Fragile New Economy: The Rise of Intangible Capital and Financial Instability*

Ye Li†

April 27, 2019

Abstract

Intangible capital creates endogenous financial risk by inducing self-perpetuating savings gluts. Firms save for investments in intangibles that are unpledgeable but essential for the creation of new assets. Intermediaries profit from channeling firms' savings into asset price bubbles. The bubbly value in turn stimulates firms' asset creation and savings for intangibles. Fragility builds up as banks' debt accumulates and funding cost declines. The model offers a coherent account of intangible investment, corporate savings, intermediary leverage, interest rate, and collateral asset price in the decades leading up to the Great Recession. It generates booms with rising downside risks and stagnant recessions.

*I am grateful to helpful comments from Tobias Adrian, Juliane Begenau, Jonathan Berk, Patrick Bolton, Diana Bonfim, Maria Chaderina, Guojun Chen, Itay Goldstein, Gur Huberman, Jianjun Miao, Kjell Nyborg, Martin Oehmke, Adriano Rampini, Tano Santos, Thomas Sargent, José Scheinkman, Suresh Sundaresan, Neng Wang, and Pengfei Wang. I am also grateful to conference and seminar participants at CEPR ESSFM Gerzensee, Columbia University, CMU/OSU/Pittsburgh/Penn State conference, European Finance Association annual meeting, European Winter Finance Summit, Federal Reserve Bank of New York, Finance Theory Group summer school, HKUST Annual Macro Workshop, LBS Trans-Atlantic Doctoral Conference, North America Summer Meeting of Econometric Society, and Western Finance Association annual meeting. I acknowledge the generous financial support from the Macro Financial Modeling Group of Becker Friedman Institute at the University of Chicago. All errors are mine.

†The Ohio State University. E-mail: li.8935@osu.edu

1 Introduction

In the two decades leading up to the Great Recession, the U.S. economy exhibited five trends:

(1) The economy was transforming to an intangible-intensive economy. Investment in intangibles, such as marketing, proprietary technologies, and organizational capital, has overtaken physical investment as the largest source of growth before 2007 (Corrado and Hulten (2010)).

(2) The nonfinancial corporate sector holds an increasing amount of cash (Bates, Kahle, and Stulz (2009); Gao, Whited, and Zhang (2018)), $2.1 trillion by 2018, which is largely in the form of financial intermediaries' debts, such as deposits and money-market instruments.[1] A key driver is the growth of intangible-intensive industries and their large cash holdings (Begenau and Palazzo (2015); Falato, Kadyrzhanovaz, Sim, and Steri (2018); Pinkowitz, Stulz, and Williamson (2015)).

(3) The interest rates in money markets declined due to a rising demand for liquid assets (Del Negro, Giannone, Giannoni, and Tambalotti (2017)). While foreign savings feature prominently in the current narrative (Bernanke (2005); Caballero, Farhi, and Gourinchas (2008); Caballero and Krishnamurthy (2009); Gourinchas and Rey (2016)), domestic corporate savings, which are comparable in magnitude (Greenwood, Hanson, and Stein (2016)), received little attention.

(4) The financial sector grew dramatically (Adrian and Shin (2010a); Gorton, Lewellen, and Metrick (2012); Greenwood and Scharfstein (2013)), financed by money-market instruments (Adrian and Shin (2010b); Gorton (2010); Pozsar (2014)). Schularick and Taylor (2012) find that in advanced economies, bank loan-to-GDP ratio doubled in the last two decades.

(5) The prices of collateral assets, such as commercial real estate, rose steadily before 2007 (Campello, Connolly, Kankanhalli, and Steiner (2019); Chaney, Sraer, and Thesmar (2012)).

This paper has two goals. First, with a minimum amount of ingredients, it builds a dynamic macro-finance model where intangible capital is essential and the five phenomena arise jointly in booms. Second, the model features a novel mechanism of financial instability, and thus, reveals the vulnerability of an intangible-intensive economy. The core is an intermediated liquidity supply.

[1] This includes currency, checking deposits, other bank deposits, money market fund shares, security repurchase agreements, commercial papers, and the government-issued securities (including Treasury securities). Government-issued and backed securities, including checking deposits that are largely insured, account for around 50% of the $2.1 trillion (source: Financial Accounts of the United States, Federal Reserve Statistical Release Z.1).

To finance investments in unpledgeable intangibles, firms hold liquidity in the form of bank debt and attach a liquidity premium to it. Banks back their debt by holding claims on firms' tangible capital, transmitting part of the liquidity premium into a bubbly value of tangible capital. Banks' capacity to do so depends on their equity capital that absorbs the risks involved in intermediation.

The bubbly value of tangible capital rises in booms as banks accumulate equity. For firms to create new tangible capital and profit from this bubbly value, they must also invest in the essential intangibles, so they hold more liquidity for intangible investment, and thereby, feed banks a larger liquidity premium (lower debt cost), allowing them to bid up the value of tangible capital even further. Next, the model is described in more detail to show how fragility arises from this self-perpetuating corporate savings glut. First, we consider a setting without banks where firms demand liquidity and supply liquidity themselves as in Holmström and Tirole (1998), and then, banks are introduced to intermediate liquidity supply and endogenous risk arises.

The continuous-time economy has a unit mass of infinite-lived agents ("entrepreneurs") who manage two types of capital, tangible and intangible, to produce generic goods for consumption and investment. Capital depreciates stochastically, loading on a Brownian shock, which is the only source of aggregate risk. Following bad shocks, a larger fraction of capital is destroyed.

Liquidity shocks are introduced following Holmström and Tirole (1998). Entrepreneurs face idiosyncratic Poisson shocks that entails a restart of their business – their existing capital is destroyed, but they may create new tangible and intangible capital instantaneously and proportionately. In particular, intangibles are essential – tangible capital cannot be created without intangible capital. The project requires goods as inputs. Ideally, new capital is pledged to entrepreneurs who are not hit by the restart shock, so goods they produce are lent to the entrepreneurs who need to invest. However, only tangible capital can be pledged while intangible capital cannot.

Entrepreneurs may sell the ownership of their tangible capital in a competitive market at price q_t^T per unit (with goods as the numeraire). After the sale, they dutifully manage the tangible capital on behalf of the buyers and deliver the goods it produces. The ownership of tangible capital is freely traded in secondary markets. In other words, tangible capital is perfectly pledgeable and liquid. Therefore, for the restart project, entrepreneurs can obtain a loan of q_t^T per unit of new

tangible capital. Once the tangible capital is created, its ownership is given to the creditors who break even, so the entrepreneur enjoys the full investment surplus.

Intangible capital is illiquid, representing human capital, organizational capital, and certain technologies that are inalienable and difficult for creditors to repossess. It is not pledgeable or tradable. Therefore, when the required intangible investment is large relative to tangible investment, which is the case of interest, the loans backed by tangible capital are not sufficient to cover the entire restart project. As a result, entrepreneurs want to hold liquidity.

As in Holmström and Tirole (1998), a mutual fund can be formed with shares distributed to entrepreneurs. The fund owns all entrepreneurs' tangible capital, the ultimate source of liquid assets, and diversifies away the idiosyncratic restart shock. For an individual entrepreneur, when the restart shock hits, even if her own capital is destroyed, her holdings of fund shares are still valuable because others' tangible capital still exists. Entrepreneurs can thus sell fund shares in exchange for goods as inputs for the creation of new capital. Now investment is financed by both the liquidity holdings in the form of fund shares and the loans backed by new tangible capital.

An immediate result is that entrepreneurs assign a liquidity premium to fund shares, which is essentially what entrepreneurs pay to insure against the idiosyncratic restart shocks. The liquidity premium is passed along by the mutual funds to tangible capital, and translates into a bubbly value of tangible capital that is beyond the present value of goods it produces. Moreover, the liquidity supply is stable. The equilibrium features constant values of tangible capital (q_t^T) and investment.

However, such diversification service that funds provide typically require expertise and a specialized intermediation sector. Moreover, in reality, firms mainly hold money-market instruments issued by financial intermediaries in their liquidity portfolios instead of direct claims on other firms. Hence a unit mass of bankers are introduced to intermediate the liquidity supply.[2]

Bankers acquire tangible capital and finance it by their own wealth (equity) and short-term debts issued to entrepreneurs (deposits). Following negative shocks, bankers lose wealth and shrink balance sheets, leaving more tangible capital owned by entrepreneurs who face the idiosyncratic restart shocks and issuing less deposits that entrepreneurs hold to pay for investment inputs. The

[2]Introducing intermediaries can also be motivated by their expertise in monitoring (Diamond (1984)), restructuring (Bolton and Freixas (2000)), or enforcing collateralized claims (Rampini and Viswanathan (2019)).

destruction of bankers' balance-sheet capacity and the consequent contraction of liquidity supply hurt the real economy by compromising the resource reallocation towards investing entrepreneurs. As typical in macro-finance models, for example Brunnermeier and Sannikov (2014), bankers cannot recapitalize (i.e., issue outside equity) due to potential agency frictions, so their balance-sheet capacity is procyclical; otherwise, the equilibrium would be the same as the mutual-fund equilibrium with all tangible capital owned by bankers and asset (tangible capital) price, liquidity supply, and firms' investment all being constant.

The intermediated liquidity supply leads to procyclical asset (tangible capital) price, firms' investment and savings waves, and banks' cyclical risk-taking and liquidity creation. In particular, it generates a mechanism of endogenous risk accumulation in booms. The liquidity premium that entrepreneurs assign to bank deposits lowers bankers' cost of debt and pushes their required rate of return below entrepreneurs', effectively making bankers the "natural buyers" (Shleifer and Vishny (2011)) of tangible capital. Therefore, the market value of tangible capital increases in bankers' wealth. In booms, bankers accumulate wealth, so the value of tangible capital increases, making the creation of new tangible capital more profitable. Entrepreneurs want more liquidity for the companion intangible investment and assign a higher liquidity premium to deposits. This further lowers bankers' cost of debt, so banks further bid up the market price of tangible capital.

What makes bankers the natural buyers of tangible capital is their low funding cost (discount rate), which is from the liquidity premium that firms attach to deposits. Such advantage increases in booms, as firms' investment needs and liquidity demand increase, driven by the procyclical profits from creating new tangible capital. As a result of the widening discount-rate gap, the market value of tangible capital becomes increasingly sensitive to shocks hitting the natural buyers' wealth. In other words, the fire sale risk increases as booms prolong.

Interestingly, the accumulation of endogenous risk in booms is asymmetric. Negative shocks cause continuing reallocation of tangible capital to entrepreneurs with high discount rates. However, positive shocks trigger the reallocation of tangible capital to bankers with low discount rates but eventually cause bankers to consume their wealth. Therefore, the downside risk is more prominent in prolonged booms. Such asymmetry sheds light on the recent findings that longer booms

precede more severe crises.[3] The mechanism is consistent with banks' procyclical payout in data.[4]

An average boom lasts around twenty years in the model and produces the following patterns that we see in the U.S. economy in the two decades leading up to the Great Recession. The firms (entrepreneurs) hold an increasing amount of cash in the form of financial intermediaries' debts, and their liquidity demand pushes down the interest (deposit) rate. The banking sector expands by acquiring more assets (tangible capital) and issuing more money-market instruments (deposits).

When the negative shocks hit, the positive feedback mechanism flips into a downward spiral with the downside risks rise faster than the upside, in line with the evidence in Adrian, Boyarchenko, and Giannone (2019). Banks become undercapitalized and the value of tangible capital falls, which discourages entrepreneurs from holding bank deposits for investment. As a result, bankers' cost of debt rises, so they recover their wealth slowly. An average recession lasts for nine years under the benchmark calibration. The slow recovery is consistent with the U.S. experience after the Great Recession and is in contrast with the existing models (e.g., Brunnermeier and Sannikov (2014)) that feature constant costs of bank debt and relatively quick recoveries.

Lastly I examine the rise of intangible capital in two different forms and show that both strengthen the instability mechanism of intermediated liquidity supply. The first is an exogenous increase of intangibles' productivity relative to that of tangible capital. As a result, a larger fraction of output becomes unpledgeable, which generates a stronger demand for liquidity and a more procyclical wedge between entrepreneurs' and bankers' discount rates in asset (tangible capital) markets. A 10% increase of intangibles' productivity doubles the peak level of asset price volatility.

Second, the rise of intangible capital is captured by the endogenous evolution of capital composition. So far, since tangible and intangible investments are made proportionate, the capital composition has been fixed. This restriction is now relaxed. The model shows that banks' liquidity supply can outpace the creation of tangible capital in booms, tilting the aggregate investment towards intangibles, so the relative scarcity of tangible capital increases. This creates a larger liquidity premium, a more volatile tangible capital value, and faster expansion of banks in booms.

[3]Please refer to Baron and Xiong (2017), Jordà, Schularick, and Taylor (2013), Krishnamurthy and Muir (2016), and López-Salido, Stein, and Zakrajšek (2017) among others.

[4]Baron (2014) and Adrian, Boyarchenko, and Shin (2016) document banks' payout cyclicality.

The main model omits an alternative source of liquidity supply, the government debt, and the demand for liquidity from households (consumers). Appendix III presents the extended models that have these two ingredients and shows that the mechanism of intermediated liquidity supply remains effective. Appendix I provides an alternative specification of production technology. Proofs and solution methods are provided in Appendix II and IV respectively.

Literature. This paper extends the liquidity-based asset pricing framework of Holmström and Tirole (2001) by incorporating financial intermediaries, and also contributes to a broader literature on the emergence of bubbles under financial frictions (Caballero and Krishnamurthy (2006); Farhi and Tirole (2012); Hirano and Yanagawa (2017); Kiyotaki and Moore (2012); Martin and Ventura (2012)). Giglio and Severo (2012) study how bubbles arise from asset shortages in intangible-intensive economies. Miao and Wang (2018) study bubbles attached to productive collateral assets. This paper differs by linking the bubbly value of tangible capital to the banks' procyclical balance-sheet capacity, and thus, reveals endogenous risks from the *intermediated* liquidity premium.

The rise of intangible capital has attracted enormous attention.[5] It is a key ingredient in the explanation of trends in corporate profits and investment (Crouzet and Eberly (2018); Gutiérrez and Philippon (2017)). Several recent papers study the interaction between firms and banks. Dell'Ariccia, Kadyrzhanova, Minoiu, and Ratnovski (2018) and Döttling and Perotti (2017) emphasize the decline of firms' borrowing from banks. This paper instead focuses on the liability side of banks' balance sheets, i.e., firms holding banks' liabilities as liquidity buffer.

A recent literature revives the money view of banking – banks' liabilities serve as inside money that facilitates resource reallocation (Kiyotaki and Moore (2000); Hart and Zingales (2014); Piazzesi and Schneider (2016)).[6] This paper is most related to Brunnermeier and Sannikov (2016) and Quadrini (2017) who model inside money holdings as insurance against idiosyncratic risks. Here the source of idiosyncratic (liquidity) risk is capital intangibility. A mapping is created from an economy's intangible intensity to the level of endogenous risk induced by producers' liquidity

[5]The literature studies the implications of intangible capital on productivity (Atkeson and Kehoe (2005)), current account (McGrattan and Prescott (2010a)), stock valuation (Hansen, Heaton, and Li (2005); Ai, Croce, and Li (2013); Eisfeldt and Papanikolaou (2014)), and investment (Daniel, Naveen, and Yu (2018); Peters and Taylor (2017)).

[6]There is a related literature on assets' information insensitivity (e.g., banks' safe debt) and their monetary services (Gorton and Pennacchi (1990); Holmström (2012); Dang, Gorton, Holmström, and Ordonez (2014)).

demand. As a theoretical contribution, the liquidity premium creates a procyclical wedge between bankers' and entrepreneurs' discount rates that is key to the accumulation of fire-sale risk in booms and is in contrast with the constant wedge of asset-management expertise among heterogeneous agents (e.g., He and Krishnamurthy (2013); Brunnermeier and Sannikov (2014)).

The liquidity premium on banks' liabilities has been well recognized in the literature (DeAngelo and Stulz (2015); Krishnamurthy and Vissing-Jorgensen (2015); Moreira and Savov (2017); Phelan (2016); Sundaresan and Wang (2014)) and incorporated into quantitative models (e.g, Begenau (2018); Egan, Lewellen, and Sunderam (2018); Van den Heuvel (2018)). This paper reveals a novel channel through which the liquidity premium on tradable assets depends on banks' capital and the liquidity premium on bank liabilities interacts with asset prices via firms' liquidity demand.

The banks in this paper share several features with those in Klimenko, Pfeil, Rochet, and Nicolo (2016), for example, the equity issuance friction. In their paper, banks intermediate between households' endowments and short-term projects, but here, banks intermediate between producers' liquidity demand and long-term assets (tangible capital), creating excess volatility in asset price through the bubbly (liquidity) value. The intermediated fund flow reminisces that in Rampini and Viswanathan (2019). What differs is that instead of household savings, the fund flow originates from firms' investment-driven liquidity demand, which is an essential ingredient for the shock amplification mechanism and, in particular, the accumulation of downside risk in booms.

Entrepreneurs' liquidity management in this paper, especially the Poisson liquidity shock, resembles that in He and Kondor (2016) who also model firms' procyclical liquidity demand but focus on the pecuniary externalities through liquidity premia and asset prices. Typical in models of firms' cash holdings (e.g., Bolton, Chen, and Wang (2011); Décamps, Mariotti, Rochet, and Villeneuve (2011); Froot, Scharfstein, and Stein (1993); Riddick and Whited (2009)), they assume a perfectly elastic supply of liquidity (storage). In contrast, this paper explicitly models the capacity of liquidity supply as a function of tangible capital and banks' capital. Corporate treasuries are major cash pools that lend to financial intermediaries in money markets (Pozsar (2011)). Understanding the price and quantity of liquidity requires jointly modeling firms and banks.[7]

[7] In models of macroeconomy and asset pricing, liquidity demand mostly arise from households (e.g., Krishnamurthy and Vissing-Jørgensen (2012)). However, as shown by Eisfeldt and Rampini (2009), corporate liquidity de-

The model predicts that prolonged booms precede severe crises (e.g., asset price collapse). The literature documents a similar pattern – the growth of financial sector often predates crises – but focuses exclusively on the credit-market dynamics for explanation (Baron and Xiong (2017); Gomes, Grotteria, and Wachter (2018); Jordà, Schularick, and Taylor (2013); Krishnamurthy and Muir (2016); López-Salido, Stein, and Zakrajšek (2017)). This paper calls for careful examination of liquidity premium and quantities. While much progress has been made in this direction (e.g., Greenwood, Hanson, and Stein (2010); Gorton, Lewellen, and Metrick (2012); Kacperczyk, Pérignon, and Vuillemey (2018); Krishnamurthy and Vissing-Jørgensen (2012); Nagel (2016); Sunderam (2015)), the interaction between banks and firms receives little attention.

The declining interest rate features prominently in the literature on long-run macroeconomic trends (Caballero, Farhi, and Gourinchas (2017); Eggertsson, Robbins, and Wold (2018); Farhi and Gourio (2018); Marx, Mojon, and Velde (2018)). This paper differs by characterizing a mapping from intangible intensity to the dynamics of interest rate and linking interest rate to corporate savings, the size of financial sector, and collateral asset price. Moreover, the focus is on the endogenous risk from intermediated liquidity supply, so financial intermediaries play a central role.

2 Model

Consider a continuous-time, infinite-horizon economy. We first introduce only one type of agents ("entrepreneurs") to focus on their liquidity demand driven by intangible investment, and later, we introduce bankers. We fix a probability space and an information filtration that satisfy the usual regularity conditions (Protter (1990)). Agents make decisions under rational expectation.

2.1 Intangible Capital and Liquidity Demand

Preferences. There are a unit mass of agents ("entrepreneurs"). Let c_t^E denote the representative entrepreneur's *cumulative* consumption up to time t. Throughout this paper, subscripts denote time, and superscripts denote types, with "E" for entrepreneurs (and later, "B" is for bankers).

mand plays an important role in explaining quantitatively the dynamics of liquidity premium.

Agents are risk-neutral and maximize expected utility with discount rate ρ:

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t} dc_t^E\right]. \tag{1}$$

Capital and production. Each entrepreneur manages a firm that produces non-durable generic goods using tangible and intangible capital. In aggregate, the economy has K_t^T units of tangible capital and K_t^I units of intangible capital at time t. One unit of tangible capital produces constant α units of goods per unit of time. The productivity of intangible capital is $\alpha+\phi$ (where $\phi > -\alpha$). So from t to $t+dt$, the aggregate output is $\alpha K_t^T dt + (\alpha+\phi) K_t^I dt$. Appendix I shows the equivalence between this setup and a Cobb-Douglas production function that combines two types of capital.

The two types of capital differ in liquidity. Tangible capital is perfectly liquid. Entrepreneurs may sell the ownership of their firms' tangible capital in a competitive market at price q_t^T per unit (denominated in goods). After the sale, they dutifully manage the capital on behalf of the buyers and deliver the goods produced. Tangible capital is free from frictions that compromise the cash-flow pledgeability or secondary-market liquidity. We may think of tangible capital as inventory, equipments, and plants in the production sector. In reality, physical assets are not actively traded, but securities backed by them are. We will consider land and housing markets later as an extension.

In contrast, intangible capital is illiquid. It is attached to the firm and its entrepreneur. The ownership of it cannot be sold or traded in secondary markets. The goods it produces cannot be pledged to outside investors for external funds. Intangible capital may represent entrepreneurs' inalienable human capital and other intangibles, such as organizational capital, proprietary technologies, and brand names, that are difficult to repossess for creditors.

Aggregate shock. The only source of aggregate uncertainty is from a Brownian motion Z_t. As in Brunnermeier and Sannikov (2014), a fraction $\delta dt - \sigma dZ_t$ of capital, both tangible and intangible, are destroyed over dt. Given the constant return-to-scale production technology, capital destruction shocks can be interpreted as productivity shocks. Capital represents efficiency units and is counted by its output. For example, a certain number of machines constitute one unit of tangible capital if they are responsible for α units of goods per unit of time. Intangible capital is counted likewise.

Even if the actual units of capital may not change, negative productivity shocks reduce the amount of effective capital. This is what the capital destruction shocks capture.

Liquidity shock and investment. Entrepreneurs face idiosyncratic liquidity shocks. The arrival of such shocks is independent across entrepreneurs, and follows a Poisson process with intensity λ. When hit by this shock, an entrepreneur's firm loses all capital, but she is endowed with a technology to transform goods into new capital instantaneously.[8]

Tangible and intangible investments are made simultaneously: for every θ units of new intangible capital, $1-\theta$ units of new tangible capital must be created, and vice versa. This assumption reflects the necessity of having both tangible and intangible capital in place for a firm's expansion.

The parameter θ determines the pledgeability of investment projects. Given i_t^E units of goods invested, $\theta\kappa i_t^E$ units of intangible capital and $(1-\theta)\,\kappa i_t^E$ units of tangible capital are created, where $\kappa(>0)$ denotes the investment efficiency. Tangible capital can be pledged for financing. Capital is created immediately, so the entrepreneur repays investors of this project instantaneously with the ownership of new tangible capital. The investment is constrained by the pledgeable value:

$$i_t^E \leq \underbrace{\kappa i_t^E\,(1-\theta)}_{\textit{units of new tangible capital}} q_t^T. \tag{2}$$

Dividing both sides by i_t^E, we have the condition of self-financing:

$$1 \leq \kappa\,(1-\theta)\,q_t^T. \tag{3}$$

If this condition holds, investment is unconstrained.[9] The market value of tangible capital is,

$$q_{FB}^T = \frac{\alpha}{\rho+\delta+\lambda}, \tag{4}$$

[8]This specification reflects the lumpiness of investment at micro level (e.g., Doms and Dunne (1998)). Due to the idiosyncratic nature of investment opportunities, the aggregate investment is smooth, in line with Thomas (2002).

[9]Note that when investment is unconstrained, an entrepreneur prefers to scale up the investment infinitely as long as the value of new capital created exceeds the cost, i.e., $\left(q^I\theta + q_{FB}^T\,(1-\theta)\right)\kappa > 1$ where $q^I = (\alpha+\phi)\,/\,(\rho+\delta+\lambda)$. The maximum investment scale can be reasonably restricted to rule out infinite investment. This paper focuses on the case where the liquidity constraint binds, so the infinite investment will not be a concern in the analysis.

where the subscript "FB" is for the "first-best". The numerator is the production flow, and the denominator contains the discount rate (ρ) and the expected rate of destruction from the stochastic depreciation (δ) and liquidity shocks (λ). To study entrepreneurs' liquidity demand, we impose the following restriction to rule out self-financing.

Assumption 1 *Investment projects are not self-financed:* $1 > \kappa (1-\theta) \left(\frac{\alpha}{\rho+\delta+\lambda}\right)$.

Liquidity supply within the production sector. Under Assumption 1, entrepreneurs would prefer to hold liquidity, i.e., assets other than capital of their own firms, which are immune to liquidity shocks and can be traded for goods as investment inputs when liquidity shocks arrive. It has been well documented that intangible investments rely heavily on firms' internal liquidity (for example, R&D investments in Hall (1992), Himmelberg and Petersen (1994), and Hall and Lerner (2009)).

As in Holmström and Tirole (1998), one solution is to pool all liquid assets (i.e., all firms' tangible capital) in mutual funds with shares distributed to entrepreneurs. Let m_t^E denote an entrepreneur's liquidity (mutual fund) holdings. The investment constraint is now

$$i_t^E \leq \kappa i_t^E (1-\theta) q_t^T + m_t^E. \tag{5}$$

Optimal liquidity holdings. We shall focus on the case where the liquidity constraint (5) binds, so κ is assumed to be sufficiently large such that the value of new capital created exceeds the cost, $\left(q^I \theta + q_t^T (1-\theta)\right) \kappa > 1$. Let q^I denote the value of intangible capital,

$$q^I = \frac{\alpha + \phi}{\rho + \delta + \lambda}. \tag{6}$$

Because entrepreneurs are indifferent across states and over time for consumption, we can view intangible capital as a stream of goods for consumption and its value is the discounted sum of the goods. The value of tangible capital, q_t^T, can be time-varying, as it carries a state-dependent liquidity premium (to be shown later). Due to this liquidity premium, q_t^T is larger than the value of given by Equation (4), so a sufficient condition for a binding liquidity constraint is the following.

Assumption 2 *The liquidity constraint binds if* $\left[\left(\frac{\alpha+\phi}{\rho+\delta+\lambda}\right)\theta + \left(\frac{\alpha}{\rho+\delta+\lambda}\right)(1-\theta)\right]\kappa > 1$.

Rearranging the binding liquidity constraint (Equation (5)), we can solve the investment as a function of m_t^E and the leverage obtained by pledging tangible capital:

$$i_t^E = \left(\frac{1}{1 - \kappa (1 - \theta) q_t^T} \right) m_t^E. \tag{7}$$

For one more dollar of liquidity holdings, entrepreneurs can invest $1/\left[1 - \kappa (1 - \theta) q_t^T\right]$ units of goods. And, because external funds are raised against tangible capital at fair price, entrepreneurs enjoy the full investment surplus, $q^I \theta \kappa + q_t^T (1 - \theta) \kappa - 1$. Therefore, when the liquidity shock hits, the marginal benefit of liquidity holdings is the profit on investment multiplied by the leverage. In equilibrium, the risk-neutral entrepreneurs' required rate of return on the liquidity holdings, r_t, is lower than their discount rate ρ. The wedge $\rho - r_t$, which is a *liquidity premium* or carry cost of liquidity holdings, is equal to the expected marginal benefit of liquidity holdings.

Proposition 1 (Liquidity Premium) *The entrepreneurs' required expected return on liquidity is*

$$r_t = \rho - \underbrace{\lambda}_{\textit{probability of liquidity shock}} \underbrace{\left[\left(q^I \theta + q_t^T (1 - \theta)\right) \kappa - 1\right]}_{\textit{profit on investment}} \underbrace{\left(\frac{1}{1 - \kappa (1 - \theta) q_t^T} \right)}_{\textit{leverage on liquidity holdings}} \tag{8}$$

A higher value of tangible capital increases the premium that entrepreneurs assign to liquidity holdings, because q_t^T shows up in both the profit on investment and the leverage on liquidity.

Capital valuation. Mutual funds purchase tangible capital and entrepreneurs acquire fund shares in competitive markets, so funds act simply as pass-through. If the expected return of tangible capital exceeds r_t (investors' required return on mutual-fund holdings), mutual funds expand, bidding up the tangible capital price and lowering the expected return of tangible capital; otherwise, funds shrink. Therefore, r_t is also the expected return of tangible capital.

To express the return on tangible capital holdings, we conjecture that tangible capital price follows a diffusion process in equilibrium,

$$dq_t^T = q_t^T \mu_t^T dt + q_t^T \sigma_t^T dt. \tag{9}$$

Let k_t^{TM} a representative mutual fund's tangible capital holdings, which depreciate stochastically:

$$dk_{i,t}^{TM} = -\left(\delta dt - \sigma dZ_t\right) k_{i,t}^{TM} - \lambda dt k_{i,t}^{TM}, \tag{10}$$

where the extra superscript "M" indicates mutual fund. The last term is from the λdt entrepreneurs who are hit by the liquidity shocks. Under Itô's lemma, the return on tangible capital, dr_t^T, is

$$dr_t^T = \underbrace{\frac{\alpha k_t^{TM} dt}{q_t^T k_t^{TM}}}_{\textit{dividend gain}} + \underbrace{\frac{d\left(q_t^T k_t^{TM}\right)}{q_t^T k_t^{TM}}}_{\textit{capital gain}} = \left(\frac{\alpha}{q_t^T} + \mu_t^T - \delta - \lambda + \sigma_t^T \sigma\right) dt + \left(\sigma_t^T + \sigma\right) dZ_t, \tag{11}$$

where $\sigma_t^T \sigma$ is Itô's quadratic covariation term. The expected return is equal to r_t in equilibrium,

$$\mathbb{E}_t\left[dr_t^T\right] = \frac{\alpha}{q_t^T} + \mu_t^T - \delta - \lambda + \sigma_t^T \sigma = r_t. \tag{12}$$

In principle, the two stock variables, K_t^T and K_t^I, are the state variables for a Markov equilibrium. However, given the proportionality of intangible and tangible investments, $K_t^I / \left(K_t^I + K_t^T\right)$ is fixed at θ given the initial condition $K_0^I / \left(K_0^I + K_0^T\right) = \theta$. This leaves the total capital, $K_t = K_t^I + K_t^T$ as the only state variable. The constant return-to-scale production and investment technologies imply that the economy is scale-free, so the equilibrium prices, such as q_t^T and r_t, are constant. Using Equation (12) to substitute out r_t in Equation (8), we price tangible capital.

Proposition 2 (Asset Pricing) *Tangible capital, the ultimate source of liquidity, has a unit price*

$$q^T = \frac{\alpha}{\rho + \delta + \lambda - \underbrace{\lambda\left[\left(q^I \theta + q^T\left(1-\theta\right)\right)\kappa - 1\right]\left(\frac{1}{1-\kappa\left(1-\theta\right)q^T}\right)}_{\textit{liquidity premium}}}. \tag{13}$$

In comparison with the "fundamental value" of tangible capital in an unconstrained economy (Equation (4)), the illiquidity of intangible capital translates into a liquidity premium that lowers the effective discount rate, giving rise to a "bubbly" value as in Giglio and Severo (2012). Here

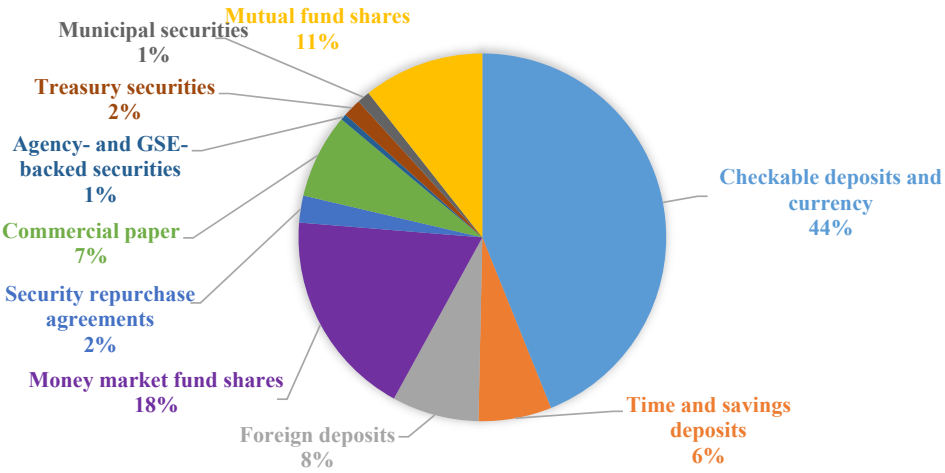

Figure 1: **Corporate Liquidity Holdings (the U.S. Financial Accounts 2017)**.

tangible capital serves two purposes: (1) it produces goods; (2) a diversified portfolio of tangible capital is held by entrepreneurs to relax the liquidity constraint on investment.

2.2 Intermediated Liquidity Supply

As shown in Figure 1, firms rarely hold direct claims on other firms, but instead hold debt securities largely issued by financial intermediaries. As documented by Pozsar (2014), corporate treasuries are among the major cash pools that feed leverage to intermediaries. Next, we introduce bankers who intermediate the supply of liquidity. Entrepreneurs are assumed to hold liquidity in the form of bank debt (referred to as "deposits") backed by banks' holdings of tangible capital.

There are several reasons why firms hold *intermediated liquidity*. Previously, mutual funds provide liquidity by diversifying aways the idiosyncratic liquidity shocks, but such service is likely to require expertise and a specialized intermediation sector.[10] Agency frictions may arise and limit the issuance of outside equity (e.g., He and Krishnamurthy (2013)), so firms hold intermediaries' debt instead of equity as their liquidity buffer. Intermediated liquidity supply is also motivated by the theoretical literature on banks as inside money creators (e.g., Kiyotaki and Moore (2000)).

[10]Introducing intermediaries can also be motivated by their expertise in monitoring (Diamond (1984)), restructuring (Bolton and Freixas (2000)), or enforcing collateralized claims (Rampini and Viswanathan (2019)).

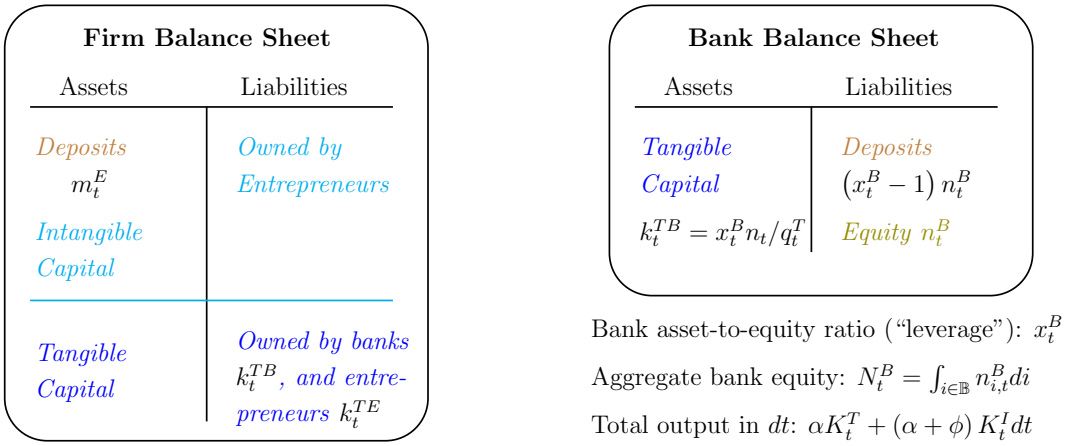

Figure 2: **Model Overview**.

The banking sector. With a slight abuse of notation, m_t^E now represent entrepreneurs' holdings of short-term bank debts ("deposits") that are issued at time t and mature at $t + dt$ with interests $r_t dt$. As will be shown later, we study a Markov equilibrium where banks never default, so bank debt is safe and the promised interest rate is its realized return.[11] Entrepreneurs use deposits to buy goods as investment inputs when hit by liquidity shocks, so banks add value to the economy because their debt serves as a liquidity buffer that facilitates goods reallocation to those with investment needs.

Let n_t^B denote the wealth of a representative banker who invests in firms' tangible capital and issues debt (deposits). A banker maximizes the same risk-neutral utility given by Equation (1) subject to the following budget (flow-of-funds) constraint

$$dn_t^B = x_t^B n_t^B dr_t^T + (1 - x_t^B) n_t^B r_t dt - dc_t^B, \tag{14}$$

where x_t^B is the fraction of wealth allocated to tangible capital, i.e., the asset-to-equity ratio or bank leverage and c_t^B is the cumulative consumption. Let k_t^{TB} denote a representative banker's tangible capital holdings (with the extra superscript "B" indicating banker), so $x_t^B n_t^B = q_t^T k_t^{TB}$. In equilibrium, $x_t^B > 1$ because banks issue debt that is held by entrepreneurs as liquidity.

[11] Macro-finance models that are built upon diffusion processes often do not feature bank default (e.g., Brunnermeier and Sannikov (2014)). Default may be introduced through an aggregate Poisson shock that destroys productive capital. The resulting sudden evaporation of liquidity and potential policy intervention can be directions for future research.

Figure 2 summarizes the structure. Entrepreneurs own intangible capital and may own tangible capital (i.e., $k_t^{TE} > 0$ in Figure 2 with the extra superscript "E" indicating entrepreneur). They manage firms to produce goods for themselves and outside investors who own tangible capital.

An undercapitalized banking sector cannot fulfill its role as liquidity supplier. To capture this idea, I assume that banks face equity issuance friction (Holmström and Tirole (1997); Bolton and Freixas (2000); Van den Heuvel (2002)). Such friction may arise because bankers' inalienable expertise is required for managing the diversified portfolio of tangible capital (He and Krishnamurthy (2013)). For simplicity, I assume that banks cannot issue equity at all, i.e., $dc_t^B \geq 0$ as in Brunnermeier and Sannikov (2014). By inspecting banks' budget constraint, we can see that negative consumption is equivalent to issuing equity and replenishing net worth.[12] This friction links firms' liquidity and intangible investment to banks' balance-sheet condition.

Entrepreneurs cannot diversify away the liquidity shocks that hit their firms' capital, so bankers pool tangible capital, the ultimate source of liquidity, and issue deposits. Intermediated liquidity supply depends on bankers' net worth. Here entrepreneurs' liquidity demand in Holmström and Tirole (1998) meets banks' limited balance-sheet capacity in Holmström and Tirole (1997).[13]

Intermediated liquidity premium in asset prices. Given the homogeneity property of bankers' problem, their value function is $q_t^B n_t^B$, linear in wealth, where q_t^B is the marginal value of wealth. Bankers do not consume when q_t^B is greater than one (the marginal value of consumption). Without the equity issuance constraint, $q_t^B = 1$, because retaining wealth by forgoing consumption does not add value when it is free to raise equity and replenish net worth. Under the issuance constraint, q_t^B varies over time in $[1, +\infty)$, following a conjectured diffusion process in equilibrium,

$$\frac{dq_t^B}{q_t^B} = \mu_t^B dt + \sigma_t^B dZ_t. \tag{15}$$

[12]See also Phelan (2016) and Klimenko, Pfeil, Rochet, and Nicolo (2016) for similar specifications. Note that negative consumption is allowed for entrepreneurs except when liquidity shocks hit. In other words, entrepreneurs are only financially constrained at such Poisson times. Allowing negative consumption is equivalent to assuming large endowments of goods – if goods are nondurable, entrepreneurs always consume to clear the goods market, indifferent between consuming and saving. This fixes their marginal value of wealth at one and required return at ρ.

[13]See also, Holmström and Tirole (2001), Eisfeldt and Rampini (2009), and Farhi and Tirole (2012) for investment-driven liquidity demand. Eisfeldt (2007) show that the liquidity premium of Treasury bills cannot be explained by the liquidity demand from consumption smoothing under standard preferences.

The proofs in Appendix II confirm these conjectures of value function and the process of q_t^B.

As will be confirmed by the solution, $\sigma_t^B < 0$ in equilibrium – negative shocks reduce bankers' net worth and increase their marginal value of wealth, q_t^B.[14] Negative shocks also lead to lower realized returns on tangible capital due to the exogenous depreciation and the decline of tangible capital price in equilibrium. Therefore, bankers require a risk premium of $-\sigma_t^B \left(\sigma_t^T + \sigma\right) dt$, the covariance between the return on tangible capital and the change of bankers' marginal value of wealth. Here $-\sigma_t^B$ is the price of risk and $\left(\sigma_t^T + \sigma\right)$ is the quantity of risk (i.e., the sum of endogenous volatility from tangible capital price change, σ_t^T, and exogenous volatility, σ).

Proposition 3 (Tangible Capital Pricing under Intermediated Liquidity Supply) *The expected return, or the discount rate, in the tangible capital market, is given by:*

$$\mathbb{E}_t \left[dr_t^T\right] = r_t + \left(-\sigma_t^B\right) \left(\sigma_t^T + \sigma\right) \tag{16}$$

The proof is in Appendix II. In the mutual-fund equilibrium, $\mathbb{E}_t \left[dr_t^T\right] = r_t$, and the gap between agents' time-discount rate ρ and the discount rate for tangible capital is precisely the liquidity premium. When the liquidity supply is intermediated, the gap narrows by $\left(-\sigma_t^B\right) \left(\sigma_t^T + \sigma\right)$ to compensate banks' risk exposure. The transmission of liquidity premium is now imperfect, because it depends on banks' limited balance-sheet capacity. Only if there were no frictions on equity issuance, $q_t^B = 1$, so $\sigma_t^B = 0$ and the full liquidity premium would be passed to tangible capital.

Intermediated liquidity premium generates a feedback mechanism as will be shown in Section 3. Following positive shocks, banks become better capitalized and require a smaller risk compensation, so they transmit the liquidity premium more effectively, lowering the discount rate for tangible capital and boosting its price. As q_t^T increases, investment becomes more profitable through higher profits on investment and leverage on internal liquidity, so entrepreneurs demand

[14] Due to the negative shocks and their persistent effects under the equity issuance constraint, the whole banking sector becomes undercapitalized and shrinks for a sustained period of time. To clear the markets of tangible capital and deposits, the spread between the expected return on tangible capital and deposit rate will have to increase so that banks would hold tangible capital and issue deposits. Like Tobin's Q, q_t^B, is a forward looking measure of profits per unit of equity. As the expected future profits rise, q_t^B increases.

more liquidity to prepare for investments (Proposition 1). A higher liquidity premium is transmitted by banks to further lower the discount rate for tangible capital and further increase q_t^T. In the process, entrepreneurs' liquidity holdings and investments increase. This mechanism of *intermediated liquidity premium* is the key to understand the accumulation of endogenous risk in booms and the severity and duration of crises, as will be shown in Section 3 after the model is fully solved.

The real-financial linkage. Let intervals $\mathbb{B} = [0,1]$ and $\mathbb{E} = [0,1]$ denote the sets of bankers and entrepreneurs respectively. $N_t^B = \int_{i\in\mathbb{B}} n_{i,t}^B di$, is the aggregate wealth of bankers, and $M_t^E = \int_{i\in\mathbb{E}} m_{i,t}^E di$ is entrepreneurs' aggregate deposits. The deposit market clears:

$$M_t^E = \left(x_t^B - 1\right) N_t^B. \tag{17}$$

Here we utilize the homogeneity of bankers, that is every banker has the same x_t^B.

Recall that $K_t = K_t^I + K_t^T$ denote the total productive capital, which evolves as:

$$dK_t = \underbrace{-\left(\delta dt - \sigma dZ_t\right) K_t}_{\textit{stochastic depreciation}} \underbrace{-\lambda dt K_t}_{\textit{liquidity shocks}} + \underbrace{\left[\kappa\left(\frac{1}{1-\kappa\left(1-\theta\right) q_t^T}\right) M_t^E\right]\lambda dt}_{\textit{investment}} + \underbrace{K_t \chi dt}_{\textit{endowments}} \tag{18}$$

The first component is the stochastic depreciation, and the second component is the capital lost due to liquidity shocks. The third component is from the λdt entrepreneurs who invest deposits, $M_t^E \lambda dt$, with a leverage, $1/\left[1-\kappa\left(1-\theta\right) q_t^T\right]$ (Equation (7)). Finally, the economy is endowed with a flow of new capital (with θ fraction being intangible and $(1-\theta)$ tangible) from χdt measure of newly born entrepreneurs. It capture sources of growth beyond the liquidity-constrained investments. To fix the population, entrepreneurs exit at idiosyncratic Poisson time with intensity χ, with their wealth evenly distributed among the rest of population. So, entrepreneurs' total discount rate ρ is essentially the sum of exit probability χ and the time-discount rate $\rho - \chi$.

Substituting the deposit market clearing condition into Equation (18), we have:

$$\frac{dK_t}{K_t} = \underbrace{\left\{\left[\kappa\left(\frac{1}{1-\kappa\left(1-\theta\right) q_t^T}\right)\left(x_t^B - 1\right)\left(\frac{N_t^B}{K_t}\right)\right]\lambda - \delta - \lambda + \chi\right\}}_{\mu_t^K} dt + \sigma dZ_t. \tag{19}$$

The instantaneous expected growth rate, μ_t^K, is directly linked to banks' equity, i.e., their capacity to intermediate liquidity supply. When banks are well-capitalized and issue abundant deposits, the economy grows fast because entrepreneurs are able to hold liquidity and goods flow to those with investment needs. To meet the liquidity demand from intangible investments, banks issue deposits backed by holdings of tangible capital, and thereby, increase the economic growth rate and welfare.

Solving the Markov equilibrium. At time t, the economy has three stock variables, K_t^I, K_t^T, and N_t^B, that in principle, would be the state variables in a time-homogeneous Markov equilibrium. As previously discussed, the economy is scale-free and the mix of intangible and tangible capital is fixed given the initial condition $K_0^I / \left(K_0^I + K_0^T\right) = \theta$, so the ratio of bank equity to total capital,

$$\eta_t = \frac{N_t^B}{K_t^I + K_t^T},$$

becomes the only state variable that drives the tangible capital price, the interest rate, the banks' leverage, and the entrepreneurs' deposit holdings. The model will be extended in Section 3.4 to have the *endogenous* capital composition as the second state variable of the economy.

The state variable η_t measures banks' liquidity creation capacity relative to the size of real sector that demands liquidity. By Itô's lemma, η_t follows a regulated diffusion process:

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ_t - dy_t^B, \tag{20}$$

where $dy_t^B = dc_t^B / n_t^B$ is bankers' consumption-to-wealth ratio. As will be shown later, bankers' consumption imposes an upper bound, $\overline{\eta}$. The drift term, μ_t^η, is $\mu_t^N - \mu_t^K - \sigma_t^N \sigma + \sigma^2$. μ_t^N is the drift of N_t^B. μ_t^K is defined in Equation (19). The last two terms are quadratic covariation of Itô's calculus. Bankers are homogeneous, so N_t^B follows the same dynamics as n_t^B. Given the return on tangible capital (Equation (11)) and banks' budget constraint (Equation (14)),

$$\frac{dn_t^B}{n_t^B} = \left[r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T\right] - r_t\right)\right] dt + x_t^B \left(\sigma_t^T + \sigma\right) dZ_t. \tag{21}$$

In equilibrium, entrepreneurs hold deposits and bankers issue debt, so $x_t^B > 1$. Following positive

shocks, better capitalized banks pass a larger liquidity premium to tangible capital, lowering the discount rate and driving up the tangible capital price, so q_t^T responds positively to dZ_t, i.e., $\sigma_t^T > 0$. Therefore, the shock elasticity of n_t^B, i.e., $\sigma_t^N = x_t^B \left(\sigma_t^T + \sigma\right)$, is larger than σ, and thus, the shock elasticity (diffusion) of η_t, $\sigma_t^\eta = \sigma_t^N - \sigma$, is positive. Following positive shocks, η_t increases.

To solve the Markov equilibrium, the optimality and market-clearing conditions are converted into a system of ordinary differential equations (ODEs). Specifically, bankers' F.O.C. for tangible capital holdings and their HJB equation form a pair of ODEs for the forward-looking variables, $q^B(\eta_t)$ and $q^T(\eta_t)$. Once they are solved as functions of η_t, other endogenous variables are derived as functions of their values and derivatives. The details are provided in Appendix III.

Proposition 4 (Markov Equilibrium) *For any initial endowments of entrepreneurs' intangible capital $\{k_{i,0}^I, i \in \mathbb{E}\}$ and tangible capital $\{k_{i,0}^{TE}, i \in \mathbb{E}\}$, and bankers' tangible capital $\{k_{j,0}^{TB}, j \in \mathbb{B}\}$ such that*

$$\int_{i\in\mathbb{E}} k_{i,0}^I di = K_0^I, \quad \int_{i\in\mathbb{E}} k_{i,0}^{TE} di + \int_{j\in\mathbb{B}} k_{j,0}^{TB} dj = K_0^T,$$

and $K_0^I / \left(K_0^I + K_0^T\right) = \theta$, there exists a Markov equilibrium on the filtered probability space generated by the Brownian motion $\{Z_t, t \geq 0\}$. The state variable. η_t, follows an autonomous law of motion (Equation (20)) in $(0, \overline{\eta}]$ that maps any path of shocks $\{Z_s, s \leq t\}$ to the current state.

Agents take as given the processes of price variables, such as q_t^T and r_t, and optimally consume, invest, trade the ownership of tangible capital, and hold or issue deposits. Prices adjust to clear all markets with goods as the numeraire. Bankers' first-order condition for tangible capital holdings and HJB equation form two ODEs for $q^B(\eta_t)$ and $q^T(\eta_t)$ with five boundary conditions:

As η_t approaches zero: (1) $\lim_{\eta_t \to 0} \frac{dq^T(\eta_t)}{d\eta_t} = 0$; (2) $\lim_{\eta_t \to 0} q^B(\eta_t) = +\infty$.

At $\overline{\eta}$: (3) $\frac{dq^T(\eta_t)}{d\eta_t} = 0$; (4) $q^B(\overline{\eta}) = 1$; (5) $\frac{dq^B(\eta_t)}{d\eta_t} = 0$,

where $\overline{\eta}$, the upper bound of η_t, is pinned down by the optimality of bankers' consumption.

The boundaries are explained below. Tangible capital has constant cash flow, α per unit of time, so what causes its price to vary is the discount-rate changes. Around $\eta_t = 0$, an absorbing

state, the banking sector is extremely small, so the discount rate (expected return) is fixed at ρ to induce entrepreneurs to own tangible capital and clear the market. Thus, q_t^T should not vary as η_t approaches zero (Condition (1)). Moreover, when banks are extremely undercapitalized, the their marginal value of equity approaches infinity (Condition (2)). The upper bound, $\overline{\eta}$, is a reflecting state. Condition (3) guarantees that q_t^T does not jump at $\underline{\eta}$. Condition (4) and (5) are the value-matching and smooth-pasting conditions respectively for the optimality of bankers' consumption.

Stationary distribution and recovery time. To study the long-run behavior of the economy, it is useful to characterize the stationary probability distribution of η_t and the expected time it takes for η_t to travel from a region of negative economic growth to a region of positive growth ("recovery time"). Since we study a time-homogeneous Markov equilibrium, time subscripts are suppressed.

Proposition 5 *The stationary probability density of state variable η_t, $p(\eta)$ is a solution to*

$$\mu^{\eta}(\eta)\, p(\eta) - \frac{1}{2}\frac{d}{d\eta}\left(\sigma^{\eta}(\eta)^2\, p(\eta)\right) = 0,$$

where $\mu^{\eta}(\eta)$ and $\sigma^{\eta}(\eta)$ are the drift and diffusion of η_t defined in Equation (20). The expected time to reach $\eta \in (0, \overline{\eta}]$ from any given value of $\underline{\eta}$, $g(\eta)$ is a solution to

$$1 - g'(\eta)\, \mu^{\eta}(\eta) - \frac{\sigma^{\eta}(\eta)^2}{2} g''(\eta) = 0,$$

with the boundary conditions $g\left(\underline{\eta}\right) = 0$ and $g'\left(\underline{\eta}\right) = 0$.

3 Solution

3.1 Parameter Choices

The model is solved numerically. One unit of time is one year. Entrepreneurs' time-discounting rate is 6%, in line with historic average returns of stocks and corporate bonds. Entrepreneurs' exit rate, χ, is set to 4%, so their overall discount rate, ρ, is 10%. The exit rate is also the entry rate of new entrepreneurs and their capital, which captures sources of growth outside of the model and generates a 1.8% long-run mean of economic growth rate under the stationary density.

We may interpret δ and σ as the average and standard deviation of project failure rate. The choices of $\delta = 4\%$ and $\sigma = 2\%$ are in line with the time-series mean and standard deviation of delinquency rates of commercial and industrial loans (source: FRED). The volatility of capital failure rate, σ, is a key parameter for banks' risk-taking, so whether the model-implied bank leverage matches data serves as another check on the choice of σ's value. With $\sigma = 2\%$, the long-run mean of bank leverage is 19, in line with what Adrian, Boyarchenko, and Shin (2016) document.

Liquidity shocks arrive every ten years on average, i.e., $\lambda = 1/10$, and this parameter choice is disciplined by the variation of economic growth rate. The economy has a constant source of growth from the entry of new entrepreneurs, governed by χ, and a state-dependent source of growth from liquidity-constrained investments. Therefore, λ is a pivotal parameter for the range of variation of μ_t^K, which is between -1% to 4%. The parameter of investment efficiency, κ, is critical for the marginal benefit of liquidity holdings. It is set to 3 to generate a 15% long-run mean of firms' cash-to-asset ratio, in line with the average in Compustat.

The annual production of tangible capital, α, is set to 0.05 to generate a 7.5 long-run mean of q_t^T/α, in line with the EV/EBITDA ratio of tangible industries such as construction, mining, and materials (Compustat). A key parameter is ϕ. It measures the productivity gap between tangible and intangible capital. The rise of intangible capital is captured by an increase of ϕ, meaning that more output is attributed to intangible capital. Comparative statics reveal how the equilibrium dynamics respond as the economy becomes more intangible-intensive (Section 3.3). In the baseline case, $\phi = 0.01$, i.e., 20% productivity advantage of intangibles over tangible capital. In the baseline model, θ, the fraction of capital and investment that are intangible, is set to 70%. Later in Section 3.4, the model is extended to allow endogenous capital composition.

3.2 Equilibrium Dynamics

The model reveals a feedback mechanism that accumulates endogenous risk in booms and amplifies the shock impact on both real and financial variables in crises. At the center is the intermediated liquidity premium. The equilibrium dynamics offer a coherent account of asset price and interest rate variation, the expansion and contraction of banking sector, and the variation of corporate cash

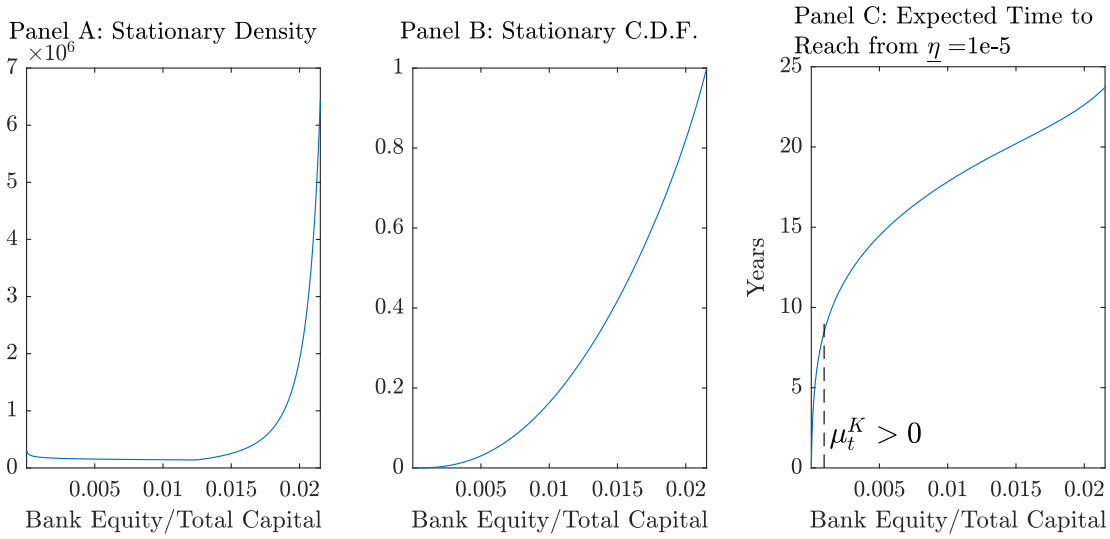

Figure 3: **Stationary Distribution and Recovery Time**.

holdings in the decades around the financial crisis of 2007–2008.

In the following, variables from the numeric solution (details in Appendix III) are plotted against the state variable, η_t, the ratio of bank equity to total capital. Graphs start at $\eta_t = 0.00001$ (chosen to be close to zero) and ends at the reflecting boundary, $\overline{\eta}$, at which bankers consume. Following positive shocks, η_t moves to the right and the banking sector expands relative to the real sector; following negative shocks, η_t moves left and the banking sector shrinks.

Panel A and B of Figure 3 show the stationary probability density and cumulative probability function of η_t respectively. They measure the amount of time the economy spends at different levels of η_t over the long run. A key feature is that recessionary states (low η_t and $\mu_t^K < 0$) are rare events. However, recessions are stagnant. Panel C shows the expected time to reach different levels of η_t from $\eta_t = 0.00001$. It takes nine years to reach the lowest value of η_t with $\mu_t^K > 0$. The complete cycle spans more than two decades, consistent with the focus of this paper on a longer time frame than a typical business cycle.

In this economy, q_t^T capitalizes all the liquid cash flows, so an increase of q_t^T corresponds to a broad increase of asset prices in reality. Panel A of Figure 4 plots q_t^T/α, the price-to-cash flow ratio of tangible capital. Recall that the dividend of tangible capital is fixed at α per year, so what

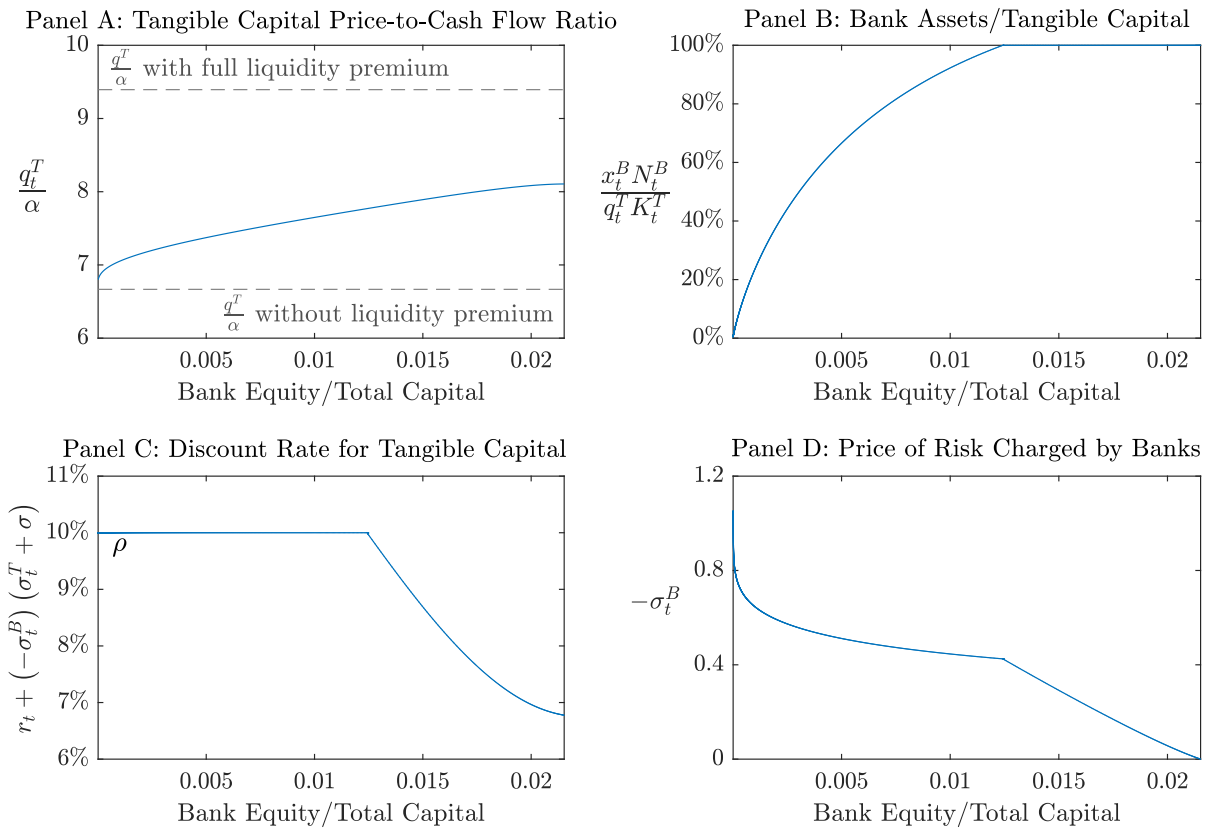

Figure 4: **Asset Pricing and the Intermediated Liquidity Premium**.

drives the changes in q_t^T is the discount rate, and in particular, the intermediated liquidity premium.

As shown in Panel B and C of Figure 4, when banks are relatively undercapitalized and hold tangible capital together with entrepreneurs, the expected return or discount rate is ρ as required by entrepreneurs. When banks are wealthy, they hold all the tangible capital and charge a small price of risk, $-\sigma_t^B$ (Panel D), pushing the discount rate below ρ by transmitting the liquidity premium on deposits. Therefore, q_t^T increases in η_t because as banks become wealthier, the economy is more likely to stay in the region where the discount rate for tangible capital is below ρ.

Panel A of Figure 4 also plots two benchmark values of tangible capital price. The dashed line above is from the mutual-fund equilibrium, where the discount rate for tangible capital fully reflects the liquidity premium (Proposition 2). The dashed line below is from the first-best case where intangible investment is pledgeable and the liquidity premium is zero. The liquidity pre-

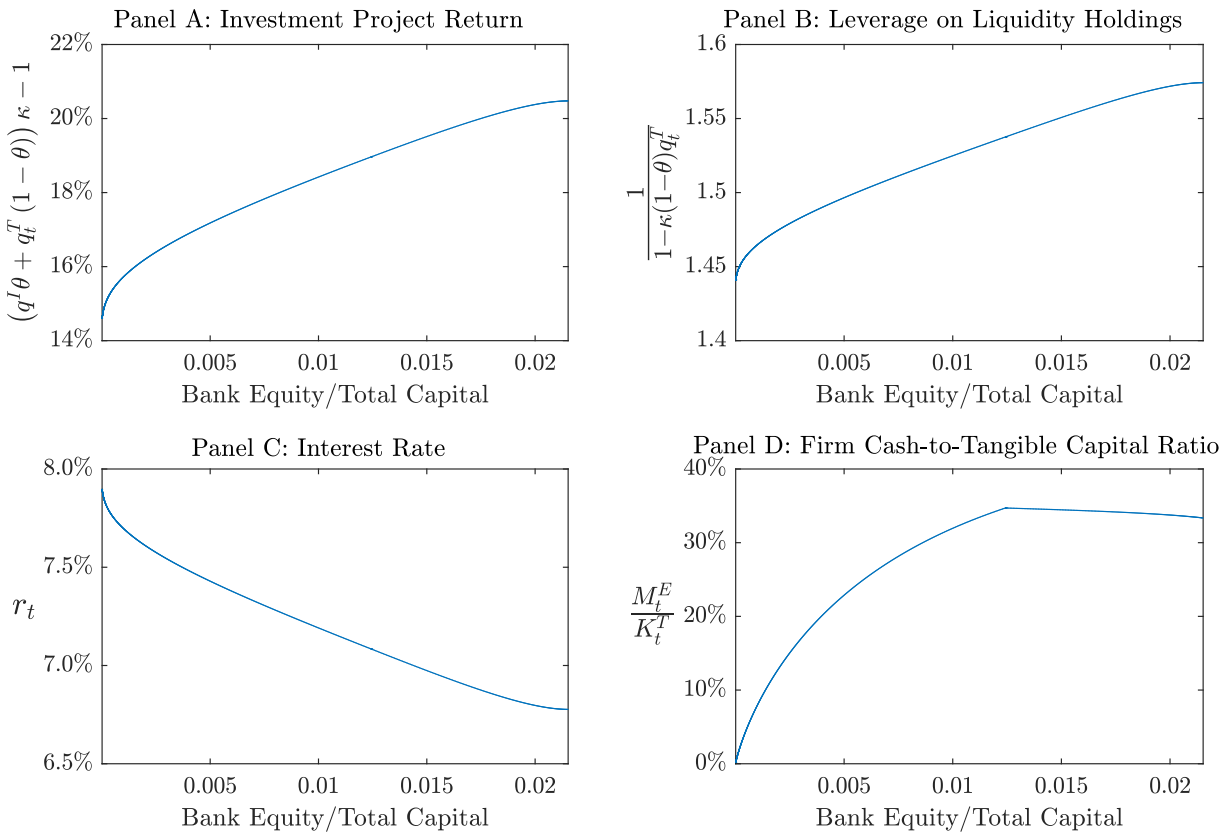

Figure 5: **Endogenous Corporate Savings Glut**.

mium, $\rho - r_t$, arises because intangible capital is illiquid (Proposition 1). If banks' equity issuance constraint were removed, banks' balance-sheet capacity would be unlimited and the transmission of liquidity premium would be perfect as in the mutual-fund equilibrium. Under the equity issuance constraint, banks take a share of the liquidity premium, i.e., $\left(-\sigma_t^B\right)\left(\sigma_t^T + \sigma\right)$, to compensate their risk exposure and only transmit the remaining part to the tangible capital market (Proposition 3).

As η_t increases and banks transmit an increasingly larger share of liquidity premium, the price of tangible capital rises, which in turn increases the liquidity premium through its impact on the entrepreneurs' incentive to hold liquidity for investments (Proposition 1).[15] First, as in the Q-model of Hayashi (1982), when q_t^T is higher, the profits per unit of goods invested become

[15]Investment need is a key determinant of firms' cash holdings (Denis and Sibilkov (2010); Duchin (2010)). Firms with less collateral also tend to hold more cash (Almeida and Campello (2007); Li, Whited, and Wu (2016)).

higher. Second, the leverage on liquidity also becomes higher, because the external financing capacity increases in q_t^T. Panel A and B of Figure 5 plot the two variables. A stronger liquidity demand pushes down the interest (deposit) rate, r_t (Panel C), feeding banks with cheap financing and lowering their discount rate. As a result, q_t^T can increase even further.

This feedback mechanism generates an endogenous savings glut in the production sector that resembles the rise of U.S. corporate cash holdings in the decades leading up to the financial crisis (e.g., Bates, Kahle, and Stulz (2009)). Panel D of Figure 5 plots the ratio of firms' deposits ("cash") to tangible capital, K_t^T. Tangible capital is a closer counterpart to firms' book assets in data because intangible capital is often ignored in book assets (e.g., Peters and Taylor (2017)). In the empirical literature, many have attributed the enormous corporate cash holdings to an increasing share of firms that heavily rely on intangible capital (Begenau and Palazzo (2015); Pinkowitz, Stulz, and Williamson (2015); Graham and Leary (2015); Falato et al. (2018)). In the model, the run-up of firms' cash holdings stops when banks are very large and a further growth of bank equity outpaces the growth of tangible capital value, and thus, crowds out bank debt (deposits).

The corporate savings glut supplies cheap leverage to banks, so the banking sector expands as it did before the financial crisis (Adrian and Shin (2010b); Greenwood and Scharfstein (2013); Schularick and Taylor (2012)). Many have argued that such expansion fed on the liquidity premia on intermediaries' money-like liabilities (Adrian and Shin (2010a); Gorton (2010); Pozsar (2014)). Next, I show that this endogenous savings glut leads to a unique mechanism of financial instability, which stands in contrast with the current literature that focuses on exogenous savings (e.g., foreign savings) and their implications on interest rate and asset prices (Caballero, Farhi, and Gourinchas (2008); Caballero and Krishnamurthy (2009); Bolton, Santos, and Scheinkman (2018)).

The feedback mechanism of intermediated liquidity premium not only affects the level of asset price but also amplifies its volatility. Let us consider the economy moving from $\eta_t = 0.005$ to the right in Panel A of Figure 6 (which reproduces Panel C of Figure 4). Initially the discount rate stays at ρ unless the economy is hit by extremely large positive shocks. But as we move to the right, η_t approaches the cutoff point where the discount rate falls below ρ. As a result, even small shocks can cause a discount-rate change, so the asset price, q_t^T, becomes more sensitive to shocks (i.e.,

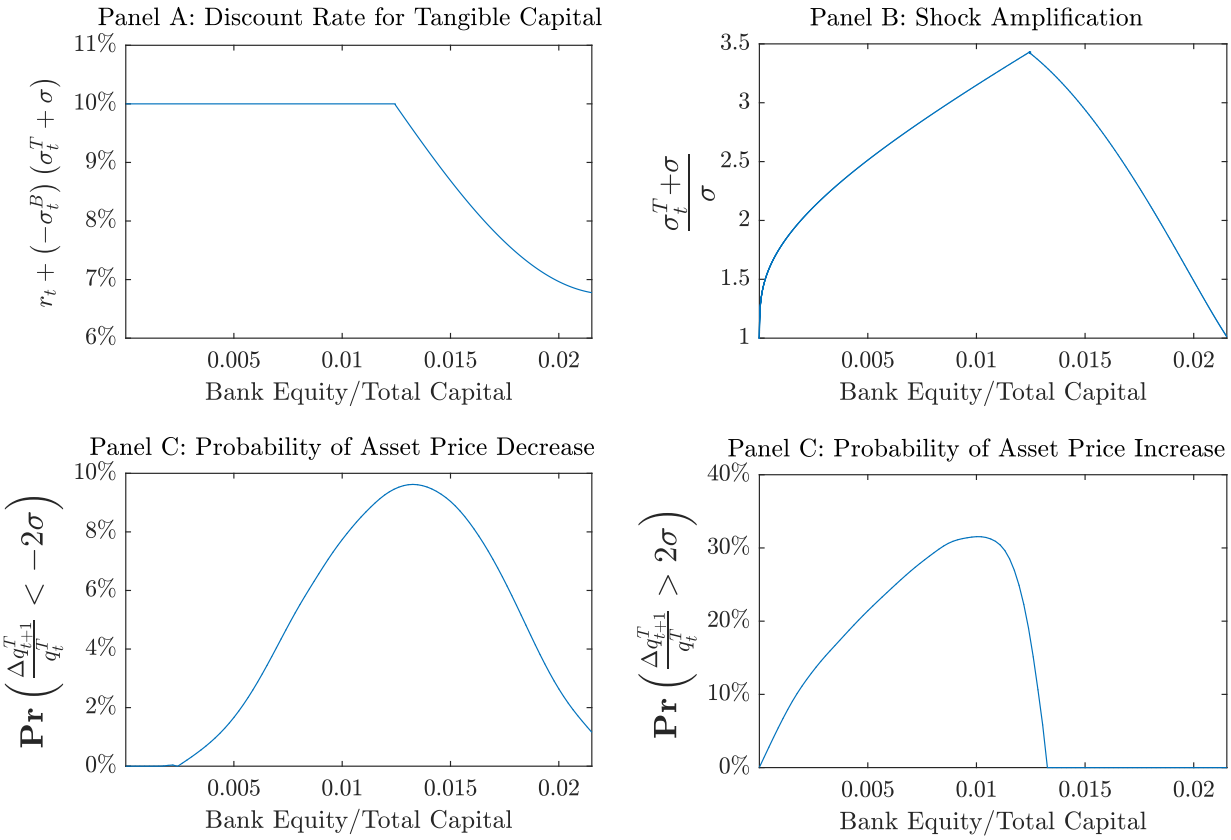

Figure 6: **Endogenous Risk Accumulation**.

higher σ_t^T). Therefore, in Panel B of Figure 6, the ratio of total volatility of tangible capital return, $\sigma_t^T + \sigma$, to the exogenous volatility from capital depreciation, σ, increases as η_t increases, meaning that the shock amplification mechanism becomes stronger as booms prolong. The endogenous risk eventually declines as the sensitivity of discount rate in η_t becomes increasingly smaller.

An alternative perspective on the accumulation of endogenous risk is to examine the state-dependent difference between entrepreneurs' and bankers' discount rates. When η_t is small, both require an expected return of ρ, but as η_t increases, the prospect of a discount-rate divergence rises. When bankers' required return falls below ρ, they become the natural buyers of tangible capital. Negative shocks deplete bank equity and trigger reallocation of tangible capital to entrepreneurs whose discount rate is higher. Such reallocation depresses the value of tangible capital, q_t^T, through the fire sale channel (Shleifer and Vishny (2011)). Different from typical fire sale dynamics (e.g.,

Brunnermeier and Sannikov (2014)), here the difference between entrepreneurs and bankers is time-varying and state-dependent. The longer a boom lasts (i.e., η_t increases), the sharper a difference exists between entrepreneurs' and bankers' discount rate due to the intermediated liquidity premium. As a result, endogenous risks accumulate and the economy becomes increasingly fragile.

The endogenous volatility in the form of asset price variation is important from a welfare perspective. As shown in Equation (19), the economic growth rate is directly tied to q_t^T through the scale of investment because an increase of q_t^T enlarges entrepreneurs' financing capacity. Therefore, the volatility of asset price translates into the volatility of economic growth rate.

The accumulation of endogenous risk is asymmetric. Panel C and D of Figure 6 plot respectively the probabilities of 2σ decrease and increase of q_t^T in one year.[16] Note that at sufficiently low (high) values of η_t, a further decrease (increase) by 2σ is impossible because it goes beyond the range of q_t^T. During booms, the probability of a drop in q_t^T increases as η_t increases, and reaches its peak after eighteen years from $\eta_t = 0.00001$ according to Panel C of Figure 3. It eventually declines as the shock amplification weakens (Panel B of Figure 6). The probability of an increase in q_t^T also increases but declines earlier, suggesting that the risk accumulation due to intermediated liquidity premium is asymmetric, biased towards the downside. Such asymmetry helps explain the findings that long periods of boom and banking expansion precede severe crises (e.g., Jordà, Schularick, and Taylor (2013); Baron and Xiong (2017)).

When negative shocks hit, the feedback mechanism of intermediated liquidity premium turns into a vicious cycle. Banks' equity is depleted, so asset price declines, which in turn discourages entrepreneurs from saving for investments and, thereby, causes an increase of r_t. As the debt cost increases, bankers require a higher return on tangible capital, which further depresses asset price. The negative impact of declining asset price on banks' equity is amplified by leverage.[17]

Over the long run, the economy spends more of time in booms, close to the banker consumption (right) boundary, according to Figure 3. In response to negative shocks, the economy moves

[16] Given the model solution, these probabilities can be calculated using the FeynmanKac formula.

[17] Without equity issuance friction, the equilibrium of intermediated liquidity supply is the same as the mutual-fund equilibrium that features constant asset price and zero endogenous risk. Partially relaxing the equity issuance constraint (as in He and Krishnamurthy (2013)) may improve the quantitative performance of the model, but as long as there are certain frictions on banks' equity issuance, the qualitative implications carry through.

from the very right end (e.g., $\eta_t = 0.15$) to the left (e.g., $\eta_t = 0.02$) in sFigure 6. As the economic and financial conditions deteriorate, the downside risk of q_t^T (and economic growth, μ_t^K) rises in Panel C of Figure 6, while the upside risk is relatively insensitive (Panel D of Figure 6). This prediction speaks directly to the findings of Adrian, Boyarchenko, and Giannone (2019) – upside risks are low in most periods while downside risks increase as financial conditions deteriorate.

As shown in Figure 3, recovery from deep crises (e.g., $\eta_t = 0.005$) is slow. In crises, the entrepreneurs' incentive to invest is low, so they assign a small liquidity premium to deposits and demand a high deposit rate. Given a high cost of debt and low return on equity, banks accumulate equity slowly, so the economy grows out of recession slowly. This is in contrast with the relatively speedy recovery in He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) due to the high return on equity in bad states. Their results rely on a stable source of debt financing for banks (from households), while here, the entrepreneurs' demand for bank debt is procyclical.

3.3 Intangible-Intensive Economy

The productivity wedge between intangible and tangible capital, ϕ, determines the relative importance of intangible capital in production. An increase of ϕ captures the transition towards a more intangible-intensive economy. Here we consider a 10% increase of the productivity of intangible capital, i.e., an increase of ϕ from 0.01 to 0.016. For the comparison with the baseline model, other parameters are fixed. Specifically, the productivity of tangible capital, α, is *not* adjusted downward to fix the level of output, so any difference between the prices of tangible capital in the two cases is fully attributed to the changes of liquidity premium due to the increase of ϕ.

Panel A of Figure 7 plots the deposit rate. The basic pattern still holds – as the banking sector expands and asset price increases, the liquidity premium increases and the interest rate declines. However, the level of interest rate is now lower because the liquidity premium is larger – intangible capital is more valuable, so entrepreneurs prefer to hold more liquidity for intangible investments (Panel D of Figure 7). To meet the stronger liquidity demand and earn the liquidity premium, bankers postpone consumption to a higher level of bank equity-to-capital ratio than the baseline case, so overall, the banking sector becomes larger (i.e., the upper bound of η_t rises). A higher

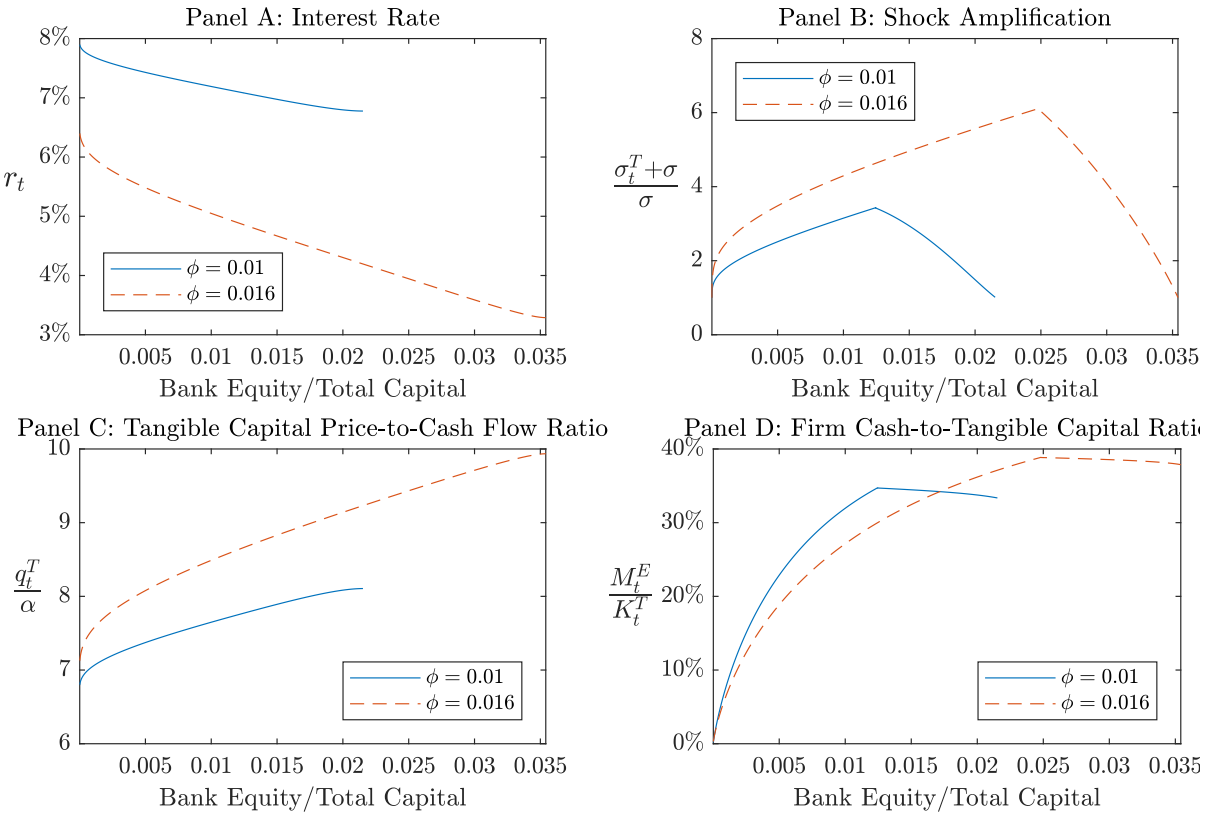

Figure 7: **Intangible-Intensive Economy**.

liquidity premium also leads to a higher price of tangible capital as shown in Panel C of Figure 7. Thus, the model predicts that a transition towards intangible-intensive economy features a lower interest rate, more cash held by firms, the expansion of banking sector, and higher asset prices.

A more intangible-intensive economy has a stronger shock amplification mechanism, as shown in Panel B of Figure 7. The ratio of total return volatility of tangible capital to the exogenous volatility rises above six as the economy goes through a booming period of bank expansion. With a stronger liquidity demand from entrepreneurs, the economy now has a more functioning feedback mechanism driven by the intermediated liquidity premium. Related, Panel A and C show that interest rate and asset price are both more sensitive to bank equity.

3.4 Endogenous Capital Composition

So far, intangible and tangible investments have been made proportionately, so the intangible capital share is fixed at $K_t^I / \left(K_t^I + K_t^T\right) = \theta$. This investment technology captures the necessity of having both tangible and intangible capital in place for production. While in reality, most intangibles (e.g., organizational capital) cannot be productive without tangibles (e.g., equipments and plants), the creation of certain tangible capital does not require intangible investments. One example is real-estate investment. It can be externally financed, and real estate generates production flows (e.g., rental revenues) without requiring substantial intangible capital.

To make the model more realistic, it is assumed that entrepreneurs can transform an *extra* $i_t^T K_t^T$ units of goods into $\beta\sqrt{i_t^T} K_t^T$ units of new tangible capital, where i_t^T is the investment *rate*, when the Poisson shock arrives. This opportunity is in addition to the proportionate creation of tangible and intangible capital. This investment is not liquidity-unconstrained, as tangible capital can be pledged for external financing, so i_t^T is driven by q_t^T as in the q-theory (Hayashi (1982)):

$$i_t^T = \arg\max_i \left\{ q_t^T \beta \sqrt{i} K_t^T - i K_t^T \right\} = \frac{\beta^2}{4} \left(q_t^T\right)^2, \tag{22}$$

where β measures the investment efficiency. The investment technology is chosen for analytical convenience as this extension is to show that qualitatively, the mechanism of intermediated liquidity premium is strengthened in the presence of endogenous capital composition. Also note that the model does not capture the heterogeneity of tangible capital. All tangible capital, whether created independently or with intangible capital, produces α unites of goods per unit of time.

In this extended model, the capital composition ("intangible share"),

$$\omega_t = \frac{K_t^I}{K_t^I + K_t^T}, \tag{23}$$

becomes a meaningful endogenous state variable that drives the tangible capital price, the interest rate, banks' leverage, and entrepreneurs' liquidity holdings in the Markov equilibrium. As documented by Begenau and Palazzo (2015) and Peters and Taylor (2017) among others, capital composition evolves over time in data. The Markov equilibrium is defined similarly as in Section

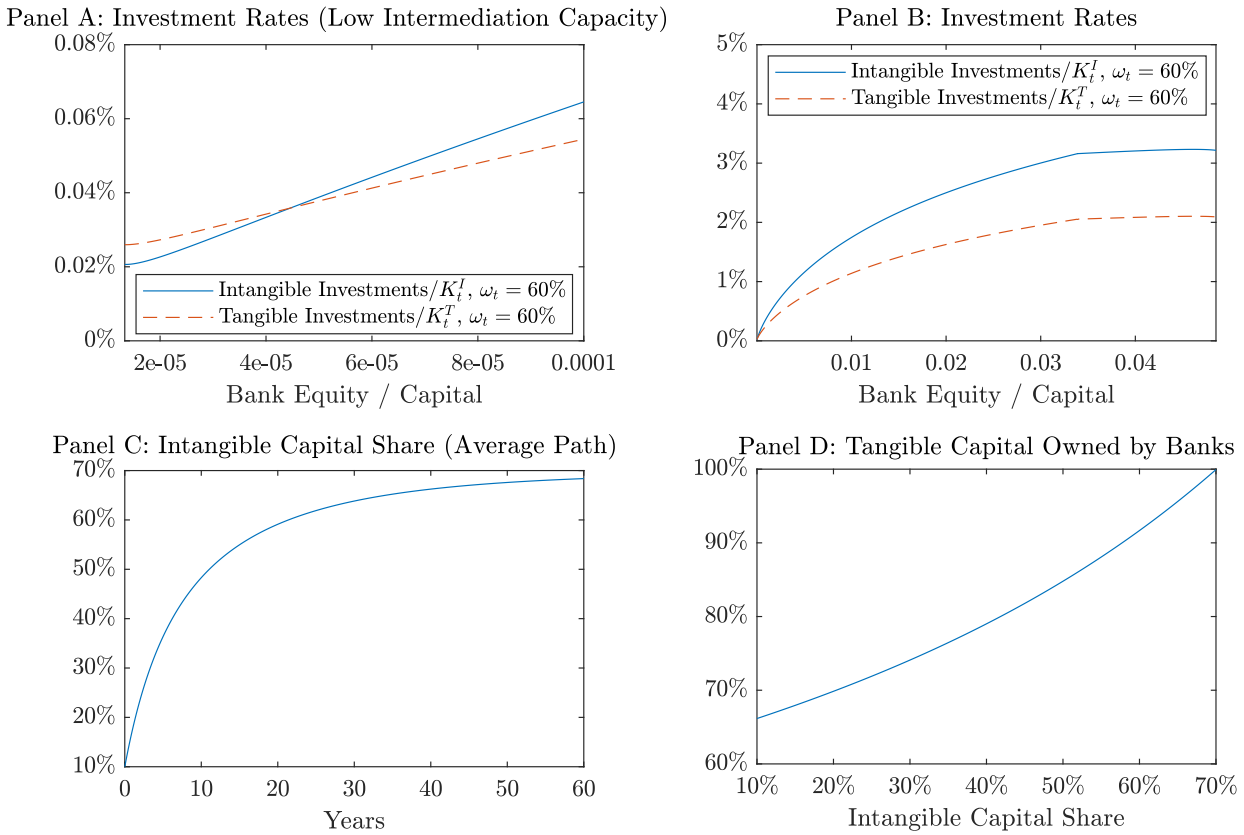

Figure 8: **The Rise of Intangible Capital**.

2 with an extra state variable ω_t, and the solution method is explained in Appendix III.

The interaction between ω_t and η_t leads to reinforcing dynamics. Consider positive shocks. Tangible capital price, q_t^T, increases as bankers become wealthier, i.e., η_t increases. Banks issue more deposits, so entrepreneurs hold more liquidity and invest more in intangibles. The increase of q_t^T also drives up tangible investment, but as will be shown, the force of liquidity supply dominates under the current parameter values, so the intangible share, ω_t, increases. As tangible capital becomes relatively more scarce, the liquidity premium increases and the deposit rate decreases. Given a lower debt cost, banks expand and bid up the price of tangible capital even further.

Liquidity supplied by banks leads to the creation of more illiquid intangible capital. Panel A of Figure 8 plots the tangible and intangible investment rates against η_t, the ratio of bank equity to total capital, in a range of low values of η_t. The intangible share, ω_t, is fixed at 60%. Panel

B compares the two types of investment in the full range of η_t. The values of parameters are the same in the baseline model, and β is set to 0.2 for illustrative purpose. Reading the graphs from the left to the right, we follow a period of boom where following positive shocks, banks expand and issue more deposits that entrepreneurs hold for intangible investments, and as the banks' liquidity supply increases, more intangible capital is created making tangible capital relatively more scarce.

In contrast to the analysis in Section 3.3 that focuses on an exogenous increase of intangible productivity, here the rise of intangible capital is captured by the endogenous evolution of capital composition and is driven by the liquidity supplied by the banking sector. Panel C of Figure 8 plots the average path of intangible capital share, ω_t, starting at 10%. The average is calculated using the joint stationary distribution of state variables.

As the economy becomes increasingly intangible-intensive, the banking sector becomes more important as liquidity suppliers and grows by earning the liquidity premium that increases as tangible capital becomes relatively more scarce. Panel D of 8 plots the average share of tangible capital owned by banks against different levels of ω_t.

The model creates a mapping from intangible intensity to corporate cash holdings, interest rate, asset price, and endogenous risk. Figure 9 plots the average paths of firms' cash-to-tangible asset ratio, the deposit rate, the price-to-cash flow ratio of tangible capital, and the ratio of total return volatility to exogenous depreciation volatility that measures the strength of shock amplification. Figure 9 helps us identify the long-run average values of these variables given a level of intangible intensity. For example, Corrado, Hulten, and Sichel (2005) show that the ratio of the income accrued to intangible capital to the income accrued to tangible capital is 3/5.[18] Given $\alpha = 0.05$ and $\phi = 0.01$, this maps to $\omega_t = 1/3$, and according to the Panel D of Figure 9, the model generates a return volatility of tangible capital that is 4.5 times the exogenous volatility.

As the economy becomes more intangible-intensive, banks grow by issuing more deposits and acquiring more tangible capital. In the process, firms hold more cash (Panel A), and tangible capital price increases (Panel B). As investment becomes more profitable, entrepreneurs assign a

[18]Using data on national accounts and estimates by Corrado, Hulten, and Sichel (2005), Corrado, Hulten, and Sichel (2009) show that for the years 2000-2003, the share of income earned by the owners of intangible capital reaches 15%, while the owners of physical capital obtain 25% and the remaining 60% is absorbed by labor. Since my model does not feature labor input, I consider the relative output share of intangible capital relative to that of tangible capital.

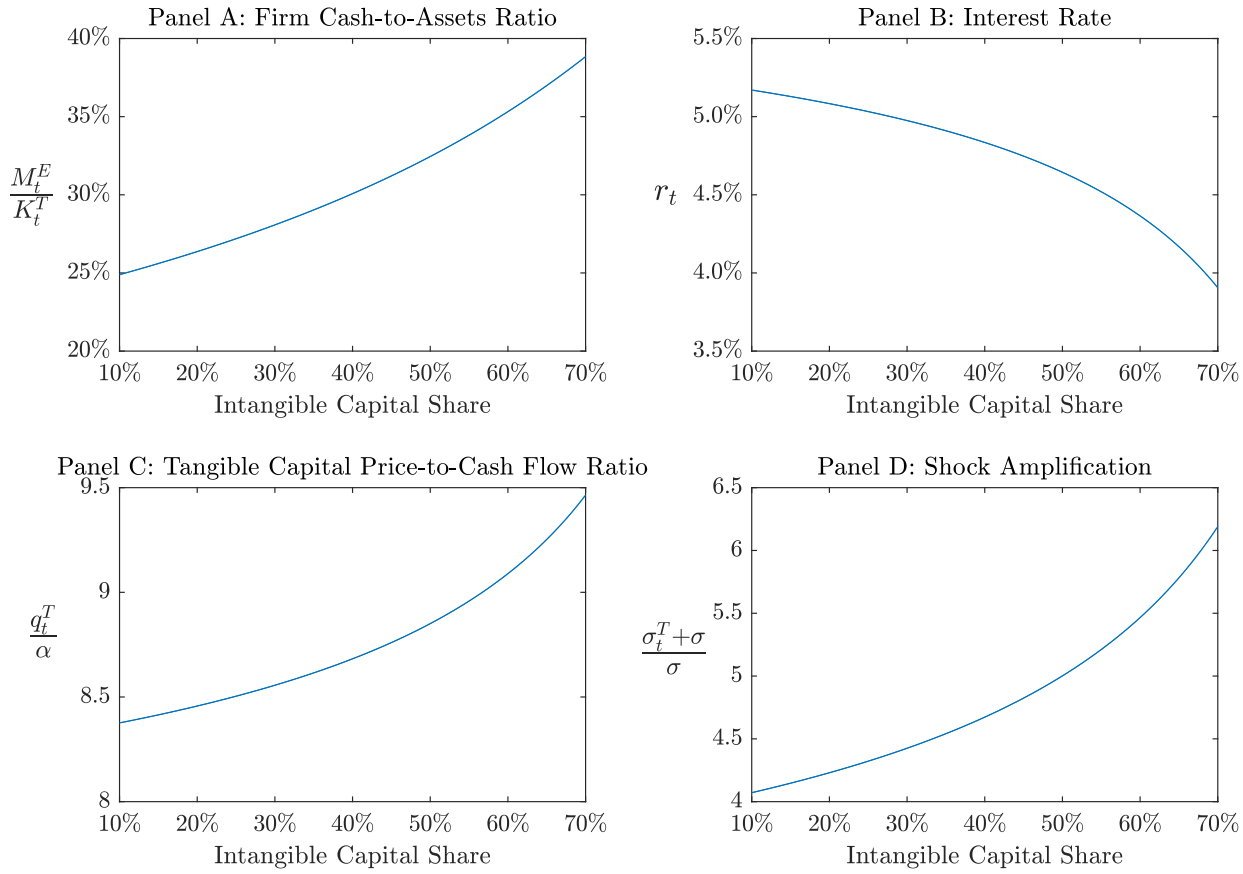

Figure 9: **Transition to Intangible-Intangible Economy**.

larger liquidity premium to deposits and accept an increasingly lower interest rate (Panel C). The mechanism of intermediated liquidity premium becomes stronger as tangible capital, the ultimate source of liquidity, is increasingly scarce and banks' capacity to intermediate the liquidity supply increasingly important. As shown in Panel D, the shock amplification mechanism is strengthened, generating a larger endogenous volatility of asset price as the intangible share increases.

4 Conclusion

This paper aims to provide a coherent account of several trends in the two decades leading up to the Great Recession, such as the rising corporate cash holdings, the expansion of financial sector, the

declining interest rate, and the rising prices of risky assets. Moreover, it characterizes a mapping from the intangible intensity of a production economy to the level of endogenous risk. At the center is the dynamic interaction between the liquidity suppliers (banks) and demanders (entrepreneurs).

The liquidity demand of firms arises from investment needs and capital illiquidity, motivated by the increasing reliance on intangible capital in the advanced economies. The financial stability implications of this structural change has not yet been explored in the existing literature. This paper proposes a channel of intermediated liquidity supply that links a rising asset price and a declining interest rate with the accumulation of endogenous risk in booms. It sheds light on the recent empirical findings that a long period of banking expansion precedes severe crises (e.g., Jordà, Schularick, and Taylor (2013)) and that the downside risk rises faster than the upside risk as the financial conditions deteriorate (e.g., Adrian, Boyarchenko, and Giannone (2019)). This new channel of instability creates a quantitatively significant mechanism that amplifies macro shocks.

Even though this paper briefly discusses government debt as alternative source of liquidity in Appendix IV, it leaves out the study of more sophisticated strategy of government debt management for future research (e.g., Li (2017)). A *procyclical* provision of liquidity through government debt issuance may achieve stabilizing effects by weakening the channel of intermediated liquidity.

The model also leaves out banks' default. The empirical literature on financial crises commonly use banks' default or a high possibility of default as a crisis indicator. A theoretical model of crisis should ideally accommodate default, and by doing so, it opens up the question of optimal government intervention, for instance, through equity injection into the banking sector, in order to prevent a sudden evaporation of liquidity. To finance the intervention in bad times, the government may increase the issuance of government debt, which implies *countercyclical* debt issuance.

Given the government intervention that restores ex post efficiency, potential concerns over banks' moral hazard may arise, which call for an analysis of ex ante banking regulations, such as capital and liquidity requirements, in intangible-intensive economies. In the existing literature, the linkage between financial regulation and the composition of production capital is still missing.

References

Adrian, T., N. Boyarchenko, and D. Giannone (2019). Vulnerable growth. *American Economic Review*, forthcoming.

Adrian, T., N. Boyarchenko, and H. S. Shin (2016). On the scale of financial intermediaries. Staff Reports 743, Federal Reserve Bank of New York.

Adrian, T. and H. S. Shin (2010a). The changing nature of financial intermediation and the financial crisis of 2007-09. Staff Reports 430, Federal Reserve Bank of New York.

Adrian, T. and H. S. Shin (2010b). Liquidity and leverage. *Journal of Financial Intermediation 19*(3), 418 – 437. Risk Transfer Mechanisms and Financial Stability.

Ai, H., M. M. Croce, and K. Li (2013). Toward a quantitative general equilibrium asset pricing model with intangible capital. *The Review of Financial Studies 26*(2), 491–530.

Almeida, H. and M. Campello (2007). Financial constraints, asset tangibility, and corporate investment. *Review of Financial Studies 20*(5), 1429–1460.

Atkeson, A. and P. J. Kehoe (2005). Modeling and measuring organization capital. *Journal of Political Economy 113*(5), 1026–1053.

Baron, M. (2014). Countercyclical bank equity issuance. Working paper, Princeton University.

Baron, M. and W. Xiong (2017). Credit expansion and neglected crash risk. *The Quarterly Journal of Economics 132*(2), 713–764.

Bates, T. W., K. M. Kahle, and R. M. Stulz (2009). Why do U.S. firms hold so much more cash than they used to? *The Journal of Finance 64*(5), 1985–2021.

Begenau, J. (2018). Capital requirements, risk choice, and liquidity provision in a business cycle model. Working paper, Stanford GSB.

Begenau, J. and B. Palazzo (2015). Firm selection and corporate cash holdings. 2015 Meeting Papers 1047, Society for Economic Dynamics.

Bernanke, B. S. (2005). The global saving glut and the U.S. current account deficit. Remarks given at the sandridge lecture, Virginia Association of Economists, Richmond, March 10.

Bolton, P., H. Chen, and N. Wang (2011). A unified theory of Tobin's q, corporate investment, financing, and risk management. *The Journal of Finance 66*(5), 1545–1578.

Bolton, P. and X. Freixas (2000). Equity, bonds, and bank debt: Capital structure and financial market equilibrium under asymmetric information. *Journal of Political Economy 108*(2), 324–351.

Bolton, P., T. Santos, and J. A. Scheinkman (2018). Savings gluts and financial fragility. Working paper, Columbia University.

Brunnermeier, M. K. and Y. Sannikov (2014). A macroeconomic model with a financial sector. *American Economic Review 104*(2), 379–421.

Brunnermeier, M. K. and Y. Sannikov (2016). The I theory of money. Working paper, Princeton University.

Caballero, R. J., E. Farhi, and P. Gourinchas (2008). An equilibrium model of "global imbalances" and low interest rates. *American Economic Review 98*(1), 358–93.

Caballero, R. J., E. Farhi, and P. Gourinchas (2017). Rents, technical change, and risk premia: Accounting for secular trends in interest rates, returns on capital, earning yields, and factor shares. Working Paper 23127, National Bureau of Economic Research.

Caballero, R. J. and A. Krishnamurthy (2006). Bubbles and capital flow volatility: Causes and risk management. *Journal of Monetary Economics 53*(1), 35–53.

Caballero, R. J. and A. Krishnamurthy (2009). Global imbalances and financial fragility. *American Economic Review 99*(2), 584–88.

Campello, M., R. A. Connolly, G. Kankanhalli, and E. Steiner (2019). How collateral affects borrowing: Evidence from U.S. micro-level data. Working paper, Cornell SC Johnson College of Business.

Chaney, T., D. Sraer, and D. Thesmar (2012). The collateral channel: How real estate shocks affect corporate investment. *American Economic Review 102*(6), 2381–2409.

Corrado, C., C. Hulten, and D. Sichel (2005). Measuring capital and technology: An expanded framework. In C. Corrado, J. Haltiwanger, and D. Sichel (Eds.), *Measuring Capital in the New Economy*, pp. 11–46. San Diego: University of Chicago Press.

Corrado, C., C. Hulten, and D. Sichel (2009). Intangible capital and U.S. economic growth. *Review of Income and Wealth 55*(3), 661–685.

Corrado, C. A. and C. R. Hulten (2010). How do you measure a "technological revolution"? *American Economic Review 100*(2), 99–104.

Crouzet, N. and J. Eberly (2018). Understanding weak capital investment: the role of market concentration and intangibles. Working paper, Northwestern University.

Dang, T. V., G. Gorton, B. Holmström, and G. Ordonez (2014). Banks as secret keepers. Working Paper 20255, National Bureau of Economic Research.

Daniel, N. D., L. Naveen, and J. Yu (2018). Changing U.S. economy and investment-cash flow sensitivity. Working paper, 2018 CityU of Hong Kong International Finance Conference.

DeAngelo, H. and R. M. Stulz (2015). Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks. *Journal of Financial Economics 116*(2), 219 – 236.

Décamps, J.-P., T. Mariotti, J.-C. Rochet, and S. Villeneuve (2011). Free cash flow, issuance costs, and stock prices. *The Journal of Finance 66*(5), 1501–1544.

Del Negro, M., D. Giannone, M. Giannoni, and A. Tambalotti (2017). Safety, liquidity, and the natural rate of interest. Staff Reports 812, Federal Reserve Bank of New York.

Dell'Ariccia, G., D. Kadyrzhanova, C. Minoiu, and L. Ratnovski (2018). Bank lending in the knowledge economy. Working paper, International Monetary Fund.

Denis, D. J. and V. Sibilkov (2010). Financial constraints, investment, and the value of cash holdings. *Review of Financial Studies 23*(1), 247–269.

Diamond, D. W. (1984). Financial intermediation and delegated monitoring. *The Review of Economic Studies 51*(3), 393–414.

Doms, M. and T. Dunne (1998). Capital adjustment patterns in manufacturing plants. *Review of Economic Dynamics 1*(2), 409 – 429.

Döttling, R. and E. C. Perotti (2017). Secular Trends and Technological Progress. Discussion Papers 12519, CEPR.

Duchin, R. (2010). Cash holdings and corporate diversification. *The Journal of Finance 65*(3), 955–992.

Egan, M., S. Lewellen, and A. Sunderam (2018). The cross section of bank value. Working paper, Harvard Business School.

Eggertsson, G. B., J. A. Robbins, and E. G. Wold (2018). Kaldor and piketty's facts: The rise of monopoly power in the united states. Working Paper 24287, National Bureau of Economic Research.

Eisfeldt, A. and A. Rampini (2009). Financing shortfalls and the value of aggregate liquidity. Working paper, Duke University.

Eisfeldt, A. L. (2007). Smoothing with liquid and illiquid assets. *Journal of Monetary Economics 54*(6), 1572 – 1586.

Eisfeldt, A. L. and D. Papanikolaou (2014). The value and ownership of intangible capital. *American Economic Review 104*(5), 189–94.

Falato, A., D. Kadyrzhanovaz, J. Sim, and R. Steri (2018). Rising intangible capital, shrinking debt capacity, and the us corporate savings glut. Technical report, Board of Governors of the Federal Reserve System (U.S.).

Farhi, E. and F. Gourio (2018). Accounting for macro-finance trends: Market power, intangibles, and risk premia. Working Paper 25282, National Bureau of Economic Research.

Farhi, E. and J. Tirole (2012). Bubbly liquidity. *The Review of Economic Studies*.

Froot, K. A., D. S. Scharfstein, and J. C. Stein (1993). Risk management: Coordinating corporate investment and financing policies. *The Journal of Finance 48*(5), 1629–1658.

Gao, X., T. M. Whited, and N. Zhang (2018). Corporate money demand. Working paper, University of Michigan Stephen M. Ross School of Business.

Giglio, S. and T. Severo (2012). Intangible capital, relative asset shortages and bubbles. *Journal of Monetary Economics 59*(3), 303 – 317.

Gomes, J. F., M. Grotteria, and J. A. Wachter (2018). Foreseen risks. Working paper, The Wharton School, University of Pennsylvania.

Gorton, G. (2010). *Slapped by the Invisible Hand: The Panic of 2007*. New York, N.Y.: OxfordUniversity Press.

Gorton, G., S. Lewellen, and A. Metrick (2012). The safe-asset share. *American Economic Review 102*(3), 101–06.

Gorton, G. and G. Pennacchi (1990). Financial intermediaries and liquidity creation. *The Journal of Finance 45*(1), 49–71.

Gourinchas, P. and H. Rey (2016). Real interest rates, imbalances and the curse of regional safe asset providers at the zero lower bound. Working Paper 22618, National Bureau of Economic Research.

Graham, J. R. and M. T. Leary (2015). The evolution of corporate cash. Working paper, Duke University.

Greenwood, R., S. Hanson, and J. C. Stein (2010). A gap-filling theory of corporate debt maturity choice. *The Journal of Finance 65*(3), 993–1028.

Greenwood, R., S. G. Hanson, and J. C. Stein (2016). The federal reserve's balance sheet as a financial-stability tool. 2016 economic policy symposium proceedings, Federal Reserve Bank of Kansas City, Jackson Hole.

Greenwood, R. and D. Scharfstein (2013). The growth of finance. *Journal of Economic Perspectives 27*(2), 3–28.

Gutiérrez, G. and T. Philippon (2017). Declining competition and investment in the u.s. Working Paper 23583, National Bureau of Economic Research.

Hall, B. H. (1992). Investment and research and development at the firm level: Does the source of financing matter? Working Paper 4096, National Bureau of Economic Research.

Hall, B. H. and J. Lerner (2009). The financing of R&D and innovation. Working Paper 15325, National Bureau of Economic Research.

Hansen, L. P., J. C. Heaton, and N. Li (2005). Intangible Risk. In *Measuring Capital in the New Economy*, NBER Chapters, pp. 111–152. National Bureau of Economic Research, Inc.

Hart, O. and L. Zingales (2014). Banks are where the liquidity is. Working paper, Harvard University.

Hayashi, F. (1982). Tobin's marginal q and average q: A neoclassical interpretation. *Econometrica 50*(1), 213–224.

He, Z. and P. Kondor (2016). Inefficient investment waves. *Econometrica 84*(2), 735–780.

He, Z. and A. Krishnamurthy (2013). Intermediary asset pricing. *American Economic Review 103*(2), 732–70.

Himmelberg, C. P. and B. C. Petersen (1994). R&D and internal finance: A panel study of small firms in high-tech industries. *The Review of Economics and Statistics 76*(1), 38–51.

Hirano, T. and N. Yanagawa (2017). Asset bubbles, endogenous growth, and financial frictions. *The Review of Economic Studies 84*(1), 406–443.

Holmström, B. (2012). The nature of liquidity provision: When ignorance is bliss. In *Presidential Address, Econometric Society, ASSA meetings, Chicago*.

Holmström, B. and J. Tirole (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics 112*(3), 663–691.

Holmström, B. and J. Tirole (1998). Private and public supply of liquidity. *Journal of Political Economy 106*(1), 1–40.

Holmström, B. and J. Tirole (2001). LAPM: A liquidity-based asset pricing model. *The Journal of Finance 56*(5), 1837–1867.

Jordà, O., M. Schularick, and A. M. Taylor (2013). When credit bites back. *Journal of Money, Credit and Banking 45*(s2), 3–28.

Kacperczyk, M., C. Pérignon, and G. Vuillemey (2018). The private production of safe assets. Research Paper FIN-2017-1212, HEC Paris.

Kiyotaki, N. and J. Moore (2000). Inside money and liquidity. Working paper, London School of Economics.

Kiyotaki, N. and J. Moore (2012). Liquidity, business cycles, and monetary policy. Working paper, Princeton University.

Klimenko, N., S. Pfeil, J.-C. Rochet, and G. D. Nicolo (2016). Aggregate bank capital and credit dynamics. Swiss Finance Institute Research Paper Series 16-42.

Krishnamurthy, A. and T. Muir (2016). How credit cycles across a financial crisis. Working paper, Stanford University Graduate School of Business.

Krishnamurthy, A. and A. Vissing-Jørgensen (2012). The aggregate demand for treasury debt. *Journal of Political Economy 120*(2), 233–267.

Krishnamurthy, A. and A. Vissing-Jorgensen (2015). The impact of treasury supply on financial sector lending and stability. *Journal of Financial Economics 118*(3), 571 – 600.

Li, S., T. M. Whited, and Y. Wu (2016). Collateral, taxes, and leverage. *Review of Financial Studies 29*(6), 1453–1500.

Li, Y. (2017). Procyclical finance: The money view. Charles A. Dice Center Working Paper 2017-24, The Ohio State University Fisher College of Business.

López-Salido, D., J. C. Stein, and E. Zakrajšek (2017). Credit-Market Sentiment and the Business Cycle*. *The Quarterly Journal of Economics 132*(3), 1373–1426.

Martin, A. and J. Ventura (2012). Economic growth with bubbles. *American Economic Review 102*(6), 3033–58.

Marx, M., B. Mojon, and F. Velde (2018). Why have interest rates fallen far below the return on capital. Working Paper 2018-01, Federal Reserve Bank of Chicago.

McGrattan, E. R. and E. C. Prescott (2010a). Technology capital and the us current account. *American Economic Review 100*(4), 1493–1522.

McGrattan, E. R. and E. C. Prescott (2010b). Unmeasured investment and the puzzling us boom in the 1990s. *American Economic Journal: Macroeconomics 2*(4), 88–123.

Miao, J. and P. Wang (2018). Asset bubbles and credit constraints. *American Economic Review 108*(9), 2590–2628.

Moreira, A. and A. Savov (2017). The macroeconomics of shadow banking. *The Journal of Finance 72*(6), 2381–2432.

Nagel, S. (2016). The liquidity premium of near-money assets. *The Quarterly Journal of Economics*, forthcoming.

Peters, R. H. and L. A. Taylor (2017). Intangible capital and the investment-q relation. *Journal of Financial Economics 123*(2), 251 – 272.

Phelan, G. (2016). Financial intermediation, leverage, and macroeconomic instability. *American Economic Journal: Macroeconomics 8*(4), 199–224.

Piazzesi, M. and M. Schneider (2016). Payments, credit and asset prices. Working paper, Stanford University.

Pinkowitz, L., R. M. Stulz, and R. Williamson (2015). Do U.S. firms hold more cash than foreign firms do? *Review of Financial Studies*.

Poterba, J. M. and J. J. Rotemberg (1986). Money in the utility function: An empirical implementation. Working Paper 1796, National Bureau of Economic Research.

Pozsar, Z. (2011). Institutional cash pools and the Triffin dilemma of the U.S. banking system. Working paper WP/11/190, International Monetary Fund.

Pozsar, Z. (2014). Shadow banking: The money view. Working paper 14-04, Office of Financial Research.

Protter, P. E. (1990). *Stochastic Integration and Differential Equations*. New York: Springer-Verlag.

Quadrini, V. (2017). Bank liabilities channel. *Journal of Monetary Economics 89*, 25 – 44. Carnegie-Rochester-NYU Conference Series on the Macroeconomics of Liquidity in Capital Markets and the Corporate Sector.

Rampini, A. A. and S. Viswanathan (2019). Financial Intermediary Capital. *The Review of Economic Studies 86*(1), 413–455.

Riddick, L. A. and T. M. Whited (2009). The corporate propensity to save. *The Journal of Finance 64*(4), 1729–1766.

Schularick, M. and A. M. Taylor (2012). Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. *American Economic Review 102*(2), 1029–61.

Shleifer, A. and R. Vishny (2011). Fire sales in finance and macroeconomics. *Journal of Economic Perspectives 25*(1), 29–48.

Sidrauski, M. (1967). Inflation and economic growth. *Journal of Political Economy 75*(6), 796–810.

Sundaresan, S. and Z. Wang (2014). Bank liability structure. Research Paper No. 14-41, Columbia Business School.

Sunderam, A. (2015). Money creation and the shadow banking system. *Review of Financial Studies 28*(4), 939–977.

Thomas, J. (2002). Is lumpy investment relevant for the business cycle? *Journal of Political Economy 110*(3), 508–534.

Van den Heuvel, S. J. (2002). The bank capital channel of monetary policy. Working paper, The Wharton School of the University of Pennsylvania.

Van den Heuvel, S. J. (2018). The welfare effects of bank liquidity and capital requirements. Working paper, Federal Reserve Board.

Woodford, M. (1990). Public debt as private liquidity. *The American Economic Review 80*(2), 382–388.

Appendix I: Alternative Production Technology

In the following, it is shown that the typical Cobb-Douglas production function that combines tangible and intangible capital (e.g., Giglio and Severo (2012) and McGrattan and Prescott (2010b)) is equivalent to the model in the main text when the capital composition is fixed.

There is a unit mass of final good producer who has access to a Cobb-Douglas production technology that combines tangible capital (k_t^{TP}) and intangible capital (k_t^{IP}) as follows:

$$y_t = \hat{\alpha} \left(k_t^{TP}\right)^{1-\hat{\phi}} \left(k_t^{IP}\right)^{\hat{\phi}}, \tag{24}$$

where $\hat{\alpha} > 0$, $\hat{\phi} \in (0, 1)$, y_t is the output per unit of time at t, and the extra superscript "P" indicates the final good producer. The "ˆ" differentiates these parameters from α and ϕ in the main text. The aggregate output, Y_t, is $\int_{i\in[0,1]} y_t\,(i)\,di$ where i indexes a representative producer.

A representative producer rents capital in competitive markets from the entrepreneurs who own capital by paying the rent r_t^T and r_t^I for tangible and intangible capital respectively. Therefore, the producer solves the following profit maximization problem:

$$\max_{k_t^{TP}, k_t^{IP}} \hat{\alpha} \left(k_t^{TP}\right)^{1-\hat{\phi}} \left(k_t^{IP}\right)^{\hat{\phi}} - r_t^T k_t^{TP} - r_t^I k_t^{IP}, \tag{25}$$

with the following first-order conditions

$$\left(1 - \hat{\phi}\right) y_t = r_t^T k_t^{TP}, \tag{26}$$

and

$$\hat{\phi} y_t = r_t^I k_t^{IP}. \tag{27}$$

It is clear that $\hat{\phi}$ is the fraction of output attributed to intangible capital, a common measure of intangible intensity (e.g., Corrado, Hulten, and Sichel (2005)).

At time t, the aggregate stocks of tangible and intangible capital are K_t^T and K_t^I respectively. After the tangible and intangible capital markets clear, the aggregate output is given by

$$Y_t = \hat{\alpha} \left(K_t^T\right)^{1-\hat{\phi}} \left(K_t^I\right)^{\hat{\phi}}. \tag{28}$$

So, aggregating the first-order conditions over producers, we have

$$r_t^T = \left(1 - \hat{\phi}\right) \frac{Y_t}{K_t^T} = \left(1 - \hat{\phi}\right) \hat{\alpha} \left(\frac{K_t^I}{K_t^T}\right)^{\hat{\phi}}, \tag{29}$$

and

$$r_t^I = \hat{\phi} \frac{Y_t}{K_t^I} = \hat{\phi}\hat{\alpha} \left(\frac{K_t^T}{K_t^I}\right)^{1-\hat{\phi}}. \tag{30}$$

When the capital composition is fixed, i.e., $K_t^I / \left(K_t^I + K_t^T\right) = \theta$, we have

$$r_t^T = \left(1 - \hat{\phi}\right) \hat{\alpha} \left(\frac{\theta}{1-\theta}\right)^{\hat{\phi}}, \tag{31}$$

and

$$r_t^I = \hat{\phi}\hat{\alpha} \left(\frac{1-\theta}{\theta}\right)^{1-\hat{\phi}}. \tag{32}$$

which are exactly the constant flow revenues from tangible capital (α) and intangible capital ($\alpha+\phi$) respectively in the model in the main text.

Moreover, there exists a mapping from (α, ϕ) in the main text to $\left(\hat{\alpha}, \hat{\phi}\right)$ in the Cobb-Douglas production function. In particular, the output share of intangible is given by

$$\hat{\phi} = \frac{1}{1 + \left(\frac{\alpha}{\alpha+\phi}\right)\left(\frac{1-\theta}{\theta}\right)}. \tag{33}$$

In Section 3.3, the increased importance of intangible capital in production is captured by a higher value of ϕ, which in the current setting, translates into a higher value of $\hat{\phi}$.

In Section 3.4, the intangible share of total capital, $\omega_t = K_t^I / \left(K_t^I + K_t^T\right)$, evolves endogenously. In the current setting, this leads to state-dependent revenue flows of capital, i.e.,

$$r_t^T = \left(1 - \hat{\phi}\right) \hat{\alpha} \left(\frac{\omega_t}{1-\omega_t}\right)^{\hat{\phi}}, \tag{34}$$

and

$$r_t^I = \hat{\phi}\hat{\alpha} \left(\frac{1-\omega_t}{\omega_t}\right)^{1-\hat{\phi}}, \tag{35}$$

which differ from the constant revenue flows of capital in the main text.

The model in the main text is the preferred setup for two reasons. First, in the main model, an increase of ω_t leads to stronger liquidity demand of entrepreneurs, while in the current setup, the impact of an increase of ω_t on entrepreneurs' liquidity demand is ambiguous. when ω_t increases, the relative scarcity of tangible capital increases, which leads to an increase in r_t^T and a decrease in r_t^I. As a result, q_t^T increases, not only because bankers who have relatively low discount rates accumulate wealth but also because the cash flow from tangible capital increases. However, the value of intangible capital declines due to the lower rental revenue, r_t^I. The overall effect on investment profits and entrepreneurs' liquidity demand is unclear. Second, different from the main model, even though the capital composition evolves endogenously, the output share of intangible capital is always fixed at $\hat{\phi}$ in the current setup. To capture the endogenous change of intangible intensity, one has to model $\hat{\phi}$ as a function of entrepreneurs' investment in the modification of production technology. This much complicates the exposition of model mechanism.

Appendix II: Proofs of Propositions

Proof of Proposition 1. Entrepreneurs ("firms") maximize life-time utility, $\mathbb{E}\left[\int_{t=0}^{+\infty} e^{-\rho t} dc_t^E\right]$, subject to the following wealth (equity) dynamics:

$$dw_t^E = -dc_t^E + \mu_t^w w_t^E dt + \sigma_t^w w_t^E dZ_t + \left(\widehat{w}_t^E - w_t^E\right) dN_t,$$

$\mu_t^w w_t^E$ and $\sigma_t^w w_t^E$ are the drift and diffusion terms that depend on choices of tangible capital and deposit holdings and will be elaborated later. dN_t is the increment of the idiosyncratic counting (Poisson) process. $dN_t = 1$ if a liquidity shock arrives. At the Poisson time, an entrepreneur's wealth jumps to

$$\widehat{w}_t^E = w_t^E + \left(\frac{q^I \theta\kappa + q_t^T (1-\theta)\kappa - 1}{1 - \kappa(1-\theta) q_t^T}\right) m_t^E.$$

Note that w_t^E is the *liquid* wealth of entrepreneurs. When analyzing entrepreneurs' decisions, we can simply regard the production flows from intangible capital as streams of consumption, and because entrepreneurs are risk-neutral (indifferent across states and over time for consumption), we discount the production flows by ρ and deal with the present value of intangible capital.

We conjecture that the value function is linear in equity w_t^E: $v_t^E = \zeta_t^E w_t^E + v^I$, where ζ_t^E

is the marginal value of liquid wealth, and v^I is the present value of consumption flows from intangible capital. In equilibrium, ζ_t^E follows a diffusion process:

$$d\zeta_t^E = \zeta_t^E \mu_t^\zeta dt + \zeta_t^E \sigma_t^\zeta dZ_t,$$

where $\zeta_t^E \mu_t^\zeta$ and $\zeta_t^E \sigma_t^\zeta$ are the drift and diffusion terms respectively. Entrepreneurs' marginal value of wealth, ζ_t^E, is a summary statistic of their investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual is hit by liquidity shocks.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho v_t^E dt = \max_{dc_t^E \in \mathbb{R}, k_t^E \geq 0, m_t^E \geq 0} dc_t^E - \zeta_t^E dc_t^E + \{w_t^E \zeta_t^E \mu_t^\zeta + w_t^E \zeta_t^E \mu_t^w + w_t \zeta_t^E \sigma_t^\zeta \sigma_t^w + \lambda \zeta_t^E \left[\widehat{w}_t - w_t\right]\} dt.$$

Entrepreneurs can choose any $dc_t^E \in \mathbb{R}$, so ζ_t^E must be equal to one, and thus, I have also confirmed the value function conjecture.

Since ζ_t^E is a constant equal to one, μ_t^ζ and σ_t^ζ are both zero. The HJB equation is simplified:

$$\rho v_t^E dt = \max_{k_t^E \geq 0, m_t^E \geq 0} \mu_t^w w_t^E dt + \lambda dt \left(\frac{q^I \theta \kappa + q_t^T (1-\theta) \kappa - 1}{1 - \kappa (1-\theta) q_t^T} \right) m_t^E. \tag{36}$$

Wealth drift has production, value change of tangible capital holdings, and deposit return:

$$\mu_t^w w_t^E dt = \underbrace{\alpha k_t^E dt + \mathbb{E}_t \left(q_{t+dt}^T k_{t+dt}^E - q_t^T k_t^E \right)}_{\mathbb{E}_t \left[dr_t^T \right] q_t^T k_t^E} + r_t m_t^E dt.$$

Let $d\psi_t^E$ denote the Lagrange multiplier of the budget constraint, $q_t^T k_t^T + m_t^E \leq w_t^E$. The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: $m_t^E \geq 0$, and

$$m_t^E \left\{ r_t dt + \lambda dt \left(\frac{q^I \theta \kappa + q_t^T (1-\theta) \kappa - 1}{1 - \kappa (1-\theta) q_t^T} \right) - d\psi_t^E \right\} = 0.$$

The F.O.C. for optimal tangible capital holdings is : $k_t^E \geq 0$, and

$$-\mathbb{E}_t \left[dr_t^T \right] + d\psi_t^E = 0.$$

Substituting these optimality conditions into the HJB equation, we have

$$\rho v_t^E dt = w_t^E d\psi_t^E.$$

Because $\zeta_t^E = 1$, $v_t^E = w_t^E$, and $d\psi_t^E = \rho dt$. Substituting $d\psi_t^E = \rho dt$ into the F.O.C. for m_t^E, we have

$$\rho - r_t = \lambda \left(\frac{q^I \theta \kappa + q_t^T (1-\theta) \kappa - 1}{1 - \kappa (1-\theta) q_t^T} \right).$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for k_t^E and rearranging the equation, we have

$$\mathbb{E}_t \left[dr_t^T \right] = \rho dt.$$

Proof of Proposition 2. Please refer to the main text.

Proof of Proposition 3. Conjecture that the bank's value function takes the linear form: $v_t^B = q_t^B n_t^B$. In equilibrium, the marginal value of equity, q_t^B, evolves as follows

$$dq_t^B = q_t^B \mu_t^B dt + q_t^B \sigma_t^B dZ_t.$$

Define $dy_t^B = dc_t^B / n_t^B$, the consumption-to-wealth ratio of bankers. Under the conjectured functional form, the HJB equation is

$$\begin{aligned} \rho v_t^B dt &= \max_{dy_t^B \in \mathbb{R}} \left\{ \left(1 - q_t^B\right) \mathbb{I}_{\{dy_t^B > 0\}} n_t^B dy_t^B \right\} + \mu_t^B q_t^B n_t^B + \\ & \max_{x_t^B \geq 0} \left\{ r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T \right] - r_t \right) + x_t^B \sigma_t^B \left(\sigma_t^T + \sigma \right) \right\} q_t^B n_t^B, \end{aligned}$$

where $\gamma_t^B = -\sigma_t^B$. Dividing both sides by $q_t^B n_t^B$, we eliminate n_t^B in the HJB equation,

$$\begin{aligned} \rho &= \max_{dy_t^B \in \mathbb{R}} \left\{ \frac{\left(1 - q_t^B\right)}{q_t^B} \mathbb{I}_{\{dy_t^B > 0\}} dy_t^B \right\} + \mu_t^B + \\ & \max_{x_t^B \geq 0} \left\{ r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T \right] - r_t \right) + x_t^B \sigma_t^B \left(\sigma_t^T + \sigma \right) \right\}, \end{aligned}$$

and thus, confirm the conjecture of linear value function. The indifference condition for x_t^B gives the equation in Proposition 3.

Proof of Proposition 4. The system of ordinary differential equations is constructed in detail in Appendix III. The boundary conditions are explained in the main text.

Proof of Proposition 5. Following Brunnermeier and Sannikov (2014), I derive the stationary probability density. Probability density of η_t at time t, $p(\eta, t)$, has Kolmogorov forward equation

$$\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta}\left(\eta \mu^{\eta}(\eta) p(\eta, t)\right) + \frac{1}{2}\frac{\partial^2}{\partial \eta^2}\left(\eta^2 \sigma^{\eta}(\eta)^2 p(\eta, t)\right).$$

Note that in a Markov equilibrium, μ_t^{η} and σ_t^{η} are functions of η_t. A stationary density is a solution to the forward equation that does not vary with time (i.e. $\frac{\partial}{\partial t} p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over η, $p(\eta)$ solves the following first-order ordinary differential equation within the two reflecting boundaries:

$$0 = C - \eta \mu^{\eta}(\eta) p(\eta) + \frac{1}{2}\frac{d}{d\eta}\left(\eta^2 \sigma^{\eta}(\eta)^2 p(\eta)\right), \quad \eta \in \left[\underline{\eta}, \overline{\eta}\right].$$

The integration constant C is zero because of the reflecting boundaries. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_{\underline{\eta}}^{\overline{\eta}} p(\eta)\, d\eta = 1$).

Next, I solve the expected time to reach from $\underline{\eta}$. Define $f_{\eta_0}(\eta)$ the expected time it takes to reach η_0 starting from $\eta \leq \eta_0$. Define $g(\eta_0) = f_{\eta_0}\left(\underline{\eta}\right)$ the expected time to reach η_0 from $\underline{\eta}$. One has to reach $\eta \in \left(\underline{\eta}, \eta_0\right)$ first and then reach η_0 from η. Therefore, $g(\eta) + f_{\eta_0}(\eta) = g(\eta_0)$. Since $g(\eta_0)$ is constant, we differentiate both sides to have $g'(\eta) = -f'_{\eta_0}(\eta)$ and $g''(\eta) = -f''_{\eta_0}(\eta)$.

From η_t, the expected time to reach η_0, denoted by $f_{\eta_0}(\eta_t)$, is decomposed into $s - t$, and $E_t\left[f_{\eta_0}(\eta_s)\right]$, i.e., the expected time to reach η_0 from η_s ($s \geq t$) after $s - t$ has passed. We have $f_{\eta_0}(\eta_t)$ equal to $E_t\left[f_{\eta_0}(\eta_s)\right] + s - t$. Therefore, $t + f_{\eta_0}(\eta_t)$ is a martingale, so f_{η_0} satisfies the ordinary differential equation: $1 + f'_{\eta_0}(\eta)\mu^{\eta}(\eta) + \frac{\sigma^{\eta}(\eta)^2}{2} f''_{\eta_0}(\eta) = 0$. Therefore, $g(\eta)$ must satisfy

$$1 - g'(\eta)\mu^{\eta}(\eta) - \frac{\sigma^{\eta}(\eta)^2}{2} g''(\eta) = 0.$$

It takes no time to reach $\underline{\eta}$, so $g\left(\underline{\eta}\right) = 0$. Moreover, since $\underline{\eta}$ is a reflecting boundary, $g'\left(\underline{\eta}\right) = 0$.

Appendix III: Extensions

III.1: Government as Liquidity Transformer

The model is extended to incorporate government as a liquidity transformer. The state power allows for taxation on the non-pledgeable output of intangible capital, and then the government may use tax revenues to repay debts that entrepreneurs hold as liquidity for intangible investments, effectively transforming illiquid output into liquid securities. In reality, the production sector holds government debt in its liquidity portfolio, for example Treasury bills, together with liabilities of the financial sector (e.g., deposits). Henceforth in the model entrepreneurs can hold both bank deposits and government debt as liquidity.

To highlight the role government debt as alternative source of liquidity and to make the mechanism transparent, several realistic fiscal distortions are assumed away. Specifically, we consider a simple government debt strategy – government debt to output ratio is set at a perpetual level of 40% with interests paid by lump-sum tax levied on all agents. Since government debt and bank deposits are perfect substitutes, both pay interests $r_t dt$. As explained in Appendix III, solving the model with government debt only requires a small adjustment of the baseline algorithm.

First and foremost, introducing government debt alleviates the liquidity shortage as in Woodford (1990) and Holmström and Tirole (1998). Entrepreneurs' liquidity holdings, m_t^E, now contain both bank deposits and government debts, and thus, intangible investments increase. In the expression of economic growth rate, Equation (18), bank deposits are replaced by the sum of government debt and bank deposits. Therefore, the growth effect of government debt is straightforward.

How does government debt affects financial stability, i.e., the volatility of asset price and the frequency of banking crisis (low η_t states)? The liquidity demand in this paper arises from intangible investments that have a constant returns to scale of technology, so the marginal value of liquidity holdings does not decline when entrepreneurs hold more liquidity. Therefore, introducing government debt does not have significant influence on the equilibrium liquidity premium, banks' funding cost, and the mechanism of intermediated liquidity premium that causes endogenous risk in asset prices and cyclical banking crises.

Figure 10 compares the deposit rate, volatility multiplier, the stationary cumulative probability function, and economic growth rate in the baseline economy and the economy with 40% government debt to output ratio. The first thing we notice from Figure 10 is that the size of the banking sector stays roughly the same. Bankers consume at similar levels of aggregate equity to

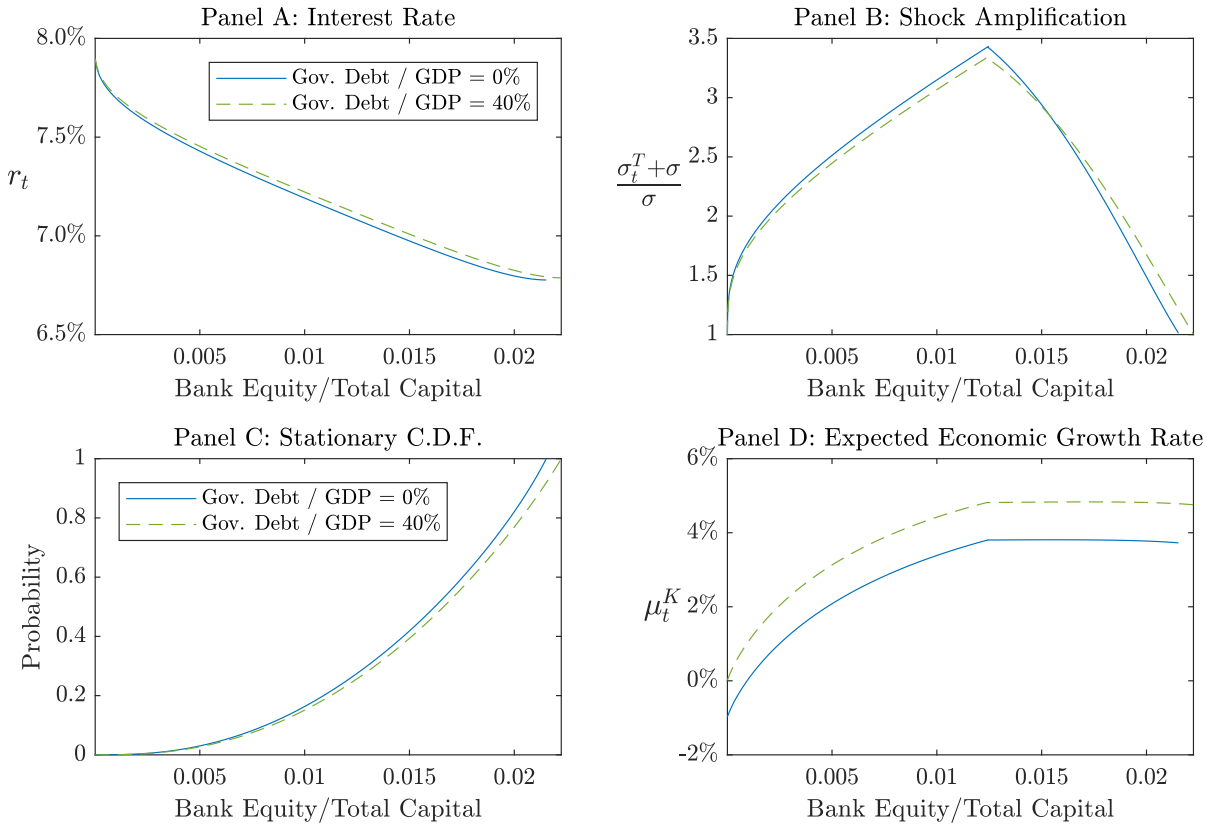

Figure 10: **Government Debt as Alternative Liquidity Supply**.

output ratio (the right boundary) with or without government debt. The reason is that the return on bank equity is not significantly affected by government debt, which is in turn due to the fact that the equilibrium liquidity premium and banks' funding costs are almost immune to the additional supply of liquidity from government debt issuance (as shown in the Panel A of 10).

Another key message from Panel A of 10 is that more government debt does not raise the interest rate level. This is consistent with what we observe in the last few decades – the U.S. has seen a rising level of government debt, but at the same time, the interest rate on deposits and other money market instruments stayed at low levels.

Panel B of Figure 10 shows that the mechanism of endogenous volatility in tangible capital price is robust to the addition of government debt. As previously discussed, the key to the endogenous risk accumulation is the widening gap in the funding cost between bankers (the natural asset buyers) and entrepreneurs (the second-best asset buyers) as η_t increases. This mechanism is

not affected by government debt given that the equilibrium dynamics of liquidity premium stays roughly the same. Similarly, Panel C of Figure 10 shows that the probability of low η_t states (i.e., states with an undercapitalized banking sector) does not change much after government debt is introduced. However, note that even though the frequency of banking crisis is not responsive to government debt, the severity of crisis has been much reduced. As shown in Panel D of Figure 10, government debt lifts up the growth rates uniformly across states of the world. The reason is that even in crisis states where banks retreat from liquidity supply, entrepreneurs can still hold government debt to sustain intangible investments and economic growth.

III.2: Household Money Demand

A missing element in the model is the money demand of households. By the end of 2017, the U.S. households hold \$ 11.4 trillion dollars of deposits and money market fund shares, an equivalent of 59% U.S. GDP (source: Financial Accounts of the United States). Next, I examine how the model mechanism is affected by the addition of households' money demand. For simplicity, it is assumed that households have an inelastic demand of deposits that is equal to 59% of total output (i.e., the ratio of the U.S. in 2017).[19]

Adding households' money demand reduces the economic growth rate because it crowds out entrepreneurs' liquidity holdings and investments. However, it does not affect the instability mechanism of intermediated liquidity premium. The intuition is similar to the case of government debt. In the model, given that the investment technology in the production sector has constant return-to-scale, even if households' money demand reduces the quantity of liquidity held by entrepreneurs, it does not significantly affect the marginal value of liquidity.

Figure 11 compares the deposit rate, volatility multiplier, the stationary cumulative probability function, and economic growth rate in the baseline economy (solid lines) and the economy with 59% household deposits to output ratio (dashed lines). As shown in Panel A, adding the households' demand for bank deposits does not significantly affect the interest rate level (and $\rho - r_t$, the liquidity premium). Given a similar dynamics of liquidity premium as in the baseline model, the extended model with households' deposit holdings amplifies shocks to a similar level as the

[19] An alternative approach is to specify money-in-utility as in, for example, Sidrauski (1967). However, it complicates the mechanism by adding another dimension of time variation in the liquidity premium that confounds the mechanism of intermediated liquidity premium. Moreover, how to specify the money-in-utility function is still a debatable empirical issue (e.g., Poterba and Rotemberg (1986)).

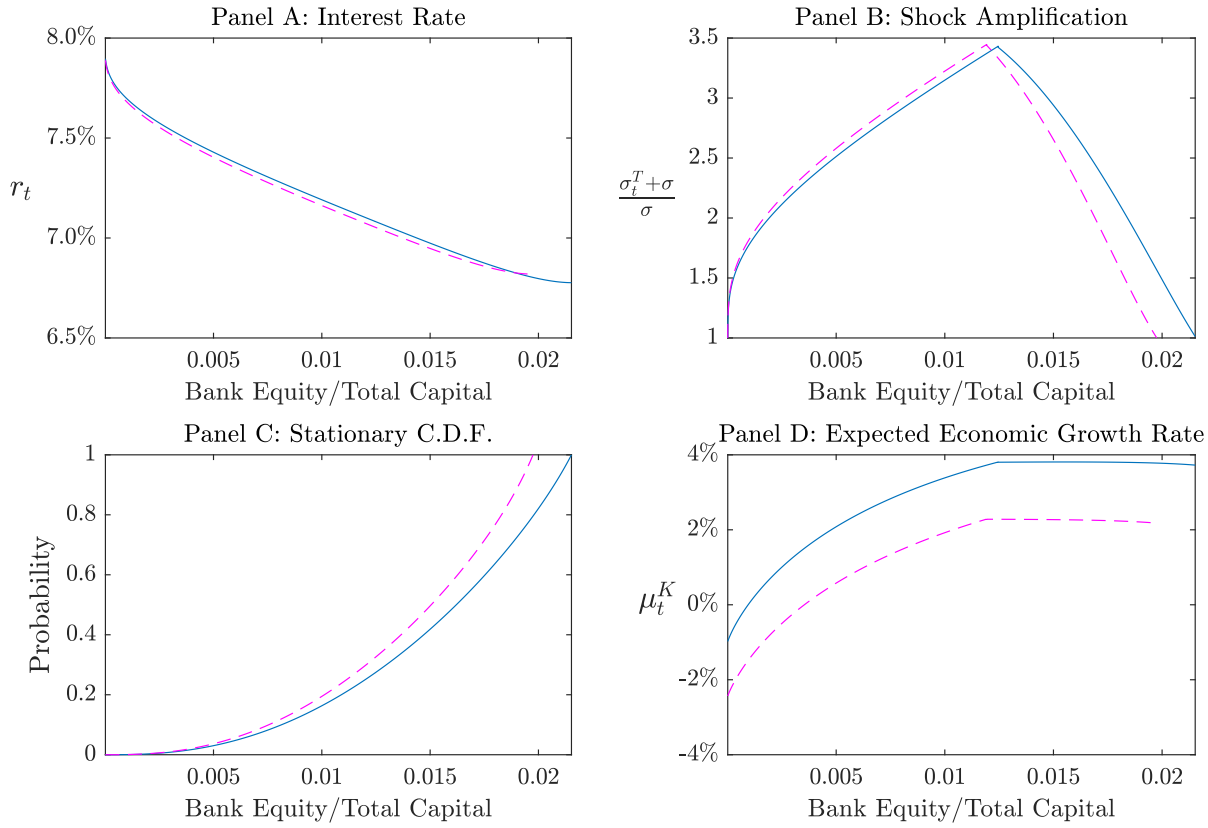

Figure 11: **Introducing Households' Demand for Bank Deposits**.

baseline model does (Panel B). In other words, the instability of asset price remains unaffected.

Households crowd out entrepreneurs in the deposit market, so, as shown in Panel D of Figure 11, the economic growth rate declines across states of the world because entrepreneurs are crowded out in the money market by households and hold less liquidity. The dynamics of state variable, η_t, is changed because its denominator, the total amount of capital, grows slower. As a result, the stationary distribution of η_t is affected, and in particular, the probability mass is more concentrated in lower values of η_t (Panel C of Figure 11).

Appendix IV: Solving the Equilibrium

The baseline model. The fully solved time-homogeneous Markov equilibrium is a set of functions that map η_t to the values of endogenous variables, such as tangible capital price, interest rate,

bank leverage, and entrepreneurs' deposit holdings. In the description of solution method, time subscripts are suppressed to save notations.

First, we construct a mapping from η, $q^B(\eta)$, $q^T(\eta)$, $dq^B(\eta)/d\eta$, and $dq^T(\eta)/d\eta$ to the second-order derivatives, $d^2q^B(\eta)/d\eta^2$ and $d^2q^T(\eta)/d\eta$, i.e., a system of second-order ordinary differential equations.

Given q^T, Proposition 1 solves the liquidity premium, $\rho - r$ and deposit rate r. For small values of η, consider the case where banks do not hold all tangible capital, so $\rho = r - \sigma^B\left(\sigma^T + \sigma\right)$. By Itô's lemma,

$$\sigma^T = \epsilon^T \sigma^\eta \text{ and } \sigma^B = \epsilon^B \sigma^\eta.$$

where $\epsilon^T = \frac{dq^T/q^T}{d\eta/\eta}$ and $\epsilon^B = \frac{dq^B/q^T}{d\eta/\eta}$. Since $\rho - r$ is a function of q^T, we substitute the decomposition of σ^T and σ^B into $\rho = r - \sigma^B\left(\sigma^T + \sigma\right)$ to obtain a quadratic equation of σ^η, and the roots are

$$\sigma^\eta = \frac{-\epsilon^B\sigma \pm \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T\left(\rho - r\right)}}{2\epsilon^B\epsilon^T}.$$

Because $\epsilon^B < 0$, $\epsilon^T > 0$, and $\rho \geq r$, the only positive root is

$$\sigma^\eta = \frac{-\epsilon^B\sigma - \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T\left(\rho - r\right)}}{2\epsilon^B\epsilon^T}. \tag{37}$$

Because $\sigma^\eta = x^B\left(\sigma^T + \sigma\right) - \sigma$, we can solve

$$x^B = \frac{\sigma^\eta + \sigma}{\sigma^\eta\epsilon^T + \sigma}. \tag{38}$$

Next, we need to check whether $x^B N^B \leq q^T K^T$, or equivalently, $x^B\eta \leq q^T(1-\theta)$. If this condition does not hold, we are at point where banks are large enough to hold all tangible capital, so we set $x^B = q^T(1-\theta)/\eta$, and solve σ^η using Equation (38). Once x^B and σ^η are solved, σ^B and σ^T can be solved by Itô's lemma, and bankers' required risk compensation, $-\sigma^B\left(\sigma^T + \sigma\right)$.

Next, we solve the drift terms of bankers' wealth,

$$\mu^N = x^B\mathbb{E}\left[dr^T\right] + \left(1 - x^B\right)r,$$

and the diffusion term

$$\sigma^N = x^B \left(\sigma^T + \sigma\right).$$

From Equation (19), we can solve μ^K. Note that for the model with government debt, we only need to replace the total bank deposits in Equation (19) with the sum of bank deposits and government debt (which is proportional to K_t). Then from Equation (20),

$$\mu^\eta = \mu^N - \mu^K - \sigma^N \sigma + \sigma^2.$$

To solve the second-order derivatives, $d^2 q^B(\eta)/d\eta^2$ and $d^2 q^T(\eta)/d\eta$, we substitute bankers' optimality conditions into bankers' HJB equation to obtain

$$\mu^B = \rho - r,$$

and bankers' optimality condition for tangible capital holdings in Proposition 3,

$$\mu^T = r - \sigma^B \left(\sigma^T + \sigma\right) - \sigma^T \sigma + \delta + \lambda - \frac{\alpha}{q^T},$$

so from Itô's lemma,

$$\frac{d^2 q^T}{d\eta^2} = 2q^T \frac{\left(\mu^T - \epsilon^T \mu^\eta\right)}{\left(\sigma^\eta \eta\right)^2}, \text{ and, } \frac{d^2 q^B}{d\eta^2} = 2q^B \frac{\left(\mu^B - \epsilon^B \mu^\eta\right)}{\left(\sigma^\eta \eta\right)^2}.$$

The procedure constructs a mapping from η, $q^B(\eta)$, $q^T(\eta)$, $dq^B(\eta)/d\eta$, and $dq^T(\eta)/d\eta$ to the second-order derivatives, $d^2 q^B(\eta)/d\eta^2$ and $d^2 q^T(\eta)/d\eta$. The five boundary conditions in Proposition 4 pin down a unique solution to the two second-order ODEs and one endogenous boundary (bankers' consumption boundary $\overline{\eta}$).

Government debt as liquidity. The solution of model with government debt as an alternative source of liquidity follows the same steps except that firms' liquidity is higher and equal to the sum of bank deposits and government debt. The resulting higher investment only changes the dynamics (and specifically the drift) of K_t. Let M_t^G denote the total amount of government debt.

We have

$$dK_t = \underbrace{-\left(\delta dt - \sigma dZ_t\right) K_t}_{\textit{stochastic depreciation}} \underbrace{-\lambda dt K_t}_{\textit{liquidity shocks}} + \underbrace{\left[\kappa\left(\frac{1}{1-\kappa\left(1-\theta\right) q_t^T}\right)\left(M_t^E + M_t^G\right)\right]\lambda dt}_{\textit{investment}} + \underbrace{K_t \chi dt}_{\textit{endowments}}. \tag{39}$$

We set a constant government debt to output ratio at 40%, so

$$\frac{M_t^G}{K_t} = \left(\frac{M_t^G}{\alpha K_t^T + \left(\alpha+\phi\right) K_t^I}\right)\left[\alpha\left(1-\theta\right) + \left(\alpha+\phi\right)\theta\right] = 0.4\left(\alpha\left(1-\theta\right) + \left(\alpha+\phi\right)\theta\right). \tag{40}$$

Substituting the deposit market clearing condition into Equation (18), we have:

$$\frac{dK_t}{K_t} = \underbrace{\left\{\kappa\left(\frac{1}{1-\kappa\left(1-\theta\right) q_t^T}\right)\left[\left(x_t^B - 1\right)\left(\frac{N_t^B}{K_t}\right) + \frac{M_t^G}{K_t}\right]\lambda - \delta - \lambda + \chi\right\}}_{\mu_t^K} dt + \sigma dZ_t. \tag{41}$$

To solve the model with government debt, the modification on baseline algorithm is adjusting μ_t^K by adding the extra liquidity term, M_t^G/K_t.

Household money demand. The solution of model with household money demand is similar to that of model with government debt. Let M_t^H denote the total amount of deposits that household hold. Both entrepreneurs and households hold bank deposits. The deposit market clearing condition is

$$M_t^E + M_t^H = \left(x_t^B - 1\right) N_t^B. \tag{42}$$

Substituting it into the evolution of capital, we have

$$\frac{dK_t}{K_t} = \underbrace{\left\{\kappa\left(\frac{1}{1-\kappa\left(1-\theta\right) q_t^T}\right)\left[\left(x_t^B - 1\right)\left(\frac{N_t^B}{K_t}\right) - \frac{M_t^H}{K_t}\right]\lambda - \delta - \lambda + \chi\right\}}_{\mu_t^K} dt + \sigma dZ_t, \tag{43}$$

where

$$\frac{M_t^H}{K_t} = \left(\frac{M_t^H}{\alpha K_t^T + \left(\alpha+\phi\right) K_t^I}\right)\left[\alpha\left(1-\theta\right) + \left(\alpha+\phi\right)\theta\right] = 0.59\left(\alpha\left(1-\theta\right) + \left(\alpha+\phi\right)\theta\right), \tag{44}$$

as we set a constant household money demand to output ratio at 59%. To solve the model, the modification on baseline algorithm is adjusting μ_t^K by subtracting the deposits held by households.

Endogenous capital composition. Solving the Markov equilibrium requires solving a system of partial differential equations because now there are two state variables. In the follow, it is shown that the state variables, (η_t, ω_t), can be monotonically transformed to one variable with autonomous law of motion and the other variable whose evolution depends on both itself and the first variable. Moreover, it can be shown that q_t^T and q_t^B only depend on the autonomous state variable, so solving q_t^T and q_t^B becomes a problem of solving a pair of ordinary differential equations. Then we can transform the solution back into the space of (η_t, ω_t).

First, the evolution of ω_t is solved. The stock of tangible capital, K_t^T, has the following law of motion

$$\frac{dK_t^T}{K_t^T} = \left[\kappa\left(\frac{\left(x_t^B - 1\right)\eta_t}{1-\kappa\left(1-\theta\right)q_t^T}\right)\left(\frac{1-\theta}{1-\omega_t}\right)\right]\lambda dt + \beta\sqrt{i_t^T}\lambda dt - \left(\delta dt - \sigma dZ_t\right) - \lambda dt + \chi dt. \quad (45)$$

The evolution given by Equation (19) is augmented by the extra investment. The last term is from the endowments as before. A measure χdt of entrepreneurs enter, carrying $K_t^T\chi dt$ tangible and $K_t^I\chi dt$ intangible capital. Intangible capital follows the law of motion given by Equation (19),

$$\frac{dK_t^I}{K_t^I} = \left[\kappa\left(\frac{\left(x_t^B - 1\right)\eta_t}{1-\kappa\left(1-\theta\right)q_t^T}\right)\left(\frac{\theta}{\omega_t}\right)\right]\lambda dt - \left(\delta dt - \sigma dZ_t\right) - \lambda dt + \chi dt, \quad (46)$$

except that the intangible capital share deviates from θ. By Itô's lemma, the intangible share, ω_t, has the following law of motion in $(0,1)$:

$$d\omega_t = \omega_t\left(1-\omega_t\right)\left\{\left[\left(\frac{\theta}{\omega_t}\right) - \left(\frac{1-\theta}{1-\omega_t}\right)\right]\kappa\left(\frac{\left(x_t^B - 1\right)\eta_t}{1-\kappa\left(1-\theta\right)q_t^T}\right) - \beta_0\left(i_t^T\right)^{\beta_1}\right\}\lambda dt. \quad (47)$$

Next, we can define a new variable $\widetilde{\eta}_t = \eta_t/\left(1-\omega_t\right)$. By Iô's lemma, it evolves as

$$\frac{d\widetilde{\eta}_t}{\widetilde{\eta}_t} = \frac{d\eta_t}{\eta_t} + \frac{d\omega_t}{\left(1-\omega_t\right)}, \quad (48)$$

because ω_t does not load on the Brownian shock. Using Equation (20) and (47), we have

$$\frac{d\widetilde{\eta}_t}{\widetilde{\eta}_t} = \mu_t^{\widetilde{\eta}} dt + \sigma_t^{\widetilde{\eta}} dZ_t, \tag{49}$$

where

$$\begin{aligned}\mu_t^{\widetilde{\eta}} = \left[r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T\right] - r_t\right)\right] - \left[\kappa \left(\frac{\left(x_t^B - 1\right)\widetilde{\eta}_t}{1 - \kappa\left(1-\theta\right) q_t^T}\right)(1-\theta)\right]\lambda - \beta\sqrt{i_t^T}\lambda \\ + \delta - \lambda - \chi + \left[\sigma^2 - x_t^B\left(\sigma_t^T + \sigma\right)\sigma\right]\end{aligned} \tag{50}$$

and

$$\sigma_t^{\widetilde{\eta}} = x_t^B\left(\sigma_t^T + \sigma\right) - \sigma, \tag{51}$$

and the expected return on tangible capital is given by Equation (16). This gives the dynamics of $\widetilde{\eta}_t$ when banks payout ratio is zero. Similarly there exists $\widetilde{\eta}^*$, a payout boundary, such that when $\widetilde{\eta} \geq \widetilde{\eta}^*$, bankers consume. In the following, the model is solved in the space of $(\widetilde{\eta}_t, \omega_t)$ instead of (η_t, ω_t), where $\widetilde{\eta}_t$ will be shown as an autonomous state variable and the evolution of ω_t depends on both $\widetilde{\eta}_t$ and ω_t.

In an equilibrium where q_t^T and q_t^B only depend on $\widetilde{\eta}_t$, the interest rate, r_t, only depends on $\widetilde{\eta}_t$ (Equation (8)), and thus, so does x_t^B because when $x_t^B N_t^B < q_t^T K_t^T$, x_t^B is solved by Equation (38) with σ_t^η replaced by $\sigma_t^{\widetilde{\eta}}$ and when $x_t^B N_t^B \geq q_t^T K_t^T$ x_t^B, $x_t^B = q_t^T/\widetilde{\eta}_t$. Also, given Equation (22), i_t^T only depends on $\widetilde{\eta}_t$. As a result, $\widetilde{\eta}_t$ indeed has an autonomous law of motion. The boundary conditions are the same as the case with time-invariant capital composition (Proposition 4). Therefore, the conjecture that q_t^T and q_t^B only depend on $\widetilde{\eta}_t$ is internally consistent. Same as before, bankers' HJB equation,

$$\mu_t^B = \rho - r,$$

and bankers' optimality condition for tangible capital holdings in Proposition 3,

$$\mu_t^T = r_t - \sigma_t^B\left(\sigma_t^T + \sigma\right) - \sigma_t^T\sigma + \delta + \lambda - \frac{\alpha}{q_t^T},$$

form a pair of differential equations for q_t^T and q_t^B, so given that r_t and x_t^B (and therefore $\sigma_t^{\widetilde{\eta}}$) only depend on $\widetilde{\eta}_t$, there are a pair of ordinary equation in the space of $\widetilde{\eta}_t \in (0, \widetilde{\eta}^*)$ that solve q_t^T and q_t^B as functions of $\widetilde{\eta}_t$. Therefore, we confirm the conjecture that q_t^T and q_t^B only depend

on $\widetilde{\eta}_t$. As previously discussed, given q^T and q^B, we can solve r_t, x_t^B, and i_t^T as functions of $\widetilde{\eta}_t = \eta_t / (1 - \omega_t)$. Once these variables are solved, the evolution of ω_t (and K_t^T and K_t^I) can be solved. Therefore given any (η_t, ω_t), we have a unique $(\widetilde{\eta}_t, \omega_t)$ and a unique set of values for endogenous variables given any initial condition, for example, as stated in Proposition 4.