Dynamic Banking and the Value of Deposits

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Abstract

We propose a dynamic theory of banking where deposits play the role of productive capital as in the classical Q-theory of investment for non-financial firms. A key conceptual innovation of our theory is that the stock of deposits cannot be perfectly controlled by the bank. Demand deposit accounts commit the bank to allow holders to withdraw or deposit funds at will. The resultant uncertainty in deposit flows exposes the bank to the risk of violating regulatory restrictions on leverage. Deposits create value for the bank except when it is close to hitting the leverage restrictions, because sudden deposit inflows can force the bank into costly equity issuance. We show that the bank is endogenously risk averse with respect to both the deposit flow risk and standard loan return risk. Our model predictions on dynamic bank valuation and asset-liability management are broadly consistent with the evidence. Moreover, our model lends itself to a quantitative evaluation of the costs and benefits of leverage regulations.

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1 Introduction

During the global financial crisis, and more recently during the unfolding COVID-19 pandemic, banks have been subject to large deposit inflows and outflows. The largest and safest banks have undergone large increases in their balance sheets as a result of massive inflows of funds into deposit accounts, while weaker banks have suffered from deposit outflows. Most dramatically, just in April 2020 alone, deposits of US banks increased by $865 billion. From Q4 2019 to Q1 2020, JPMorgan Chase experienced an increase of 18% percent of its deposit base, and the deposit liabilities of Citigroup and Bank of America increased by 11% and 10%, respectively.[1]

Large deposit flows are both an opportunity and a risk for banks. As the literature has emphasized, the bank business model rests on deposit taking. Demand deposit account is a source of cheap funding that banks rely on to finance their lending and trading activities. Depositors accept relatively low rates for the convenience of using deposits as means of payment. But the consequence of depositors' freedom to move funds in and out of their deposit accounts is that banks cannot perfectly control the size of their deposit base and balance sheet. To the extent that the banking literature is modeling such risk, it has done so only in terms of the bank run risk. Such models assume that, in the no-run equilibrium, banks face no risk with respect to the size of deposit base so that deposits can be treated as fixed-maturity debts.

We depart from this narrow framing and treat banks' deposit base more generally as randomly evolving and subject to shocks of both inflows and outflows. In our dynamic model of the banking firm, deposits have random maturity, and the deposit base is a sticky and stochastic variable that is only partially controllable. The size of deposit liabilities as a randomly evolving variable is one of the key features that distinguishes a bank from a non-financial firm. The other important distinguishing feature is that banks are subject to capital regulations. To be able to continue operating, a bank must make sure that its equity capital remains above a minimum required amount. Importantly, due to the deposit risk, a bank does not have full control over the ratio of equity capital to deposits, which is the key endogenous state variable in our model. Unexpected

[1] See "U.S. Banks are 'Swimming in Money' as deposits increase by 2 trillion dollars amid the coronavirus" by Hugh Son, CNBC June 21, 2020. https://www.cnbc.com/2020/06/21/banks-have-grown-by-2-trillion-in-deposits-since-coronavirus-first-hit.html

deposit inflows increase the denominator, so when the bank is undercapitalized, it has to raise equity to avoid hitting the regulatory restriction on leverage.

We add two important frictions to this model. First, we assume in line with the evidence that it is costly for banks to issue new equity. Second, we assume that there is a lower bound for the remuneration of deposits. A natural lower bound is zero, but we can also allow for a negative lower bound that accounts for various fees. In practice, banks are loath to impose negative rates on deposits even if this could help stem an inflow of new deposits and the associated involuntary expansion of leverage. Empirically, this deposit rate lower bound has become increasingly binding in the current low-rate environment.

The combined effect of these two frictions is that a bank is endogenously risk-averse. When the bank's equity capital-to-deposit ratio is low, a sudden increase in deposits could cause a violation of the leverage restriction, which forces the bank to pay the issuance (dilution) costs and raise equity. In this region, the marginal value of additional deposits is low, even sharply negative. In contrast, when the bank is well capitalized (i.e., when it has a high equity capital-to-deposit ratio), the marginal value of deposits is high, as deposits represent cheap financing for lending with a low risk of violating the regulatory constraint.

We do not add any other frictions that may be relevant in practice. One important friction that is absent from our model is fire-sale pricing (or other forms of asset adjustment costs), which happens when a bank seeks to quickly shrink its asset base. In our model, the bank can costlessly change the size of its asset portfolio, so our focus is entirely on the lack of control of deposit liabilities rather than assets. This is a reasonable assumption for financial securities, which can be quickly adjusted at low costs. Admittedly, this is a stronger assumption as far as the bank's loan book is concerned. Adding an adjustment cost to the loan book, however, is a major complication as this would introduce a second state variable, rendering the mechanism much less transparent.

We characterize the bank's dynamic asset-liability management decisions including optimal deposit remuneration, short-term borrowing, lending, equity issuance, and dividend payout in the presence of regulatory constraints. We solve for the franchise value of the bank and how it varies with the equity capital-to-deposit ratio. We also solve for the marginal value of deposits and show that it decreases as the bank approaches the regulatory constraints.

Because it is costly to issue equity, the marginal value of equity capital is larger than one, and depends on the value of equity capital-to-deposit ratio, denoted by k, which is bounded by two endogenous reflecting boundaries. The marginal value of equity capital varies over a wide range: it is equal to one at the dividend payout boundary, when k is high and the bank is indifferent between retaining an extra unit of capital or paying it out (that is how the endogenous upper bound is defined), and when k is low, it can rise to above fifty at the equity issuance boundary (the lower bound of k), even under conservative values for the issuance costs from the empirical literature. When the bank is close to hitting its regulatory leverage restriction, any additional unit of equity capital is very valuable as it reduces the likelihood of costly equity issuance.

The marginal value of equity capital does not decline with k linearly. The bank's franchise value is strictly concave in k, so that the bank is endogenously risk-averse even though shareholders are assumed to be risk neutral. We show that it is optimal for the bank to substantially reduce lending as k declines and the bank approaches the regulatory constraint. This is consistent with empirical findings linking changes in bank equity capital to bank lending. When k increases and does approach the payout boundary, it is more likely to stay around than not. Indeed, at the peak of the stationary density of k the marginal value of equity is only slightly above one, so that, for the majority of time, the bank does not seem to be financially constrained. This nonlinearity captures a sharp contrast between the normal times and the crisis times when k is low, close to the equity issuance boundary, and the marginal value of equity capital shoots up dramatically.

In our model, deposits are valuable because depositors are willing to accept a deposit rate that is below the prevailing risk-free rate. This can be motivated by the convenience of payment services offered by deposit accounts or banks' deposit market power Drechsler, Savov, and Schnabl (2017). When the bank has sufficient equity capital, deposits create value by allowing the bank to finance risky lending with cheap sources of funds. The deposit stock in effect serves as a form of productive capital for the bank. However, when the bank's equity capital is depleted, the marginal value of deposits can be negative. The reason is that, near the costly issuance boundary, uncontrollable deposit inflows destroy value for the bank's shareholders by increasing the bank's leverage and amplifying the likelihood of costly equity issuance in compliance of leverage restriction. The bank then wants to deleverage and turn away deposits. However, the bank can only go as far as setting the deposit rate at the lower bound; it cannot turn down deposits by further lowering the rate.

Losing control of its deposit base is a problem when the bank faces equity issuance costs, for then, in effect, it also loses control of its leverage. Indeed, we show that without the equity issuance costs the bank can costlessly offset any increase in deposits with a commensurate amount of newly raised equity capital, thereby maintaining its leverage below the regulatory cap. The bank's franchise value is then linear in k, so that the bank is no longer endogenously risk averse.

In our model, a sharp distinction is drawn between deposits and short-term debt. With short-term debt, the bank can always choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debt. Deposits do not have a well-defined maturity. Deposits leave the bank only when depositors chose to withdraw funds (for example, to pay other banks' depositors or for cash withdrawal). Hence, deposits add value only if the bank has sufficient equity capital and thus is not at the risk of costly equity issuance in compliance of regulatory requirements.

Given that the bank is endogenously risk averse, it wants to hold safe assets for precautionary reasons when the equity capital-to-deposit ratio, k, is low, but when k is high, the bank switches its position in risk-free assets, issuing short-term debt to obtain an even higher leverage than the leverage already obtained through deposit-taking. Such procyclicality is again due to the risk with respect to uncontrollable deposit flows. Specially, when k is small and the bank is close to violating the regulatory constraint, it deleverages by simultaneously reducing short-term debt and lowering the deposit rate to turn away deposits. When the bank has eliminated short-term debt and is close to the deposit rate lower bound, it almost loses control of its liability structure. As a result, to avoid costly equity issuance, the bank has to work on the asset-side of its balance sheet, tuning down its exposure to loan risk by holding risk-free assets. When k is large and the bank is sufficiently away from the equity issuance boundary, its risk-taking incentive is strong and results in a switch from the bank holding risk-free assets to raising short-term debt for risky lending.

Literature. In the banking literature, the focus is on bank runs when it comes to banks' commitment to allow depositors to withdraw funds without prior notice (see, e.g., Diamond and Dybvig, 1983; Allen and Gale, 2004; Goldstein and Pauzner, 2005). However, the deposit flow risk is more ubiquitous than the dramatic bank runs and influences banks' daily operation. Moreover, it is not just deposit outflow that poses a challenge to banks. Deposit inflow can also be a risk, especially in the presence of regulatory constraints and equity issuance costs.

Depositors are willing to accept relatively low rates because of the convenience of using deposits as means of payment. What enables deposits as money is precisely banks' commitment to allow depositors to move funds in and out of their accounts. Banks are thus exposed to large payment inflows and outflows at high frequencies (Denbee, Julliard, Li, and Yuan, 2018). The maturity of deposit liabilities is not chosen by the bank. It often depends on depositors' payment needs that are uncertain. With a diversified depositor base, a bank essentially views deposits as debts that retire at a stochastic rate. Bianchi and Bigio (2014) and De Nicolò, Gamba, and Lucchetta (2014) also recognize such payment shocks. However, in their models, deposits are one-period contracts (with intra-period shocks), so banks can freely adjust the deposit base every period. In contrast, deposit contract in reality (and in our model) is infinite-horizon – depositors can freely hold deposits as long as they want. To adjust deposit base, banks can change deposit remuneration but completely loses control of deposit base when its deposit rate hits the lower bound. The key to our main results is precisely banks' lack of control of the deposit base, which exposes banks to the risk of paying equity issuance costs to stay in compliance with the regulatory requirements.

Deposits are inside money of the private sector – stores of value and means of payment issued by banks to depositors. This feature has been well recognized in the recent macro-finance literature. However, deposits are modelled as short-term debts (e.g., Piazzesi and Schneider, 2016; Drechsler, Savov, and Schnabl, 2018b) with interest rates below the prevailing rate by a money premium (Stein, 2012; DeAngelo and Stulz, 2015; Krishnamurthy and Vissing-Jørgensen, 2015; Greenwood, Hanson, and Stein, 2015; Li, 2019; Begenau, 2019). Brunnermeier and Sannikov (2016) and Drechsler, Savov, and Schnabl (2018a) are notable exceptions. Brunnermeier and Sannikov (2016) model deposits as infinite-maturity nominal liabilities and study the general equilibrium implications. Our model is more related to Drechsler, Savov, and Schnabl (2018a) who find that banks face little interest rate risk since both their assets and sticky deposit liabilities have long duration. Beyond the long duration, we incorporate the random evolution of deposit liabilities and highlight the impact of deposit risk on bank valuation and asset-liability management. Finally, dynamic banking models typically differentiate short-term debts and deposits in their interest rate and operation costs (Hugonnier and Morellec, 2017; Van den Heuvel, 2018; Begenau, 2019). However, in all the models, banks do not face uncertainty in the size of deposit liabilities.

2 Model

We model a single bank's decisions under the risk-neutral measure, effectively assuming no arbitrage and taking as exogenous the pricing kernel (stochastic discount factor) that depends on the aggregate dynamics of the broader economy. Let r denote the risk-free rate, which is also the expected return of all financial assets under the risk-neutral measure.

Risky Assets. We use A_t to denote the value of the bank's holdings of loans and other investments at time t.[2] It has the following law of motion:

$$dA_t = A_t \left(r + \alpha_A \right) dt + A_t \sigma_A d\mathcal{W}_t^A, \tag{1}$$

The parameter α_A reflects the return from the bank's expertise. Because we set up our model under the risk-neutral measure, α_A is the risk-adjusted value-added.[3] The second term in (1) describes the Brownian shock, where σ_A is the diffusion-volatility parameter and $\mathcal{W}^A$ is a standard Brownian motion. Examples of these shocks include unexpected charge-offs of delinquent loans. At any time t, the bank may adjust its risky assets and the liability structure (i.e., deposits, bonds, and equity).

[2]The bank's assets include not only loans but also other assets that generate revenues of trading and services such as cash management, trade credit, derivatives, structured products, and underwriting of securities (Bolton, 2017).

[3]The bank may have expertise in loan screening (Ramakrishnan and Thakor, 1984), monitoring (Diamond, 1984), relationship lending (Boot and Thakor, 2000), restructuring (Bolton and Freixas, 2000), and serving local markets (Gertler and Kiyotaki, 2010). More generally, in the macro-finance literature, banks are often modelled as agents with expertise in asset management (He and Krishnamurthy, 2012, 2013; Brunnermeier and Sannikov, 2014, 2016).

Deposits. Deposits are at the core of our model. Let X_t denote the value of deposits at time t on the liability side of the bank's balance sheet. It has the following law of motion:

$$dX_t = -X_t \left(\delta_X dt - \sigma_X d\mathcal{W}_t^X\right) + X_t n\left(i_t\right) dt. \tag{2}$$

where $\mathcal{W}_t^X$ is a standard Brownian motion. Given a diversified depositor base, a $\left(\delta_X dt - \sigma_X d\mathcal{W}_t^X\right)$ fraction are withdrawn in dt because depositors may need cash or pay agents who hold accounts at other banks. If $\left(\delta_X dt - \sigma_X d\mathcal{W}_t^X\right) > 0$, the bank's own depositors receive payments into their accounts. Deposits thus have an average *effective duration* of $1/\delta_X$, and σ_X captures payment flow uncertainty.[4] The stochastic withdrawal is in line with the three-dates models (see, e.g., Diamond and Dybvig, 1983; Allen and Gale, 2004), where agents' stochastic preferences over early and late consumption translate into uncertainty in the deposit outflow. The deposit flow shock, $d\mathcal{W}_t^X$, is likely to be positively correlated with the loan repayment shock, $d\mathcal{W}_t^A$, as a healthy asset portfolio can attract depositors. Let ϕdt denote the instantaneous covariance between $d\mathcal{W}_t^X$ and $d\mathcal{W}_t^A$.

In the presence of diffusive shocks (instead of jump shocks), the bank can avoid default by adjusting the balance sheet locally and thus preserve a positive continuation value for equityholders. Therefore, deposits are risk-free for depositors. The deposit rate is i_t, chosen by the bank. The spread, $r - i_t$, can be positive if agents value the convenience of deposits as means of payment (e.g., DeAngelo and Stulz, 2015; Nagel, 2016; Piazzesi and Schneider, 2016; Li, 2019, 2018). For deposits to function as means of payment, depositors must be able to move funds in and out of their accounts freely. By allowing this, the bank exposes itself to the deposit-base risk in (2).

The bank can adjust the growth rate of deposits via i_t in $n\left(i_t\right) dt$, where the deposit demand elasticity depends on the bank's deposit market power (Drechsler, Savov, and Schnabl, 2017). Reducing the deposit rate causes deposit outflow, i.e., $n'\left(i_t\right) < 0$. Moreover, following Hugonnier and Morellec (2017) and Drechsler, Savov, and Schnabl (2018a), we assume that to maintain the existing deposits and attract new deposits, the bank pays a flow cost $C\left(n\left(i_t\right), X_t\right) dt$, which captures the expenses of maintaining branches, marketing products, and servicing customers.

[4] The value of δ_X largely depends on where the bank sits in the payment network, and the payment flow uncertainty σ_X can be significant in data (see, e.g., Denbee, Julliard, Li, and Yuan, 2018).

Deposits are essentially long-term debts with stochastic maturity and controllable increments. Not all depositors withdraw at the same time, and withdrawal depends on depositors' payment needs. Therefore, a diversified depositor base implies an effective duration of deposits that depends on the average rate of withdrawal.

Our treatment of deposits stands in contrast with the macro-finance literature and dynamic banking literature that generally treats deposits simply as short-term debts (motivated by depositors' right to withdraw at any time). We share with Drechsler, Savov, and Schnabl (2018a) the view that the right to withdrawal does not necessarily translate into a low duration of deposits. We emphasize that the right to withdrawal imposes a lower bound on the feasible deposit rate – the bank cannot set a negative deposit rate because depositors will withdraw en masse and earn the zero return on dollar bills that also work as means of payment. We assume that in such a bank run, the shareholders' equity is wiped out, so the bank always avoids such scenario. In reality, depositors may tolerate an effective negative deposit rate (often in the form of fees) due to the inconvenience of carrying dollar bills, but as long as there exists a lower bound, our results hold.

Bonds. The bank issues short-term bonds (e.g., financial commercial papers), and it is costless to do so. Let B_t denote the value of bonds issued at t that will mature at $t + dt$. Without default risk, the contractual rate of return for short-term debt initiated at t is the risk-free rate r. The bank's bond interest payment over time interval dt is $B_t r dt$. The bank may choose not to issue bonds but instead invest in risk-free bonds issued by other entities in the economy (e.g., the government). In this case, we have $B_t < 0$. Whether the bank issues or holds risk-free bonds will depend on its risk-taking capacity, which in turns depends on the existing deposit liabilities and equity capital.

Equity, Dividend, and Costly Issuance. Let K_t denote the bank's equity (or "capital"), so the following identity summarizes all the balance-sheet items:

$$K_t = A_t - (B_t + X_t) \ . \tag{3}$$

The bank can pay out dividends that reduce K_t. We use U_t to denote the (undiscounted) cumulative dividends, so the amount of (non-negative) incremental payout is dU_t.

The bank may find it optimal to issue external equity. In reality, banks face significant external financing costs due to asymmetric information and incentive issues.[5] A large empirical literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction in response to the announcement of a new equity issue.[6] Let F_t denote the bank's (undiscounted) cumulative net external equity financing up to time t and H_t to denote the corresponding (undiscounted) cumulative costs of external equity financing up to time t.

The bank's equityholders are protected by limited liability. Let τ denote the stochastic stopping time when the bank defaults. Therefore, the bank maximizes the equityholders' value,

$$V_0 = \mathbb{E}\left[\int_{t=0}^{\tau} e^{-\rho t}\left(dU_t - dF_t - dH_t\right)\right].^{7} \tag{4}$$

Because the bank only faces (locally continuous) diffusive shocks, it can avoid default as long as the continuation value is positive. In our numeric solution, this is indeed the case, so $\tau = +\infty$. We assume that the discount rate ρ, i.e., the equityholders' required return, is greater than r. This impatience can be microfounded by a Poisson death rate that is equal to $\rho - r$.

Capital Requirement. The bank must meet a capital requirement. For example, the Basel III accords stipulate that banks must back a specific percentage of risk-weighted assets with equity.[8]

[5]Explicitly modeling informational asymmetry would result in a substantially more involved analysis. Lucas and McDonald (1990) provides a tractable analysis by making the simplifying assumption that the informational asymmetry is short lived, i.e. it lasts one period.

[6]Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs) of equity, the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), the direct costs average 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was -3% and the loss in equity value as a percentage of the size of the new equity issue was as high as -31% (see Eckbo, Masulis, and Norli, 2007, for a survey).

[7]We assume that the bank manager's incentive is aligned with equityholders. Becht, Bolton, and Röell (2011) discuss the issues of corporate governance in the banking sector.

[8]See Thakor (2014) for a review of the debate on bank capital and its regulations.

As in Begenau (2019), Davydiuk (2017), Nguyen (2015), and Van den Heuvel (2018), we introduce

$$\frac{A_t}{K_t} \leq \xi_K \,. \tag{5}$$

In accordance with Basel III capital standards, banks must maintain the Tier 1 capital ratio (Tier 1 capital divided by total risk-weighted assets) of 6% (increased to 7% from 2019 onward) (see, e.g., Begenau, 2019; Davydiuk, 2017). We set ξ_K equal to $1/0.07 = 14.3$.[9]

Leverage Restriction. Since January 1, 2018, banks in the U.S. face a supplementary leverage ratio restriction (SLR). It supplements the capital requirement that can be vulnerable to manipulation (Plosser and Santos, 2014). Banks are required to maintain a ratio of tier 1 capital to total consolidated assets at a minimum level of 3%. The U.S. bank holding companies that have been identified as global systemically important banks ("U.S. G-SIBs") must maintain an SLR of greater than 5%, and if they fail to do so, they will be subject to increasingly stringent restrictions on its ability to make capital distributions and discretionary bonus payments.[10]

The ratio of its total assets (or liabilities) to equity capital cannot exceed ξ_L. When the bank has short-term debts, i.e., $B > 0$, the leverage ratio requirement is given by

$$\frac{A}{K} = \frac{K + X + B}{K} \leq \xi_L \,, \tag{6}$$

[9]Begenau (2019) and Davydiuk (2017) set ξ_K to be the sample average of the ratio of Tier 1 equity to risky assets in their models for the reason that banks typically maintain a buffer over the regulatory thresholds in order to prevent regulatory corrective action. In our model, the buffer arises endogenously, driven by banks' precaution to avoid paying the equity issuance costs, so we set ξ_K to the regulatory threshold. In other studies on banking regulations, De Nicolò, Gamba, and Lucchetta (2014) calibrates the capital requirements to 4% and 12%, Hugonnier and Morellec (2017) calibrates the threholds to 4% , 7%, 9%, and 20% to investigate the effects of the proposal by Admati and Hellwig (2013), and Phelan (2016) calibrates the threshold to 7.7% and 10.6% in a macroeconomic model.

[10]During the Covid-19 crisis, the U.S. regulators excluded Treasury securities and reserve held at the Federal Reserve System from the denominator of SLR, as banks faced an influx of deposits on the liability side of their balance sheets and, on the asset side, acquired significant amounts of U.S. Treasury securities and news loans (especially due to customers drawing on credit lines).

and when $B < 0$, the leverage ratio requirement is given by

$$\frac{A - B}{K} = \frac{K + X}{K} \leq \xi_L \,. \tag{7}$$

SLR capital requirement of 5% translates into $\xi_L = 20$. When $B > 0$, both capital requirement and SRL restriction become a restriction of A/K.

3 Dynamic Banking without Equity Issuance Costs

A key friction in our model is the equity issuance cost. Next, we show that without such costs, the value function is linear in the deposit stock, X, and capital, K. As a result, the bank does not exhibit endogenous risk aversion and the marginal value of deposits is constant.

Without the issuance costs, the marginal value of capital is equal to one, i.e., $V_K(X, K) = 1$, because if $V_K(X, K) > 1$, the bank raises equity, and the bank pays out dividend if $V_K(X, K) < 1$. In Appendix A, we show that there exists a constant Q such that

$$V(X, K) = QX + K \,. \tag{8}$$

A key result is that Q does not depend on any of the risk parameters, i.e., σ_A and σ_X. Without the equity issuance costs, the bank is not concerned about risks, because when it needs capital following adverse shocks, it can always raise capital. Intuitively, as long as $\alpha_A > 0$, i.e., lending generates excess return, the bank will borrow short-term debt, increasing leverage beyond what is already obtained through deposit-taking. As previously discussed, when $B > 0$, both capital requirement and SLR restriction become a restriction on A/K. Because $\xi_L > \xi_K$, capital requirement binds and the bank's optimal lending is proportional to equity capital, i.e., $A/K = \xi_K$. Moreover, the bank sets a constant deposit rate.

4 Dynamic Banking under Equity issuance Costs

4.1 Bank Optimization

We derive the optimality conditions for the bank's control variables and the Hamilton-Jacobi-Bellman (HJB) equation for the bank's value function. In the next subsections, we parameterize the deposit maintenance cost and provide intuitive characterizations of the bank's optimal policies.

State and Control Variables. The bank solves a dynamic optimization problem with two state variables, the deposit stock X_t and the equity capital K_t. We denote the shareholders' value at time t as V_t. This present value results from the bank's optimal control of the stochastic processes of loan portfolio size A_t, short-term borrowing B_t, the deposit rate i_t, the payout of dividends dU_t, and the value of newly issued equity shares dF_t:

$$V_t = V\left(X_t, K_t\right) = \max_{\{A,B,i,U,F\}} \mathbb{E}\left[\int_{t=0}^{\tau} e^{-\rho t}\left(dU_t - dF_t - dH_t\right)\right]. \tag{9}$$

The value function is a function of the state variables, i.e., $V_t = V\left(X_t, K_t\right)$. Every instant, given the state variables, X_t and K_t, the bank optimizes the control variables before the realization of diffusion shocks, taking into consideration the impact on the evolution of state variables (and through such impact, the continuation value). To solve the bank's optimal choices and value function, we need the laws of motion of state variables that show how the choice variables affect their evolution. The law of motion for X_t is given by (2). For the equity capital K_t, we have

$$dK_t = A_t\left[\left(r + \alpha_A\right)dt + \sigma_A d\mathcal{W}_t^A\right] - B_t r dt - X_t i_t dt - C\left(n\left(i_t\right), X_t\right)dt - dU_t + dF_t\,. \tag{10}$$

The first three terms on the right side record the return on loans, bond interest expenses, and deposit interest expenses. The fourth term is the operation cost associated with adjusting and maintaining the deposit stock. The last two terms are the dividend payout and capital raised via equity issuance.

Given X_t and K_t, the bank's choices of A_t and B_t resemble a portfolio problem (Merton, 1969).[11] Let π_t^A denote the portfolio weight on loans, i.e., $\pi_t^A (X_t + K_t) = A_t$, so the weight on bonds is $\left(\pi_t^A - 1\right)$ because $B_t = A_t - (X_t + K_t)$. Note that if $A_t > X_t + K_t$, the bank issues bonds, $B_t > 0$, paying the interest rate r; if $A_t < X_t + K_t$, the bank lends in the short-term debt market (i.e., $B_t < 0$) and earns the interest rate r. We can rewrite the law of motion for K_t as

$$dK_t = (X_t + K_t) \left[\left(r + \pi_t^A \alpha_A\right) dt + \pi_t^A \sigma_A d\mathcal{W}_t^A\right] - X_t i_t dt - C\left(n\left(i_t\right), X_t\right) dt - dU_t + dF_t\,. \tag{11}$$

Given the Markov nature of the bank's problem, we suppress the time subscript of X and K going forward to simplify the notations wherever it does not cause confusion.

Payout and Equity Issuance. The bank pays out dividends, i.e, $dU_t > 0$, only if the decrease of continuation value is equal to or less than the consumption value of dividends, $V(X, K) - V(X, K - dU_t) \leq dU_t$, i.e.,

$$V_K(X, K) \leq 1\,. \tag{12}$$

The optimality of payout also requires the following smooth-pasting condition:

$$V_{KK}(X, K) = 0\,. \tag{13}$$

Following Bolton, Chen, and Wang (2011), we assume that the bank incurs proportional and fixed costs of issuing equity. Let M_t denote the amount raised. $\psi_1 M_t$ is the proportional equity-issuance cost. We further assume that the fixed cost is linear in X_t, so that $\psi_0 X_t$ denotes the fixed equity-issuance cost. This makes the bank's problem homogeneous in X_t and, as will be shown shortly, significantly simplifies the solution. Moreover, as the bank grows geometrically with X_t, modeling the fixed cost as increasing in X_t avoids it becoming eventually negligible.

The bank raises equity, i.e., $dF_t > 0$, only if the increase of shareholders' value after issuance

[11] The bank may adjust the loan amount A_t by selling loans in the secondary market. The technological progress on the reduction of information asymmetries between loan buyers and loan sellers facilitate the trading of loans, and and the design of contract between the loan buyers and originators can alleviate the moral hazard problem (reduced monitoring incentive) on the part of loan originators (e.g., Pennacchi, 1988; Gorton and Pennacchi, 1995).

is equal to or greater than the cost,

$$V\left(X,K+dF_{t}\right)-V\left(X,K\right)\geq dF_{t}+dH_{t}=\psi_{0}X+\left(1+\psi_{1}\right)M_{t}\,. \tag{14}$$

where $dF_t = M_t$ is the capital raised and, as previously discussed, the issuance costs have a fixed and a proportional components, $dH_t = \psi_0 X + \psi_1 M_t$. The optimal amount of issuance is given by the following condition:

$$V_{K}\left(X,K+M_{t}\right)=1+\psi_{1}\,. \tag{15}$$

HJB Equation. Given the laws of motion (2) for X and (11) for K, in the interior region where $dU_t = 0$ and $dF_t = 0$, the bank's HJB equation is

$$\begin{aligned}\rho V\left(X,K\right)=\max_{\{\pi^{A},\pi^{R},i\}}\; & V_{X}\left(X,K\right)X\left[-\delta_{X}+n\left(i\right)\right]+\frac{1}{2}V_{XX}\left(X,K\right)X^{2}\sigma_{X}^{2} \\ & +V_{K}\left(X,K\right)\left(X+K\right)\left(r+\pi^{A}\alpha_{A}\right)+\frac{1}{2}V_{KK}\left(X,K\right)\left(X+K\right)^{2}\left(\pi^{A}\sigma_{A}\right)^{2} \\ & -V_{K}\left(X,K\right)\left[Xi+C\left(n\left(i\right),X\right)\right]+V_{XK}\left(X,K\right)X\left(X+K\right)\pi^{A}\sigma_{A}\sigma_{X}\phi\,.\end{aligned} \tag{16}$$

Lending. The first-order condition for π^A gives the following solution:

$$\pi^{A}=\frac{\alpha_{A}+\epsilon\left(X,K\right)\sigma_{A}\sigma_{X}\phi}{\gamma\left(X,K\right)\sigma_{A}^{2}\left(\frac{X+K}{K}\right)}\,, \tag{17}$$

where we define the endogenous risk aversion parameter based the value function

$$\gamma\left(X,K\right)\equiv\frac{-V_{KK}\left(X,K\right)K}{V_{K}\left(X,K\right)}\,, \tag{18}$$

and the elasticity of marginal value of capital, $V_K(X,K)$, to the stock of deposit liabilities

$$\epsilon\left(X,K\right)\equiv\frac{V_{XK}\left(X,K\right)X}{V_{K}\left(X,K\right)}\,. \tag{19}$$

Even though the bank evaluates the equityholders' payoffs with a risk-neutral objective in (4), it can be effectively risk-averse, i.e., $\gamma(X,K) > 0$, due to the equity issuance cost. When $\epsilon(X,K) > 0$, deposits and capital are complementary in creating value for banks' shareholders.

While setting up $\pi^A = A/(X+K)$ as the control variable is convenient for solving the model, it is intuitive to express the solution in loan-to-capital ratio, i.e., $A/K = \pi^A(X+K)/K$:

$$\frac{A}{K} = \frac{\alpha_A + \epsilon(X,K)\,\sigma_A\sigma_X\phi}{\gamma(X,K)\,\sigma_A^2}\,, \tag{20}$$

This solution resembles Merton's portfolio choice. In the numerator, a higher excess return, α_A, increases lending. The bank's incentive to lend is also strengthened when deposits are natural hedge – the asset-side shock, $d\mathcal{W}^A$, and the liability-side (deposit) shock, $d\mathcal{W}^A$ are positively correlated ($\phi > 0$) and more deposits make capital more valuable (i.e., $\epsilon(X,K) > 0$). [12].

Deposit Rate. The bank sets the deposit rate, i, to equate the marginal value of new deposits, $V_X(X,K)\,n'(i)\,X$, and the marginal costs from reducing the shareholders' profits (i.e., return on equity capital) by paying interests on the existing deposits, $V_K(X,K)\,X$, and by paying the costs of maintaining a larger deposit franchise, $V_K(X,K)\,C_n(n(i),X)\,n'(i)$:

$$V_X(X,K)\,n'(i)\,X = V_K(X,K)\left[X + C_n(n(i),X)\,n'(i)\right]. \tag{21}$$

4.2 Optimal Deposit Rate

We parameterize the deposit-flow function and the deposit cost function to obtain more intuitive solutions. First, we specify the deposit demand as a simple linear function of the deposit rate

$$n(i) = \omega i\,, \tag{22}$$

[12]While different in mechanism, this feature of our model echoes the literature on the synergy between lending and deposit-taking (see, e.g., Calomiris and Kahn, 1991; Berlin and Mester, 1999; Kashyap, Rajan, and Stein, 2002; Gatev and Strahan, 2006; Hanson, Shleifer, Stein, and Vishny, 2015). Our mechanism also echoes the recent finding of Drechsler et al. (2018a) that deposit liabilities serve as a natural hedge (against interest-rate risk) for long-term loans.

where, as shown in (2), ω is the semi-elasticity of deposits X_t with respect to i, which we will calibrate to the estimate from Drechsler, Savov, and Schnabl (2017) in our numeric solution. Next, we specify the deposit maintenance/adjustment cost as follows,

$$C(n(i), X) = \frac{\theta}{2} n(i)^2 X \,. \tag{23}$$

The cost is increasing in the existing amount of deposits, X_t, and is increasing and convex in the flow of new deposits $n(i)$, reflecting the increasing marginal cost of expanding the depositor base.

This functional form leads to a Hayashi style optimal policy of deposit rate. In Hayashi (1982), firms make investments in productive capital, while, in our model, the bank attracts depositors by raising the deposit rate, building up its customer capital. Using (21), we obtain

$$i = \frac{\frac{V_X(X,K)}{V_K(X,K)} - \frac{1}{\omega}}{\theta} \,. \tag{24}$$

Consistent with the evidence in Drechsler, Savov, and Schnabl (2017), the deposit rate is higher when the demand is more elastic, i.e., ω is high. The bank also sets a higher rate to attract more deposits when the marginal adjustment cost increases slowly, i.e., θ is low.

The bank sets a high deposit rate when the marginal value of deposits, $V_X(X, K)$, is high relative to the marginal value of equity capital, $V_K(X, K)$. Paying a higher deposit rate attracts more deposits but paying more interest expenses reduce earnings and equity. Section 3 presents the solution of the bank's problem without the equity issuance costs. In that case, the marginal value of equity is always equal to one and the marginal value of deposits is a constant Q. Therefore, the optimal rate is a constant:

$$i = \frac{Q - \frac{1}{\omega}}{\theta} \,. \tag{25}$$

In the presence of equity issuance cost, the optimal policy of deposit rate depends on X and K.[13]

[13] The difference between (24) and (25) is akin to the difference in a firm's optimal investment in Hayashi (1982) and Bolton, Chen, and Wang (2011). In Bolton, Chen, and Wang (2011), the cost of raising equity induces a state-dependent value of liquidity, so the ratio of marginal value of capital to that of liquidity drives the firm's investment.

An interesting feature of the optimal deposit rate is that it hits the zero lower bound when

$$\frac{V_X(X,K)}{V_K(X,K)} \le \frac{1}{\omega}\,. \tag{26}$$

Once the deposit rate reaches zero, the bank cannot further decrease the deposit rate to reduce deposits. Later we show that this restriction makes deposits undesirable, especially when the bank is undercapitalized, and thus, is concerned of a high leverage from large deposits that amplifies the impact of negative shocks on equity, increasing the likelihood of costly equity issuance.

When the deposit demand is more elastic, i.e., ω is high, the bank has to pay a higher deposit rate, as shown in (24), which turns to decrease the shareholders' value. However, given the value function, it is less likely for the condition (26) to hold, because a high demand elasticity allows the bank to control the deposit flow more effectively and thereby to avoid hitting the zero lower bound. This result suggests that the deposit-rate lower bound is more acute a problem for larger banks with greater deposit market power or stickier deposit base (i.e., smaller ω). Smaller banks with less deposit market power are less concerned of the deposit-rate lower bound, but they have to pay higher interest rates to attract depositors.

4.3 Optimal Risk-Taking

Given the functional forms of deposit flows and costs, the bank's problem is homogeneous in X and its value function $V(X,K) = v(k)\,X$, where

$$k = \frac{K}{X}\,, \tag{27}$$

Therefore, instead of working with X and K as the state variables, we will work with X and k. The capital-to-deposit ratio, k, captures the composition of *long-term* funding on the liability side of the bank's balance sheet. We will show that the choice variables are functions of k only.

Next, we simplify the expression of loan-to-capital ratio, a measure of the bank's risk-taking.

First, note that the expression of the effective risk aversion in (18) can be simplified to

$$\gamma(k) = \frac{-V_{KK}(X,K)\,K}{V_K(X,K)} = -\frac{v''(k)\,k}{v'(k)}\,. \tag{28}$$

And, the elasticity of marginal value of capital to deposits in is given by (19)

$$\epsilon(k) = \frac{V_{XK}(X,K)\,X}{V_K(X,K)} = -\frac{v''(k)\,k}{v'(k)}\,, \tag{29}$$

which happens to be equal to $\gamma(k)$.

Using $\epsilon(k) = \gamma(k)$, we simplify the optimal loan-to-capital ratio from (20):

$$\frac{A}{K} = \frac{\alpha_A}{\gamma(k)\,\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi\,, \tag{30}$$

The bank's risk-taking is state-dependent and only depends on k through $\gamma(k)$. When the effective risk aversion is low, the bank chooses a high loan-to-capital ratio; when the effective risk aversion is high, the bank reduces its risk exposure. In our numeric solution, we show that $\gamma(k)$ decreases in k, so the loan-to-capital ratio increases when the bank has a high equity buffer relative to its deposit liabilities. The correlation between the loan return shock and the deposit flow shock, ϕ, induces a hedging demand. The risk of deposit flow is essentially the bank's background risk from the perspective of portfolio management and a natural hedge when $\phi > 0$.

4.4 Solving the Value Function

Value Function ODE. To solve the bank's value function, we simplify the HJB equation to obtain an ordinary differential equation for $v(k)$. First, given that $V(X,K) = v(k)\,X$, we obtain

$$\begin{aligned} &V_K(X,K) = v'(k)\ ,\ V_X(X,K) = v(k) - v'(k)\,k \\ &V_{KK}(X,K) = v''(k)\,\frac{1}{X}\,,\ V_{XX}(X,K) = v''(k)\,\frac{k^2}{X}\,,\ V_{XK}(X,K) = -v''(k)\,\frac{k}{X}. \end{aligned} \tag{31}$$

Using these expressions, we can rewrite the HJB equation (16) as

$$\begin{aligned}\rho v\left(k\right) = & \max_{\pi^A, i} \ \left[v\left(k\right) - v'\left(k\right)k\right]\left(-\delta_X + \omega i\right) + \frac{1}{2}v''\left(k\right)k^2\sigma_X^2 \\ & + v'\left(k\right)\left(1+k\right)\left(r + \pi^A\alpha_A\right) + \frac{1}{2}v''\left(k\right)\left(1+k\right)^2\left(\pi^A\sigma_A\right)^2 \\ & - v'\left(k\right)\left[i + \frac{\theta}{2}\left(\omega i\right)^2\right] - v''\left(k\right)k\left(1+k\right)\pi^A\sigma_A\sigma_X\phi\,. \end{aligned} \tag{32}$$

To show that (32) is an ODE for $v\left(k\right)$, we need to show that the control variables only depend on k and the level and derivatives of $v\left(k\right)$. First, by definition, $\pi^A = A/\left(X+K\right)$, so we obtain the following simplified expression for π^A from (30):

$$\pi^A = \left(\frac{A}{K}\right)\left(\frac{K}{K+X}\right) = \left(\frac{\alpha_A}{\gamma\left(k\right)\sigma_A^2} + \frac{\sigma_X}{\sigma_A}\phi\right)\left(\frac{k}{1+k}\right). \tag{33}$$

The optimal deposit rate given by (24) only depends on $V_X\left(X,K\right) = v\left(k\right) - v'\left(k\right)k$ and $V_K\left(X,K\right) = v'\left(k\right)$. Then we can substitute these optimal choices into the HJB equation to obtain an second-order ODE for $v\left(k\right)$ that contains only k and the level and derivatives of $v\left(k\right)$. Fully solving the model then takes two steps, first, solving the ODE to obtain $v\left(k\right)$, and second, using the solved $v\left(k\right)$ and its derivatives to solve the bank's optimal choices.

Boundary Conditions. Let $\overline{k}$ and $\underline{k}$ denote respectively the dividend payout and issuance boundaries, and let $m \equiv M/X$ denote the amount financing raised via issuance (scaled by X). The boundary conditions implied the optimality condition on payout (12) and (13) are

$$v'\left(\overline{k}\right) = 1\,, \tag{34}$$

and the smooth-pasting condition,

$$v''\left(\overline{k}\right) = 0\,. \tag{35}$$

The boundary conditions implied by the optimality condition on issuance (14) and (15) are

$$v\left(\underline{k}+m\right)-v\left(\underline{k}\right)=\psi_0+\left(1+\psi_1\right)m\,, \tag{36}$$

and

$$v'\left(\underline{k}+m\right)=1+\psi_1\,. \tag{37}$$

Our numerical solution of $v(k)$ features global concavity, so (34) and (37) imply that $\overline{k}>\underline{k}$. Given $\underline{k}$, the four boundary conditions above solve the second-order ODE for $v(k)$ (i.e., the HJB equation), the upper boundary $\overline{k}$, and the endogenous issuance amount m. However, we still need one condition to pin down $\underline{k}$. In our numerical solution, $v(k)$ is globally concave, so $\underline{k}=0$, i.e., the bank does not pay the issuance costs unless its capital drops to zero. However, in the presence of leverage ratio requirement, $\underline{k}$ must be positive. In our numerical solution, when k is small, $B<0$ so the leverage ratio constraint implies that $(K+X)/K\leq\xi_L$, i.e.,

$$k\geq\underline{k}=\frac{1}{1-\xi_L^{-1}}-1\,. \tag{38}$$

Therefore, when k is small (and $B<0$), SLR requirement becomes a lower bound on the state variable and triggers equity issuance. When k is large, the capital requirement binds before the SLR requirement (due to $\xi_L>\xi_K$) and restricts the control variable $A/K=\pi^A\left(\frac{1+k}{k}\right)\leq\xi_K$. We highlight that the newly introduced SLR requirement is more effective a tool to motivate bank recapitalization (that happens in bad times, i.e., when k is low), while the traditional capital requirement is a restriction on risk-taking (and binds in good times, i.e., when k is high).

5 Quantitative Analysis

5.1 Parameter Choices

We set the unit of time to one year and choose r to be equal to 1% in line with the average of the Fed funds rate in the last decade. Shareholders' discount rate ρ is set to 4.5% in line with the

Table 1: PARAMETER VALUES

This table summarizes the parameter values for our baseline analysis. One unit of time is one year.

Parameters	Symbol	Value
risk-free rate	r	1%
discount rate	ρ	4.5%
bank excess return	α_A	0.2%
asset return volatility	σ_A	10%
deposit flow (mean)	δ_X	0
deposit flow (volatility)	σ_X	5%
deposit maintenance cost	θ	0.007
deposit demand semi-elasticity	ω	5.3
corr. between deposit and asset shocks	ϕ	0.8
equity issuance fixed cost	ψ_0	0.1%
equity issuance propositional cost	ψ_1	5.0%
SLR requirement parameter	ξ_L	20
capital requirement parameter	ξ_K	14.3

commonly used discount rate in dynamic corporate finance models (e.g., Bolton, Chen, and Wang, 2011).[14]. We set α_A to 0.2%, so that the model generates an average return on assets (ROA) equal to 1.05%, approximately the average of US banks in the last decade (source: FRED). Note that when k is large, the bank only holds risky assets, but when k is small, the bank holds risk-free assets ($B < 0$). Therefore, the ROA is state-dependent. To calculate the average value of ROA (and other metrics later), we use the stationary distribution of k. We set asset-value volatility, σ_A, to 10% following Sundaresan and Wang (2014) and Hugonnier and Morellec (2017) who in turn refer to the calculation of Moody's KMV Investor Service.

For the deposit dynamics, we set δ_X to 0% and σ_X to 5% following Bianchi and Bigio (2014). We further set ω, the semi-elasticity of deposits to the deposit rate, to 5.3, which is an estimate from Drechsler, Savov, and Schnabl (2017). The correlation between asset-side shock and liability-side (deposit) shock is set to 0.8. Under this value, the model generates an average deposit-to-total

[14]This is also consistent with the dynamic contracting literature (e.g., DeMarzo and Fishman, 2006; DeMarzo and Sannikov, 2006; Biais, Mariotti, Plantin, and Rochet, 2007; DeMarzo and Fishman, 2007).

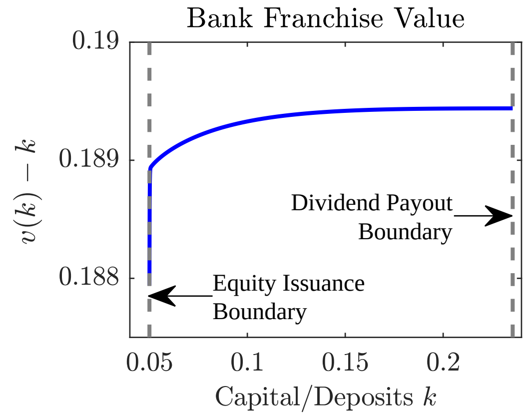

Figure 1: Bank Franchise Value

liabilities ratio equal to 96% in line with the evidence (Drechsler, Savov, and Schnabl, 2017). As for the cost of maintaining deposit franchise, we set θ, the parameter in the cost function, to 0.007, so that the model generates an average deposit rate equal to 0.07%, approximately the average rate of checking deposits in the recent decade (source: FRED). We set the proportional issuance cost is 5% according to (Boyson, Fahlenbrach, and Stulz, 2016). The fixed cost is set to 0.1%. Under this value, the model generates an issuance-to-equity ratio of 1% in line with the evidence (Baron, 2020). The regulatory parameters were discussed in Section 2.

5.2 Bank Franchise Value

In the frictionless world of Modigliani and Miller (1958) the bank shareholders' value V_t is equal to book equity, K_t. In our model, two forces creates a wedge, giving rise to a strictly positive bank franchise value $(V_t - K_t)$. First, as shown in Section 3, deposit-taking can create value for shareholders. Even if the bank is subject to a deposit maintenance cost, it can adjust the deposit rate to exploit the downward-sloping demand for deposits and make a profit. Second, the equity issuance cost contributes to the wedge. The bank has to maintain a positive level of profits, the present value of which allows it to raise equity financing net of issuing costs. In Figure 1, we plot the franchise value, $(V_t - K_t)/X_t = v(k) - k$, against the key state variable k (the deposit-to-

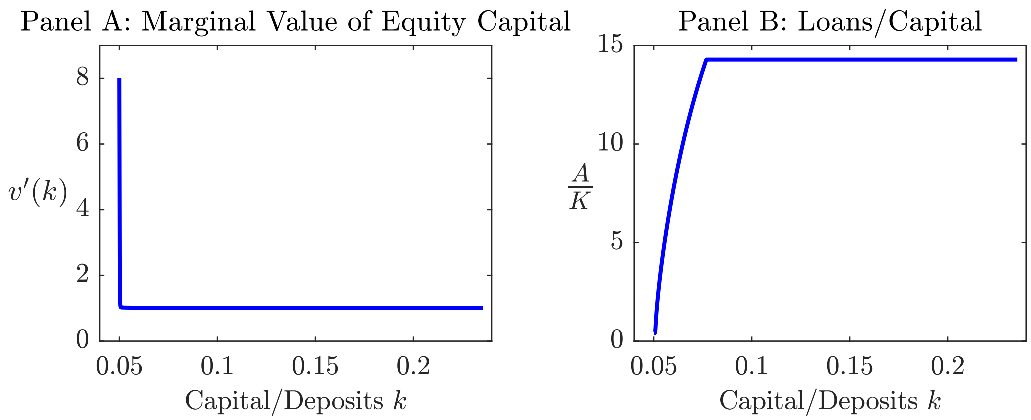

Figure 2: Marginal Value of Equity Capital and Bank Risk-Taking.

capital ratio). The franchise value increases when capital accumulates relative to deposits, which is consistent with the evidence from Minton, Stulz, and Taboada (2019).

At the lower boundary $\underline{k}$, the bank is required to raise new equity and then must pay an issuance cost. The further away k is from this issuance boundary, the lower the likelihood of hitting the boundary and paying an issuance cost. Therefore, the franchise value $(V_t - K_t)/X_t$ increases in k. The interior region ends at the upper (dividend payout) boundary $\overline{k}$. At that point the bank has sufficient retained earnings, so that it is safe to pay out dividends to impatient shareholders. Note that near $\overline{k}$ the franchise value is flat and relatively insensitive to small variations in k because, at that point, the likelihood of a large loss of equity or a large deposit inflow that dramatically decrease k and force the bank to raise costly new equity is low.

These results have several implications on the empirical analysis of bank valuation (e.g., Atkeson, d'Avernas, Eisfeldt, and Weill, 2019; Minton, Stulz, and Taboada, 2019). First, the deposit stock is a more natural normalization than book equity because of homogeneity property of the bank's problem. Second, shareholder value (scaled by deposits) increases in the capital-to-deposit ratio, reaching its highest level when the bank pays out dividends, and falling to its lowest level when the bank raises equity. This is consistent with the evidence that payout is procyclical while equity issuance is countercyclical (Adrian, Boyarchenko, and Shin, 2015; Baron, 2020).

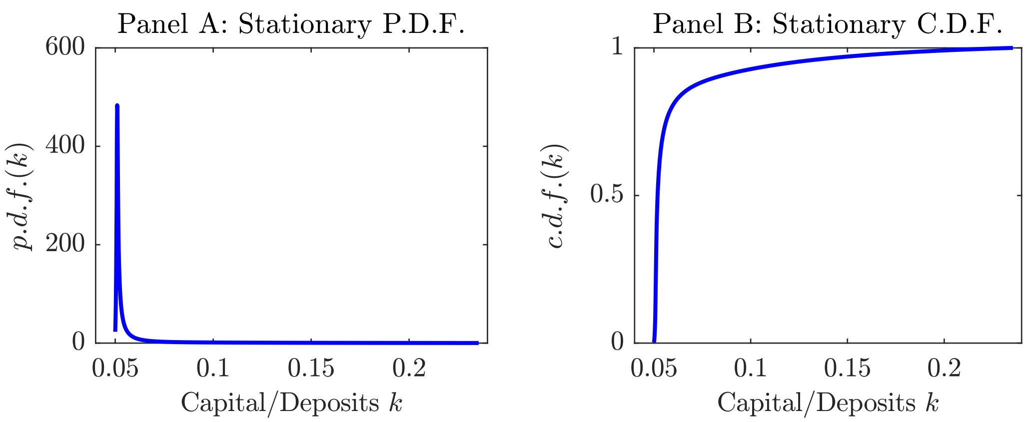

Figure 3: Stationary Probability Density and Cumulative Distribution Function.

5.3 Marginal Value of Equity Capital and Risk-Taking

Panel A of Figure 2 plots the marginal value of bank capital, $V_K(K, X) = v'(k)$. Without financial frictions (costly equity issuance) this variable should always be equal to one. When it is costly to issue equity, however, the marginal value of equity capital rises sharply above one near the issuance boundary. At $\underline{k}$, a value of $v'(k)$ close to eight means that one dollar of equity is worth eight dollars because of the imminence of costly equity issuance.[15]

In Panel B of Figure 2, we describe the bank's optimal risk-taking behavior. Panel A plots the target ratio of loan value to capital, A_t/K_t, given by (30). The bank obviously cannot exceed the regulatory capital requirement (i.e., $A/K \leq \xi_K = 14.3$), but it can expand its balance sheet up to that target. Risk-taking is procyclical. As capital accumulates relative to deposits (as k increases), the bank expands its balance sheet, financing the expansion through deposits and wholesale (short-term bond) funding. But when capital is depleted relative to deposits, the bank deleverages. This is consistent with the findings of Ben-David, Palvia, and Stulz (2020) that distressed banks deleverage and decrease observable measures of riskiness. A natural measure of risk-taking incentives in our model is $\gamma(k)$ the endogenous relative risk aversion coefficient of the bank defined in (18). $\gamma(k)$

[15]Note that even though the proportional cost of issuance is only 5%, due to the fixed cost, the value of one dollar equity can be much higher than 1.05.

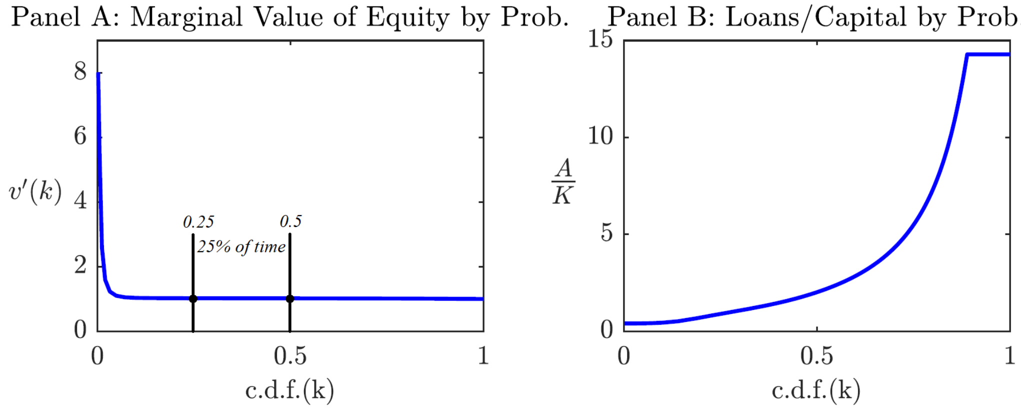

Figure 4: The Long Run Distribution of Marginal Value of Equity Capital and Bank Risk-Taking.

decreases in k because, as shown in Panel A of Figure 2, the value function is extremely concave near the equity issuance (lower) boundary $\underline{k}$ but, as k increases, the concavity subdues quickly.

Panel A of Figure 3 plots the stationary probability density of the state variable k, and Panel B plots the corresponding cumulative probability function. It shows how much time the bank spends in various regions of k after the loan return and deposit flow shocks are realized over the long run. Note how the probability mass is highly concentrated in the area where k is close to the lower boundary k, but still large enough that the marginal value of equity at that point is low. In fact, the marginal value of equity is 1.02 where the density function peaks. Importantly, even if for the majority of time the bank does not seem to be financially constrained, the shadow value of equity rises dramatically when equity is depleted to the point where the bank may be forced into a costly equity issuance, as shown in Panel A of Figure 2. These results illustrate the sharp contrast between normal times, when the bank is comfortably meeting its leverage requirements, and crisis times, when it is in danger of violating its leverage requirements and triggering equity issuance.

In Panel A of Figure 4, we plot the marginal value of equity against the cumulative distribution function (c.d.f.) of the stationary distribution of k (with $c.d.f.\left(\underline{k}\right) = 0$ and $c.d.f.\left(\overline{k}\right) = 1$). In the graph, the horizontal span represents the amount of time the bank spends in a region on the long run. We illustrate that the bank spends 25% of the time with its marginal value of equity between 1.019 and 1.022. The bank spends less than 5% of the time in the region where it is in danger

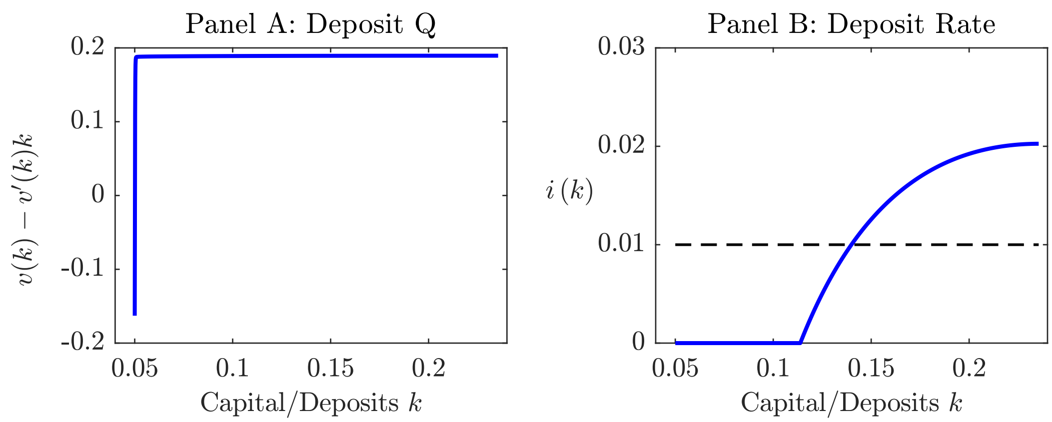

Figure 5: Marginal Value of Deposits and Deposit Rate.

of violating the leverage requirement with a marginal value of equity is above 1.022. In other words, crisis states are rare but they cast a long shadow over the bank's management of its balance sheet. As the bank becomes better capitalized relative to its deposit liabilities (i.e., k increases), the marginal value of equity declines dramatically, so that the bank's value $v(k)$ is concave and the bank is effectively risk averse. In Panel B of Figure 4, we plot the loan-to-capital ratio, A_t/K_t, against the c.d.f. of k, and show that around 10% of the time, the capital requirement binds.

5.4 The Value of Deposits

Panel A of Figure 5 plots the marginal value of deposits, i.e., $V_X(X, K) = v(k) - v'(k)k$, which we call the Deposit Q. When the bank has ample capital relative to deposits, i.e., k is large, the deposit Q is positive. However, it turns sharply negative when k nears the lower boundary of costly equity issuance. In Panel B of Figure 5, we plot the optimal deposit rate and compare it with the risk-free rate. When the bank has sufficient capital to buffer risk, it is willing to set a deposit rate that is above $r = 1\%$, i.e., the interest rate it pays to borrow in short-term debt. The bank sacrifices return on equity but, through a high deposit rate, gains a greater depositor base, which then can help the bank borrow at a deposit rate below $r = 1\%$ when its capital becomes lower (i.e., k decreases).

Fundamentally, deposits create value by allowing the bank finance risky lending with relative

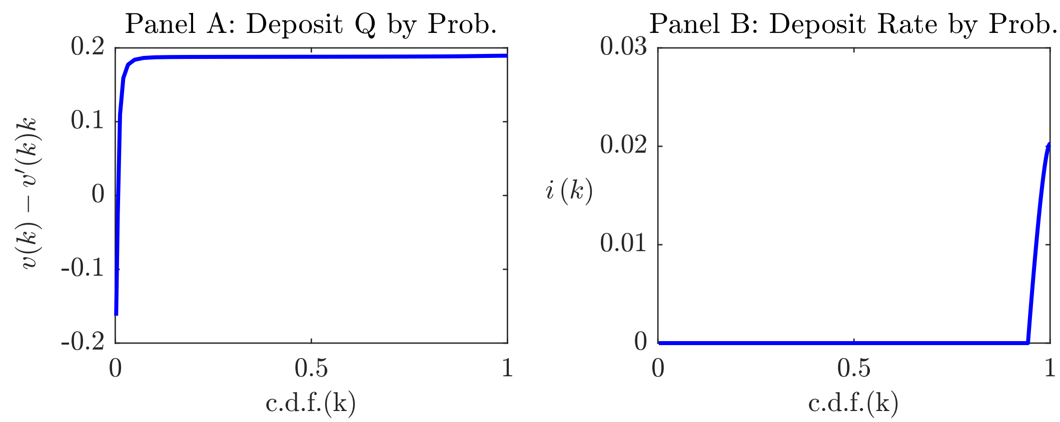

Figure 6: The Long Run Distribution of Marginal Value of Deposits and Deposit Rate.

cheap sources of funds. The deposit stock serves as a form of productive capital for the bank. The comovement of loan growth (Panel B of Figure 2) and deposit rate increase (Panel B of Figure 5) is consistent with the finding of Ben-David, Palvia, and Spatt (2017).

A key finding is that the deposit Q can turn negative when a bank's capital is low relative to its deposit liabilities. The reason is that when k is near the equity issuance boundary, deposits destroy value for the bank's shareholders by forcing the bank to sustain a high level of leverage that amplifies the impact of shocks on bank capital and makes it more likely to incur costly equity issuance. The bank may want to delever, turning away deposits by lowering the deposit rate. However, as shown by Panel B of Figure 5, doing so has a limit, that is the zero lower bound of deposit rate. Setting a deposit rate below zero causes depositors to withdraw deposits en masse and hoard dollar bills (which has a zero return). While we do not explicitly model the consequence of a run, the value destroyed through liquidation of loans and fire sale is likely to make the zero lower bound a binding constraint for the bank.

In Figure 6, we plot the deposit Q and deposit rate against the stationary c.d.f. of k. The deposit Q is positive, around 0.188 in 98.8% of the time, but near the issuance boundary (i.e., $c.d.f.(\underline{k}) = 0$), it can drop to -0.163. The deposit rate lower bound binds 94.4% of time.

Deposits are very different from short-term debts. For short-term debts, the bank can always

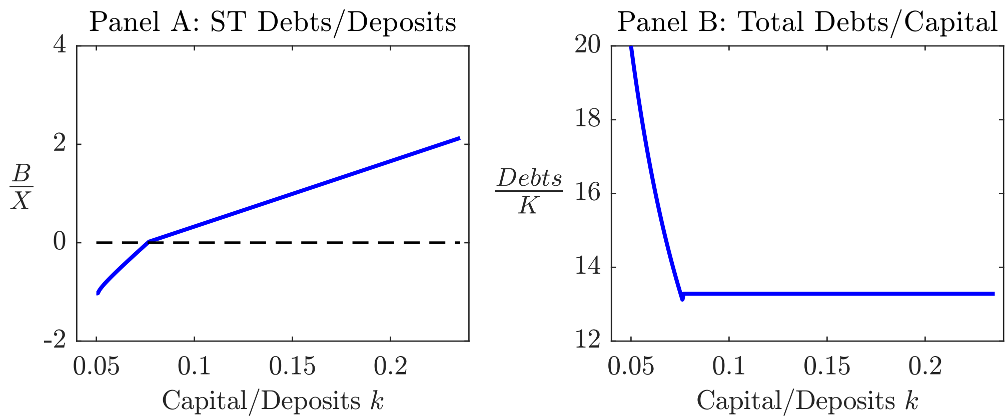

Figure 7: Short-Term Debt and Total Leverage

choose to stop borrowing at maturity, and therefore, does not face the problem of unwanted debts. However, deposit contracts do not have maturity. Deposits leave the bank only when depositors withdraw dollar bills or make payments to those who do not hold accounts at the bank. As long as depositors are willing to hold deposits, the bank cannot turn away the existing depositors. After hitting the zero lower bound, the bank loses control of its leverage, and when the bank is sufficiently close to incur costly equity issuance, the marginal value of deposits is negative.

5.5 Short-Term Borrowing vs. Safe Asset Demand

Figure 7 analyzes the the bank's debt structure. Panel A plots the ratio of short-term debts to deposits. When capital is abundant relative to deposits, the bank raises funds from short-term debts for risky lending, i.e., $B_t > 0$. In Panel B, we plot the total leverage. The capital requirement limits the loan-to-capital ratio, so, as the bank issues more short-term debts when k increases, the total leverage increases initially and quickly reaches the regulatory limit. Once the capital requirement binds, a further increase of k induces a substitution from deposits to short-term debts.

When the bank's equity capital is scarce relative to its deposit liabilities, the bank switches its short-term debt position, holding risk-free debts to reduce the overall riskiness of its asset portfolio. When k is small and the deposit rate reaches the lower bound, the bank loses control of its leverage, and therefore, has to work on the asset-side of its balance sheet to de-risk (and to reduce the likelihood of costly equity issuance) by holding risk-free assets, i.e., $B_t < 0$.

As shown in Panel B of Figure 7, once the bank has stopped borrowing short-term debts and deposits become the only type of debts, a further decline of $k = K/X$ induces a lock-step increase of leverage X/K that resembles what the large U.S. banks have gone through during the Covid-19 crisis. In fact, Federal Reserve Board temporarily relaxes the supplementary leverage ratio requirement in April 2020 to alleviate banks' stress, precisely in response to the influx of deposits. Deposit inflows push banks towards costly equity issuance by reducing k, and close to the issuance boundary, $\underline{k}$, the deposits become burdensome and have a negative marginal value.

Capital requirement and leverage regulation play distinct roles in our model. The former is defined on the ratio of risky lending to equity capital, which translates into a constraint on the bank's control variable, A_t/K_t (see (30)). Instead of mandating a loss buffer (as the model does not feature bankruptcy), capital requirement acts as a macro-prudential regulation that limits risk-taking when the bank becomes better capitalized relative to its deposit liabilities, i.e., k increases. In contrast, the supplementary leverage ratio requirement (SLR) is a constraint on the bank's state variable, k, because when k is small, deposits are the bank's only liabilities and k is the total debt-to-equity ratio. As shown in (38), it is the SLR that triggers equity issuance.

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A Solving the Model without Equity Issuance Costs

By inspecting the HJB equation (16), we know that when the value function is linear in K, the bank maximizes π^A (and borrow short-term debts) as long as $\alpha_A > 0$. Therefore, A is the bank's total assets and the capital requirement and SLR restriction are both a restriction on A/K. Because $\xi_L > \xi_K$, capital requirement binds, i.e., $A/K = \xi_K$ (or $\pi_A\left(1+\frac{1}{k}\right) = \xi_K$). The optimal deposit rate is given by (24):

$$i = \frac{Q - \frac{1}{\omega}}{\theta}\,. \tag{39}$$

Next we solve Q using the HJB equation (16) under the conjecture of value function $(k+Q)\,X$:

$$\rho k + \rho Q = Q\left(-\delta_X + \omega i\right) + r + rk + \xi_K \alpha_A k - \left[i + \frac{\theta}{2}\left(\omega i\right)^2\right].$$

For this equation to hold, we need the following quadratic equation to hold

$$\frac{\omega}{\theta}\left(1 - \frac{\omega}{2}\right)Q^2 - \left(\frac{2}{\theta} + \delta_X + \rho\right)Q + r + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta} = 0\,, \tag{40}$$

which solves Q, and the coefficient on k is equal to zero, i.e.,

$$\rho = r + \xi_K \alpha_A\,. \tag{41}$$

Equation (41) requires that the bank is indifferent between paying out dividend and retaining equity. If $\rho > r + \xi_K \alpha$, the bank prefers paying out dividends because the expected return on equity capital is below the shareholders' required rate of return. If $\rho < r + \xi_K \alpha$, the bank never pays dividend and prefers to raise an infinite amount of equity because the expected return on equity is greater than the shareholders' required return.

Under the condition,

$$\left(\frac{2}{\theta} + \delta_X + \rho\right)^2 \geq \frac{4\omega}{\theta}\left(1 - \frac{\omega}{2}\right)\left[r + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta}\right], \tag{42}$$

the roots of Equation (40) exist and are given by

$$Q = \frac{\left(\frac{2}{\theta} + \delta_X + \rho\right) \pm \sqrt{\left(\frac{2}{\theta} + \delta_X + \rho\right)^2 - \frac{4\omega}{\theta}\left(1 - \frac{\omega}{2}\right)\left[r + \left(\frac{1}{\omega} + \frac{1}{2}\right)\frac{1}{\theta}\right]}}{\frac{2\omega}{\theta}\left(1 - \frac{\omega}{2}\right)}\,. \tag{43}$$