering the funding constraint, $x \leq \bar{d} + \bar{g}$, we solve the first-best level of investment, denoted by $x_t^{*j}(\zeta)$: for $x_t^{*j}(\zeta) > 0$, we have

$$F'(x_t^{*j}(\zeta) + \zeta)q_t^j = 1; \quad (12)$$

and $x_t^{*j}(\zeta) = 0$ for any ζ such that $F'(x_t^{*j}(\zeta) + \zeta)q_t^j = F'(\zeta)q_t^j < 1$ under the natural constraint that the amount of financing cannot be negative. To simplify the discussion, we introduce $\bar{\zeta}_t^j$

$$F'(\bar{\zeta}_t^j)q_t^j = 1. \quad (13)$$

Under $q_t^H > q_t^L$ and a strictly concave $F(\cdot)$, we have $\bar{\zeta}_t^H > \bar{\zeta}_t^L$ from (13). Type- H capital is

¹⁷Credit freeze happens for various reasons, such as lenders' lack of capital (Bernanke and Lown, 1991), information decay in booms (Gorton and Ordoñez, 2014; Asriyan, Laeven, and Martin, 2018), foreigners' withdrawal (Van Nieuwerburgh and Veldkamp, 2009; Koijen, Koulischer, Nguyen, and Yogo, 2020), and ambiguity in risk evaluation (Boyarchenko (2012); Caballero and Simsek (2013); Drechsler (2013)). A market crash happened during the global financial crisis (Acharya, Schnabl, and Suarez, 2013; Brunnermeier, 2009; Gorton, Laarits, and Metrick, 2017; Kacperczyk and Schnabl, 2010; Krishnamurthy, 2010). Credit markets were under tremendous stress during the Covid-19 pandemic (Falato, Goldstein, and Hortaçsu, 2020; Haddad, Moreira, and Muir, 2020; Halling, Yu, and Zechner, 2020; Kargar, Lester, Lindsay, Liu, Weill, and Zúñiga, 2020; Ma, Xiao, and Zeng, 2020).

¹⁸If a firm's debt is not publicly traded, the government can "free-ride" banks' information production in pricing or screening (for example, the government relied on banks to screen borrowers in MSLP during the Covid-19 pandemic).

naturally more valuable than type- L capital because type- H firm has a higher productivity, i.e., $A^H > A^L$. We will later formally show $q_t^H > q_t^L$ after fully solve the equilibrium and endogenous capital value. The following lemma summarizes the first-best levels of financing.

Lemma 1 (First best) *The first-best level of financing for a type- j firm with realized ζ is given by*

$$x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+, \quad (14)$$

where $j \in \{H, L\}$. Under $q_t^H > q_t^L$, we have $\bar{\zeta}_t^H > \bar{\zeta}_t^L$.

A firm's optimal level of financing is the minimum of first-best level and the available funding:

$$x_t^j(\zeta) = \min\{x_t^{*j}(\zeta), \bar{d} + \bar{g}\}. \quad (15)$$

Government intervention improves efficiency by bringing $x_t^j(\zeta)$ closer to $x_t^{*j}(\zeta)$. Here the government acts as a financial intermediary in crises (Lucas, 2016). It finances lending with lump-sum taxes on deep-pocket households and transfers the repayments to households.

So far, the only source of inefficiency is the limited funding supply in the private sector, and the optimal intervention simply requires \bar{g} to be sufficiently large so that $x_t^j(\zeta) = x_t^{*j}(\zeta)$. However, funding in crises is limited because of frictions not only on the credit supply side but also on the demand side. Introducing borrowers' moral hazard allows credit intervention to have potentially harmful effects. Specifically, a large scale intervention can lead to over-investment in crises.

We model borrowers' moral hazard in the form of strategic default. So far, we have assumed that firms repay debts if they survive. However, as well-known in the corporate finance literature (Bolton and Scharfstein, 1990; Hart and Moore, 1998), there are two types of default. First, a firm defaults because it cannot survive, which happens with a probability $1 - F(x_t^j(\zeta) + \zeta)$. Second, a firm survives but its owners extract value from creditors through strategic default and renegotiation. The owners may threaten to divert resources, engage in wasteful spending, or withhold human capital (Aghion and Bolton, 1992; Hart and Moore, 1994). After renegotiation, the firm owners obtain β while the private-sector creditors seize the capital and obtain $q_t^j - \beta$ as the control right is typically transferred to creditors when default happens. The recovery value is zero for the government, which is in line with evidence on the prevalent abuse of government funding during crises (Hanson et al., 2020; Lynch, 2021; Griffin et al., 2023). Also it is rare for the government

to engage in bankrupt firms' business restructuring (seizing the capital and continuing operating). For firms that strategically default, government funding is essentially a subsidy on survival.

We summarize the timing in a crisis. The private-sector creditors set a menu of interest rates for all firms according to their break-even condition (11) so that a type- j firm with a realized ζ faces interest rate $r_t^j(\zeta, x_t^j(\zeta))$ given by (11). Next, according to the survival probability function, a certain fraction of firms survive the crisis. A firm that has survived then compares the value per unit of capital from strategic default, β , and the value under debt repayment, $q_t^j - [1 + r_t^j(\zeta, x_t^j(\zeta))] x_t^j(\zeta)$. Firms will strategically default if and only if it is strictly better than honoring debt repayment.

Let $\underline{\zeta}_t^j$ denote the solution to the following indifference condition over ζ ,

$$q_t^j - \left[1 + r_t^j(\underline{\zeta}_t^j, x_t^j(\underline{\zeta}_t^j)) \right] x_t^j(\underline{\zeta}_t^j) = \beta, \quad (16)$$

if a solution exists. Note that if β is sufficiently high, for example, $\beta = q_t^j$, then all firms prefer strategic default over debt repayment and the indifference condition may never hold for any ζ . If $\beta = 0$, then all firms repay their debts. Therefore, the equation (16) may not have a solution. The following lemma shows that when the solution of equation (16) exists, it is unique, and there are less type- H firms that strategically default than type- L firms. The full proof is in the appendix.¹⁹

Lemma 2 (Strategic default) *A type- j firm with realized ζ strategically defaults if and only if $\zeta < \underline{\zeta}_t^j$, where $\underline{\zeta}_t^j$ as a solution to (16) is unique. Furthermore, under $q_t^H > q_t^L$, we have $\underline{\zeta}_t^H < \underline{\zeta}_t^L$.*

Firms with a lower ζ have more liquidity needs and borrow more, so they tend to strategically default. Intuitively, if a firm has more debt obligations, default saves a large repayment. Default also causes the firm owners to lose capital ownership, so when capital is more valuable (q_t^j is higher), firms tend not to strategically default. Given that $q_t^L < q_t^H$ implies $\underline{\zeta}_t^L > \underline{\zeta}_t^H$, there are more type- L firms that strategically default ($\zeta \in (0, \underline{\zeta}_t^L)$) than type- H firms.

For firms that strategically default, their objective function is $F(x_t^j(\zeta) + \zeta)\beta$ without concern over repayment. Therefore, they simply borrow as much as possible to maximize survival probability. Private-sector creditors know q_t^j (and hence the strategic-default threshold $\underline{\zeta}_t^j$), and they know firms' draw of ζ , so private-sector creditors know which firm will repay debts and which

¹⁹The intuition behind the proof is as follows. Given q_t^j , we compute the value under debt repayment, $q_t^j - [1 + r_t^j(\zeta, x_t^j(\zeta))]x_t^j(\zeta)$ for all values of ζ , which we prove is a monotonic function of ζ . This function incorporates the interest-setting function $r_t^j(\zeta, x)$ given by (11) and a firm's optimal borrowing $x_t^j(\zeta)$ function given by (15). The threshold $\underline{\zeta}_t^j$ is where this value under debt repayment as a function of ζ crosses β , the value under strategic default.

will strategically default. When lending to firms that will strategically default, private-sector creditors understand that such firms will borrow as much as possible. Therefore, beyond the interest rate $r_t^j(\zeta, x_t^j(\zeta))$, private-sector creditors also specify a debt limit, denoted by $\hat{d}_t^j(\zeta)$,

$$\hat{d}_t^j(\zeta) = F(\hat{d}_t^j(\zeta) + \zeta + \bar{g})(q_t^j - \beta), \quad (17)$$

to make sure that they break even.²⁰ Note that the interest rate is relevant for firms that strategically default even though they do not repay their debts, because the decision to default or not is based on comparing the value in strategic default and the value after debt repayment, as shown in (16).

Lemma 3 (Endogenous debt capacity) *For any ζ , $\hat{d}_t^j(\zeta)$ in (17) exists and is unique. It is strictly increasing and concave in ζ . Furthermore, under $q_t^H > q_t^L$, we have $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$.*

In summary, a type- j firm's optimal amount of financing is given by

$$x_t^j(\zeta) = \underbrace{\mathbf{1}_{\zeta \geq \underline{\zeta}_t^j} \min\{(\bar{\zeta}_t^j - \zeta)^+, \bar{d} + \bar{g}\}}_{\text{no strategic default}} + \underbrace{\mathbf{1}_{\zeta < \underline{\zeta}_t^j} \left(\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g} \right)}_{\text{strategic default}}, \quad (18)$$

Firms that repay debt raise funding given by (15). Firms that strategically default max out borrowing. Based on Lemma 1, 2, and 3, the following proposition summarizes firms' financing strategy.

Proposition 1 (Equilibrium financing) *The financing amount of a type- j firm, $j \in \{H, L\}$, with a realized ζ , $x_t^j(\zeta)$, is given by (18), with $\bar{\zeta}_t^H > \bar{\zeta}_t^L$, $\underline{\zeta}_t^H < \underline{\zeta}_t^L$, and $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q_t^H > q_t^L$. It also has the following properties: (1) $x_t^j(\zeta)$ is increasing and concave in ζ for $\zeta < \underline{\zeta}_t^j$ and $\hat{d}_t^j(\zeta) \leq \bar{d}$; (2) $x_t^j(\zeta)$ is decreasing and linear in ζ for $\zeta \in [\underline{\zeta}_t^j, \bar{\zeta}_t^j]$ and $(\bar{\zeta}_t^j - \zeta)^+ \leq \bar{d} + \bar{g}$.*

For a type- j firm, $j \in \{H, L\}$ with ζ above the default threshold $\underline{\zeta}_t^j$, a lower ζ (i.e., a stronger liquidity need) leads to more borrowing. For this firm, the only source of potential inefficiency is the funding limit. For a type- j firm with $\zeta < \underline{\zeta}_t^j$, it faces an additional problem of endogenous debt limit. As $\hat{d}_t^j(\zeta)$ increases in ζ , a firm with a stronger liquidity need (i.e., a lower ζ) actually has a smaller debt capacity $\hat{d}_t^j(\zeta)$ and thus can borrow less. Moreover, given the concavity of $\hat{d}_t^j(\zeta)$ in ζ

²⁰Note that if $\hat{d}_t^j(\zeta) \geq \bar{d}$, $x_t^j(\zeta) = \bar{d} + \bar{g}$, in which case the creditors earn positive profits because, under a strictly increasing and concave $F(\cdot)$, $F(\bar{d} + \bar{g} + \zeta)(q_t^j - \beta) > \bar{d}$. The creditors may want to lend more but they are constrained by the amount of available funds, \bar{d} . Such funding shortage and the positive profits earned by those who can provide funding is a common feature of liquidity crises that we capture in our model.

(see Lemma 3), the further ζ decreases, the faster debt capacity shrinks. This is a classic insight from the corporate finance literature: A borrower's own lack of commitment against strategic default, which is more severe when ζ is lower and the financing need is stronger (see Lemma 2), compromises her financing capacity. The financing strategy characterized in Proposition 1 has a rich set of features that allow us to discuss different channels through which government intervention can affect firms' financing in crises and may alleviate or exacerbate inefficiencies.

Discussion: Productivity and credit risk. Firms differ in productivity, A^j (type) and ζ (a smaller ζ indicates stronger liquidity needs in crises), where ζ drives a wedge between firm type and credit quality. A type- H firm may default by failing to survive, which happens with probability $1 - F(x_t^H(\zeta) + \zeta)$. A type- H firm may also strategically default. In contrast, a type- L firm that survives the liquidity crisis may choose to repay its debt if its ζ is above the strategic default threshold, ζ_t^L . This distinction between firm productivity (type) and credit risk is consistent with the evidence that we present in Appendix C. To summarize, in both data and the model, firm quality and credit risk are not perfectly aligned. Therefore, although the government may differentiate credit quality by following the private-sector pricing of credit risk, it cannot obtain a perfect indicator of firm productivity from credit markets. Moreover, some credit programs do not even differentiate firms by credit risk, for example, PPP during the Covid-19 crisis (see Appendix B.3 for our analysis).

2.3 Efficiency and Equilibrium

Intervention and efficiency. The efficiency criterion in our model is whether firms' investment in survival is at the first-best level given by (14). There are two frictions, and both contribute to the financial constraints on investment. The first friction is on the funding supply side, i.e., the limit on private-sector funding, denoted by \bar{d} . The second is on the funding demand side, i.e., borrowers' strategic default, which gives rise to the endogenous limit on private-sector debt capacity, $\hat{d}_t^j(\zeta)$.

For firms that repay debts (i.e., with $\zeta \geq \zeta_t^j$), government intervention enlarges financing capacity and unequivocally improves efficiency by relaxing the funding constraint. These firms repay their debts after surviving the crisis, so, under the disciplinary effect of debt repayment, they do not over-invest. They invest up to $x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+$, a level that balances the marginal benefit of improving the survival probability and the marginal cost of investment (see Lemma 1).

For firms that strategically default (i.e., with $\zeta < \zeta_t^j$), government intervention also enlarges

financing capacity and increases the financing amount, $x_t^j(\zeta)$, but the efficiency implications are more complex. Government intervention enlarges firms' financing capacity through two channels, direct liquidity injection and private funding crowding-in. First, an increase in \bar{g} provides more funding to these firms, as shown in (18). This additional source of funds increases the survival probability and thus lowers the interest rate charged by private-sector creditors as determined by (11). Second, as shown in (17), an increase in \bar{g} raises the survival probability, making the creditors more willing to lend, resulting in a higher $\hat{d}_t^j(\zeta)$ (a crowding-in effect).

Lemma 4 (Private funding crowding-in) *For any ζ and $j \in \{H, L\}$, and given q_t^j , government financing crowds in private lending, $\partial \hat{d}_t^j(\zeta) / \partial \bar{g} > 0$, and reduces interest rate, $\partial r_t^j(\zeta) / \partial \bar{g} \leq 0$.*

To analyze the efficiency implication of credit intervention among firms that strategically default, first notice that the goal of these firms is to maximize the survival probability. After surviving the crisis, firm owners renege on debt repayment and extract rent by holding up the creditors in renegotiation. These firms over-invest if the amount they borrow, $\min\{\hat{d}_t^j(\zeta), \bar{d}\} + \bar{g}$, is above the first-best level, $x_t^{*j}(\zeta) = (\bar{\zeta}_t^j - \zeta)^+$, and any further increase in \bar{g} exacerbates such inefficiency.

Among firms that strategically default, over-investment tends to happen to those with high ζ . Formally, if there exists a threshold $\zeta_t^{*j} < \underline{\zeta}_t^j$ such that $\min\{\hat{d}_t^j(\zeta_t^{*j}), \bar{d}\} + \bar{g} = (\bar{\zeta}_t^j - \zeta_t^{*j})^+$ then any firm with $\zeta \in (\zeta_t^{*j}, \underline{\zeta}_t^j)$ over-invests, because the endogenous debt capacity, $\hat{d}_t^j(\zeta)$, is increasing in ζ while the first-best level of financing, $(\bar{\zeta}_t^j - \zeta)^+$, is decreasing in ζ . For these firms, a higher \bar{g} exacerbates over-investment. For firms that have $\zeta < \zeta_t^{*j}$ and under-invest, an increase in \bar{g} improves efficiency by alleviating their under-investment, whether the firms strategically default or not. The next proposition summarizes the efficiency implications of credit intervention.

Proposition 2 (Intervention and efficiency) *Credit intervention improves efficiency by alleviating under-investment for firms that repay debts (i.e., $\zeta \geq \underline{\zeta}_t^j$) and face a binding funding constraint (i.e., $\bar{d} + \bar{g} < \bar{\zeta}_t^j - \zeta$) and also for firms that strategically default (i.e., $\zeta < \underline{\zeta}_t^j$) and under-invest (i.e., $\hat{d}_t^j(\zeta) + \bar{g} < \bar{\zeta}_t^j - \zeta$). Credit intervention reduces efficiency by exacerbating over-investment among firms that strategically default (i.e., $\zeta < \underline{\zeta}_t^j$) and over-invest (i.e., $\hat{d}_t^j(\zeta) + \bar{g} > \bar{\zeta}_t^j - \zeta$).*

It is clear that the detrimental effects of credit intervention through over-investment depends on the scale of intervention. When the government increases \bar{g} , the investment of firms that strategically default increases through higher \bar{g} and through a higher $\hat{d}_t^j(\zeta)$ due to the crowding-in effect

in Lemma 4. This enlarges the over-investment region of ζ and thus exacerbates the inefficiency from over-investment. On the other hand, when \bar{g} declines, the over-investment region shrinks.

Moreover, a sufficiently large scale of intervention can guarantee a net negative impact. There is a limit to efficiency improvement from relaxing the financial constraint on under-investing firms: When \bar{g} is sufficiently large, all firms that repay debts have reached their first-best level of financing and would not invest more. However, there is no limit on over-investment: The goal of firms that strategically default is to maximize borrowing and survival probability, so they will keep borrowing as long as the government is willing to lend. The following corollary summarizes this result.

Corollary 1 (Excessive credit intervention) *The cost of over-investment dominates the benefit from financing the under-investing firms when the scale of credit intervention is sufficiently large.*

So far, we have separately discussed the efficiency implications of credit intervention on firms with $\zeta \geq \underline{\zeta}_t^j$ and on firms with $\zeta < \underline{\zeta}_t^j$. Credit intervention may also affect the strategic default threshold, $\underline{\zeta}_t^j$. While a higher \bar{g} does not affect β , the value that firms owners can extract through strategic default and renegotiation, it increases firm value in the debt repayment region by relaxing the financial constraint, allowing profitable investment to be funded.²¹ This force lowers the default threshold, $\underline{\zeta}_t^j$ and reduces the measure of firms that strategically default. However, as we will show in the next section, this channel of efficiency improvement is not quantitatively significant.

We have analyzed the efficiency implications of intervention within a firm type. Lemma 1, 2, and 3 also reveal the difference in the impact of intervention across firm types. In Proposition 2, credit intervention improves efficiency by relaxing the financial constraint on firms that repay debts. According to Lemma 1, among these firms, type- H firms have a higher first-best level of investment so this positive impact is more prominent among type- H firms. In Proposition 2, credit intervention exacerbates over-investment among firms that strategically default. According to Lemma 2, there are more type- L firms that strategically default than type- H firms. This suggests that the negative impact through over-investment tends to be more reflected among type- L firms.

By incorporating the funding supply- and demand-side frictions, our model allows both under- and over-investment to emerge in equilibrium. While under-investment can happen to both firms that repay debts and those that strategically default, over-investment happens to the latter. Credit intervention improves efficiency among firms that under-invest and exacerbates inefficiency among

²¹Recall that once the firm survives the liquidity shock, a strategic default is a comparison between the firm value under debt repayment, which increases in \bar{g} , and the value from strategic default β .

those that over-invest. The goal of our model is to capture such intricate effects of credit intervention and, after calibration, to allow the dominant channels to emerge in the quantitative analysis.

In our model, credit intervention is size-dependent but not dependent on an individual firm's type, $j \in \{H, L\}$, or the severity of its liquidity crisis, ζ . A firm with k_t units of capital can borrow from the government up to $k_t \bar{g}$. There are several motivations behind this setup. First and foremost, the government may not have information on firms' types, in line with the tradition in economics that emphasizes the informational disadvantage of central authorities (Hayek, 1945). Importantly, our setup captures policy design in reality. The funding limit is set proportional to accounting measures of operation scale (k_t in our model) such as the programs during Covid-19 pandemic. MSLP set a borrowing limit to six times the borrower's operating income rather than profits or productivity per unit of resources deployed. Similarly, the limit in PPP was a multiple of the borrower's payroll rather than tied to labor productivity. Furthermore, the corporate bond purchase programs, i.e., PMCCF and SMCCF, imposed limits proportional to an issuer's existing debts (a measure of liability size rather than productivity).²²

In the next subsection, we discuss how the differential effects of credit intervention on type- L and H firms translate into channels that may affect the cleansing effect of crises and thereby generate intertemporal dependence in intervention scale across crises (i.e., the slippery slope of intervention). In the following, we close this subsection and complete the equilibrium characterization by defining the stationary equilibrium and solving the endogenous capital values.

Capital value and equilibrium. Capital value plays an important role in our model. It is the key distinguishing factor that separates type- L and type- H firms. As shown in Lemma 1, capital values drive the first-best level of investment in crises. According to Lemma 2 and 3, capital values affect the strategic default thresholds and firms' endogenous debt capacity in the private funding market.

To simplify notation, we define the expected value per unit of type- j capital to the owner of a

²²Primary market corporate bond purchase facility: The amount of outstanding bonds or loans of an eligible issuer that borrows from the Facility may not exceed 130 percent of the issuer's maximum outstanding bonds and loans on any day between March 22, 2019, and March 22, 2020. Secondary market corporate bond purchase facility: The amount of bonds that the Facility will purchase from the secondary market of any eligible issuer is capped at 10 percent of the issuers maximum bonds outstanding on any day between March 22, 2019, and March 22, 2020.

type- j firm with a realized ζ as follows

$$\pi_t^j(\zeta) = \underbrace{(F(x_t^j(\zeta) + \zeta)q_t^j - x_t^j(\zeta)) \mathbf{1}_{\zeta \geq \zeta_t^j}}_{\text{no strategic default}} + \underbrace{F(x_t^j(\zeta) + \zeta)\beta \mathbf{1}_{\zeta < \zeta_t^j}}_{\text{strategic default}}, \quad (19)$$

where $F(\cdot)$ is the survival probability as a function of crisis severity, ζ , and funds available $x_t^j(\zeta)$. Note that, in case of no strategic default, we have substituted out the interest rate using the creditors' break-even condition (17). In equilibrium, capital value, q_t^j , satisfies the following equation that equates investors' required rate of return over dt , i.e., the discount rate, $r dt$, and the sum of expected return on holding capital from price appreciation, cash-flow yield net off investment costs in normal times, capital growth via investment net off depreciation, and return in crises calculated as the difference between the ζ -averaged capital value in a crisis and the pre-crisis capital value:

$$r = \mathbb{E}_t \left[\frac{dq_t^j / dt}{q_{t-}^j} \right] + \frac{A^j - \phi(\iota_{t-}^j)}{q_{t-}^j} + (\iota_{t-}^j - \delta) + \lambda \frac{\int_0^\infty \pi_t^j(\zeta) dH(\zeta) - q_{t-}^j}{q_{t-}^j}, \quad (20)$$

where we use the subscript $t-$ in q_{t-}^j to denote the pre-crisis capital value and accordingly ι_{t-}^j to denote the investment rate driven by q_{t-}^j (see (7)). The value $\pi_t^j(\zeta)$ is defined in equation (19), and as reminder, $H(\cdot)$ is the c.d.f. for ζ . The equilibrium can be characterized below.

Proposition 3 (Stationary equilibrium) *There exists a stationary equilibrium where firms' investment rate in normal times, ι^j , borrowing in crises, $x^j(\zeta)$, borrowing cost in crises, $r^j(\zeta)$, debt capacity under strategic default, $\hat{d}^j(\zeta)$, and capital value, q^j , are time-invariant. Furthermore, the equilibrium is unique with $q^H > q^L$, under the following conditions: (1) $\iota^j - \delta \leq r$; (2) $\frac{\beta \lambda_F}{4[1 - \lambda_F(q^j - \beta)]} < 1$, where λ_F is the intensity parameter of exponential distribution c.d.f. $F(\cdot)$.*

The first parameter restriction is standard in the asset pricing literature: $\iota^j - \delta$, the growth rate from investment net off depreciation cannot be higher than the discount rate, r ; otherwise capital value is infinite. The second parameter restriction limits the degree of moral hazard: Given λ_F in $F(x_t^j(\zeta) + \zeta) = 1 - e^{-\lambda_F(x_t^j(\zeta) + \zeta)}$, the firm owners' bargaining power in strategic default cannot be too large, i.e., β cannot be too large relative to $q^j - \beta$ (the creditors' recovery value). And, given the degree of moral hazard, the survival probability cannot be too responsive to liquidity injection, i.e., λ_F cannot be too large. We will show that the parameter restrictions (1) and (2) do not bind in our calibration, that is, the interior values deliver close match between model and data moments.

In our baseline model, the scale of credit intervention, \bar{g} , is time-invariant. We characterize an equilibrium with constant capital values as it provides a transparent presentation of key economic mechanisms. Later we will consider an economy where the government employs a dynamic strategy of credit intervention, in which case the value of capital varies over time.

2.4 The Cleansing Effect of Crises and Slippery Slope of Intervention

In this subsection, we characterize the cleansing effect of crises, which is key to the intertemporal linkage that generates the slippery slope of credit intervention. To characterize the aggregate dynamics, we use (8) and (9) to derive the law of motion for two state variables, capital quality ω_t defined in equation (3), and capital quantity K_t defined in equation (4):

$$\frac{dK_t}{K_{t-}} = \underbrace{[-\delta + (\omega_{t-}\iota_{t-}^H + (1 - \omega_{t-})\iota_{t-}^L) + \eta]}_{\mu_t^K(\omega_{t-})} dt + \underbrace{(\omega_{t-}\kappa_t^H + (1 - \omega_{t-})\kappa_t^L - 1)}_{\Delta_t^K(\omega_{t-})} dN_t, \quad (21)$$

where κ_t^j , the fraction of type- j capital that survives a crisis, is defined as

$$\kappa_t^j \equiv \int F(x_t^j(\zeta) + \zeta) dH(\zeta), \quad (22)$$

and

$$d\omega_t = \underbrace{\omega_{t-} (1 - \omega_{t-}) \left(\iota_{t-}^H - \iota_{t-}^L - \frac{\eta}{1 - \omega_{t-}} \right)}_{\mu_t^\omega(\omega_{t-})} dt + \underbrace{\left(\frac{\omega_{t-}\kappa_t^H}{\omega_{t-}\kappa_t^H + (1 - \omega_{t-})\kappa_t^L} - \omega_{t-} \right)}_{\Delta_t^\omega(\omega_{t-})} dN_t. \quad (23)$$

From Proposition 3, the normal-time capital growth rate, ι_{t-}^j , and capital surviving rate in crises, κ_t^j , are time-invariant in the stationary equilibrium. Given (7), we have $\iota^H > \iota^L$ under $q^H > q^L$, which then implies that ω_t has a tendency to drift upward. As previously discussed, we preserve the stationarity of ω_t via the exogenous entry rate of type- L firms, η , introduced in (9).

The cleansing effect refers to an increase in ω_t in crises, i.e., $\Delta_t^\omega(\omega_{t-}) > 0$, which is equivalent to type- H firms having a higher survival rate than type- L firms, i.e., $\kappa_t^H > \kappa_t^L$. Therefore, the cleansing effect of crises is simply about which type of firms in aggregate invest more in survival. Consider the first-best scenario where firms are not financially constrained and invest in survival at the level given by (14). Here the cleansing effect emerges because type- H firms have a higher first-

best level of investment than type- L firms, i.e., $x^{H^*}(\zeta) > x^{L^*}(\zeta)$ (see Lemma 1). Type- H firms want to borrow more and invest more in survival because their capital is more valuable.

Proposition 4 (The cleansing effect in the first-best economy) *In the first-best economy where firms invest at the level given by (14), crises have a cleansing effect, i.e., $\Delta_t^\omega(\omega_{t-}) > 0$.*

As previously discussed, once we introduce the funding supply- and demand-side frictions, firms may under- or over-invest in survival. Next, we first discuss how introducing only the funding-supply friction to the first-best economy affects the cleansing effect of crises. The funding supply-side friction, that is the private-sector creditors cannot lend beyond \bar{d} , causes firms of both types to under-invest and thereby dampens the cleansing effect in Proposition 4. Intuitively, when this funding-supply constraint binds, i.e., $\bar{d} < x^{j^*}(\zeta)$, both types of firms invest at the same level, \bar{d} . Credit intervention, by expanding funding supply and allowing more firms to invest at the first-best level, reverses this dampening effect. The next lemma summarizes this result.

Corollary 2 (Funding-supply friction and the cleansing effect) *Introducing a sufficiently tight funding constraint ($\bar{d} \rightarrow 0$) to the first-best economy weakens the cleansing effect of crises. Credit intervention, by expanding funding supply, reverses this force and strengthens the cleansing effect.*

Next, we discuss the impact of introducing funding demand-side friction on the cleansing effect of crises. The demand-side friction, i.e., firms' inability to commit against strategic default, gives rise to a new channel of cleansing effect. As shown in Lemma 3, firms that strategically default borrow as much as they can to maximize their survival probability, and as a result, the private-sector creditors impose a debt limit. Type- H firms face a higher endogenous debt limit, i.e., $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ and thus can borrow more than type- L firms. This is a new channel of cleansing effect. While the cleansing effect in Proposition 4 arises from type- H firms wanting to invest more in survival than type- L firms, this new channel is about type- H firms being able to obtain more funding. Credit intervention can dampen this channel of cleansing effect by offering funding to both types of firms. Intuitively, under a concave $F(\cdot)$, the marginal impact of \bar{g} on survival probability is greater for type- L firms because type- H firms already have a higher level of private-sector funding.

Corollary 3 (Funding-demand friction and the cleansing effect) *Given any measure of ζ where both types of firms strategically default and the private-sector debt limit binds before the private-sector funding supply constraint (i.e., $\hat{d}_t^j < \bar{d}$), a cleansing effect emerges in crises, i.e., the share of surviving firms of type H is greater than that of type L . Credit intervention weakens this channel.*

In sum, two channels of cleansing effects emerge in crises: (1) Among firms that repay debts, type- H firms want to borrow more to preserve capital; (2) among firms that strategically default, type- H firms can borrow more. Credit intervention strengthens the first channel of cleansing effect while can weaken the second channel. However, there is a limit to the impact of credit intervention on the first channel: If \bar{g} is sufficiently large to allow the first-best investment in survival for both types, then any further increase in \bar{g} no longer changes the survival probability for firms that repay debts. In contrast, there is no limit to the impact of credit intervention on the second channel: No matter how large \bar{g} is, any further increase in \bar{g} always improve the survival probability for firms that strategically default as they maximize their borrowing. Therefore, for a sufficiently large \bar{g} , the weakening of cleansing effect by credit support for firms that strategically default dominates the strengthening of cleansing effect by credit support for firms that repay debts.

So far, we have discussed that a type- H firm has a higher survival probability than a type- L firm if either both firms repay their debts, in which case type- H firms want to borrow more under a higher investment target, or if both firms strategically default, in which case type- H firms can borrow more under a higher debt capacity. There is also the case where a type- H firm repays its debt while the type- L firm with the same ζ strategically defaults.²³ In the following, to sharpen the analytical results, we introduce a parameter restriction so that we can focus on characterizing the quantitatively relevant region of parameter values. The following restriction is not a binding constraint in our parameter calibration; in other words, we do not impose the restriction in calibration, but we find that the restriction is satisfied in our baseline calibration and holds in a much broader set of parameter values. We will discuss calibration in details in the next section.

Condition 1 q^H is sufficiently large and q^L is sufficiently low so that the equilibrium has the following properties: (1) $\underline{\zeta}^H = 0$, (2) $\bar{\zeta}^L < \bar{d} + \bar{g}$, and (3) $\bar{\zeta}^H > \hat{d}^L(\underline{\zeta}^L) + \underline{\zeta}^L + \bar{g}$.

Condition 1 focuses on capital values. As discussed in Subsection 2.3, q^j drives $\bar{\zeta}^j$ and, given the severity of a firm's liquidity crisis, ζ , it determines the first-best level of investment through $\bar{\zeta}^j$. Moreover, q^j determines the strategic default threshold $\underline{\zeta}^j$ and the endogenous debt limit, $\hat{d}^j(\zeta)$.

This condition is sufficiently general, as one can view the binary types in our model as a discretized version of a continuum of firm types, and type- H firms represent the right tail of the most productive while the firms with low productivity are represented by type L in the model.

²³Note that from Lemma 2, type- H firms have a lower default threshold for ζ , so given the same ζ , we can rule out the case where the type- H firm strategically defaults while the type- L firm repays its debt.

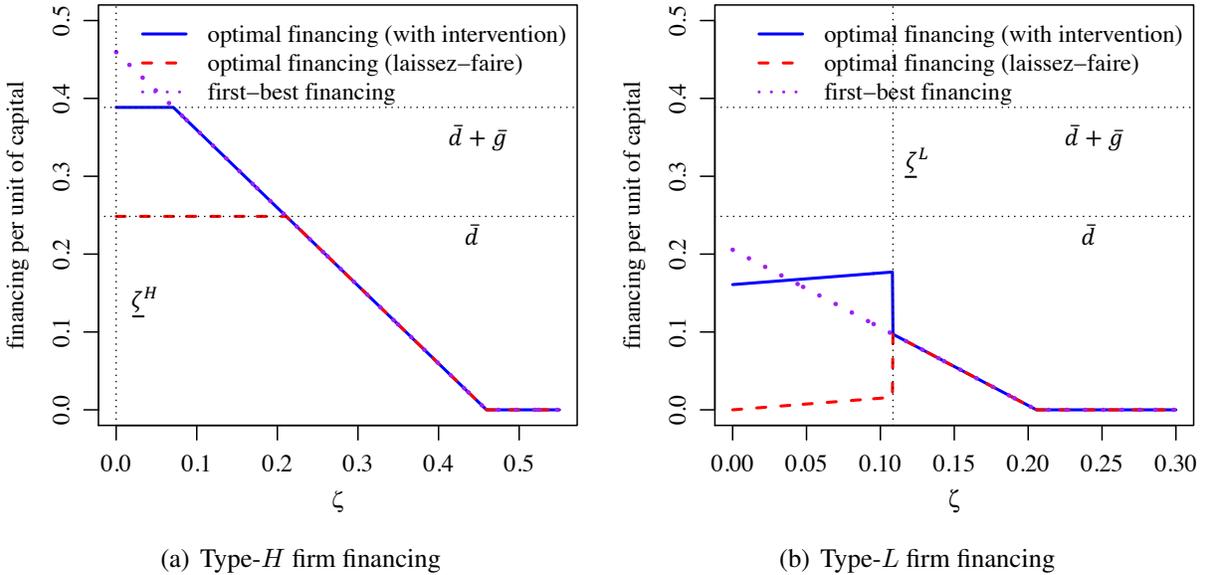


Figure 1: Optimal firm financing. This figure illustrates the optimal firm financing in different scenarios, under the calibrated model parameters that we discuss in Section 3. The solid line shows the optimal financing under credit intervention. The dashed line shows the optimal financing without credit intervention ($\bar{g} = 0$). The dotted line shows the first-best financing amount without funding supply- and demand-side frictions.

Under Condition 1, type- H firms do not strategically default ($\zeta^H = 0$) because the firm owners find it rather costly to lose control right and capital value. Firms may still face a lack of funding supply, so, for type H , the relevant form of inefficiency is under-investment. Panel A of Figure 1 illustrates the optimal amount of financing for a type- H firm with different values of ζ . This graph is based on the calibrated parameter values that will be discussed in the next section. When a firm has a sufficiently large ζ , the first-best level of financing is zero (see (14)). As ζ declines, the liquidity crisis becomes more severe, and the first-best level of financing increases (dotted line). Financing is constrained by funding supply, which is equal to \bar{d} without government intervention (dashed line) and is equal to $\bar{d} + \bar{g}$ with government intervention (solid line).

The second property, $\bar{\zeta}^L < \bar{d} + \bar{g}$, implies that type- L firms do not under-invest due to the lack of funding supply even when the liquidity shock is most severe ($\zeta = 0$ and the first-best investment is $\bar{\zeta}^L$). This is due to the fact that a low value of type- L capital leads to a low first-best investment level. As shown in Panel B of Figure 1, the first-best level (dotted line) is below $\bar{d} + \bar{g}$. Note that type- L firms still face the endogenous debt limit due to strategic default. The dashed line in Panel B of Figure 1 shows that when ζ is small and the liquidity needs are strong, the endogenous debt

limit binds. As ζ increases from zero, the debt limit, $\hat{d}^L(\zeta)$, increases (see Lemma 3), so the firm is able to raise more funds. However, as long as ζ is below the strategic default threshold, $\underline{\zeta}^L$, the financing level is below the first-best. The solid line shows the situation with credit intervention. The firm is able to raise more funds from the government and from the private-sector creditors due to the crowding-in effect in Lemma 4. However, over-investment emerges among firms with relatively high ζ (still below $\underline{\zeta}^L$), even though those with ζ close to zero still under-invest.

As previously discussed, crises generate a cleansing effect among firms that repay debts and among firms that strategically default. However, given the same ζ , when type- H firms repay debts and type- L firms strategically default (i.e., $\zeta \in (\underline{\zeta}^H, \underline{\zeta}^L)$), it is ambiguous which type invest more. The last property delivered by Condition 1 resolves such ambiguity. For any $\zeta < \underline{\zeta}^L$ (type- L 's strategic default region), the last property implies $\bar{\zeta}^H - \zeta > \hat{d}^L(\underline{\zeta}^L) + \bar{g}$ so that type- H firms invest more in survival than type- L firms, which leads to the cleansing effect of crises in this region of ζ .

Proposition 5 (Capital destruction and the cleansing effect in crises) *Crises feature capital destruction, $\Delta_t^K < 0$, and under Condition 1, have a cleansing effect, $\Delta_t^\omega > 0$.*

Our focus on crises with cleansing effects is empirically motivated. For example, during the Covid-19 crisis, firms with lower productivity were more likely to cease operations permanently (Muzi et al., 2023). And, during the Covid-19 crisis, economic activity was reallocated toward firms with higher pre-crisis labor productivity, and such reallocation is stronger compared with pre-crisis times (Bruhn et al., 2023). These studies also document that credit intervention dampens the cleansing effect. Moreover, Dörr et al. (2022) find that credit intervention during the Covid-19 pandemic disproportionately benefited firms that are already financially vulnerable pre-crisis.²⁴ Consistent with the evidence, the proposition shows that credit intervention dampens the cleansing effect in our model. Moreover, credit intervention has a positive effect on firm survival and the total quantity of capital preserved, which is also consistent with the evidence (Bartik et al., 2020; Bartlett and Morse, 2020; Hubbard and Strain, 2020; Denes et al., 2021; Kawaguchi et al., 2021). Therefore, the government faces a trade-off between capital quantity, K_t , and quality ω_t .

²⁴Muzi et al. (2023) analyze data from firms in 34 countries. Bruhn et al. (2023) utilizes the World Banks Enterprise Surveys Covid-19 Follow-up Surveys, encompassing around 8,000 firms in 23 emerging and developing countries across Europe and Asia. Dörr et al. (2022) examine 1.5 million German companies.

Proposition 6 (The quantity-quality trade-off) *Credit intervention alleviates capital destruction*

$$\frac{\partial \Delta_t^K}{\partial \bar{g}} > 0,$$

but dampens the cleansing effect

$$\frac{\partial \Delta_t^\omega}{\partial \bar{g}} < 0.$$

The slippery slope of intervention. The cleansing effect of crises, $\Delta_t^\omega > 0$, and the impact of government intervention, $\partial \Delta_t^\omega / \partial \bar{g} < 0$ are important for analyzing the intertemporal linkages. Firm quality, ω_t , is a key state variable. By dampening the cleansing effect of the current crisis, credit intervention begets an intervention of greater scale in the next crisis. The following proposition characterizes a critical connection between pre-crisis firm quality (ω_{t-}) and intervention scale.

Proposition 7 (Pre-crisis firm quality and intervention scale) *To contain the output drop at any given level, the required scale of intervention is larger if the pre-crisis firm quality is lower.*

Output drop in crises is caused by the decline of capital quantity. Since type- L firms have a lower capital productivity, they need more units of capital to produce the same level of output than type- H firms. Furthermore, type- L firms have less private-sector financing capacity so they need to rely more on the government. Therefore, when an economy enters into a crisis with more type- L firms and less type- H firms (i.e., a lower ω_{t-}), it requires the government to provide a larger scale of intervention if the goal is to contain the drop in output at a certain level.

This connection between the pre-crisis firm quality and intervention scale is key to establish the slippery slope of intervention. In Proposition 6, we show that credit intervention dampens the cleansing effect and reduces the post-crisis average firm quality, ω_t . When the economy enters into the next crisis with a lower firm quality, the scale of intervention has to increase, if the policy maker aims to contain the output drop at any given level. In the next section, we calibrate our model and show that the slippery slope of intervention is quantitatively important.

Discussion: Low productivity firms vs. zombie firms. Zombie firms are permanently impaired firms whose operation relies on external financial resources (Caballero et al., 2008; Acharya et al., 2019, 2021). Type- L firms are not zombies. In a crisis, both type- H and L firms may over-invest and can thus be viewed as engaging in a negative-NPV transaction, but after they survive the crisis,

these firms will invest at the efficient level in normal times, guided by their Tobin's q (capital value), and use their capital to produce. Firms' problem in crises is of a temporary nature. We model a liquidity crisis, not a solvency crisis. Relative to the literature on zombie firms, our paper offers a different and complementary perspective on the distortions from credit intervention. Moreover, our mechanism does not rely on zombies crowding out normal firms in product or factor markets, which is a key ingredient in models on crisis cleansing effect, zombie firms, and the efficiency implications of policy intervention (e.g., Caballero and Hammour, 1994; Acharya et al., 2021).

3 Quantitative Analysis

3.1 Model Calibration

We summarize the calibration method. First, we parameterize the survival probability function as $F(x) = 1 - \exp(-\lambda_F x)$ and, for the cumulative distribution function of ζ that indexes the baseline level of survival probability, we use $H(\zeta) = 1 - \exp(-\zeta/l_\zeta)$ (with an average ζ equal to l_ζ). Including λ_F and l_ζ , we have a total of twelve parameters. The other ten parameters include the investment cost parameter θ , crisis frequency λ , debt restructuring rent β , capital depreciation rate δ , discount rate r , capital productivities A^H and A^L , entry rate of new firms η , the private-sector credit-supply limit in crises \bar{d} , and a baseline value for government credit support \bar{g} . The parameter values and moment conditions are summarized in Table 1 and Table 2, respectively.

The investment cost function controls the average investment rate of the economy, and we set the parameter θ to generate an average annual investment-to-capital ratio of 10% following the literature (e.g., Gertler et al. (2020) target a 2.5% quarterly investment-to-capital ratio).

We set the parameter governing the survival probability function, λ_F to generate an average output drop in crises that matches the average GDP decline in crises. According to Reinhart and Rogoff (2009), the peak-to-trough decline in GDP across a large sample of crises is 9.3%.

The parameter l_ζ in the distribution of ζ affects the sensitivity of firm survival to credit intervention. When l_ζ is small, there is a large density of type- L firms that rely on government funding support, so the impact of intervention on firm survival is stronger. Bartlett III and Morse (2020) used a survey of 1,000 small businesses to study how PPP loans impacted their survival likelihoods. They find that PPP application success increased a firm's medium-run survival probability by 20.5%, but only for microbusinesses (those with 1-5 employees). Kawaguchi et al. (2021) sur-

Table 1: Parameter values. This table shows the parameter values in our quantitative analysis and the corresponding calibration targets. Five model parameters are directly set by the observed data, including λ , δ , r , η , and \bar{g} . The rest of parameters, θ , λ_F , l_ζ , β , A^H , A^L , and \bar{d} are solved to match the moment targets.

Parameter	Description	Value	Data Source or Targeted Moment
θ	Investment cost	9.2	Average investment/capital ratio
λ_F	Survival rate	5.8	Average GDP drop in crises
l_ζ	Average ζ	0.16	Impact of credit intervention on firm survival
λ	Crisis frequency	0.06	Crisis frequency in the data
β	Debt restructuring rent	0.43	Average creditor recovery rate
δ	Capital depreciation rate	0.2	Capital depreciation and firm exit rate
r	Real discount rate	0.06	Average real bond return plus equity premium
A^H	Productivity of type- H firms	0.57	Average output-to-capital ratio
A^L	Productivity of type- L firms	0.15	TFP inter-quartile ratio
η	Entry rate of new firms	0.062	Firm entry rate
\bar{d}	Private-sector credit availability	0.25	Private-sector debt/GDP ratio
\bar{g}	Government credit support	0.14	Covid-19 credit support in the U.S.

Table 2: Moment matching. We report the moment matching results for calibrating θ , λ_F , l_ζ , β , A^H , A^L , and \bar{d} .

Moment Description	Model	Data
Average investment-to-capital ratio	0.1	0.1
Average GDP drop in crises	-9.3%	-9.3%
Average impact of credit intervention on firm survival likelihood	10%	10%
Average creditor recovery rate	49%	49%
Average output-to-capital ratio	45%	45%
TFP ratio between 90% and 10% percentiles	3.7	3.7
Average private-sector debt/GDP ratio	36%	36%

veyed small businesses during the Covid-19 pandemic and found that lump-sum subsidies in Japan increased firms' self-reported prospects of survival by 19%. However, there is also evidence that the impact is much smaller. For example, Hubbard and Strain (2020) used an intent-to-treat model with data on private firms from Dun & Bradstreet. They find that PPP eligibility reduced business closure odds by 0.22% with no significant effect for firms closer to the 500-employee cutoff. Given the mixed evidence, we target a 10% survival likelihood improvement.

Next, for crisis frequency λ , we map to the empirical counterpart in Taylor (2015), i.e., the frequency of crises is about 6% in a panel of 17 countries from 1800 to 2012.

The parameter β , which represents the borrowers' value after debt renegotiation and restructuring, governs the firm owners' incentive to strategically default. We set the parameter to generate

a bankruptcy recovery rate for creditors that match the empirical counterpart. Below we explain how we calculate the creditors' recovery rate in our model. For type- j firm, $j \in \{H, L\}$, there are two cases of bankruptcy: (1) Liquidity-induced bankruptcy, which happens with probability $1 - F(\zeta + x^j(\zeta))$, and the recovery rate is zero; (2) Strategic default, which happens when a firm survives the crisis and has $\zeta < \underline{\zeta}^j$, generates a recovery rate for creditor equal to $(q^j - \beta)/x^j(\zeta)$. Therefore, the creditors' recovery rate of type j firm in bankruptcy (averaging over ζ) is

$$\mathcal{R}^j = \int \left((1 - F(\zeta + x^j(\zeta))) \cdot 0 + F(\zeta + x^j(\zeta)) 1_{\zeta < \underline{\zeta}^j} \frac{q^j - \beta}{x^j(\zeta)} \right) dH(\zeta)$$

with a conditional probability mass

$$\mathcal{M}^j = \int \left((1 - F(\zeta + x^j(\zeta))) + F(\zeta + x^j(\zeta)) 1_{\zeta < \underline{\zeta}^j} \right) dH(\zeta)$$

Let w_{avg} denote the average of ω (the fraction of firms being type- H) in the model. Then total (cross-type) average creditor recovery rate is

$$\bar{R} = \frac{w_{avg} \mathcal{R}^H + (1 - w_{avg}) \mathcal{R}^L}{w_{avg} \mathcal{M}^H + (1 - w_{avg}) \mathcal{M}^L}$$

The moment target for \bar{R} is the empirical average creditor recovery in Chapter 11 bankruptcies. According to Antill (2022), average recovery rate from Moody's Ultimate Recovery database is 49%, and according to Dou et al. (2021), the average recovery rates from across different classes of creditors is 46%.²⁵ We use 49% recovery rate as our targeted value.

The parameter, δ in the model can be interpreted as the depreciation rate of capital plus exits of businesses in normal times. According to Business Dynamics Statistics, the average firm establishment exit annual shutdown rate is about 10% from 1982 to 2020. With a quarterly depreciation of 2.5% (Gertler and Karadi, 2011b), we set $\delta = 0.025 * 4 + 0.1 = 0.2$.²⁶

The discount rate r reflects how the firm owners discount cash flows. Since productive capital is a long-term asset, we consider the long-term real rate plus an unlevered equity premium. In the U.S., the historical average real return of government bond is 2.85% and the historical average real

²⁵See Row 1 of Table 1 in Antill (2022). Table 1 Panel B in Dou et al. (2021) reports junior and senior creditor recovery rates and their fractions of total debt. The weighted average is $(0.559)*0.204 + (1-0.559)*0.788 = 0.46$.

²⁶We note that a larger depreciation parameter δ in the literature is not uncommon, especially when it incorporates the business exit rate. For example, Gertler et al. (2020) set $\delta = 0.33$ to match the investment to capital ratio.

return of equity is 8.46%.²⁷ The proper discount rate on capital is the unlevered cost of equity, which is between the two discount rates, depending on average leverage. With a historical average debt/equity ratio of 0.5, the unlevered real return of equity is 6.6%.²⁸ Therefore, we set $r = 0.066$.

We choose the productivity parameters A^H and A^L jointly to generate a ratio A^H/A^L that matches the ratio between 90% and 10% percentiles of TFP distribution²⁹ and an average output-to-capital ratio close to 1/2.24 in Elenev et al. (2021).³⁰ And, we calibrate the firm entry rate to the data counterpart in Clementi and Palazzo (2016), setting $\eta = 0.062$. For \bar{d} that determines private-sector credit availability, we target the ratio of total (non-financial) private-sector debt to GDP in the data. We match the actual private-sector lending to GDP ratio in the model to the data counterpart. This ratio is on average 0.36 in the historical sample of 1952–2023.³¹

Finally, we set the baseline case \bar{g} according to the size of intervention (not total takeover, which is smaller) during Covid-19, including programs initiated by both the central bank and government in the U.S. The Fed set up around 2.3 trillion credit support for firms (c.11% of 2020 GDP) decomposed into 0.6 trillion in MSLP, 0.75 trillion in PMCCF and SMCCF, and 0.95 trillion in PPP Liquidity Facility. The total fiscal response that supports businesses is 4 trillion.³² Taken together, the amount is about 30% of the GDP in 2020. Thus, we set the ratio of credit support-to-capital stock $\bar{g} = 0.45 * 30\% \approx 0.14$ where, as previously discussed, 0.45 is the output-to-capital ratio.

Before we conclude the calibration, we define the average value of ω_t as $\bar{\omega}$, which is long-run average implied by the stationary distribution of ω_t , determined by both the drift and Poisson components of law of motion. For our simulation analysis, we will initiate ω_t at this average value.

3.2 The Trade-Off between Quantity and Quality

In Proposition 6, we show that an increase in the scale of credit intervention preserves more units of capital in crises, i.e., $\partial\Delta_t^K/\partial\bar{g} > 0$, but dampens the cleansing effect of crises, i.e., $\partial\Delta_t^\omega/\partial\bar{g} < 0$.

²⁷See Tables VII and X of Jordà et al. (2019). Note that the world-average real return of long-term government bond is 2.61%, close to the U.S. average. The world-average real equity return is 7.12%, also similar to the U.S. average.

²⁸We download the flow-of-funds data on non-financial corporate business debt (ticker “BCNSDODNS”) and equity (ticker “NCBEILQ027S”), and the average debt/equity ratio from 1951-2022 is about 0.5. We ignore tax and calculate the unlevered required return on equity as $(0.0846 + 0.5 \times 0.0285)/(1 + 0.5) = 6.6\%$.

²⁹See Appendix C for details of how we measure the TFP distribution.

³⁰An alternative target the Kaldor facts in growth models, i.e., an average capital to output ratio of 2.5.

³¹See Table “Debt of Nonfinancial Sectors, 1952–2023” under “Z.1-Financial Accounts” issued by the Fed.

³²For the decomposition of the programs offered by the Fed, see this Brookings article, “What did the Fed do in response to the Covid-19 crisis?”, and for the decomposition of fiscal response, see this IMF report, “Fiscal Monitor Database of Country Fiscal Measures in Response to the Covid-19 Pandemic”.

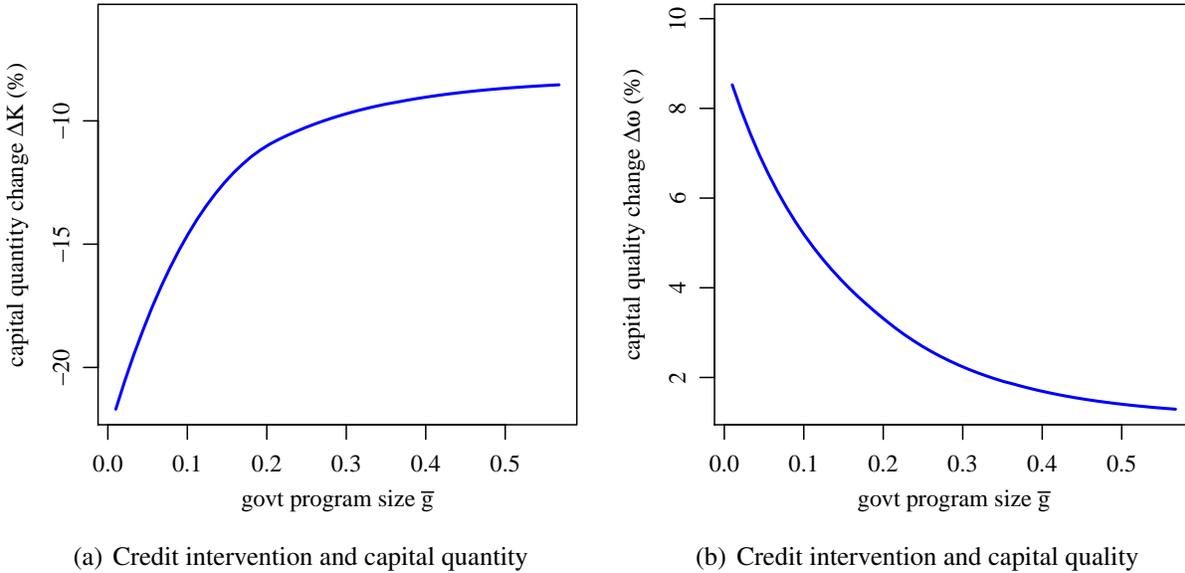


Figure 2: Credit intervention impact: Capital quantity vs. quality. This figure illustrates how \bar{g} affects ΔK_t and $\Delta \omega_t$ in a crisis. The calculation requires the pre-crisis ω_{t-} which we set to the average value $\bar{\omega}$ defined in Section 3.1.

Capital quantity, K_t , and quality, ω_t are the two state variables that drive aggregate dynamics. Figure 2 illustrates the quantity-quality trade-off. We fix the pre-crisis ω_t at the average value $\bar{\omega}$ defined in Section 3.1. In Panel A, we plot the percentage change of K_t against \bar{g} . The figure shows that as intervention changes from $\bar{g} = 0$ to $\bar{g} = 0.14$ (i.e., 30% of GDP as discussed in Section 3.1), the percentage destruction of capital shrinks from -22% to -13% . In Panel B, as intervention changes from $\bar{g} = 0$ to $\bar{g} = 0.14$, the cleansing effect $\Delta \omega_t$ falls from about 9% to 4%.

An increase in the scale of intervention, \bar{g} , improves efficiency by reducing the gap between financially constrained firms' investment and the first-best level. A higher \bar{g} can also lead to over-investment among firms that strategically default. While both forces preserve capital quantity, their impact on capital quality differs. Lemma 2 shows that there are more type- L firms that choose to strategically default than type- H firms. The calibrated parameter values satisfy Condition 1 in Section 2: Type- H firms do not strategically default, while strategic default and over-investment can happen among type- L firms. Therefore, when \bar{g} increases, its positive impact on type- H firms' survival is limited, as type- H firms do not invest beyond the first-best level. In contrast, among type- L firms that strategically default, investment always increases in \bar{g} . As a result, as the size of intervention increases, type- L firms benefit more. This dampens the cleansing effect of crises.

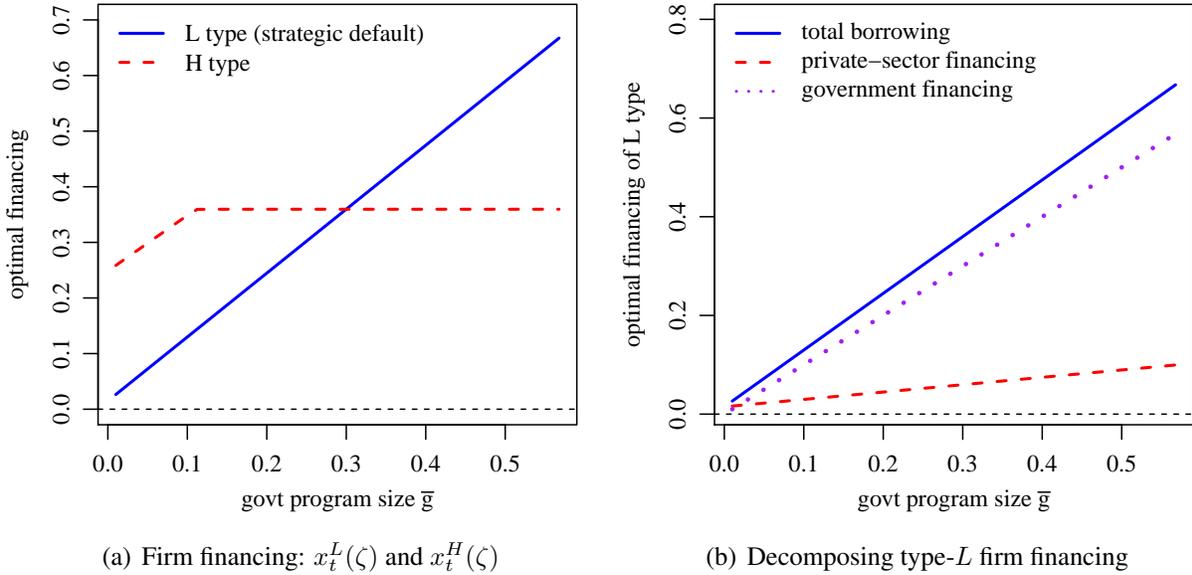


Figure 3: Credit intervention and firm financing. This figure illustrates how intervention affects the optimal amount of financing by H and L type firms, accounting for the endogenous responses of q^H and q^L . In both panels, we choose $\zeta = 0.1$, which is a case of strategic default for L-type firms. In panel (b), we decompose the actual borrowing of L-type firms into private-sector financing (which reaches the endogenous limit \hat{d}^L), and government financing (which reaches the limit \bar{g}). As \bar{g} changes, all equilibrium variables change accordingly, including capital values.

Panel A of Figure 3 illustrates this mechanism. Fixing a value of ζ , we plot the optimal financing and investment of a type- H firm (dashed line) against \bar{g} , which flattens out at the first-best level, and that of a type- L firm that strategically defaults, which is always increasing in \bar{g} . In Panel B, we decompose the type- L firm's investment into funding from the government (dashed line) and the private sector (dotted line). A higher \bar{g} allows the type- L firm to invest more through both the direct liquidity provision and crowding in the private-sector funding (see Lemma 4 in Section 2).

So far, our discussion of the welfare and efficiency implications of credit intervention, and has focused on the impact in crises, i.e., the quantity-quality trade-off, $\partial\Delta_t^K/\partial\bar{g} > 0$ and $\partial\Delta_t^\omega/\partial\bar{g} < 0$. The impact of credit intervention also spills over to the normal times. By improving survival probability for both types, credit intervention increases capital values, q^L and q^H , and thereby raises normal-time investment rates, ι^H and ι^L (see (7)), lifting upward the growth trajectory of capital quantity, K_t . Moreover, as previously discussed, the positive impact on survival probability is greater for type- L firms (which is why credit intervention dampens the cleansing effect of crises), so a higher \bar{g} increases q^L more than q^H , introducing a downward bias in the drift of ω_t .

Next, we introduce the welfare function as a criterion for the overall impact of credit intervention. At time t , the social welfare is defined as the present value of life-time consumption flows and is a function of the two state variables, K_t and ω_t . Since the economy is scalable with respect to capital, we conjecture the welfare at time t as $W(\omega_t)K_t$. It can be written as follows:

$$\mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} (\omega_s A^H + (1 - \omega_s) A^L) K_s ds - (\omega_s \iota_s^H + (1 - \omega_s) \iota_s^L) K_s ds - I_s K_{s-} dN_s \right], \quad (24)$$

where, in the integral, we record the consumption flow as the aggregate output net off goods invested in normal times and crises times, and the investment in a crisis at time s , I_s , is given by

$$I_s \equiv \omega_{s-} \int_\zeta x_s^H(\zeta) dH(\zeta) + (1 - \omega_{s-}) \int_\zeta x_s^L(\zeta) dH(\zeta). \quad (25)$$

In the stationary equilibrium, the K_t -scaled welfare function $W(\omega)$ satisfies the following ordinary differential equation:

$$\begin{aligned} rW(\omega) &= \omega A^H + (1 - \omega) A^L - (\omega \iota^H + (1 - \omega) \iota^L) + W(\omega) \mu_K(\omega) + W'(\omega) \mu_\omega(\omega) \\ &\quad - \lambda I(\omega) + \lambda [W(\omega + \Delta^\omega(\omega)) (1 + \Delta^K(\omega)) - W(\omega)] \end{aligned} \quad (26)$$

We numerically solve for the welfare function $W(\omega)$ as determined by equation (26) and illustrate the impact of \bar{g} on welfare in Figure 4. To highlight the dependence of welfare on \bar{g} , we write the welfare as $W(\omega; \bar{g})$. Figure 4(a) shows the impact of \bar{g} on the average firm quality $\bar{\omega}(\bar{g})$, where the average is calculated based on simulated path of the economy. We find that as the government expands its scale of intervention, the average firm quality declines.

Figure 4(b) plots percentage improvement in welfare from the laissez-faire economy to the intervened economy, $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$, which increases in \bar{g} for small \bar{g} , but decreases once \bar{g} passes a threshold. The welfare curve is upward-sloping when the scale of intervention is low. The marginal improvement of welfare due to type- H firms' efficient investment dominates the loss from wasteful investment by type- L firms that strategically default. Therefore, a timid intervention almost guarantees a positive contribution to welfare at the margin. In Corollary 1 in Section 2, we show that excessive credit intervention can destroy welfare, which is on the downward-sloping component of the welfare curve. The over-investment by type- L firms comes at the expense of aggregate consumption, so even though the total capital stock, K_t , grows faster,

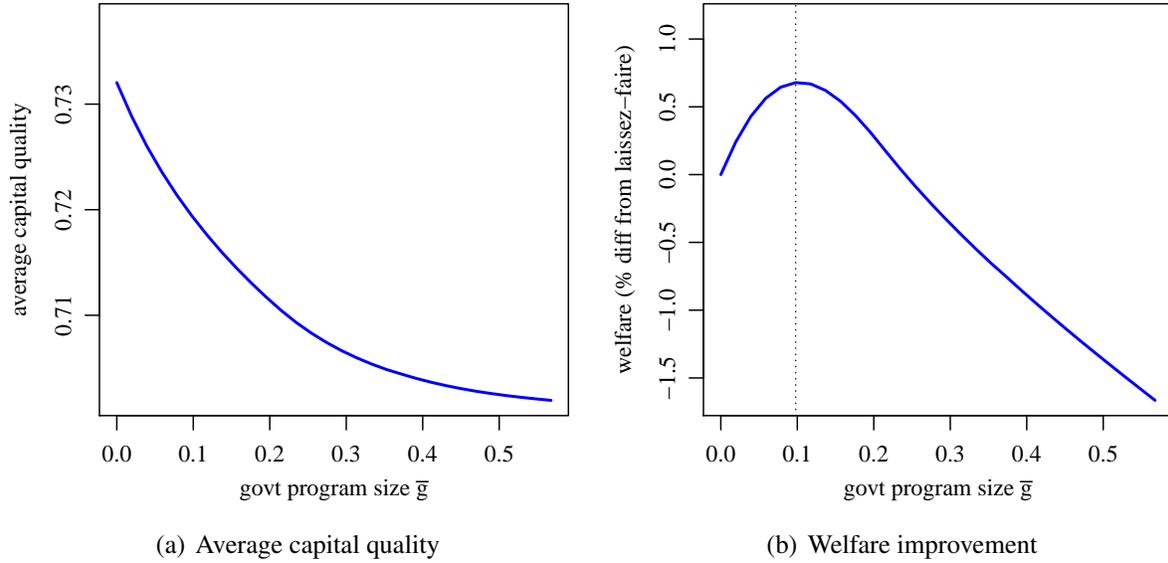


Figure 4: Credit intervention and welfare. In panel (a), we plot the average firm quality $\bar{\omega}$ as a function of government intervention \bar{g} . For each \bar{g} , we solve the model again and calculate the average of simulated ω_t as $\bar{\omega}(\bar{g})$. In panel (b), we show the welfare difference $W(\bar{\omega}(\bar{g}); \bar{g})/W(\bar{\omega}(0); 0) - 1$ as a function of government intervention \bar{g} .

the social welfare, which is the present value of households' life-time consumption, declines. Intuitively, the optimal intervention balances the benefit from relaxing the financial constraint on under-investing firms and the cost of exacerbating over-investment among type- L firms that strategically default. The vertical dotted line marks the optimal intervention size, which is equivalent to 22% of GDP. For comparison, in Section 3.1, we document that the total scale of credit support from the government and central bank in the U.S. during Covid-19 crisis is about 30% of GDP.

In the stationary equilibrium, the scale of intervention, \bar{g} , is constant, chosen at $t = 0$. In Panel A of Figure 5, we allow the government to optimize \bar{g} at $t = 0$, i.e.,

$$g^*(\omega_0) = \arg \max_{\bar{g}} W(\omega_0; \bar{g})$$

and plot the optimal \bar{g} against ω_0 (solid line). The curve is upward-sloping. Intuitively, at the left end where the economy is dominated with type- L firms, the government optimally restricts funding support because a large scale intervention is likely to result in type- L firms' over-investment. In contrast, near the right end where type- H firms dominate, the optimal scale of intervention is high. When the economy starts with a larger fraction of firms being type- H (i.e., ω_0 is higher), the

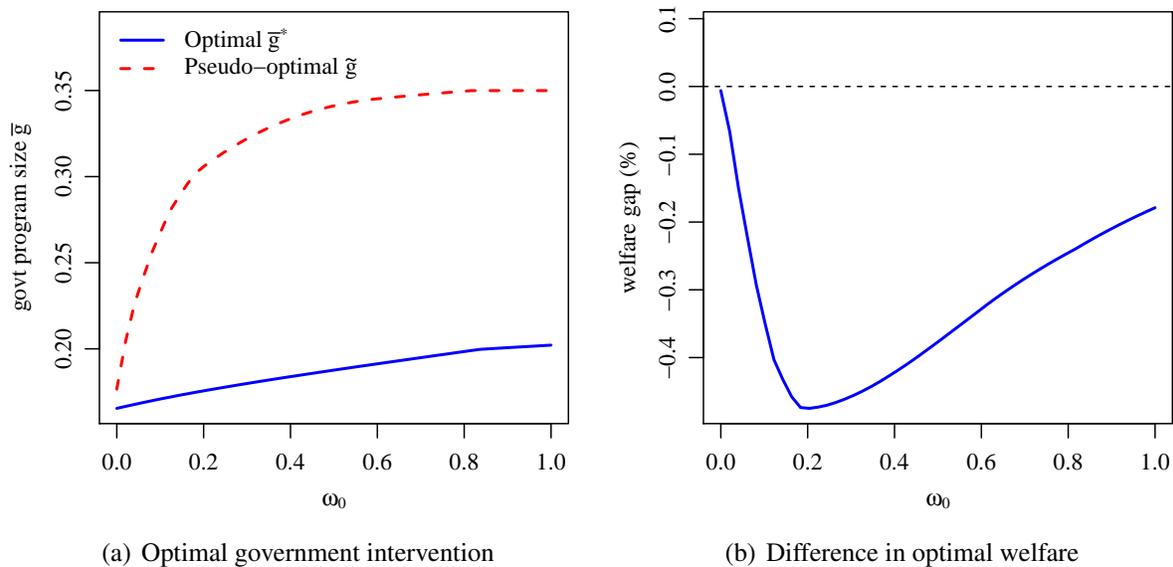


Figure 5: Optimal intervention and welfare. In panel (a), we plot the optimal intervention $\bar{g}^*(\omega_0)$ as a function of the initial state ω_0 , and the pseudo-optimal policy $\tilde{g}(\omega_0)$ that ignores the impact of intervention on firm quality change, treating $\Delta^\omega(\omega; \bar{g})$ as $\Delta^\omega(\omega; 0)$ for all \bar{g} . In panel (b), we show the percentage decline of social welfare due to using the pseudo-optimal policy instead of the optimal policy.

planner can focus more on the efficiency gain from addressing type- H firms' under-investment.

The mechanism in our model has quantitatively important implications on policy making. The dashed line in Panel A of Figure 5 plots a “pseudo-optimal” \bar{g} that is chosen at $t = 0$ without considering the negative impact of credit intervention on capital quality, ω_t . Specifically, when solving the welfare function given by (26), the planner mistakenly replaces $\Delta^\omega(\omega; \bar{g})$ with $\Delta^\omega(\omega; \bar{g} = 0)$. The resultant “pseudo welfare function”, denoted by $\tilde{W}(\omega; \bar{g})$, represents a policy-making criterion that ignores the key mechanism in our paper—credit intervention dampens the cleansing effect of crises—and thus only focuses on the positive impact on capital quantity. Formally, the associated “pseudo-optimal scale of intervention” is given by

$$\tilde{g}^*(\omega_0) = \arg \max_{\bar{g}} \tilde{W}(\omega_0; \bar{g})$$

As shown in Panel A of Figure 5, ignoring the policy impact on capital quality leads to intervention (dashed line) that is almost double the size of optimal intervention (solid line). In Panel B of Figure 5, we plot the percentage decline of welfare from using $\tilde{g}^*(\omega_0)$ rather than the optimal policy

$g^*(\omega_0)$.³³ Ignoring the impact of credit intervention on capital quality translates into a sizeable welfare loss. The welfare cost is largest at the intermediate values of ω_0 .

3.3 The Slippery Slope of Credit Intervention

The downward bias in firm quality brought by credit intervention generates a slippery slope of intervention. We first show that the impact of credit intervention on ω_t persists over time. Therefore, intervention in the current crisis leads to a lower ω_{t-} entering into the next crisis. Accordingly to Proposition 7, a lower ω_{t-} translates into a greater scale of intervention if the policy maker's goal is to contain the output drop to a certain level. Therefore, in equilibrium, credit intervention in the current crisis begets interventions of greater scales in future crises.

As our focus shifts towards the dynamics of intervention, a key issue to address is agents' expectation of the policy plan. To clearly illustrate the mechanism, we focus on the *forward propagation* of intervention impact: In the current crisis, intervention dampens the cleansing effect, reducing ω_t , and such reduction affects the scale of intervention in future crises. Agents' expectation of the dynamic policy plan confounds the mechanism by introducing a *backward propagation* of intervention impact: agents' current behavior varies with their expectation of intervention in future crises. Agents' expectation enters into the equilibrium conditions only through the capital values, q^H and q^L . We shut down this expectation channel by solving q^H and q^L under $\bar{g} = 0$. In Appendix B.1, we consider a fully dynamic model where the government optimally adjusts the scale of intervention in every crisis, and agents, under rational expectation, have perfect knowledge of the policy plan so both the forward and backward propagation are active.

The distortionary effects of credit intervention on firm quality distribution is very persistent. In Panel A of Figure 6, we plot ω_t ten years ($t = 10$) after the current crisis ($t = 0$) against the intervention size \bar{g} at $t = 0$. As in Section 3.2, the government sets a constant scale of intervention at $t = 0$. We fix the starting firm quality ω_0 at its deterministic steady state in the economy with $\bar{g} = 0$. An increase of credit intervention \bar{g} from 0 to 0.14 (intervention/GDP about 30%, which is our baseline calibration) causes ω_t to decline by around 0.02 ten years later, which is a significant decline in the percentage of firms being type- H from around 75.1% to 73.1%. Note that since we shut down agents' expectation of intervention in q^H and q^L , the persistent impact of intervention

³³The curve starts at $\omega_0 = 0$ as it is an absorbing state where the economy is populated by only type- L firms and $\omega_t = \omega_0 = 0$ so credit intervention cannot change ω_t . Note that the other extreme, i.e., $\omega_0 = 1$ where the economy has only type- H firms, is not an absorbing state due to the entry of type- L firms.

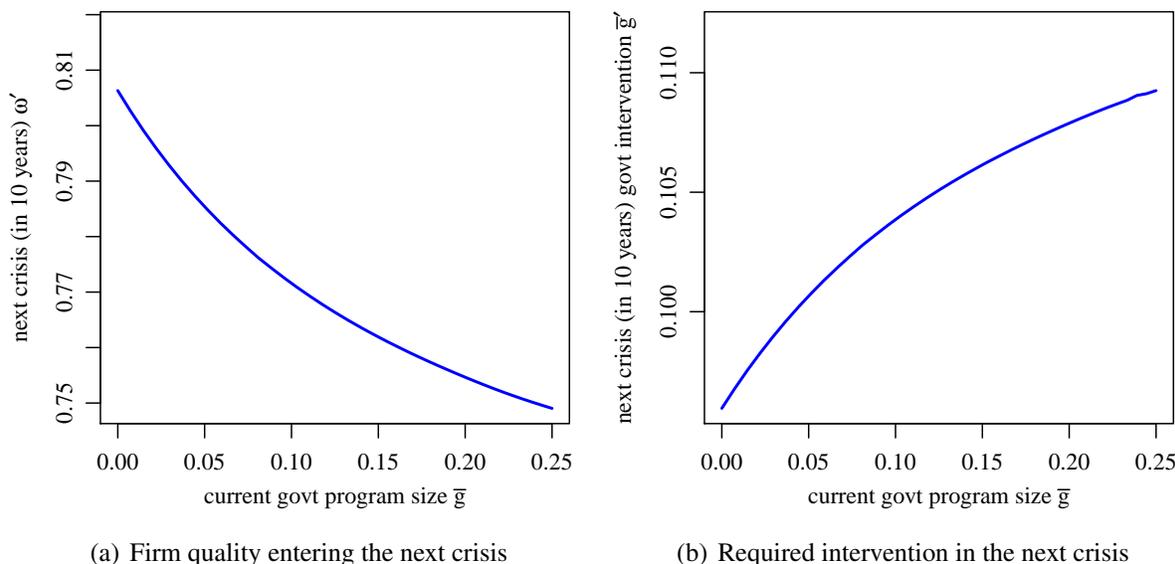


Figure 6: Intervention pass-through across crises. We show how \bar{g} in the current crisis affects capital quality entering the next crisis, ω' , and intervention needed, \bar{g}' , in the next crisis to contain output drop within -10% . The next crisis happens ten years after the current one. Agents expect no intervention (pass-through is only due to forward propagation). The current crisis happens at ω equal to the average value of ω_t in the laissez-faire economy.

on ω_t does not come from policy distortions in agents' normal-time investments and the drift of ω_t . The persistent impact on ω_t is purely generated by the reduction of ω_t in the current crisis at $t = 0$.

Accordingly to Proposition 7, such deterioration of capital quality in the future translates into a greater scale of intervention that is necessary for containing the output drop to a certain level. In Panel B of Figure 6, we consider a crisis that happens ten years from now and plot the necessary scale of intervention against the current scale of intervention. An increase of current intervention \bar{g} from 0 to 0.14 leads to an increase of intervention scale from 0.107 in the next crisis to 0.112, so the pass-through rate is about 4%, i.e., each one dollar of intervention per unit of capital in the current crisis generates 4 cents extra intervention per unit of capital in the next crisis should it happen ten years later. If we take into account the growth of capital stock over the ten-year period, the inter-crisis pass-through rate in the dollar amount is even greater.

We further illustrate the slippery slope of credit intervention with a quarterly simulation of two crises, one in Q1 of the first year and the other in Q1 of the tenth year with \bar{g} equal to 0.14 following our baseline calibration in Section 3.1. In Panel A of Figure 7, we compare the paths of ω_t in the simulation and in an economy without intervention, both starting from the average

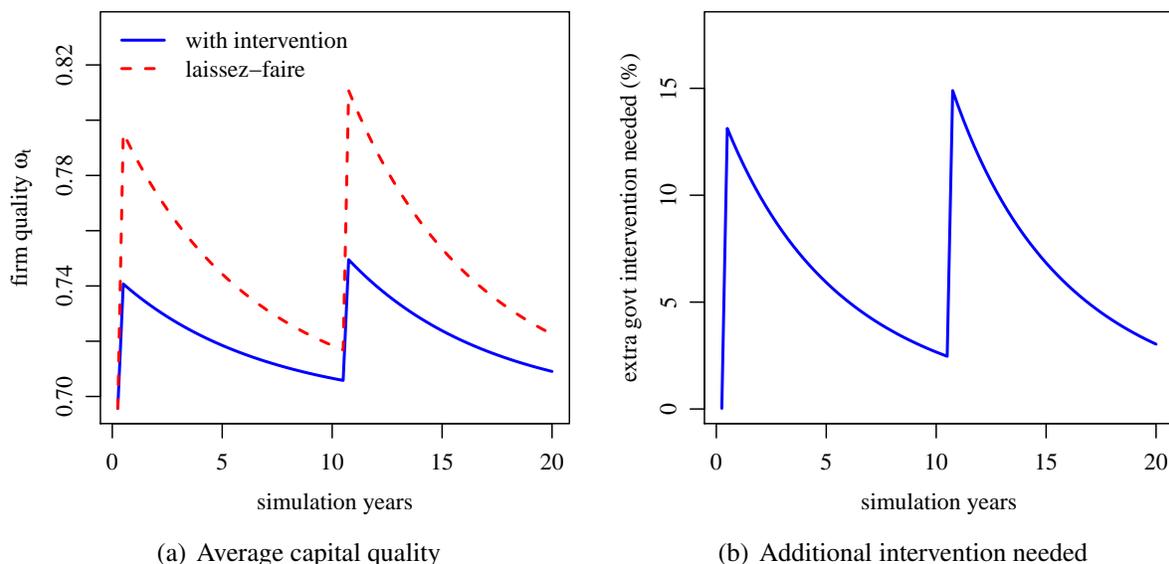


Figure 7: The slippery slope of intervention: Simulated paths. We compare the quarterly simulation of two economies, the laissez-faire economy and the intervened economy without agents expectation of intervention, where one crisis happens in Q1 of the first year and another crisis in Q1 of the tenth year. Both simulations start at the average ω_t in the laissez-faire economy. In Panel (a), we plot the path of capital quality, ω_t . In Panel (b), for both economies, we calculate the amount of intervention required to contain output drop within -10% if a crisis happens over the next instant, and then we plot the percentage increase from the laissez-faire economy to the intervened economy.

value of ω_t in the economy without intervention. After the first crisis, ω_t jumps upward due to the cleansing effect, but the quality wedge between the laissez-faire economy and intervened economy widens as intervention dampens the cleansing effect. Because we shut down agents' expectation of intervention, firms in both economies have the same normal-time investment rates driven by the same capital values, and ω_t in both economies converge to the same steady state over time. In spite of the normal-time convergence, the impact of intervention of ω_t is persistent, evidently shown by the sizable wedge ten years later when the second crisis hits the economy. In the second crisis, the cleansing effect increases ω_t , and the quality wedge widens again.

Next, we take as given the simulated paths of ω_t of the two economies in Panel A of Figure 7, and, at any point in time along the paths, we calculate the necessary scale of intervention were a crisis to happen in the very next instant. In Panel B of Figure 7, we show the extra amount of funding support in the intervened economy relative to that in the laissez-faire economy on percentage terms. Here the government's goal is to contain the output drop within -10% . Because the two economies

have the same drift of ω_t , the quality wedge is widest right after a crisis and narrows in normal times. Immediately after the first crisis, if another crisis were to happen, the intervened economy requires about 13% larger funding support from the government than the laissez-faire economy. The wedge shrinks in normal times but still quantitatively significant. For example, five years after the first crisis (i.e., at $t = 5$), if a crisis were to happen over the next instant dt , the intervened economy requires a 7% larger funding support from the government than the laissez-faire economy.

In sum, credit intervention biases ω_t downwards in crises. As a result, the economy enters into future crises with a smaller share of firms being type- H than the laissez-faire benchmark, so the credit support needed to contain output drop is larger. Our model generates a slippery slope of intervention, a trap of policy makers' own making: The past interventions cause the government to spend more should a crisis occur in the future. However, this policy trap can be a necessary evil because by relaxing firms' financial constraints in crises, policy interventions can improve welfare. In Appendix B.1, we show that the same pattern emerges even when the government optimally adjust \bar{g} in a fully dynamic fashion to maximize the social welfare.

4 Conclusion

To analyze the long-term consequences of credit intervention in crises, we develop a model of firm quality dynamics and highlight a trade-off between quantity and quality in determining the scale of intervention. Crises exhibit a cleansing effect: Firms with high productivity want to invest more in surviving a crisis, and they can invest more than firms with low productivity because their financing capacity is larger. Credit intervention relaxes the financial constraint for all firms and preserves the total production capacity in the economy. However, by benefiting firms with low productivity more, credit intervention dampens the cleansing effect of crises. Our model generates a slippery slope of intervention. As the current intervention biases downward the firm quality distribution, the economy enters the next crisis with lower total productivity, and an intervention of a greater scale becomes necessary. Larger interventions lead to stronger distortions, which in turn call for even larger interventions in the future. However, we show that when carefully designed, credit intervention improves welfare relative to the laissez-faire benchmark.

References

- Abel, A. B. and J. C. Eberly (1994). A unified model of investment under uncertainty. *The American Economic Review* 84(5), 1369–1384.
- Acemoglu, D., U. Akcigit, H. Alp, N. Bloom, and W. Kerr (2018, November). Innovation, reallocation, and growth. *American Economic Review* 108(11), 3450–91.
- Acharya, V., I. Drechsler, and P. Schnabl (2014). A pyrrhic victory? bank bailouts and sovereign credit risk. *Journal of Finance* 69(6), 2689–2739.
- Acharya, V. V., T. Eisert, C. Eufinger, and C. Hirsch (2019, 01). Whatever It Takes: The Real Effects of Unconventional Monetary Policy. *The Review of Financial Studies* 32(9), 3366–3411.
- Acharya, V. V., S. Lenzu, and O. Wang (2021). Zombie lending and policy traps. Working paper, New York University.
- Acharya, V. V., P. Schnabl, and G. Suarez (2013). Securitization without risk transfer. *Journal of Financial Economics* 107(3), 515–536.
- Aghion, P. and P. Bolton (1992, 07). An Incomplete Contracts Approach to Financial Contracting. *The Review of Economic Studies* 59(3), 473–494.
- Altman, E. I. (1968). Financial ratios, discriminant analysis and the prediction of corporate bankruptcy. *The journal of finance* 23(4), 589–609.
- Antill, S. (2022). Do the right firms survive bankruptcy? *Journal of Financial Economics* 144(2), 523–546.
- Antill, S. and C. Clayton (2021). Crisis interventions in corporate insolvency. Technical report, Harvard Business School and Yale School of Management.
- Araújo, A., S. Schommer, and M. Woodford (2015, January). Conventional and unconventional monetary policy with endogenous collateral constraints. *American Economic Journal: Macroeconomics* 7(1), 1–43.
- Asriyan, V., W. Fuchs, and B. Green (2019, November). Liquidity sentiments. *American Economic Review* 109(11), 3813–48.
- Asriyan, V., L. Laeven, and A. Martin (2018). Collateral booms and information depletion. Working Paper DP13340, Centre for Economic Policy Research.
- Balloch, C., S. Djankov, Juanita Gonzalez-Uribe, and D. Vayanos (2020). A restart procedure to deal with covid-19. Covid-19 in Developing Economies. CEPR E-Book.
- Banerjee, A. V. and B. Moll (2010, January). Why does misallocation persist? *American Economic Journal: Macroeconomics* 2(1), 189–206.
- Bartik, A. W., Z. B. Cullen, E. L. Glaeser, M. Luca, C. T. Stanton, and A. Sunderam (2020). When should public programs be privately administered? theory and evidence from the paycheck protection program. Working Paper 27623, National Bureau of Economic Research.
- Bartlett, Robert P, I. and A. Morse (2020, July). Small business survival capabilities and policy effectiveness: Evidence from oakland. Working Paper 27629, National Bureau of Economic Research.
- Bartlett III, R. P. and A. Morse (2020). Small business survival capabilities and policy effectiveness: Evidence from oakland. Technical report, National Bureau of Economic Research.

- Bassetto, M. and W. Cui (2020, December). A Ramsey Theory of Financial Distortions. Working Papers 775, Federal Reserve Bank of Minneapolis.
- Bebchuk, L. A. and I. Goldstein (2011). Self-fulfilling credit market freezes. *The Review of Financial Studies* 24(11), 3519–3555.
- Bernanke, B. and C. S. Lown (1991). The credit crunch. *Brookings Papers on Economic Activity* 22(2), 205–248.
- Bigio, S. (2015, June). Endogenous liquidity and the business cycle. *American Economic Review* 105(6), 1883–1927.
- Bolton, P., T. Santos, and J. A. Scheinkman (2016). Cream-skimming in financial markets. *The Journal of Finance* 71(2), 709–736.
- Bolton, P. and D. S. Scharfstein (1990). A theory of predation based on agency problems in financial contracting. *The American Economic Review* 80(1), 93–106.
- Bouvard, M., P. Chaigneau, and A. Motta (2015). Transparency in the financial system: Rollover risk and crises. *The Journal of Finance* 70(4), 1805–1837.
- Boyarchenko, N. (2012). Ambiguity shifts and the 2007/2008 financial crisis. *Journal of Monetary Economics* 59(5), 493–507. Carnegie-NYU-Rochester Conference Series on Public Policy - Robust Macroeconomic Policy at Carnegie Mellon University on November 11-12, 2011.
- Bruhn, M., A. Demirgüç-Kunt, and D. Singer (2023). Competition and firm recovery post-covid-19. *Small Business Economics*, 1–32.
- Brunnermeier, M. and A. Krishnamurthy (2020). Corporate debt overhang and credit policy. BPEA conference.
- Brunnermeier, M. K. (2009, March). Deciphering the liquidity and credit crunch 2007-2008. *Journal of Economic Perspectives* 23(1), 77–100.
- Brunnermeier, M. K., L. Garicano, P. R. Lane, M. Pagano, R. Reis, T. Santos, D. Thesmar, S. Van Nieuwerburgh, and D. Vayanos (2016, May). The sovereign-bank diabolic loop and esbies. *American Economic Review* 106(5), 508–12.
- Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *The American Economic Review* 104(2), 379–421.
- Caballero, R. J. and M. L. Hammour (1994). The cleansing effect of recessions. *The American Economic Review* 84(5), 1350–1368.
- Caballero, R. J., T. Hoshi, and A. K. Kashyap (2008). Zombie lending and depressed restructuring in japan. *American economic review* 98(5), 1943–1977.
- Caballero, R. J. and A. Simsek (2013). Fire sales in a model of complexity. *The Journal of Finance* 68(6), 2549–2587.
- Calomiris, C. W., M. Flandreau, and L. Laeven (2016). Political foundations of the lender of last resort: A global historical narrative. *Journal of Financial Intermediation* 28, 48 – 65. Rules for the Lender of Last Resort.
- Caramp, N. (2017). Sowing the seeds of financial crises: Endogenous asset creation and adverse selection. Technical report, University of California, Davis.

- Chari, V. V., A. Shourideh, and A. Zetlin-Jones (2014, December). Reputation and persistence of adverse selection in secondary loan markets. *American Economic Review* 104(12), 4027–70.
- Chemla, G. and C. A. Hennessy (2014). Skin in the game and moral hazard. *The Journal of Finance* 69(4), 1597–1641.
- Clementi, G. L. and B. Palazzo (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics* 8(3), 1–41.
- Crouzet, N. and F. Tourre (2020). Can the cure kill the patient? corporate credit interventions and debt overhang. Working paper, Federal Reserve Bank of Chicago, Northwestern University, and Copenhagen Business School.
- Cúrdia, V. and M. Woodford (2011). The central-bank balance sheet as an instrument of monetary policy. *Journal of Monetary Economics* 58(1), 54 – 79. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.
- Daley, B., B. Green, and V. Vanasco (2020). Securitization, ratings, and credit supply. *The Journal of Finance* 75(2), 1037–1082.
- David, J. M., L. Schmid, and D. Zeke (2018). Risk-adjusted capital allocation and misallocation. Working paper,, University of Southern California.
- David, J. M. and D. Zeke (2021). Risk-taking, capital allocation and optimal monetary policy. Working paper, Federal Reserve Bank of Chicago and University of Southern California.
- De Meza, D. and D. C. Webb (1987). Too much investment: a problem of asymmetric information. *The quarterly journal of economics* 102(2), 281–292.
- Del Negro, M., G. Eggertsson, A. Ferrero, and N. Kiyotaki (2017). The great escape? A quantitative evaluation of the Fed’s liquidity facilities. *The American Economic Review* 107(3), 824–857.
- Denes, M., S. Lagaras, and M. Tsoutsoura (2021, 01). First come, first served: The timing of government support and its impact on firms. *SSRN Electronic Journal*.
- Dörr, J. O., G. Licht, and S. Murmann (2022). Small firms and the covid-19 insolvency gap. *Small Business Economics* 58(2), 887–917.
- Dou, W., Y. Ji, D. Tian, and P. Wang (2020). Asset pricing with misallocation. Working paper, Peking University, HKUST, and University of Pennsylvania.
- Dou, W. W., L. A. Taylor, W. Wang, and W. Wang (2021). Dissecting bankruptcy frictions. *Journal of Financial Economics* 142(3), 975–1000.
- Drechsler, I. (2013). Uncertainty, time-varying fear, and asset prices. *The Journal of Finance* 68(5), 1843–1889.
- Eberly, J. C. and N. Wang (2008). Reallocating and pricing illiquid capital: Two productive trees. Working paper, Columbia Business School and Kellogg School of Management.
- Eisfeldt, A. (2004). Endogenous liquidity in asset markets. *The Journal of Finance* 59(1), 1–30.
- Eisfeldt, A. and A. Rampini (2006). Capital reallocation and liquidity. *Journal of Monetary Economics* 53(3), 369–399.

- Eisfeldt, A. and A. Rampini (2008). Managerial incentives, capital reallocation, and the business cycle. *Journal of Financial Economics* 87(1), 177–199.
- Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2020). Can the covid bailouts save the economy? *Economic Policy*.
- Elenev, V., T. Landvoigt, and S. Van Nieuwerburgh (2021). A macroeconomic model with financially constrained producers and intermediaries. *Econometrica* 89(3), 1361–1418.
- English, W. B. and J. N. Liang (2020). Designing the main street lending program: Challenges and options. Working Paper 64, The Hutchins Center on Fiscal and Monetary Policy.
- Falato, A., I. Goldstein, and A. Hortaçsu (2020, July). Financial fragility in the covid-19 crisis: The case of investment funds in corporate bond markets. Working Paper 27559, National Bureau of Economic Research.
- Farboodi, M. and P. Kondor (2021). Cleansing by tight credit: Rational cycles and endogenous lending standards. Technical report, MIT and LSE.
- Faria-e-Castro, M., J. Martinez, and T. Philippon (2016, 12). Runs versus Lemons: Information Disclosure and Fiscal Capacity. *The Review of Economic Studies* 84(4), 1683–1707.
- Fuchs, W., B. Green, and D. Papanikolaou (2016). Adverse selection, slow-moving capital, and misallocation. *Journal of Financial Economics* 120(2), 286 – 308.
- Fukui, M. (2018). Asset quality cycles. *Journal of Monetary Economics* 95, 97–108.
- Geanakoplos, J. (2010, April). *The Leverage Cycle*, pp. 1–65. University of Chicago Press.
- Gertler, M. and P. Karadi (2011a). A model of unconventional monetary policy. *Journal of Monetary Economics* 58(1), 17 – 34. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010.
- Gertler, M. and P. Karadi (2011b). A model of unconventional monetary policy. *Journal of Monetary Economics* 58(1), 17–34.
- Gertler, M. and N. Kiyotaki (2010). Financial intermediation and credit policy in business cycle analysis. *Handbook of Monetary Economics* 3(3), 547–599.
- Gertler, M., N. Kiyotaki, and A. Prestipino (2020). A macroeconomic model with financial panics. *The Review of Economic Studies* 87(1), 240–288.
- Gertler, M., N. Kiyotaki, and A. Queralto (2012). Financial crises, bank risk exposure and government financial policy. *Journal of Monetary Economics* 59, S17 – S34. Supplement issue:October 15-16 2010 Research Conference on 'Directions for Macroeconomics: What did we Learn from the Economic Crises' Sponsored by the Swiss National Bank (<http://www.snb.ch>).
- Giannetti, M. and F. Saidi (2019). Shock propagation and banking structure. *The Review of Financial Studies* 32(7), 2499–2540.
- Gilchrist, S., J. W. Sim, and E. Zakrajšek (2013). Misallocation and financial market frictions: Some direct evidence from the dispersion in borrowing costs. *Review of Economic Dynamics* 16(1), 159–176. Special issue: Misallocation and Productivity.

- Goldstein, I. and Y. Leitner (2018). Stress tests and information disclosure. *Journal of Economic Theory* 177, 34–69.
- Goldstein, I. and H. Sapra (2014). Should banks' stress test results be disclosed? an analysis of the costs and benefits. *Foundations and Trends in Finance* 8(1), 1–54.
- Goodhart, C. A. (1998). *The Evolution of Central Banks*,. Cambridge, MA.
- Gorton, G., T. Laarits, and A. Metrick (2017). The run on repo and the Fed's response. *Working paper*.
- Gorton, G. and G. Ordoñez (2014, February). Collateral crises. *American Economic Review* 104(2), 343–78.
- Gourio, F. (2012, May). Disaster risk and business cycles. *American Economic Review* 102(6), 2734–66.
- Greenwood, R., B. Iverson, and D. Thesmar (2020). Sizing up corporate restructuring in the covid crisis. *Brookings Papers of Economic Activity*, forthcoming.
- Griffin, J. M., S. Kruger, and P. Mahajan (2023). Did fintech lenders facilitate ppp fraud? *The Journal of Finance* 78(3), 1777–1827.
- Haddad, V., A. Moreira, and T. Muir (2020). When selling becomes viral: Disruptions in debt markets in the covid-19 crisis and the feds response. Working Paper 27168, National Bureau of Economic Research.
- Halling, M., J. Yu, and J. Zechner (2020). How Did COVID-19 Affect Firms' Access to Public Capital Markets? *The Review of Corporate Finance Studies* 9(3), 501–533.
- Hanson, S., J. Stein, A. Sunderman, and E. Zwick (2020). Business credit programs in the pandemic era. *Brookings Papers of Economic Activity*.
- Hanson, S. G., D. S. Scharfstein, and A. Sunderam (2018). Social Risk, Fiscal Risk, and the Portfolio of Government Programs. *The Review of Financial Studies* 32(6), 2341–2382.
- Hanson, S. G., J. C. Stein, A. Sunderam, and E. Zwick (2020). Business credit programs in the pandemic era. *Brookings Papers on Economic Activity Fall*, 3–60.
- Hart, O. and J. Moore (1994). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics* 109(4), 841–879.
- Hart, O. and J. Moore (1998, 02). Default and Renegotiation: A Dynamic Model of Debt*. *The Quarterly Journal of Economics* 113(1), 1–41.
- Hayashi, F. (1982). Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica* 50(1), 213–224.
- Hayek, F. (1945). The use of knowledge in society. *American Economic Review* 35, 519–530.
- Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy* 106(1), 1–40.
- Hu, Y. (2017). A dynamic theory of bank lending, firm entry, and investment fluctuations. Technical report, University of North Carolina at Chapel Hill.
- Hubbard, R. G. and M. R. Strain (2020, October). Has the paycheck protection program succeeded? Working Paper 28032, National Bureau of Economic Research.

- İmrohoroğlu, A. and Ş. Tüzel (2014). Firm-level productivity, risk, and return. *Management Science* 60(8), 2073–2090.
- Jermann, U. and V. Quadrini (2012, February). Macroeconomic effects of financial shocks. *American Economic Review* 102(1), 238–71.
- Jordà, Ò., K. Knoll, D. Kuvshinov, M. Schularick, and A. M. Taylor (2019). The rate of return on everything, 1870–2015. *The Quarterly Journal of Economics* 134(3), 1225–1298.
- Jovanovic, B. and P. L. Rousseau (2008, 11). Mergers as Reallocation. *The Review of Economics and Statistics* 90(4), 765–776.
- Kacperczyk, M. and P. Schnabl (2010, March). When safe proved risky: Commercial paper during the financial crisis of 2007-2009. *Journal of Economic Perspectives* 24(1), 29–50.
- Kargar, M., B. Lester, D. Lindsay, S. Liu, P.-O. Weill, and D. Zúñiga (2020). Corporate bond liquidity during the covid-19 crisis. Working Paper 27355, National Bureau of Economic Research.
- Kawaguchi, K., N. Kodama, and M. Tanaka (2021). Small business under the covid-19 crisis: Expected short- and medium-run effects of anti-contagion and economic policies. *Journal of the Japanese and International Economies* 61, 101138.
- Kehoe, T. J. and D. K. Levine (1993). Debt-constrained asset markets. *The Review of Economic Studies* 60(4), 865–888.
- Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy* 105(2), 211–248.
- Kogan, L., D. Papanikolaou, A. Seru, and N. Stoffman (2017, 03). Technological Innovation, Resource Allocation, and Growth*. *The Quarterly Journal of Economics* 132(2), 665–712.
- Koijen, R. S., F. Koulischer, B. Nguyen, and M. Yogo (2020). Inspecting the mechanism of quantitative easing in the euro area. *Journal of Financial Economics forthcoming*.
- Krishnamurthy, A. (2010, March). How debt markets have malfunctioned in the crisis. *Journal of Economic Perspectives* 24(1), 3–28.
- Kurlat, P. (2013, June). Lemons markets and the transmission of aggregate shocks. *American Economic Review* 103(4), 1463–89.
- Kurtzman, R. and D. Zeke (2020). Misallocation costs of digging deeper into the central bank toolkit. *Review of Economic Dynamics* 38, 94–126.
- Lee, M. J. and D. Neuhann (2021). A dynamic theory of collateral quality and long-term interventions. Technical report, Federal Reserve Bank of New York and University of Texas at Austin.
- Li, S., T. M. Whited, and Y. Wu (2016, 06). Collateral, Taxes, and Leverage. *The Review of Financial Studies* 29(6), 1453–1500.
- Liu, X. (2016). Interbank market freezes and creditor runs. *The Review of financial studies* 29(7), 1860–1910.
- Lucas, D. (2012). Valuation of government policies and projects. *Annual Review of Financial Economics* 4(1), 39–58.
- Lucas, D. (2016). Credit policy as fiscal policy. *Brookings Papers on Economic Activity* (1), 1–57.

- Lynch, J. (2021). Bounded support: Success and limitations of liquidity support during times of crisis. Technical report, Ohio State University.
- Ma, Y., K. Xiao, and Y. Zeng (2020). Mutual fund liquidity transformation and reverse flight to liquidity. Working paper, Columbia University and University of Pennsylvania.
- Midrigan, V. and D. Y. Xu (2014, February). Finance and misallocation: Evidence from plant-level data. *American Economic Review* 104(2), 422–58.
- Moll, B. (2014, October). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review* 104(10), 3186–3221.
- Moreira, A. and A. Savov (2017). The macroeconomics of shadow banking. *Journal of Finance* 72(6), 2381–2432.
- Muzi, S., F. Jolevski, K. Ueda, and D. Viganola (2023). Productivity and firm exit during the covid-19 crisis: Cross-country evidence. *Small Business Economics* 60(4), 1719–1760.
- Neuhann, D. (2018). Inefficient asset price booms. Technical report, University of Texas at Austin.
- Olley, S. and A. Pakes (1992). The dynamics of productivity in the telecommunications equipment industry.
- Papoutsis, M., M. Piazzesi, and M. Schneider (2021). How unconventional is green monetary policy? Working paper, European Central Bank and Stanford University.
- Philippon, T. and P. Schnabl (2013). Efficient recapitalization. *The Journal of Finance* 68(1), 1–42.
- Ramey, V. A. and M. D. Shapiro (1998). Costly capital reallocation and the effects of government spending. *Carnegie-Rochester Conference Series on Public Policy* 48, 145–194.
- Rampini, A. A. and S. Viswanathan (2010). Collateral, risk management, and the distribution of debt capacity. *The Journal of Finance* 65(6), 2293–2322.
- Reinhart, C. M. and K. S. Rogoff (2009). *This time is different: Eight centuries of financial folly*. Princeton University Press.
- Shapiro, J. and D. Skeie (2015). Information Management in Banking Crises. *Review of Financial Studies* 28(8), 2322–2363.
- Stiglitz, J. E. and A. Weiss (1981). Credit rationing in markets with imperfect information. *The American economic review* 71(3), 393–410.
- Taylor, A. M. (2015). Credit, financial stability, and the macroeconomy. *Annu. Rev. Econ.* 7(1), 309–339.
- Van Nieuwerburgh, S. and L. Veldkamp (2009). Information immobility and the home bias puzzle. *The Journal of Finance* 64(3), 1187–1215.
- Vanasco, V. (2017). The downside of asset screening for market liquidity. *The Journal of Finance* 72(5), 1937–1982.
- Wachter, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance* 68(3), 987–1035.
- Wang, J., J. Yang, B. C. Iverson, and R. Kluender (2020). Bankruptcy and the covid-19 crisis.
- Williams, B. (2015). Stress tests and bank portfolio choice. Technical report, New York University.
- Zryumov, P. (2015). Dynamic adverse selection: Time varying market conditions and endogenous entry. Technical report, University of Rochester, Simon Business School.

Internet Appendix

“Firm Quality Dynamics and the Slippery Slope of Credit Intervention”

A Proofs

Proof of Lemma 1. See proof in the main text.

Proof of Lemma 2. Using the creditors’ break-even condition (11) to substitute out the interest rate in the value under debt repayment, we obtain

$$q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta) = q_t^j - \frac{x_t^j(\zeta)}{F(x_t^j(\zeta) + \zeta)}. \quad (\text{A1})$$

Consider two cases. First, the private-market funding constraint binds, so $x_t^j(\zeta) = \bar{d}$. We have

$$q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta) = q_t^j - \frac{\bar{d}}{F(\bar{d} + \zeta)}, \quad (\text{A2})$$

which increases in ζ because $F(\cdot)$ is an increasing function. Second, the private-market funding constraint does not bind, so $x_t^j(\zeta) = \bar{\zeta}_t^j - \zeta$. We have

$$q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta) = q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)}, \quad (\text{A3})$$

which increases in ζ . The value under debt repayment is monotonic in ζ , so there exists a unique $\underline{\zeta}_t^j$ where the value under debt repayment as a function of ζ is equal to β .

Note that so far, we have characterized the value under debt repayment for the whole space of ζ , i.e., the function $q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta)$ of ζ on the domain $\zeta \in [0, +\infty]$, and we show that this function is equal to β at a unique value $\zeta = \underline{\zeta}_t^j$. Next, we prove that this $\underline{\zeta}_t^j$ is indeed the default threshold; that is, if a firm survives, it will default if $\zeta < \underline{\zeta}_t^j$ and repay its debt if $\zeta \geq \underline{\zeta}_t^j$. First, consider $\zeta \geq \underline{\zeta}_t^j$. If $\underline{\zeta}_t^j$ is the default threshold, the creditors know it and know that the firm with $\zeta \geq \underline{\zeta}_t^j$ will not default conditional on survival. Therefore, when the firm borrows, it faces an interest rate function, $r_t^j(\zeta, x)$ given by (11) and borrow $x_t^j(\zeta)$ given by (15). After survival, the firm will not default because the value from repaying its debt, $q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta)$

is higher than β . Next, consider $\zeta < \underline{\zeta}_t^j$. We prove that a firm with $\zeta < \underline{\zeta}_t^j$ will default after survival by contradiction. Suppose that this firm does not default (i.e., $\zeta < \underline{\zeta}_t^j$ is not the criterion for default). The creditors know that this firm does not default and offer an interest rate function, $r_t^j(\zeta, x)$ given by (11). Since the firm will not default conditional on survival, it will borrow $x_t^j(\zeta)$ given by (15) before survival. After survival, its value is $q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta)$, which is below β because $\zeta < \underline{\zeta}_t^j$. This is a contradiction. Therefore, for firms with $\zeta < \underline{\zeta}_t^j$, strategic default happens. And, as previously shown, for firms with $\zeta \geq \underline{\zeta}_t^j$, debts are repaid. Therefore, we first prove that $q_t^j - (1 + r_t^j(\zeta, x_t^j(\zeta))) x_t^j(\zeta)$ is an increasing function of ζ that equals β at $\zeta = \underline{\zeta}_t^j$, and then we prove that $\underline{\zeta}_t^j$ is threshold with ζ below which a firm strategically defaults.

Next, we show that given ζ , the value under debt repayment increases in q_t^j . This is obvious when the private-market funding constraint binds, i.e., $x_t^j(\zeta) = \bar{d}$, and the value under debt repayment is $q_t^j - \frac{\bar{d}}{F(\bar{d} + \zeta)}$. When the private-market funding constraint does not bind, i.e., $x_t^j(\zeta) = \bar{\zeta}_t^j - \zeta$, we calculate the derivative of the value under debt repayment with respect to q_t^j :

$$\frac{\partial \left(q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)} \right)}{\partial q_t^j} = 1 - \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{F(\bar{\zeta}_t^j)^2} \frac{d\bar{\zeta}_t^j}{dq_t^j} \quad (\text{A4})$$

Differentiate (13) with respect to q_t^j , we obtain

$$F''(\bar{\zeta}_t^j) q_t^j \frac{d\bar{\zeta}_t^j}{dq_t^j} + F'(\bar{\zeta}_t^j) = 0, \quad (\text{A5})$$

and rearranging the equation, we solve

$$\frac{d\bar{\zeta}_t^j}{dq_t^j} = - \frac{F'(\bar{\zeta}_t^j)}{F''(\bar{\zeta}_t^j) q_t^j}. \quad (\text{A6})$$

Using this expression to substitute out $\frac{d\bar{\zeta}_t^j}{dq_t^j}$ in (A4), we obtain

$$\begin{aligned} \frac{\partial \left(q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)} \right)}{\partial q_t^j} &= 1 - \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{F(\bar{\zeta}_t^j)^2} \frac{d\bar{\zeta}_t^j}{dq_t^j} = 1 + \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{F(\bar{\zeta}_t^j)^2} \frac{F'(\bar{\zeta}_t^j)}{F''(\bar{\zeta}_t^j) q_t^j} \\ &= 1 + \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{(F(\bar{\zeta}_t^j) q_t^j)^2 F''(\bar{\zeta}_t^j)} \end{aligned} \quad (\text{A7})$$

Because $F(\cdot)$ is concave and $F(0) = 0$, we have

$$F(\bar{\zeta}_t^j) = F(\bar{\zeta}_t^j) - F(0) \geq F'(\bar{\zeta}_t^j)\bar{\zeta}_t^j$$

Therefore, we obtain

$$F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta) = F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)\bar{\zeta}_t^j + F'(\bar{\zeta}_t^j)\zeta > 0, \quad (\text{A8})$$

for all $\zeta > 0$ because $F'(\cdot) > 0$. Using this result, we have shown that

$$\frac{\partial \left(q_t^j - \frac{\bar{\zeta}_t^j - \zeta}{F(\bar{\zeta}_t^j)} \right)}{\partial q_t^j} = 1 + \frac{F(\bar{\zeta}_t^j) - F'(\bar{\zeta}_t^j)(\bar{\zeta}_t^j - \zeta)}{(F(\bar{\zeta}_t^j)q_t^j)^2 F''(\bar{\zeta}_t^j)} > 1 > 0. \quad (\text{A9})$$

In sum, we have shown that given ζ , the value under debt repayment increases in q_t^j whether the private-market funding constraint binds or not. Therefore, for a higher q_t^j , the value under debt repayment is higher for all ζ . Since the value under debt repayment is increasing in ζ , this implies that $\underline{\zeta}_t^j$ is lower when q_t^j is higher, i.e., the value under debt repayment is equal to the value under strategic default, β at a lower value of ζ . Thus, $\underline{\zeta}_t^j$ is a decreasing function of q_t^j .

Proof of Lemma 3 and 4. First, we show that the following expression is monotonic in d for any $\zeta > 0$,

$$\hat{\pi}^j(d) = F(d + \zeta + \bar{g})(q_t^j - \beta) - d$$

We note that the derivative over d is

$$\hat{\pi}^j(d)' = F'(d + \zeta + \bar{g})(q_t^j - \beta) - 1$$

Replacing $q_t^j - \beta$ with equation (17), the above derivative can be simplified as

$$\hat{\pi}^j(d)' = \frac{F'(d + \zeta + \bar{g})d}{F(d + \zeta + \bar{g})} - 1$$

By strict concavity of $F(\cdot)$, we have

$$F(d + \zeta + \bar{g}) > F'(d + \zeta + \bar{g})(d + \zeta) > F'(d + \zeta + \bar{g})d$$

Therefore, we obtain $\hat{\pi}^j(d)' < 0$ so that $\hat{\pi}^j(d)$ is a decreasing function. Since for any $\zeta > 0$, $\hat{\pi}^j(0) > 0$ while $\hat{\pi}^j(\infty) = -\infty$, a solution exists and it is unique.

Next, since $\hat{\pi}^j(d)$ increases in ζ , $\hat{d}^j(\zeta)$ as the solution to $\hat{\pi}^j(d) = 0$ increases in ζ . Similarly, $\hat{d}^j(\zeta)$ also increases in q_t^j , which implies $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$, and it increases in \bar{g} (Lemma 4).

Next, we prove that $\hat{d}^j(\zeta)$ is strictly concave in ζ . Differentiating the the creditors' break-even condition (17), we obtain:

$$F'(\hat{d}^j(\zeta) + \zeta)(\hat{d}^j(\zeta) + 1)(q_t^j - \beta) = \hat{d}^j(\zeta). \quad (\text{A10})$$

Rearranging the equation, we have

$$\hat{d}^j(\zeta) = \left(\frac{1}{F'(\hat{d}^j(\zeta) + \zeta)(q_t^j - \beta)} - 1 \right)^{-1}. \quad (\text{A11})$$

From this expression, we can see that $\hat{d}^j(\zeta)$ decreases in ζ , i.e., $\hat{d}^j(\zeta)$ is strictly concave in ζ .

Finally, we show the impact of government intervention on the interest rate. The interest-rate determination equation is

$$F(x_t^j + \zeta)(1 + r_t^j) = 1$$

Since x_t^j increases with government intervention \bar{g} , the interest rate r_t^j decreases with higher \bar{g} . This decreasing relationship is strict when $\zeta_t^j - \zeta > \bar{d} + \bar{g}$.

Proof of Proposition 1 Summarizing Lemma 1, 2, and 3, we obtain $\bar{\zeta}_t^H > \bar{\zeta}_t^L$, $\underline{\zeta}_t^H < \underline{\zeta}_t^L$, and $\hat{d}_t^H(\zeta) > \hat{d}_t^L(\zeta)$ under $q_t^H > q_t^L$.

Next, property (1) and (2) are direct results of equation (18).

Proof of Proposition 2 Results directly follow the comparison between the first-best solution in equation (14) and the optimal financing in equation (18).

Proof of Corollary 1 Let's consider the case of $\bar{g} \geq \bar{\zeta}_t^H$. In this case, government credit alone can satisfy the capacity need by the first-best solution. We find that under this scenario, by further expanding government intervention \bar{g} , the efficiency gain in Proposition 2 is not affected, but the efficiency loss in Proposition 2 is further increased. When $\bar{g} \rightarrow \infty$, the survival probabil-

ity $F(\bar{d} + \bar{g}) \rightarrow 1$, so the efficiency change is approximately $F(\bar{d} + \bar{g})q_t^j - (\bar{d} + \bar{g}) \rightarrow -\infty$. Consequently, the efficiency loss dominates the efficiency gain for a sufficiently large scale of government intervention.

Proof of Proposition 3 In the capital price equation (20), the optimal borrowing decision $x_t^j(\zeta)$ and interest rate r_t^j are both time-invariant functions of ζ and q_t^j , and the optimal investment ι_t^j is a time-invariant function of q_t^j . When q_t^j is a constant, the variables above are constant, which in turn imply a constant q_t^j according to (20). Therefore, we have shown the existence of a stationary equilibrium.

Next, we prove uniqueness. We omit the subscript t since all variables are time-invariant in the stationary equilibrium. Rewriting equation (20), we obtain

$$0 = A^j - \phi(\iota^j) + \iota^j q^j - (\delta + r)q^j + \lambda \left(\int_0^\infty F(x^j(\zeta) + \zeta) \max\{q^j - (1 + r^j)x^j(\zeta), \beta\} dH(\zeta) - q^j \right)$$

With the interest-rate determination in equation (11), we can simplify the problem as

$$0 = A^j - \phi(\iota^j) + \iota^j q^j - (\delta + r)q^j + \lambda \left(\int_0^\infty \max\{F(x^j(\zeta) + \zeta)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta)\} dH(\zeta) - q^j \right) \quad (\text{A12})$$

Note that, inside the integral in the last term, $\max\{F(x_t^j(\zeta) + \zeta)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta)\}$ is the objective function of a type- j firm with a realized ζ . When the funding constraint binds, i.e., $x_t^j(\zeta) = \bar{g} + \bar{d}$, the first term in the max operator increases in q^j and the second term is non-decreasing in q^j . When the funding constraint does not bind, the envelope theorem implies that $\max\{F(x_t^j(\zeta) + \zeta)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta)\}$ is (weakly) increasing in q^j .

Uniqueness will follow if we can prove the monotonicity over q^j in the right-hand side of the above equation. First, we analyze $-\phi(\iota_t^j) + \iota^j q^j - (\delta + r)q^j$. Note that the investment cost is

$$\phi(\iota^j) = \frac{q^j - 1}{\theta} + \frac{\theta}{2} \left(\frac{q^j - 1}{\theta} \right)^2 = \frac{1}{2\theta}(q^j)^2 - \frac{1}{2\theta}$$

Thus,

$$-\phi(\iota^j) + \iota^j q^j - (\delta + r)q^j = \frac{1}{2\theta}(q^j)^2 - (r + \delta + \frac{1}{\theta})q^j + \frac{1}{2\theta}$$

Under the condition on investment $i^j \leq r + \delta$ (i.e., investment rate lower than the total discount

rate on capital), we get

$$q^j \leq 1 + \theta(r + \delta)$$

which implies that $-\phi(\iota^j) + \iota^j q^j - (\delta + r)q^j$ strictly decreases in q^j .

Next, we discuss the crisis-investment component in the large bracket. For convenience of discussion, we rewrite the integrand as

$$\begin{aligned} & \max\{(F(x_t^j(\zeta) + \zeta) - 1)q^j - x^j(\zeta), \beta F(x_t^j(\zeta) + \zeta) - q^j\} \\ & = \mathbf{1}_{\zeta \geq \underline{\zeta}^j} \left(\max_{x \leq \bar{d} + \bar{g}} \{F(x + \zeta)q^j - x - q^j\} \right) + \mathbf{1}_{\zeta < \underline{\zeta}^j} \left(\beta F(\min\{\hat{d}^j(\zeta), \bar{d}\} + \bar{g} + \zeta) - q^j \right) \end{aligned}$$

First, consider the region of $\zeta \geq \underline{\zeta}^j$. For each x and ζ , the expression $F(x + \zeta)q^j - x - q^j$ is a decreasing function of q^j . As a result, $\max_{x \leq \bar{d} + \bar{g}} \{F(x + \zeta)q^j - x - q^j\}$ decreases in q^j .

Second, consider the region of ζ where the firm strategically defaults, i.e., $\zeta < \underline{\zeta}^j$, and the private-market debt limit is above the amount of available funding, i.e., $\hat{d}^j(\zeta) \geq \bar{d}$. The firm's value in crises, $\beta F(\min\{\hat{d}^j(\zeta), \bar{d}\} + \bar{g} + \zeta) - q^j = \beta F(\bar{d} + \bar{g} + \zeta) - q^j$ decreases in q^j .

Third, consider the region of ζ where the firm strategically defaults, i.e., $\zeta < \underline{\zeta}^j$, the funding constraint does not bind, i.e., $\hat{d}^j(\zeta) < \bar{d}$, and the firm over-invests. We calculate the derivative of \hat{d}^j over q^j . Taking derivative over q^j on equation (17),

$$\begin{aligned} & F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta) \frac{\partial \hat{d}^j(\zeta)}{\partial q^j} + F(\hat{d}^j(\zeta) + \bar{g} + \zeta) = \frac{\partial \hat{d}^j(\zeta)}{\partial q^j} \\ & \Rightarrow \frac{\partial \hat{d}^j(\zeta)}{\partial q^j} = \frac{F(\hat{d}^j(\zeta) + \bar{g} + \zeta)}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)} \end{aligned}$$

Thus, we have

$$\begin{aligned}
& \frac{\partial \left(\beta F(\hat{d}_t^j(\zeta) + \bar{g} + \zeta) - q^j \right)}{\partial q^j} \\
&= \beta F'(\hat{d}_t^j(\zeta) + \bar{g} + \zeta) \frac{\partial \hat{d}_t^j(\zeta)}{\partial q^j} - 1 \\
&= \frac{\beta F'(\hat{d}_t^j(\zeta) + \bar{g} + \zeta) F(\hat{d}^j(\zeta) + \bar{g} + \zeta)}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)} - 1 \\
&= \frac{\beta F'(\hat{d}_t^j(\zeta) + \bar{g} + \zeta) F(\hat{d}^j(\zeta) + \bar{g} + \zeta) - 1 + F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)} \\
&= \frac{F'(\hat{d}_t^j(\zeta) + \bar{g} + \zeta) \beta \left(F(\hat{d}^j(\zeta) + \bar{g} + \zeta) - 1 \right) + F'(\hat{d}^j(\zeta) + \bar{g} + \zeta) q^j - 1}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)}
\end{aligned}$$

If we have

$$F'(\hat{d}^j(\zeta) + \bar{g} + \zeta) q^j - 1 < 0 \quad (\text{A13})$$

then the derivative is negative because the first item in the numerator is negative ($F'(\cdot) > 0$ and $F(\cdot) < 1$), the second item in the numerator is negative implied by (A13), and the denominator is positive also implied by (A13). Note that inequality (A13) is equivalent to

$$\hat{d}^j(\zeta) + \bar{g} + \zeta > \bar{\zeta}^j$$

Therefore in the region of ζ where over-investment happens, we have $\frac{\partial(\beta F(\hat{d}_t^j(\zeta) + \bar{g} + \zeta) - q^j)}{\partial q^j} < 0$.

Finally, consider the region of ζ where the firm strategically defaults, i.e., $\zeta < \underline{\zeta}^j$, the funding constraint does not bind, i.e., $\hat{d}^j(\zeta) < \bar{d}$, and the firm under-invests. In this region, we impose a sufficient parametric condition that guarantees the firm's value in crises decreasing in q^j . From the previous calculation, we have

$$\frac{d[\beta F(\hat{d}^j(\zeta) + \bar{g} + \zeta)]}{dq^j} = \frac{\beta F'(\hat{d}^j(\zeta) + \bar{g} + \zeta) F(\hat{d}^j(\zeta) + \bar{g} + \zeta)}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)} \quad (\text{A14})$$

Given $F(z) = 1 - e^{-\lambda_F z}$, i.e., an exponential distribution, we have $F'(z) = \lambda_F(1 - F(z))$ and

obtain

$$\frac{d[\beta F(\hat{d}^j(\zeta) + \bar{g} + \zeta)]}{dq^j} = \frac{\beta \lambda_F (1 - F(\hat{d}_G^j(\zeta) + \bar{g} + \zeta)) F(\hat{d}_G^j(\zeta) + \bar{g} + \zeta)}{1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)} \quad (\text{A15})$$

The numerator contains a quadratic form of $F(\hat{d}_G^j(\zeta) + \bar{g} + \zeta) \in (0, 1)$, i.e., $(1 - F(\hat{d}_G^j(\zeta) + \bar{g} + \zeta)) F(\hat{d}_G^j(\zeta) + \bar{g} + \zeta)$, which has a maximum equal to $1/4$, obtained at $F(\hat{d}_G^j(\zeta) + \bar{g} + \zeta) = 1/2$. Therefore,

$$\frac{d[\beta F(\hat{d}^j(\zeta) + \bar{g} + \zeta)]}{dq^j} \leq \frac{\beta \lambda_F}{4[1 - F'(\hat{d}^j(\zeta) + \bar{g} + \zeta)(q^j - \beta)]} \quad (\text{A16})$$

Moreover, using $F'(\hat{d}^j(\zeta) + \bar{g} + \zeta) = \lambda_F e^{-\lambda_F(\hat{d}^j(\zeta) + \bar{g} + \zeta)} \leq \lambda_F$, we have

$$\frac{d[\beta F(\hat{d}^j(\zeta) + \bar{g} + \zeta)]}{dq^j} \leq \frac{\beta \lambda_F}{4[1 - \lambda_F(q^j - \beta)]} \quad (\text{A17})$$

We impose the parametric restriction

$$\frac{d[\beta F(\hat{d}^j(\zeta) + \bar{g} + \zeta)]}{dq^j} \leq \frac{\beta \lambda_F}{4[1 - \lambda_F(q^j - \beta)]} < 1. \quad (\text{A18})$$

This guarantees that $\beta F(x_t^j(\zeta) + \zeta) = \beta F(\min\{\hat{d}^j(\zeta), \bar{d}\} + \bar{g} + \zeta)$, has a derivative with respect to q^j that is smaller than one, i.e., the firm value in crises, $\beta F(x_t^j(\zeta) + \zeta) - q^j$, decreases in q^j .

We have shown that the integrand in the crisis component decreases in q^j for any ζ . Therefore, the integral decreases in q^j . In sum, the right side of (A12) decreases in q^j , and therefore, the solution of q^j is unique. Moreover, $A^H > A^L$ implies $q^H > q^L$.

Proof of Proposition 4 In the first-best economy describe by equation (14), we have

$$\int F(x_t^H(\zeta) + \zeta) dH(\zeta) > \int F(x_t^L(\zeta) + \zeta) dH(\zeta) \quad (\text{A19})$$

Therefore, $\kappa_t^H > \kappa_t^L$, which leads to $\Delta_t^\omega(\omega_{t-}) > 0$ according to equation (23).

Proof of Corollary 2 First, we consider introducing only a tight constraint ($\bar{d} \rightarrow 0$) to the first-best economy (without funding constraint, $\bar{d} = \infty$, and no government intervention yet so $\bar{g} = 0$).

In this case, $\beta = 0$, so there is no strategic default and the optimal financing is reduced to

$$x_t^j(\zeta) = \min\{(\bar{\zeta}_t^j - \zeta)^+, \bar{d} + \bar{g}\} \quad (\text{A20})$$

We need to prove that

$$\Delta_t^\omega(\bar{d} = \infty; \bar{g} = 0; \beta = 0) > \Delta_t^\omega(\bar{d} \rightarrow 0; \bar{g} = 0; \beta = 0)$$

Using the definition of κ_t^j in (22), in the case of $\bar{d} = \infty$ and $\beta = 0$, we get $\kappa_t^H > \kappa_t^L$, so there is a cleansing effect,

$$\Delta_t^\omega(\bar{d} = \infty; \bar{g} = 0; \beta = 0) > 0$$

In the case of $\bar{d} \rightarrow 0$, $\bar{g} = 0$, and $\beta = 0$, we get $\kappa_t^H = \kappa_t^L$, so there is no cleansing effect,

$$\Delta_t^\omega(\bar{d} \rightarrow 0; \bar{g} = 0; \beta = 0) = 0$$

Consequently, introducing a tight financial constraint to private-sector funding supply weakens the cleansing effect.

Next, we need to prove that the marginal effect of government financing is positive on cleansing, i.e.,

$$\frac{\partial \Delta_t^\omega}{\partial \bar{g}} \Big|_{\bar{d} \rightarrow 0, \bar{g} = 0, \beta = 0} > 0 \quad (\text{A21})$$

According to the definition of Δ_t^ω in (23), $\partial \Delta_t^\omega / \partial \bar{g} > 0$ is equivalent to

$$\frac{\partial(\kappa_t^L / \kappa_t^H)}{\partial \bar{g}} < 0 \Leftrightarrow \frac{\partial \kappa_t^L}{\partial \bar{g}} / \kappa_t^L < \frac{\partial \kappa_t^H}{\partial \bar{g}} / \kappa_t^H. \quad (\text{A22})$$

Using (A20) and the definition of κ_t^j in (22), we get

$$\frac{\partial \kappa_t^j}{\partial \bar{g}} = \int_{(\bar{\zeta}_t^j - \zeta)^+ \geq \bar{d} + \bar{g}} F'(\bar{d} + \bar{g} + \zeta) dH(\zeta)$$

Therefore, for $\bar{d} + \bar{g} < \bar{\zeta}_t^H$, we have

$$\frac{\partial \kappa_t^L}{\partial \bar{g}} < \frac{\partial \kappa_t^H}{\partial \bar{g}}$$

Since we consider the expansion of government intervention at $\bar{d} \rightarrow 0$ and $\bar{g} = 0$, $\kappa_t^L = \kappa_t^H$. As a

result, we have proven

$$\frac{\partial \kappa_t^L}{\partial \bar{g}} / \kappa_t^L < \frac{\partial \kappa_t^H}{\partial \bar{g}} / \kappa_t^H,$$

which according to (A22) leads to (A21). In summary, government financing strengthens the cleansing effect in this special case of $\beta = 0$ and tight financial constraint.

Proof of Corollary 3 Consider $\zeta \in \mathcal{U}$ for $j \in \{H, L\}$ such that $\hat{d}^j(\zeta) < \bar{d}$ and $\zeta < \underline{\zeta}_t^j$. Define $\hat{H}(\zeta) = H(\zeta)$ if $\zeta \in \mathcal{U}$ and $\hat{H}(\zeta) = 0$ if $\zeta \notin \mathcal{U}$. Among these type- j firms that strategically default, the measure of surviving firms is given by

$$\hat{\kappa}_t^j = \int F(\hat{d}_t^j + \bar{g} + \zeta) d\hat{H}(\zeta)$$

Under $\hat{d}_t^H > \hat{d}_t^L$, we have $\hat{\kappa}_t^H > \hat{\kappa}_t^L$. Therefore, the endogenous debt limit contributes to the cleansing effect of crises.

Next, we consider how credit intervention affects this channel of cleansing effect. Specifically, we prove among firms that strategically default, credit intervention reduces the share of surviving firms that are type- H , i.e.,

$$\frac{\partial(\hat{\kappa}_t^H / (\hat{\kappa}_t^H + \hat{\kappa}_t^L))}{\partial \bar{g}} < 0 \Leftrightarrow \frac{\partial \hat{\kappa}_t^L}{\partial \bar{g}} / \hat{\kappa}_t^L > \frac{\partial \hat{\kappa}_t^H}{\partial \bar{g}} / \hat{\kappa}_t^H \Leftrightarrow \frac{\partial(\hat{\kappa}_t^L / \hat{\kappa}_t^H)}{\partial \bar{g}} > 0. \quad (\text{A23})$$

We note that

$$\frac{\partial \hat{\kappa}_t^j}{\partial \bar{g}} / \hat{\kappa}_t^j = \frac{\int F'(\hat{d}_t^j + \bar{g} + \zeta) d\hat{H}(\zeta)}{\int F(\hat{d}_t^j + \bar{g} + \zeta) d\hat{H}(\zeta)}.$$

Define

$$\mathcal{L}(u) = \frac{\int F'(u + \zeta) d\hat{H}(\zeta)}{\int F(u + \zeta) d\hat{H}(\zeta)}.$$

Then the derivative of $\mathcal{L}(u)$ is

$$\mathcal{L}'(u) = \frac{\left(\int F(u + \zeta) d\hat{H}(\zeta) \right) \left(\int F''(u + \zeta) d\hat{H}(\zeta) \right) - \left(\int F'(u + \zeta) d\hat{H}(\zeta) \right)^2}{\left(\int F(u + \zeta) d\hat{H}(\zeta) \right)^2}.$$

Under $F''(\cdot) < 0$, we have $\mathcal{L}'(u) < 0$, which implies $\frac{\partial \hat{\kappa}_t^L}{\partial \bar{g}} / \hat{\kappa}_t^L > \frac{\partial \hat{\kappa}_t^H}{\partial \bar{g}} / \hat{\kappa}_t^H$.

Proof of Proposition 5 First, by definition of κ_t^j in (22) and $F(x) < 1$ for $x < \infty$, we obtain $\kappa_t^j < 1$ for $j \in \{L, H\}$.

Next, under Assumption 1, we are able to provide a stronger statement that firm quality jumps up in a crisis, $\Delta_t^\omega > 0$. According to (23), $\Delta_t^\omega > 0$ is equivalent to $\kappa_t^H > \kappa_t^L$, which by definition in (22) is equivalent to

$$\underbrace{\int F(x_t^H(\zeta) + \zeta) dH(\zeta)}_{\kappa_t^H} > \underbrace{\int F(x_t^L(\zeta) + \zeta) dH(\zeta)}_{\kappa_t^L}$$

Under Assumption 1 and $\bar{g} = 0$, the optimal financing choices of H and L types are

$$x_t^H(\zeta) = \min\{(\bar{\zeta}_t^H - \zeta)^+, \bar{d} + \bar{g}\} \quad (\text{A24})$$

$$x_t^L(\zeta) = \mathbf{1}_{\zeta \geq \underline{\zeta}_t^L} \min\{(\bar{\zeta}_t^L - \zeta)^+, \bar{d} + \bar{g}\} + \mathbf{1}_{\zeta < \underline{\zeta}_t^L} \left(\min\{\hat{d}_t^L(\zeta), \bar{d}\} + \bar{g} \right) \quad (\text{A25})$$

For $\zeta \geq \underline{\zeta}_t^L$, we have

$$x_t^H(\zeta) = \min\{(\bar{\zeta}_t^H - \zeta)^+, \bar{d} + \bar{g}\} \geq \min\{(\bar{\zeta}_t^L - \zeta)^+, \bar{d} + \bar{g}\} = x_t^L(\zeta)$$

where the inequality is strict for $\zeta \geq \max\{\bar{\zeta}_t^H - (\bar{d} + \bar{g}), \underline{\zeta}_t^L\}$.

For $\zeta < \underline{\zeta}_t^L$, we need to compare $\bar{\zeta}_t^H - \zeta$ with $\hat{d}_t^L(\zeta) + \bar{g}$. In this region, according to the monotonicity of $\hat{d}_t^L(\zeta)$ in Lemma 3, we get $\hat{d}_t^L(\zeta) \leq \hat{d}_t^L(\underline{\zeta}_t^L)$. Therefore,

$$\hat{d}_t^L(\zeta) + \zeta \leq \hat{d}_t^L(\underline{\zeta}_t^L) + \underline{\zeta}_t^L < \bar{\zeta}_t^H - \bar{g},$$

which implies that

$$\bar{\zeta}_t^H - \zeta > \hat{d}_t^L(\zeta) + \bar{g}$$

Consequently,

$$x_t^H(\zeta) = \min\{(\bar{\zeta}_t^H - \zeta)^+, \bar{d} + \bar{g}\} \geq \min\{\hat{d}_t^L(\zeta), \bar{d}\} + \bar{g} = x_t^L(\zeta)$$

In summary, for any ζ , we have $x_t^H(\zeta) \geq x_t^L(\zeta)$, and the inequality is strict for a positive measure of ζ . Consequently, we have proved that $\kappa_t^H > \kappa_t^L$, which immediately leads to $\Delta_t^\omega > 0$.

Proof of Proposition 6 Next, we analyze the impact of credit intervention. Under Assumption 1, we can simplify the financing choices of H and L types as in (A24) and (A25). We first show that $\partial\Delta_t^K/\partial\bar{g} > 0$. According to (21), a sufficient condition is

$$\frac{\partial\kappa_t^j}{\partial\bar{g}} > 0.$$

According to equation (A24), $x_t^H(\zeta)$ monotonically increases in \bar{g} (given q_t^j and thus $\bar{\zeta}_t^H$). Furthermore, according to equation (A25), $x_t^L(\zeta)$ increases monotonically in \bar{g} if either $\zeta \geq \underline{\zeta}_t^L$ or $\zeta < \underline{\zeta}_t^L$. By Assumption 1, L type firms have slackness in the continuation region $\zeta \geq \underline{\zeta}_t^j$ (i.e., $x_t^L(\zeta)$ is not affected by \bar{g} for $\zeta \geq \underline{\zeta}_t^j$), so \bar{g} does not affect the cutoff $\underline{\zeta}_t^L$ (see equation (16)). Taken together, we conclude that both $x_t^H(\zeta)$ and $x_t^L(\zeta)$ monotonically increase with \bar{g} , and therefore, κ_t^H and κ_t^L , the integration of these two functions, also increase in \bar{g} . This leads to $\partial\Delta_t^K/\partial\bar{g} > 0$.

Next, we note that $\partial\Delta_t^\omega/\partial\bar{g} < 0$ is equivalent to

$$\frac{\partial(\kappa_t^L/\kappa_t^H)}{\partial\bar{g}} > 0 \Leftrightarrow \frac{\partial\kappa_t^L}{\partial\bar{g}} - \frac{\kappa_t^L}{\kappa_t^H} \frac{\partial\kappa_t^H}{\partial\bar{g}} > 0. \quad (\text{A26})$$

Since $\kappa_t^H > \kappa_t^L$, a sufficient condition for (A26) is

$$\frac{\partial\kappa_t^L}{\partial\bar{g}} > \frac{\partial\kappa_t^H}{\partial\bar{g}}.$$

Again, we look at $x_t^H(\zeta)$ and $x_t^L(\zeta)$ for each ζ . Given $\bar{d} + \bar{g} > \bar{\zeta}_t^H$, we have $(\bar{\zeta}_t^H - \zeta)^+ \leq \bar{d} + \bar{g}$ for any $\zeta \geq 0$, so government intervention does not directly affect H-type firms, with $\partial x_t^H(\zeta)/\partial\bar{g} = 0$. On the other hand, since $\hat{d}^L(\zeta)$ increases with \bar{g} (see Lemma 4), and $\underline{\zeta}_t^L$ is not affected by \bar{g} , we find that $x_t^L(\zeta)$ as in (A25) weakly increases with \bar{g} , and the increase is strict for all $\zeta < \underline{\zeta}_t^L$. As a result, we get

$$\begin{aligned} \frac{\partial \int F(x_t^H(\zeta) + \zeta) dH(\zeta)}{\partial\bar{g}} &= 0, \\ \frac{\partial \int F(x_t^L(\zeta) + \zeta) dH(\zeta)}{\partial\bar{g}} &> 0. \end{aligned}$$

The above naturally lead to the sufficient condition in (A26), and thus the result $\partial\Delta_t^\omega/\partial\bar{g} < 0$.

Proof of Proposition 7. The percentage output drop in a crisis is

$$\frac{(A^H K_t^H + A^L K_t^L) - (A^H K_{t-}^H + A^L K_{t-}^L)}{A^H K_{t-}^H + A^L K_{t-}^L} = \frac{A^H \omega_{t-} \kappa_{t-}^H + A^L (1 - \omega_{t-}) \kappa_{t-}^L}{A^H \omega_{t-} + A^L (1 - \omega_{t-})}$$

where we denote the fractional decline of type- j capital as

$$\kappa_{t-}^j \equiv \int_{\zeta} (1 - F(x_{t-}^j(\zeta) + \zeta)) dH(\zeta)$$

We note that κ_{t-}^j is not affected by ω_{t-} but affected by \bar{g} , so we express it as $\kappa_{t-}^j(\bar{g})$.

Denote the target GDP drop as Δy . For simplicity, we drop the time- t subscripts since we consider a stationary Markov equilibrium (note that ω denotes pre-crisis capital quality ω_{t-}). Then we have

$$\frac{A^H \omega \kappa^H(\bar{g}) + A^L (1 - \omega) \kappa^L(\bar{g})}{A^H \omega + A^L (1 - \omega)} = \Delta y \quad (\text{A27})$$

We are interested in how \bar{g} is affected by the firm quality ω , i.e., we want to know the derivative of $\bar{g}(\omega)$, defined by the above equation. Taking derivative over ω on both sides of A27, we obtain

$$\begin{aligned} \left((A^H \kappa^H(\bar{g}) - A^L \kappa^L(\bar{g})) + \left(A^H \omega \frac{d\kappa^H}{d\bar{g}} + A^L (1 - \omega) \frac{d\kappa^L}{d\bar{g}} \right) \frac{d\bar{g}}{d\omega} \right) (A^H \omega + A^L (1 - \omega)) \\ - (A^H - A^L) (A^H \omega \kappa^H(\bar{g}) + A^L (1 - \omega) \kappa^L(\bar{g})) = 0 \end{aligned}$$

The terms that do not involve derivatives can be simplified as

$$\begin{aligned} (A^H \kappa^H(\bar{g}) - A^L \kappa^L(\bar{g})) (A^H \omega + A^L (1 - \omega)) - (A^H - A^L) (A^H \omega \kappa^H(\bar{g}) + A^L (1 - \omega) \kappa^L(\bar{g})) \\ = A^H A^L \kappa^H(\bar{g}) (1 - \omega) - A^H A^L \omega \kappa^L(\bar{g}) - A^H A^L (1 - \omega) \kappa^L(\bar{g}) + A^H A^L \omega \kappa^H(\bar{g}) \\ = A^H A^L (\kappa^H(\bar{g}) - \kappa^L(\bar{g})) \end{aligned}$$

Therefore, we get

$$\begin{aligned} A^H A^L (\kappa^H(\bar{g}) - \kappa^L(\bar{g})) + \left(A^H \omega \frac{d\kappa^H}{d\bar{g}} + A^L (1 - \omega) \frac{d\kappa^L}{d\bar{g}} \right) (A^H \omega + A^L (1 - \omega)) \frac{d\bar{g}}{d\omega} = 0 \\ \frac{d\bar{g}}{d\omega} = - \frac{A^H A^L (\kappa^H(\bar{g}) - \kappa^L(\bar{g}))}{\left(A^H \omega \frac{d\kappa^H}{d\bar{g}} + A^L (1 - \omega) \frac{d\kappa^L}{d\bar{g}} \right) (A^H \omega + A^L (1 - \omega))} \end{aligned}$$

By assumption, there is net cleansing effect, i.e., the change of firm quality is positive,

$$\Delta\omega = \frac{\omega}{\omega + (1 - \omega)\frac{1-\kappa^L}{1-\kappa^H}} - \omega > 0$$

which is equivalent to $\kappa^H < \kappa^L$. We already show that $d\kappa^H/d\bar{g} < 0$ and $d\kappa^L/d\bar{g} < 0$ in the proof of Proposition 6. As a result, we obtain

$$\frac{d\bar{g}}{d\omega} < 0.$$

B Additional Model Properties

B.1 Dynamic Optimal Intervention

In Section 3.3, we consider the dynamics in an environment where agents do not expect intervention. This approach purges out any backward propagation of policy impact (i.e., future interventions affect the effectiveness and necessary scale of intervention in the current crisis through agents' expectations) and thereby only shows the strength of forward propagation of policy impact (i.e., intervention in the current crisis affects the scale of future interventions) which is the focus of our paper. Next, we consider an environment where the government optimally chooses the scale of intervention in every crisis, and agents have rational expectations of the policy plan. How will the backward propagation of policy impact agents' expectations and interact with the forward propagation in equilibrium? Can the government avoid the slippery slope of intervention by dynamically adjusting \bar{g} ? These questions are important because the reality, in terms of agents' expectation of intervention, likely lies in between the situation in Section 3.3 and fully rational expectation. Moreover, in practice, the policy goal may also lie between containing the output drop in crises (the least sophisticated) and dynamically maximizing welfare (the most sophisticated).

Dynamically adjusting the scale of intervention can potentially improve efficiency. In our model, firm quality distribution, represented by ω_t , evolves over time. When ω_t is low and there are many type- L firms, the government would reduce \bar{g} , because the cost of type- L firms' over-investment overweighs the benefit of relaxing the under-investing firms' financial constraints; in contrast, when ω_t is high, the government would increase \bar{g} . Below, we solve the optimal $\bar{g}_t = g^{dynamic}(\omega_t)$ through the dynamic optimization of welfare. Intuitively, the optimal policy is dependent on only the quality state variable, ω_t , because the economy is scalable in the quantity state variable, K_t , and the intervention scale is about funding support per unit of capital. We show that even though dynamically adjusting the scale of intervention improves welfare, it cannot eliminate the persistent distortions on firm quality distribution and the slippery slope of intervention.

When the scale of government funding support depends on ω_t , both q_t^H and q_t^L become time-varying and dependent on ω_t . For $j \in \{H, L\}$, the capital value has the following law of motion:

$$\frac{dq_t^j}{q_{t-}^j} = \mu_{q,t-}^j dt + \Delta_{q,t-}^j dN_t.$$

With the capital value as a function of ω_t , (i.e., $q_t^j = q^j(\omega_t)$), we obtain the drift and jump size:

$$\mu_{q,t-}^j = \frac{dq^j(\omega_{t-})}{d\omega_{t-}} \mu_t^\omega(\omega_{t-}) dt,$$

and

$$\Delta_{q,t-}^j = \frac{q^j(\omega_{t-} + \Delta_t^\omega(\omega_{t-})) - q^j(\omega_{t-})}{q^j(\omega_{t-})},$$

The capital valuation equation is the same as (20), although now $E[dq_t^j] \neq 0$, unlike the economy with static credit intervention. Solving the model requires solving the following HJB equation for the K_t -scaled welfare function, $W(\omega_t)$:

$$\begin{aligned} rW(\omega_{t-}) = & \omega_{t-} A^H + (1 - \omega_{t-}) A^L - (\omega_{t-} \bar{t}^H(\omega_{t-}) + (1 - \omega_{t-}) \bar{t}^L(\omega_{t-})) \\ & + W(\omega_{t-}) \mu^K(\omega_{t-}) + W'(\omega_{t-}) \mu^\omega(\omega_{t-}) \\ & + \lambda \max_{\bar{g}} [W(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g})) (1 + \Delta^K(\omega_{t-}, \bar{g})) - W(\omega_{t-}) - I(\omega_{t-}, \bar{g})] . \end{aligned} \quad (\text{A28})$$

In comparison with the welfare HJB equation (26) under a constant \bar{g} , the last term on the right side of (A28) reflects the optimization over \bar{g} given the firm quality distribution, represented by ω_{t-} , that the economy carries into a crisis.

The first-order condition for the optimal \bar{g} reveals the trade-off that the government faces:

$$\begin{aligned} W'(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g})) \frac{\partial \Delta^\omega(\omega_{t-}, \bar{g})}{\partial \bar{g}} (1 + \Delta^K(\omega_{t-}, \bar{g})) + \\ W(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g})) \frac{\partial \Delta^K(\omega_{t-}, \bar{g})}{\partial \bar{g}} - \frac{\partial I(\omega_{t-}, \gamma)}{\partial \bar{g}} = 0 . \end{aligned} \quad (\text{A29})$$

The first term shows the negative impact of dampening the cleansing effect, $\frac{\partial \Delta^\omega(\omega_{t-}, \bar{g})}{\partial \bar{g}} < 0$, and its long-run effects are encoded in the marginal change of the present value of future consumption (i.e., the forward-looking welfare measure) per unit of capital over change in capital quality, $W'(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))$. The second term shows the positive impact of reducing \bar{g} through the preservation of capital, $\frac{\partial \Delta^K(\omega_{t-}, \bar{g})}{\partial \bar{g}} > 0$. Each unit of capital saved by the government funding raises welfare by $W(\omega_{t-} + \Delta^\omega(\omega_{t-}, \bar{g}))$. The last term reflects the reduction of consumption due to stimulating investment through government funding.

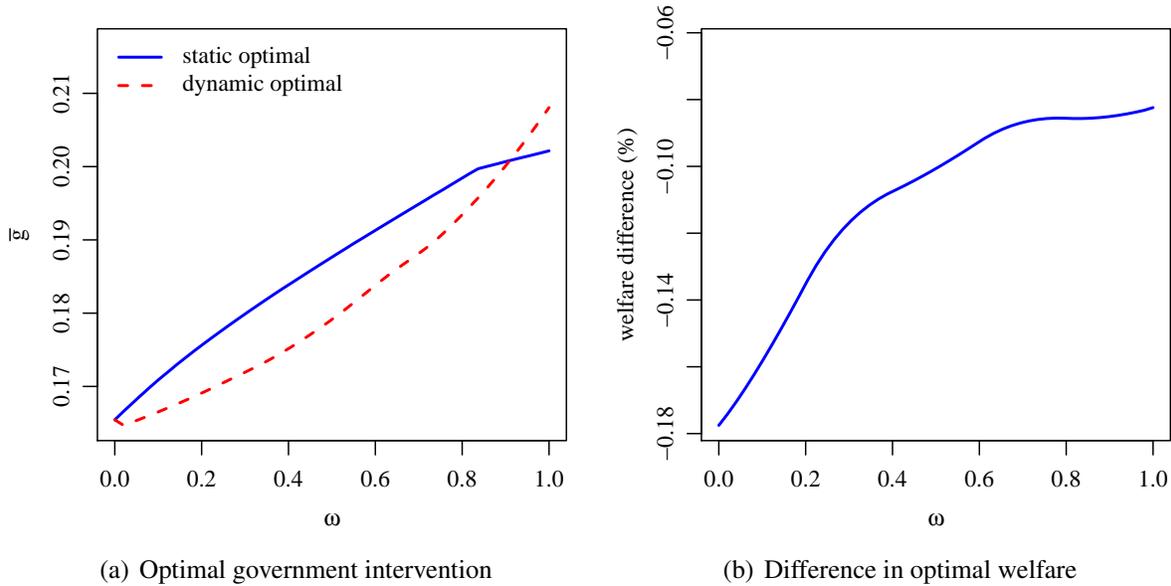


Figure A1: Optimal dynamic intervention and welfare. In panel (a), we plot the optimal dynamic government intervention $\bar{g}^*(\omega_t)$ as a function of the state ω_t at time t , and the optimal static government intervention $\bar{g}^*(\omega_0)$ as a function of initial state ω_0 . In panel (b), we show the percentage difference in welfare under the dynamic optimal policy versus the static optimal policy. A negative value indicates that welfare under dynamic optimal policy is lower.

To simplify the notation, we omit the time subscripts below. Equation (A29) implicitly defines the optimal \bar{g} as a function of ω . Once we solve the functions $q^H(\omega)$, $q^L(\omega)$, $W(\omega)$, and $\bar{g}(\omega)$, we obtain the time- t values of the other endogenous variables as functions of ω .

In equilibrium, the capital value, q_t^j ($j \in \{H, L\}$), the welfare per unit of capital, W_t , and the optimal government intervention scale \bar{g}_t , jointly satisfy the equations (20), (A28), and (A29), and firms' financing decisions are given by equation (18).

Panel A of Figure A1 compares the dynamically adjusted \bar{g} and the optimal constant \bar{g} set at $t = 0$. Overall, they are close numerically. For large ω , the dynamic \bar{g} is larger because, given the downward trajectory of firm quality for large ω , the government can reduce its funding supply when the fraction of lower-quality firms is higher. For small ω , the dynamic \bar{g} is smaller because, given the upward trajectory of firm quality for small ω , the government can increase its funding supply when the fraction of higher-quality firms is higher.

Panel B of Figure A1 illustrates the welfare difference between the scenario of committed static policy versus dynamic optimal policy. Although the dynamic policy provides extra flexibility for policy adjustment, the lack of commitment can reduce welfare through potential dynamic

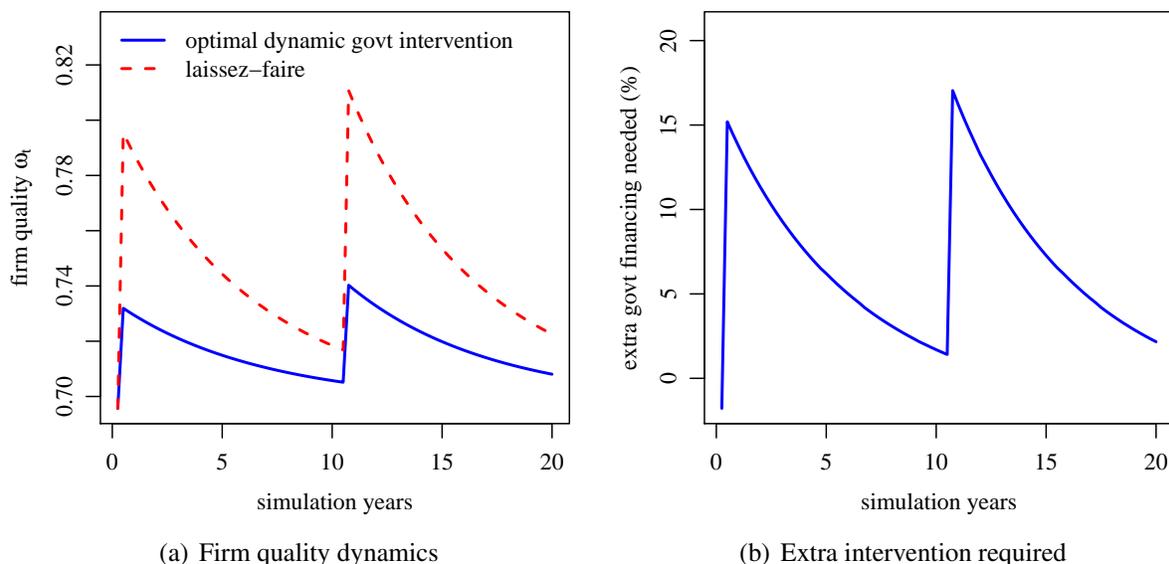


Figure A2: Slippery slope of credit intervention: Optimal dynamic policy versus laissez-faire benchmark. In this figure, we compare two economies: In the laissez-faire economy, there is no intervention, and agents believe so; the govt-intervention economy is using the optimal dynamic intervention strategy $\bar{g}_t = g^{dynamic}(\omega_t)$ and agents rationally expect so. The starting state of the simulation is the average ω in the no-intervention economy. Then, we introduce two crises on the simulation path, one at year zero and the other at year ten. Panel (a) plots the simulated paths of ω_t for the two economies respectively. Next, at each point of time, we calculate the amount of intervention needed in the laissez-faire economy that is required to achieve the same percentage of GDP drop as the economy with optimal dynamic intervention if a crisis happens at that time. Then, we show the difference in intervention scale between the optimal dynamic intervention economy and the laissez-faire economy in panel (b).

inconsistency, and the net effect from our numerical evaluation is that the lack of commitment dominates.

Finally, in Figure A2, we illustrate the comparison between an economy with optimal dynamic intervention versus an economy without government intervention. The goal is to analyze how much extra intervention is needed in the dynamic-intervention economy compared to the no-intervention economy to achieve the same goal of containing GDP drop to within 10% in crises. Different from Figure 7, in the dynamic-intervention economy, agents correctly expect interventions during future crises. However, since crisis realizations are still surprises (dN_t shocks are not predictable), each crisis still widens the gap between the intervention economy and the laissez-faire economy, causing extra intervention needed for the intervened economy.³⁴ Therefore, the optimal dynamic

³⁴Also note that in Figure A2(b) the extra intervention at the beginning is negative, because the initial state ω_0 is the same across two economics, while capital values are higher in the intervention-economy, so less intervention is needed

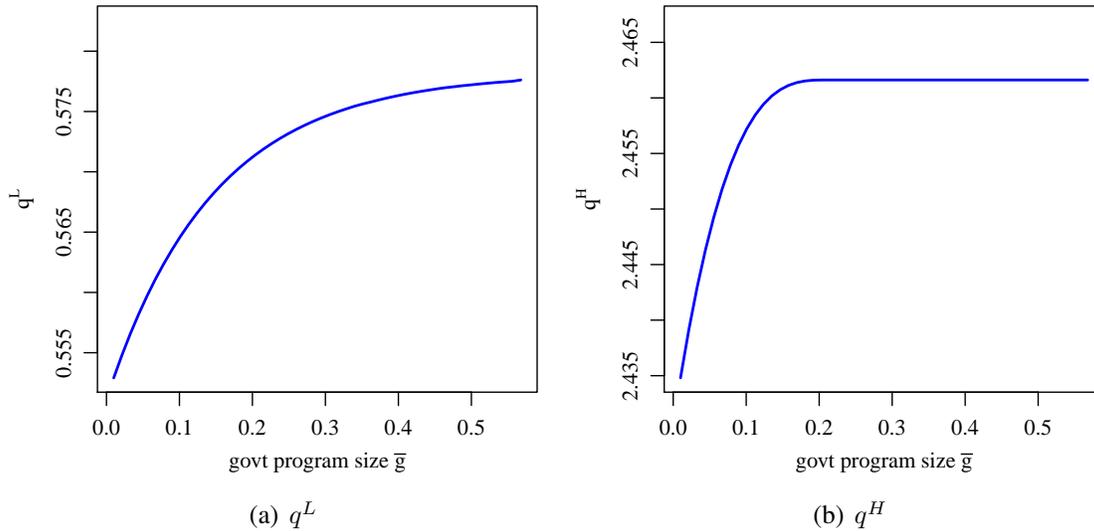


Figure A3: Credit intervention and capital values. This figure illustrates the impact of a static government intervention \bar{g} on capital value q^H and q^L .

adjustment of government policy does not eliminate the slippery slope of intervention.

B.2 Additional Results: Capital Value and the Stationary Distribution

In this subsection, we illustrate the effect of government intervention through the impact on capital values and the stationary distribution of ω_t . We note that the stationary distribution is affected not only by crisis-time jumps but also by normal-time investment that is directly driven by capital values. Figure A3 (a) and (b) illustrate the impact of government intervention on capital values. Since government intervention improves crisis-time value π^j (see equation (20)), it increases the capital value q^j . However, the impact on q^H is limited when \bar{g} expands beyond a certain threshold, because the borrowing demand from type- H firms is satiated once they reach the efficient level of investment. On the other hand, q^L further increases, because type- L firms may over-borrow to survive in future crises and the extra subsidy from a larger \bar{g} is priced in the capital valuation.

In Figure A4, we illustrate the stationary distribution of ω_t in two economies, one without intervention ($\bar{g} = 0$), and the other with baseline intervention ($\bar{g} = 0.14$). We find that the distribution of ω_t significantly shifts to the leftwards in the latter, reflecting a long-run reduction of firm quality.

in the intervention-economy. Later, the difference in ω_t becomes dominant and overcomes this capital valuation effect.

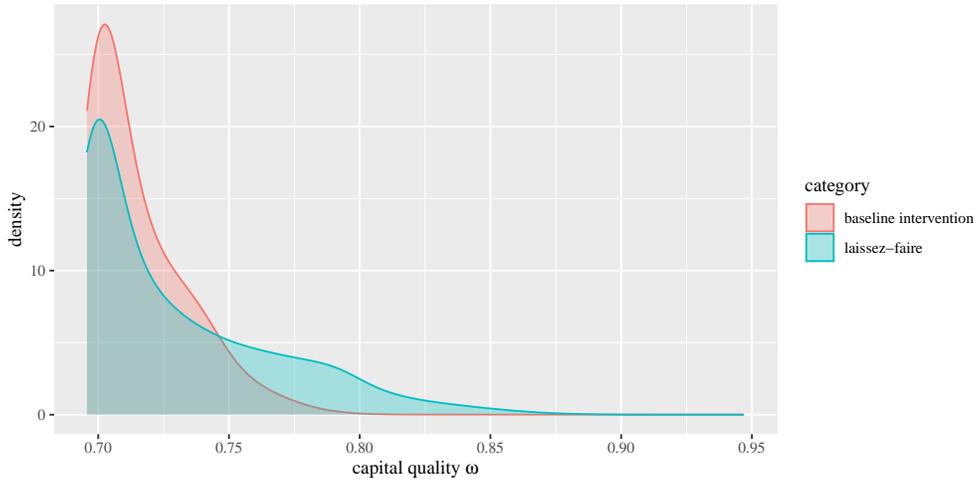


Figure A4: Stationary distribution of ω_t . This figure illustrates the stationary distribution of ω_t , under two different scenarios of government intervention: laissez-faire with $\bar{g} = 0$, and baseline intervention with $\bar{g} = 0.14$.

B.3 Intervention without Market-Based Pricing

In this section, we discuss the lack of differentiation in credit pricing and show that government intervention dampens the cleansing effect of crises and the slippery slope of credit intervention. Such lack of differentiation in credit pricing is a feature in many government programs, including the Main Street Lending Program and the Paycheck Protection Program. To clarify the channel, we demonstrate it in a simplified setting. Specifically, we shut down the funding-demand side friction (i.e., the incentive to over-borrow and strategically default) via $\beta = 0$. In our baseline model with market-based pricing discrimination, this would imply that credit intervention does not induce inefficiency. Moreover, We assume that both \bar{d} and \bar{g} are large enough so that the funding constraints do not bind (i.e., all firms can potentially invest at the first-best levels).

We first consider the laissez-faire economy. Denote the borrowing from private-sector as $d^j(\zeta)$ and borrowing from government as $g^j(\zeta)$, with type $j \in \{L, H\}$. For a type- j firm, given ζ , the optimization problem is

$$\max_{d^j} F(d^j + \zeta) [q^j - (1 + r(\zeta, d^j, q^j))d^j],$$

with private-sector break-even condition

$$F(d^j + \zeta)(1 + r(\zeta, d^j, q^j))d^j = d^j$$

Therefore, the equivalent problem is

$$\max_{d^j} F(d^j + \zeta)q^j - d^j$$

The solution is

$$d^j(\zeta) = \bar{\zeta}^j - \zeta$$

with $\bar{\zeta}^j$ defined as

$$F'(\bar{\zeta}^j)q^j = 1$$

Because $q^H > q^L$, the above solution indicates that $\bar{\zeta}^H > \bar{\zeta}^L$ so that $d^H(\zeta) > d^L(\zeta)$. By definition,

$$\kappa^j = \int F(d^j(\zeta) + \zeta)dH(\zeta)$$

Consequently, we have $\kappa^H > \kappa^L$ and $\Delta^\omega > 0$. Crises feature cleansing effect.

Next, we consider intervention. Let the interest rate charged by the government be $\bar{r} > 0$, which is the same across all firms, and for simplicity, constant over time. This implies that the equilibrium capital value is constant, similar to the stationary equilibrium in the main text. The same rate for all firms is the actual practice in government programs including MSLP and PPP. Firms may utilize the funds to (over-)invest in their own survival probability or deposit at the market-based interest rates. Note that we allow firms to lend to one another. If \bar{r} is too high for one firm, the firm may instead borrow from other firms. Forbidding such reallocation of funds results in great inefficiency induced by the lack of interest-rate discrimination in government credit support. Under the same rate set by the government, the optimization problem of the firm becomes

$$\max_{g^j \geq 0, d^j} F(d^j + g^j + \zeta) [q^j - (1 + r(\zeta, d^j, g^j, q^j))d^j - (1 + \bar{r})g^j], \quad (\text{A30})$$

with private-sector break-even condition

$$F(d^j + g^j + \zeta)(1 + r(\zeta, d^j, g^j, q^j))d^j = d^j. \quad (\text{A31})$$

With this expression, we can rewrite the optimization problem as

$$\max_{d^j \geq 0, g^j \geq 0} F(d^j + g^j + \zeta) (q^j - (1 + \bar{r})g^j) - d^j. \quad (\text{A32})$$

Denote the objective function as $J(d^j, g^j; \zeta, q^j)$. Then, the derivative over d^j is

$$J'_d(d^j, g^j; \zeta, q^j) = F'(d^j + g^j + \zeta) (q^j - (1 + \bar{r})g^j) - 1 \quad (\text{A33})$$

and the derivative over g^j is

$$J'_g(d^j, g^j; \zeta, q^j) = F'(d^j + g^j + \zeta) (q^j - (1 + \bar{r})g^j) - F(d^j + g^j + \zeta)(1 + \bar{r}) \quad (\text{A34})$$

We note that since $1 + \bar{r} > 1$, there exists d^j such that $F(d^j + g^j + \zeta)(1 + \bar{r}) = 1$ and that we can simultaneously have $J'_d = 0$ and $J'_g = 0$, i.e., an interior solution for the problem. The interior solution is $J'_g(d^j, g^j; \zeta, q^j) = J'_d(d^j, g^j; \zeta, q^j) = 0$, which implies

$$F(d^j + g^j + \zeta)(1 + \bar{r}) = 1, \quad (\text{A35})$$

$$d^j + g^j = F^{-1}\left(\frac{1}{1 + \bar{r}}\right) - \zeta. \quad (\text{A36})$$

Plugging (A36) into $J'_d = 0$, we obtain

$$g^j = \frac{1}{1 + \bar{r}} \left(q - \frac{1}{F'(F^{-1}(\frac{1}{1 + \bar{r}}))} \right)$$

which is not affected by ζ . The private-sector borrowing is then

$$d^j = F^{-1}\left(\frac{1}{1 + \bar{r}}\right) - \frac{1}{1 + \bar{r}} \left(q - \frac{1}{F'(F^{-1}(\frac{1}{1 + \bar{r}}))} \right) - \zeta.$$

Since

$$\kappa^j = \int F(d^j(\zeta) + g^j(\zeta) + \zeta) dH(\zeta)$$

According to (A36), $\kappa^H = \kappa^L$. Thus, the cleansing effect is completely eliminated, and we have the following proposition.

Proposition 8 (Lack of differentiation in pricing and the dampening of cleansing effect) *Assume $\beta = 0$ and financing capacity constraints are not binding. If the government charges the same in-*

interest rate $\bar{r} > 0$ to all firms, then the cleansing effect of crisis is completely eliminated,

$$\Delta_t^\omega = 0. \tag{A37}$$

This dampening of cleansing effect is inefficient for two reasons: First, it lacks differentiation between H and L firms, over-subsidizing L-type firms but not enough for H-type firms; Second, it lacks differentiation among same-type firms with different credit risks (due to heterogeneous ζ), causing over-borrowing for high- ζ firms but not enough borrowing for low- ζ firms.

C Credit Quality and Firm Productivity

We provide evidence that credit quality is not the same as firm productivity. Thus, the government cannot simply rely on credit pricing in the market to perfectly discriminate firms.

In our model, within each type j , there is significant variation in the survival probability that can be interpreted as “credit quality” driven by ζ . Although firm credit quality is observable via credit ratings, firm productivity requires more complex measurement. In a world where these two dimensions perfectly align, the government can rely on credit quality for its credit programs and mitigate the downside effects. Nevertheless, in the data, the correlation between credit quality and firm quality is positive but far from perfect, consistent with our model.

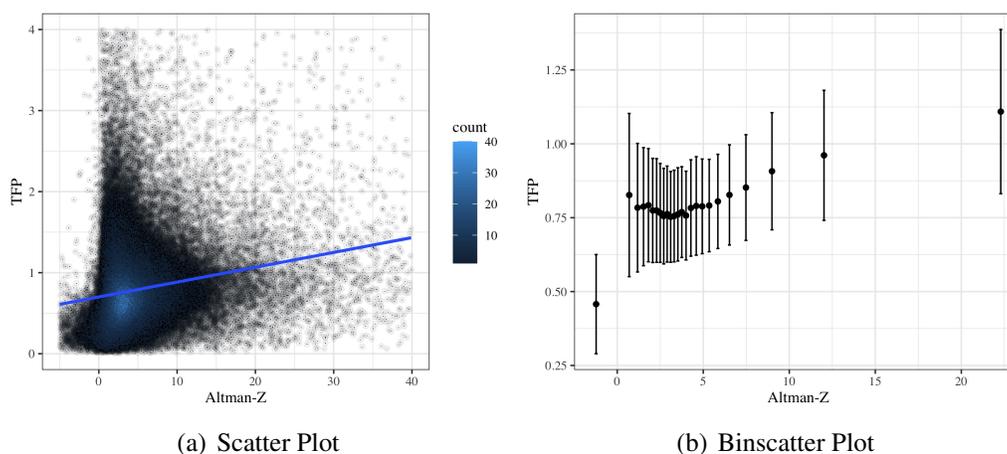


Figure A5: Credit quality v.s. firm productivity. We use the standard Altman-Z score to measure credit quality, and construct the cross-section of TFP following İmrohoroğlu and Tüzel (2014). In panel (a), we restrict the plot range to $0 \leq \text{TFP} \leq 4$ and $-5 \leq \text{Altman-Z} \leq 40$, which removes extreme outliers ($< 0.02\%$ of the sample). In panel (b), we show 25 bins of Altman-Z and plot the mean of TFP with two standard deviation bars on each side.

We measure the cross-section of firm TFP using the codes provided by İmrohoroğlu and Tüzel (2014) that apply the method in Olley and Pakes (1992). The methodology uses investment data to deal with classic simultaneity bias where the firm’s factor input decision is influenced by TFP, and it also deals with firm exit and corrects it with a selection probability estimation. We measure firm credit quality using Altman Z score (Altman, 1968), which is a combination of different accounting ratios to predict corporate bankruptcy. A higher Z score indicates better credit quality. We plot the cross-section of Z score against TFP in Figure A5. As shown by the scatter plot in panel (a), for certain levels of TFP, there is a wide distribution of credit quality. In panel (b), we report a binscatter plot. The error bars suggest that the correlation is far from perfect.