Fragile New Economy: Intangible Capital, Corporate Savings Glut, and Financial Instability*

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Abstract

The transition towards an intangible-intensive economy reshapes financial system by creating a self-perpetuating savings glut in the production sector. As intangibles become increasingly important, firms hoard liquidity to finance investment in intangibles of limited pledgeability. Firms' savings feed cheap leverage to financial intermediaries and allow intermediaries to bid up asset prices, which in turn encourages firms to save more for asset creation. This paper develops a macro-finance model that offers a coherent account of the rising corporate savings, debt-fueled growth of intermediaries, declining interest rates, and rising asset valuation. Along these secular trends, endogenous financial risk accumulates.

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1 Introduction

The development of financial markets and institutions has profound impact on industrial structure (Rajan and Zingales, 1998). Is the reverse true? Can the evolution of industrial structure shape the financial system? In this paper, I examine the transition towards an intangible-intensive economy. In the U.S., investment in intangibles has overtaken physical investment as the largest source of economic growth (Corrado and Hulten, 2010). By incorporating a defining feature of intangibles—limited pledgeability—in a dynamic model of macroeconomy with financial markets and intermediaries, I show that the rise of intangibles contributes to several secular trends in the U.S. economy, such as the accumulation of corporate savings, the downward trend in interest rates, the growth of financial intermediation sector, and the rising valuation in asset markets. Importantly, by connecting these secular trends through the rise of intangibles, my model reveals a mechanism of endogenous risk that makes the new economy financially fragile.

U.S. nonfinancial corporations have accumulated a substantial amount of cash (Bates, Kahle, and Stulz, 2009; Chen, Karabarbounis, and Neiman, 2017; Gao, Whited, and Zhang, 2020) and turned from a net borrower to a net saver (Quadrini, 2017). The connection between intangibles and corporate savings is intuitive: To finance investment in intangibles of limited pledgeability, firms cannot rely on external financing and must hold internal funds (Pinkowitz, Stulz, and Williamson, 2015; Falato, Kadyrzhanovaz, Sim, and Steri, 2018; Begenau and Palazzo, 2021).

The first innovation of this paper is to connect firms' intangible-driven demand for liquid assets to the secular decline in interest rates. Some have suggested a link between the demand for liquid assets and low interest rates (Del Negro, Giannone, Giannoni, and Tambalotti, 2017). The focus has been on foreign savings (Caballero, Farhi, and Gourinchas, 2008; Caballero and Krishnamurthy, 2009; Gourinchas and Rey, 2016). Domestic corporate savings, which are comparable in magnitude, received little attention in the literature on low interest rates.[1]

The second innovation and a distinguishing feature of my model is a general equilibrium analysis of liquid assets. What firms hold as cash are mainly deposits and other debt instruments

[1] The ratio of nonfinancial firms' liquidity holdings to foreigners' holdings has been stable since the 1990s, around 75%. Liquid assets include currency and deposits, open market papers, and repurchase agreements held directly or indirectly via mutual funds (source: Financial Accounts of the U.S.).

issued by financial intermediaries. In the decades leading up to the Great Recession, debt issuance fueled growth of the intermediation sector (Adrian and Shin, 2010; Greenwood and Scharfstein, 2013; Pozsar, 2014). Taking advantage of the low interest rate, intermediaries are able to lever up cheaply and drive up the prices of collateral assets that can back debt issuance.

In the model, these trends arise in response to an exogenous increase in intangible investment needs. To finance intangibles, firms hold savings in the form of intermediaries' debts. Intermediaries' debts are in turn backed by claims on firms' tangible capital. As firms' savings push down the interest rate, intermediaries can borrow cheaply and bid up the value of tangible capital. Tangible capital can be pledged for external financing, so a higher value of tangible capital allows firms to lever up savings for larger and more profitable investments. As a result, firms are more eager to save and the interest rate declines more, encouraging intermediaries to borrow more and to further bid up the value of tangible capital. A self-perpetuating savings glut pushes down the interest rate and pushes up the asset (tangible capital) price, allowing intermediaries to grow in the process.

This feedback mechanism also generates endogenous risk. Unlike intermediaries that play the roles as suppliers of liquid assets and hold tangible capital to back their debts, households have higher funding costs and are only willing to pay a lower price for tangible capital. As tangible capital value increases and firms accumulate savings in booms, the interest rate on liquid assets goes down, giving intermediaries an increasingly large advantage in funding cost. The longer booms last, the wider the funding-cost wedge is between intermediaries and households. When negative shocks hit and intermediaries deleverage, the reallocation of tangible capital from intermediaries to households causes a collapse of the market value of tangible capital, which discourages firms from saving for investment and exacerbates intermediaries' deleveraging. This channel, based on investment-driven demand for liquid assets, differs from the standard balance sheet channel. It offers a new explanation on why severe crises follow prolonged booms.[2]

In the model, firms' investment is financially constrained and internal funds are necessary due to the intangible component that has limited pledgeability. The tangible component makes

[2]Studies on endogenous risk accumulation focus on intermediaries as lenders rather than issuers of liquid assets (Jordà, Schularick, and Taylor, 2013; Gorton and Ordoñez, 2014; Krishnamurthy and Muir, 2016; Baron and Xiong, 2017; López-Salido, Stein, and Zakrajšek, 2017; Gorton and Ordoñez, 2020).

available external financing and a leverage on internal funds, but its endogenous market value triggers feedback effects. Importantly, when the value of tangible capital increases, firms increase savings. I provide evidence on this feature of corporate savings. In contrast, households' holdings of liquid assets decline when asset prices rise. This paper highlights the importance of firms' liquidity demand for understanding asset prices, interest rates, and financial stability. The macro-finance literature focused on households' demand for liquid assets (Kiyotaki and Moore, 2000; Stein, 2012; Moreira and Savov, 2017; Krishnamurthy and Vissing-Jørgensen, 2015; Piazzesi and Schneider, 2016; Van den Heuvel, 2018; Begenau, 2019; Begenau and Landvoigt, 2018).

Next, I provide an overview of the model and more details on the mechanism and results. The continuous-time economy has entrepreneurs, bankers, and households. Their roles are discussed sequentially. A unit mass of infinitely-lived entrepreneurs manage tangible and intangible capital to produce non-durable generic goods. Capital represents efficiency units and its output is normalized to one unit of goods per unit of time. Capital depreciates stochastically, loading on an aggregate Brownian shock. A negative shock reduces capital stocks that represent the production capacity in the economy. In spite of these common features, tangible and intangible capital differ in liquidity.

As in Holmström and Tirole (1998), entrepreneurs face liquidity shocks. Idiosyncratic Poisson shocks entail a restart of business – a firm's existing capital is destroyed, but it may create new capital. The entrepreneur chooses the amount of goods to invest (scale) and the intangible share of investment (composition). To finance the investment, the entrepreneur can sell the ownership of tangible capital at the market price and commit to dutifully managing the capital on behalf of buyers, delivering goods it produces. In other words, tangible capital is liquid (tradable and pledgeable). In contrast, intangible capital is not tradable or pledgeable, representing technological, human, and organizational capital that are inalienable or difficult for creditors to repossess.

The illiquidity of intangible capital tightens the funding constraint on investment. Investing in tangible capital relaxes the constraint, but intangible investment can be sufficiently productive such that entrepreneurs optimally choose a positive intangible share. Importantly, the productivity of intangible investment increases over time. This captures technological changes. And, as capital is essentially a stream of future consumption units, the fact that intangible investment creates increasingly more capital (production units) also captures the shift of consumers' preference towards output generated by intangibles. For example, the share of expenditure on services has been growing in the U.S., and intangibles are the key factor input in the sector (McGrattan, 2020).[3]

The funding constraint implies that entrepreneurs want to hold liquidity and finance investment with a combination of internal funds and external funds (raised against tangible capital). One solution of liquidity provision, in the spirit of Holmström and Tirole (1998), is to pool all entrepreneurs' tangible capital—the source of capitalizable output—into a mutual fund whose shares are distributed back to entrepreneurs. The fund diversifies away the idiosyncratic Poisson shocks, so when the shock hits an individual entrepreneur, her fund shares are still valuable and can be used to finance investment, even though her own capital is destroyed.

However, such diversification services require expertise. In reality, firms mainly hold money-market instruments issued by financial intermediaries in their portfolios of "cash and cash equivalents". A unit mass of infinitely-lived bankers are introduced to intermediate the supply of liquidity.

Bankers buy tangible capital with their own wealth (equity) and by issuing short-term safe debts ("deposits") that entrepreneurs hold as liquidity buffers. Bankers create value not as lenders (their typical roles in macro-finance models) but instead as the issuers of liquid assets. The model highlights bankers' role in addressing asset shortages (Caballero, 2006; Caballero, Farhi, and Gourinchas, 2017b). Entrepreneurs assign a liquidity premium to deposits, which is equal to the marginal value of liquidity due to the Poisson-arriving investment needs (Holmström and Tirole, 2001). This liquidity premium lowers the deposit rate, encouraging bankers to expand their balance sheets. However, acquiring tangible capital and issuing safe deposits involve risk-taking, so bankers' capacity to intermediate the liquidity supply depends on their wealth as the risk buffer.

Finally, households are introduced, competing with entrepreneurs to hold deposits. Following the literature, households' demand is from deposit-in-utility, motivated by the roles of deposits as means of payment. Households can also own tangible capital, but relative to bankers, they can-

[3]Two channels have been proposed to explain the growing demand of services. First, income growth, under non-homothetic preferences, makes the services sector grow faster than the rest of the economy (Kongsamut, Rebelo, and Xie, 2001; Herrendorf, Rogerson, and Ákos Valentinyi, 2013). Second, productivity growth is biased. Labor-intensive sectors benefit less from technological progress, so the relative prices of their output increase over time relative to other products, forcing an increasingly large share of consumer expenditure (Baumol, 1967; Ngai and Pissarides, 2007).

not earn the liquidity premium by issuing deposits so they face a higher funding cost and thereby require a higher expected return for holding tangible capital.

The exogenous process of intangible investment productivity and other parameters of entrepreneurs' investment technology are calibrated to match the trends and cyclical fluctuations of intangible investment and tangible investment. The arrival rate of the Poisson shock is calibrated to generate a positive response of entrepreneurs' liquidity holdings to intangible investment needs that matches the empirical estimate. The model features both firms' and households' liquidity demand, and one of the main contributions is to evaluate their relative importance in driving interest rates, asset prices, and endogenous financial risk through counterfactual analysis. Therefore, it is important to generate realistic dynamics of both firms' (i.e., entrepreneurs') and households' liquidity holdings in the baseline model. For this purpose, an exogenous trend is introduced in households' deposit-in-utility, and it is calibrated so that the magnitude of households' liquidity holdings, especially relative to those of entrepreneurs', matches data. The calibration exercise targets the evolution of quantity variables, such as investment and liquidity holdings. For price variables, such as the interest (deposit) rate and tangible capital value, the calibration exercise only targets the values at the beginning of the sample period and leaves the trends to endogenous forces.

In response to the exogenous increase in intangible investment productivity, the model generates upward trends in the intangible share of investment, entrepreneurs' liquidity holdings (and bankers' debt issuances), and tangible capital value and a downward trend in the deposit rate. To address the rising needs for intangible investment, entrepreneurs hold more deposits and push down the deposit rate, feeding bankers with cheap funding and allowing them to bid up the market value of tangible capital. A higher value of tangible capital allows entrepreneurs to lever up their liquidity holdings to larger and more profitable investment. Therefore, entrepreneurs' incentive to save is strengthened, and the deposit rate declines further. The self-enforcing mechanism successfully replicates these secular trends except that for the interest rate, it delivers a stronger downward trend into the negative territory likely due to the lack of nominal frictions and zero lower bound. Note that the feedback effects can be so strong that equilibrium multiplicity arises, in which case, the equilibrium with intangible share of investment closest to data is selected. The multiplicity is interesting by itself as it offers a potential explanation for why the rise of intangibles and the associated secular trends are more prominent in the U.S. than the rest of the world.

The feedback mechanism also amplify economic fluctuations along the trends. Endogenous financial risk accumulates after positive shocks and materializes into a downward spiral when negative shocks hit. Consider a positive shock to capital stocks. Given bankers' levered positions in tangible capital, their wealth increases significantly. The liquidity premium on deposits makes bankers' marginal costs of financing (and discount rates) lower than households'. Therefore, when bankers—the natural buyers of tangible capital—become richer, their demand drives up the market value of tangible capital, which in turn leads to a higher leverage on entrepreneurs' deposits and higher investment profits. So, entrepreneurs save more, driving down the deposit rate and bankers' discount rate, further widening the discount-rate gap between bankers and households. This makes the value of tangible capital increasingly sensitive to negative shocks that trigger reallocation of tangible capital away from the natural buyers (bankers) to households and back to entrepreneurs. Asset price volatility affects the real economy. The value of tangible capital falls significantly after negative shocks, reducing entrepreneurs' leverage on deposit holdings and their investments. By reducing bankers' wealth, the decline of tangible capital value also causes bankers to shrink balance sheets, so entrepreneurs hold fewer deposits and their investments decline further.

I construct counterfactual scenarios to highlight the quantitative importance of the rise of intangibles. In the first scenario, the trend in intangible investment productivity is muted, so parameters governing entrepreneurs' liquidity demand are fixed in the 1980s while households' liquidity demand exhibited an upward trend. In the second scenario, the trend in households' liquidity demand is shut down while the upward trend in intangible investment productivity is preserved. For interest rate and asset price (i.e., tangible capital value), these two scenarios generate weaker trends than the main model with upward trends in both entrepreneurs and households' liquidity demand.

However, when it comes to endogenous risk, the scenario with an upward trend in intangible investment productivity but no trend in households' liquidity demand dominates the main model. The reason is that entrepreneurs' incentive to save comoves with asset price (tangible capital value) as a higher tangible capital value allows entrepreneurs to lever up their savings for larger and more profitable investments. I provide evidence on such dynamics. In contrast, households' liquidity demand exhibits countercyclicality in both the model and data. In the main model, households' liquidity demand counterbalances entrepreneurs' liquidity demand, moderating the fluctuations of aggregate demand for bank deposits. Without the upward trend in households' liquidity demand, the procyclicality of entrepreneurs' savings is fully unleashed, so the hypothetical scenario where only the rise of intangibles is present features the strongest shock amplification mechanism. Therefore, despite being less than 14% of households' liquidity holdings (both in the model and data), firms' liquidity holdings, driven by the rising needs for intangible investments, have a much stronger impact on financial stability.

As the economy becomes more intangible-intensive, the pledgeability of intangible assets improves (Mann, 2018) and new markets emerge for the exchange of intangibles (Akcigit, Celik, and Greenwood, 2016) I extend the model by allowing a fraction of intangibles to be pledgeable or sellable. Note that as long as intangibles are not fully pledgeable, investment still cannot fully rely on external financing and liquidity holdings are necessary. What improved pledgeability does is to increase the leverage on liquidity holdings, which leads to a higher marginal benefit of holding liquidity. Therefore, the feedback mechanism is amplified. The trend in intangible-investment productivity triggers an increasing and convex trend in the intangible share of investment, in contrast to the linear trend in the intangible share of investment in the baseline model. As a result, entrepreneurs' savings increase more over time, resulting in a much lower deposit rate, higher tangible capital value, and a higher level of endogenous financial risk.

Literature. This paper contributes to the broad literature on the macroeconomics of intangible capital.[4] The focus is on the limited pledgeability of intangible capital and firms' savings for intangible investment, motivated by evidence on the concentration of massive cash holdings in

[4]Previous studies has shown that the rise of intangible capital is important for explaining the secular trends in corporate profits and investment (McGrattan and Prescott, 2010b; Crouzet and Eberly, 2018; Gutiérrez and Philippon, 2017; Peters and Taylor, 2017). Dell'Ariccia, Kadyrzhanova, Minoiu, and Ratnovski (2018) and Döttling and Perotti (2017) emphasize the decline of firms' borrowings from banks as a result of less collateral assets. In contrast, this paper focuses on the liability side of banks' balance sheets, i.e., firms holding banks' liabilities as liquidity buffer. Previous studies also explores broad implications of intangible capital on productivity (Atkeson and Kehoe, 2005; McGrattan, 2020), current account (McGrattan and Prescott, 2010a), stock valuation (Hansen, Heaton, and Li, 2005; Ai, Croce, and Li, 2013; Eisfeldt and Papanikolaou, 2013), and investment (Daniel, Naveen, and Yu, 2018).

intangible-intensive firms.[5] The increase of intangible investment productivity is a driving force behind the accumulation of corporate savings that is distinct from what has been proposed in the literature on corporate savings in macroeconomic dynamics (Bacchetta and Benhima, 2015; Chen, Karabarbounis, and Neiman, 2017; Quadrini, 2017).

A unique feature of the model is that liquid assets are supplied endogenously by financial intermediaries.[6] Corporate savings have become a major cash pool that lends to financial intermediaries (Adrian and Shin, 2010; Pozsar, 2011; Carlson et al., 2016). However, the existing studies on corporate savings and the shortage of saving instruments have ignored the unique roles of financial intermediaries as issuers of liquid assets (Woodford, 1990; Holmström and Tirole, 1998, 2001; Giglio and Severo, 2012; Farhi and Tirole, 2011; Martin and Ventura, 2012; Hirano and Yanagawa, 2017; Miao and Wang, 2018). The broader literature on asset shortage also studies foreign savings as sources of demand for liquid assets, but when it comes to the supply of liquid assets, the active roles of financial intermediaries are absent (Bernanke, 2005; Caballero and Krishnamurthy, 2006; Caballero, Farhi, and Gourinchas, 2008; Caballero and Krishnamurthy, 2009; Gourinchas and Rey, 2016; Maggiori, 2017; Bolton, Santos, and Scheinkman, 2018).[7]

Connecting firms' demand for liquid assets and financial intermediaries' supply delivers several unique predictions. The downward trend in interest rates has drawn enormous attention and has been studied jointly with other secular trends (Caballero, Farhi, and Gourinchas, 2017a; Eggertsson, Robbins, and Wold, 2018; Farhi and Gourio, 2018; Marx, Mojon, and Velde, 2018; Corhay, Kung, and Schmid, 2019). This paper proposes corporate savings as a driving force behind the declining interest rate and demonstrates the quantitative importance of this channel. The low interest rate allows financial intermediaries to borrow cheaply and creates a discount-rate wedge

[5]See the related findings on corporate cash holdings (Pinkowitz, Stulz, and Williamson, 2015; Graham and Leary, 2018; Falato, Kadyrzhanovaz, Sim, and Steri, 2018; Begenau and Palazzo, 2021).

[6]The literature of firms' liquidity management problem takes a partial equilibrium approach and assume a perfectly elastic supply of storage technology, leaving out the question of who issues the securities called "cash and cash equivalents" (e.g., Froot, Scharfstein, and Stein, 1993; Riddick and Whited, 2009; Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; He and Kondor, 2016).

[7]U.S. nonfinancial corporations' holdings of intermediary debts are comparable in magnitude to foreigners' holdings. The ratio of the former to the later is stable since the 1990s, around 75%. Liquid intermediary debts include currency and deposits, open market papers, and repurchase agreements held directly or indirectly via money-market or mutual funds (source: Financial Accounts of the U.S.).

between financial intermediaries and the rest of the economy, which has a destabilizing effect on the financial system: When negative shocks trigger reallocation of assets from intermediaries to the rest of the economy, asset prices collapse. Moreover, the longer a boom lasts, the wider the discount-rate wedge is and thus sharper the asset prices fall when negative shocks hit.[8] The previous literature on financial accelerators focuses on firms' borrowing rather than savings as a source of financial instability (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999).

Recent studies in the macro-finance literature highlight the value of bank liabilities in incomplete markets (Brunnermeier and Sannikov, 2016) and as liquid assets for households (Kiyotaki and Moore, 2000; Krishnamurthy and Vissing-Jørgensen, 2015; Piazzesi and Schneider, 2016; Moreira and Savov, 2017; Begenau and Landvoigt, 2018; Van den Heuvel, 2018; Begenau, 2019; Egan, Lewellen, and Sunderam, 2021).[9] This paper is the first to model both households' and firms' liquidity demand, and the model is calibrated so their relative contributions to intermediaries' funding match data. This allows for a counterfactual analysis to show the relative importance of firms' liquidity demand in affecting interest rates, asset prices, and financial instability.[10] Section 2 and 4 provide evidence on the distinct responses of households' and firms' liquidity demand to asset-price variations that are consistent with the model's predictions.[11]

2 Corporate Liquidity Demand

This section establishes a robust empirical link between intangible investment and firms' holdings of liquid assets. The intangible-liquidity link is stronger when the value of tangible capital (i.e., capitalizable or pledgeable value of future output) increases. The sample is Compustat panel (firm-

[8]This procyclical discount-rate wedge is distinct from the constant cash-flow wedge between intermediaries and households in Brunnermeier and Sannikov (2014) due to differences in production skills.

[9]See also the banking literature (Diamond and Dybvig, 1983; Gorton and Pennacchi, 1990; Goldstein and Pauzner, 2005; Dang, Gorton, Holmström, and Ordoñez, 2017; Hart and Zingales, 2014).

[10]Related, Eisfeldt (2007) show that the liquidity premium of Treasury bills cannot be explained by the liquidity demand from consumption smoothing under standard preferences. Eisfeldt and Rampini (2009) document that corporate liquidity needs are correlated with measures of liquidity premium.

[11]Except Eisfeldt and Muir (2016), the empirical literature on firms' cash holdings focuses on trends not cycles. A new finding in this paper is the comovement between corporate savings and asset prices.

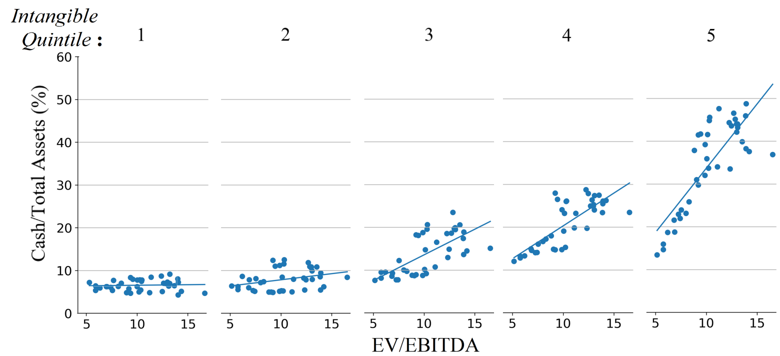

Figure 1: Capital Valuation and Cash Holdings by Intangible Quintile

year) data from 1980 to 2019.[12] Firms' liquidity holdings are given by cash and cash equivalents. Intangible intensity is measured by the ratio of intangible investment to total assets averaged over time within firm. Firms are sorted into quintiles to form the ranking variable "Intan./Assets". Following the literature, intangible investment includes R&D and organizational-capital investment that is 30% of SG&A expenses.[13] Two *aggregate* measures of tangible capital valuations are constructed. Each year, I calculate the market capitalization-weighted average ratio of enterprise value (EV) to earnings before interest, tax, depreciation, and amortization (EBITDA). EV is the present value of a firm's capitalizable output that corresponds to tangible capital value in the model.[14]

Figure 1 reports scatter charts of cash/assets against capital valuation (and regression lines) for Intan./Assets quintiles. A point is given by the quintile's market capitalization-weighted average cash/assets in a year and average EV/EBITDA in that year. More intangible firms hold more cash with a stronger correlation between cash and capital valuation. Appendix D reports similar patterns with tangible EV/EBITDA and Tobin's Q as measures of capital valuation. Tangible EV/EBITDA is the average EV/EBITDA in the lowest Intan./Assets quintile.[15]

[12] This includes Compustat firm-year observations with non-missing data for total assets and sales. All firms incorporated in the United States are included except financials (SIC 6000-6999) and utilities (SIC 4900-4999). The sample starts from 1980 because, before the 1980s, Regulation Q imposed various restrictions on deposit rates. For example, it prohibited banks from paying interest on demand deposits. This practice is inconsistent with the model specification that the deposit rate, r_t, is the price variable that clears the deposit market. Appendix C provides summary statistics.

[13] This follows a large literature on measuring intangible investment (Corrado et al., 2009; Eisfeldt and Papanikolaou, 2013; Falato et al., 2018; Peters and Taylor, 2017; Belo et al., 2014).

[14] Appendix D uses more restrictive Tangible EV/EBITDA from the lowest quintile of Intan./Assets.

[15] Two versions of Tobin's Q are calculated, the total average Tobin's Q and tangible Tobin's Q that is the average Q

Table 1: Intangible Investment, Capital Valuation, and Cash Holdings

***Panel A**: Intangibility & Corporate Cash Holdings*

$\frac{\text{Cash}}{\text{Assets}}$	Intangibility = Intan./Assets (quintile)				Intangibility = Intan./Investment			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intangibility	6.600***	6.493***	5.277***	5.009***	0.207***	0.196***	0.186***	0.170***
	(0.440)	(0.455)	(0.320)	(0.335)	(0.015)	(0.016)	(0.010)	(0.010)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,826	152,826	132,632	132,632	112,171	112,171	98,571	98,571
Adjusted R^2	0.1669	0.1903	0.2588	0.2757	0.0964	0.1185	0.2467	0.2585

***Panel B**: Capital Valuation & Intangible-Driven Corporate Cash Holdings*

$\frac{\text{Cash}}{\text{Assets}}$	Valuation = Ave. EV/EBITDA				Valuation = Tangible EV/EBITDA			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intan./Assets	-2.427**	-2.742**	-1.484	-1.846*	-1.039	-1.511	-0.277	-0.813
	(1.199)	(1.134)	(1.012)	(0.943)	(1.438)	(1.449)	(1.207)	(1.216)
Valuation	-0.731***		-0.590***		-0.789***		-0.738***	
	(0.097)		(0.066)		(0.131)		(0.082)	
Intan./Assets×	0.849***	0.881***	0.638***	0.661***	0.833***	0.884***	0.612***	0.649***
Valuation	(0.121)	(0.116)	(0.098)	(0.94)	(0.153)	(0.157)	(0.127)	(0.131)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,826	152,826	132,632	132,632	152,826	152,826	132,632	132,632
Adjusted R^2	0.2008	0.2128	0.2763	0.2883	0.1863	0.2044	0.2674	0.2832

Firm-year clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Table 1 reports regression results that correspond to the patterns in Figure 1. The explanatory variables of interest, capital valuation and the quintile ranking variable Intan./Assets, are the same as in Figure 1. Different from Figure 1 that plots the time-series variation of within-quintile average cash/assets, the regression dependent variable, cash/assets, has both time-series and cross-section variation. I consider different specifications controlling for firm characteristics and/or time fixed

of firms in the lowest Intan./Assets quintile. Averages are market capitalization-weighted.

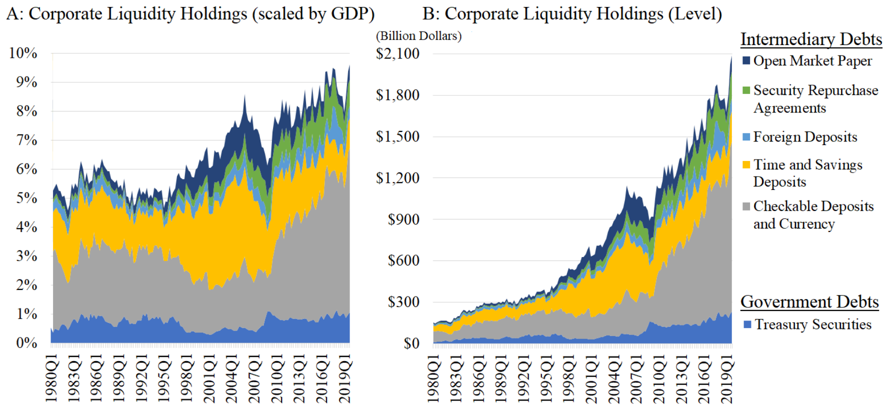

Figure 2: Decomposing Nonfinancial Firms' Holdings of Liquid Securities

effects.[16] Column (1) to (4) in Panel A shows that more intangible-intensive firms hold more cash.[17] In Column (5) to (8), the ranking variable, Intan./Assets, is replaced by intangible investment-to-total investment ratio that maps more directly to the model setup in Section 3. The estimates will guide model calibration. Column (1) to (4) in Panel B report a positive coefficient of the interaction between asset valuation and intangibility that is robust across specifications. As in Figure 1, more intangible firms' cash holdings are more sensitive to capital valuation. In Columns (5) to (8) of Panel B, I use a more restrictive measure of tangible capital valuation, tangible EV/EBITDA. Appendix C reports similar results with Tobin's Q as measure of capital valuation.[18]

Figure 2 examines the general equilibrium of liquid assets by shifting focus from demand to supply. Nonfinancial firms' liquid assets are mainly issued by financial intermediaries (source: U.S. Financial Accounts). Mutual fund and money market fund holdings are attributed to underlying assets based on sector level tables. Firms are among the major cash pools that feed leverage

[16] The control variables are selected and winsorzied following Opler et al. (1999) and Bates et al. (2009). They include (Compustat codes in parenthesis): acquisition activity (aqc/at), capex (capx/at), cash flow ([oibdp – xint – dvc – txt]/at), net working capital ([wcap – che]/at), payout dummy (equal to 1 if dvc is positive), leverage ([dlc – dltt]/at), market to book ratio ([at + prcc_f*csho – ceq]/at), R&D to sales ratio (xrd/sale), size (log of at in 2005 dollars), Tobin's Q ([at + prcc_f*csho – ceq – txdb]/[0.1*(at + prcc_f*csho – ceq – txdb) + 0.9*at]), and industry sigma, which is the 10-year mean of the cross-sectional standard deviations of firms' cash flow/assets in a two-digit SIC industry.

[17] Investment need is a key determinant of cash holdings (Denis and Sibilkov, 2010; Duchin, 2010). Firms with less collateral hold more cash (Almeida and Campello, 2007; Li, Whited, and Wu, 2016).

[18] Table D.3 reports similar results under sorting by tangible assets (PPE). Less tangible firms exhibit stronger correlation between cash and capital valuation (measured by EV/EBITDA or Tobin's Q).

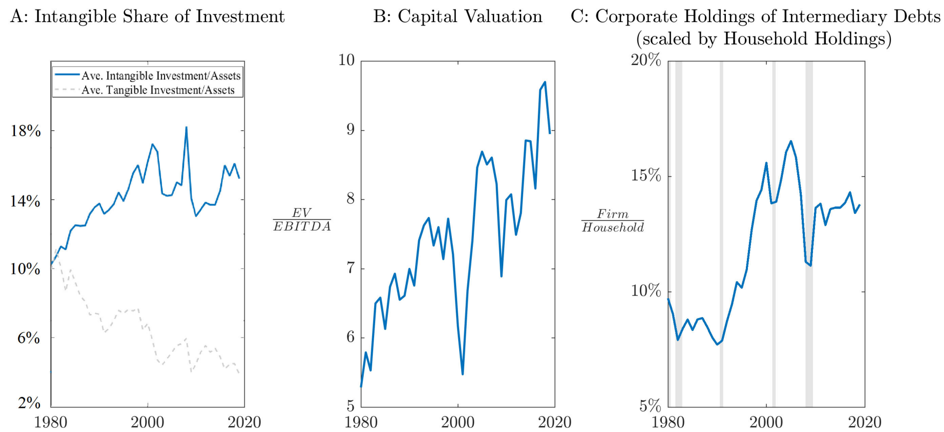

Figure 3: Intangible Investment, Capital Valuation, and Corporate Savings

to intermediaries (Carlson et al., 2016; Pozsar, 2014). Their liquid assets scaled by GDP almost doubled by 2019. The trend was interrupted by the financial crisis and firms flighted to Treasuries, but the trend resumed afterwards. However, the loss of firms' savings for intermediaries in the crisis was recognized by regulators. Retail deposits are assigned 90% to 95% stable funding factor while corporate deposits are assigned 50% (Basel Committee on Banking Supervision, 2014).

The rise of corporate savings in Figure 2 coincided with the secular increase in intangible investment especially relative to tangible investment in Panel A of Figure 3. Moreover, in Panel B of 3, capital valuation exhibits an upward trend, which, according to the evidence in Figure 1 and Table 1, reinforced the rise of intangibles in fueling the corporate savings glut. Along the secular trends, cyclical fluctuations emerge in both investment and capital valuation, feeding procyclicality to corporate savings. Panel C of Figure 3 plots the ratio of firms' holdings of intermediary debts to households' holdings (source: Financial Accounts of the U.S.). Recession years are marked by shaded areas. The ratio trends upward with cyclical drops in recessions, suggesting that, as a source of funding for intermediaries, corporate liquidity holdings are more procyclical than households'. Next, a model is built to generate both the trends and cyclical fluctuations in intangible share of investment, capital valuation, and corporate liquidity holdings. The model highlights endogenous risk that arises from the reinforcing procyclicality of these variables and becomes increasingly strong along the secular trends. The model also provides a new account of trends in interest rates and the growth of intermediation sector, which have been documented extensively.

3 Model

Consider a continuous-time, infinite-horizon economy. The model fixes an information filtration that satisfies the standard regularity conditions (Protter, 1990). The production sector is set up first with a focus on intangible-driven liquidity demand. Later, bankers and households are introduced.

3.1 The Production Sector and Liquidity Demand

Preferences. There is a unit mass of entrepreneurs. Let $\mathbb{E} = [0, 1]$ denote the set. Let c_t^E denote a representative entrepreneur's *cumulative* consumption up to time t. Throughout this paper, subscripts denote time, and whenever necessary, superscripts are used to denote agents' type, with "E" for entrepreneurs (and later, "B" for bankers and "H" for households). An entrepreneur maximizes the life-time, risk-neutral expected utility with discount rate ρ:

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t} dc_t^E\right]. \tag{1}$$

Capital and production. Each entrepreneur manages a firm that has tangible and intangible capital. Capital represents efficiency units and is counted by its output: One unit of capital produces one unit of non-durable generic goods per unit of time. In aggregate, the economy has K_t^T and K_t^I units of tangible and intangible capital, respectively, at time t that generate a flow of output, $\left(K_t^T + K_t^I\right) dt$ over dt. A fraction $\delta dt - \sigma dZ_t$ of capital are destroyed over dt. The standard Brownian motion Z_t captures aggregate shocks to production capacity.[19]

The two types of capital differ in liquidity. Tangible capital is liquid. It can be pledged for financing, and entrepreneurs may sell the capital ownership and dutifully manage the capital on behalf of investors delivering goods produced. Tangible capital represents inventory, equipment,

[19]For parsimony, the stochastic depreciation rates are the same for both types of capital. Introducing different depreciation rates for intangible and intangible capital will not change the mechanism.

plant, and property. In reality, even though certain tangible assets are not actively traded, the securities backed by their cash flows are traded. In contrast, intangible capital is illiquid. It cannot be pledged for financing, and its ownership cannot be traded. It represents human and organizational capital, customer base, and proprietary technologies that are difficult for investors to repossess.

Investment and liquidity demand. The Poisson arrival of investment needs is independent across entrepreneurs with intensity λ. When hit by the shock, an entrepreneur's firm loses all capital but is endowed with a technology to transform goods into new capital instantaneously.[20] She chooses i_t, the amount of goods invested, and θ_t, the intangible share, to create $\kappa_t^I \theta_t i_t$ units of intangible capital and $\kappa^T (1-\theta_t) i_t$ units of tangible capital. Tangible investment efficiency is constant κ^T. Intangible investment efficiency increases over time, $\kappa_t^I = \kappa^I(t)$. Capital corresponds to a stream of future goods, so an increase of κ_t^I means that intangible investment generates more production capacity. It also captures the shift of consumers' preference towards output generated by intangibles, such as professional and business services (McGrattan, 2020).[21]

Let q_t^I denote the value of intangible capital (denominated in goods). The entrepreneur is indifferent in consumption timing, so she values the goods from intangible capital simply by Gordon growth formula, accounting for normal-time depreciation and Poisson-arriving destruction

$$q_t^I = \frac{1}{\rho + \delta + \lambda}\,. \tag{2}$$

Henceforth, the time subscript is dropped for q^I. As will be emphasized later in the solution, the unit value of tangible capital, denoted by q_t^T, may vary over time and loads on the aggregate shock,

$$dq_t^T = q_t^T \mu_t^T dt + q_t^T \sigma_t^T dZ_t\,. \tag{3}$$

where the drift and diffusion terms will be solved in equilibrium.

[20]This specification reflects the lumpiness of investment at micro levels (e.g., Doms and Dunne, 1998). Due to the idiosyncratic nature of investments, the aggregate investment is smooth (Thomas, 2002).

[21]This paper takes the structural change as exogenous. The literature on the growth of services sector provides several explanations (Kongsamut et al., 2001; Herrendorf et al., 2013; Ngai and Pissarides, 2007).

Given q^I and q_t^T, an investing entrepreneur maximizes the investment profits:

$$\max_{\{i_t,\theta_t\}} \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T (1-\theta_t) - F(\theta_t)\right] i_t - i_t\,, \tag{4}$$

where a convex $F(\theta_t)$ is introduced to avoid counterfactual corner solutions (i.e., $\theta_t \in \{0,1\}$). Due to the illiquidity of intangible capital, the scale of investment is constrained by tangible value:

$$i_t \leq q_t^T \kappa^T i_t (1-\theta_t)\,. \tag{5}$$

Self-financing, $1 \leq q_t^T \kappa^T (1-\theta_t)$, is ruled out (see details in Appendix A).

Assumption: Investment projects are not self-financed: $\kappa^T \left(\frac{1}{\rho+\delta+\lambda}\right) < 1$.

Under the financial constraint, entrepreneurs would hold liquidity, i.e., assets other than their own capital, immune to the Poisson shocks.[22] Holmström and Tirole (1998) point out a solution that is to pool pledgeable assets (tangible capital) in mutual funds where idiosyncratic shocks are diversified away. Then entrepreneurs hold the mutual-fund shares and use them for investment. Let m_t^E denote an entrepreneur's liquidity holdings, so the constraint (5) becomes

$$i_t \leq q_t^T \kappa^T i_t (1-\theta_t) + m_t^E. \tag{6}$$

However, as shown in Figure 2, firms rarely hold direct claims on other firms but instead hold debt securities largely issued by financial intermediaries. Diversification may require intermediaries' expertise.[23] And, under agency frictions that limit equity issuances (e.g., He and Krishnamurthy, 2013), firms hold intermediaries' debt rather than equity. Intermediated liquidity supply is also motivated by studies on banks as inside money creators (e.g., Kiyotaki and Moore, 2000).

[22]It is well documented that intangible investments rely heavily on firms' internal liquidity (for example, R&D investments in Hall (1992), Himmelberg and Petersen (1994), and Hall and Lerner (2009)).

[23]Intermediation is also be motivated by required expertise in monitoring (Diamond, 1984), restructuring (Bolton and Freixas, 2000), or enforcing collateralized claims (Rampini and Viswanathan, 2019).

3.2 Intermediated Liquidity Supply

Bankers are introduced to intermediate the supply of liquidity. Entrepreneurs are assumed to hold liquidity in the form of short-term bank debts (referred to as "deposits") that are in turn backed by bankers' holdings of tangible capital. With a slight abuse of notation, m_t^E now represents entrepreneurs' deposit holdings that mature in dt with interests $r_t dt$. I characterize a Markov equilibrium where banks never default, so bank debt is safe and $r_t dt$ is also the realized return.[24]

When the Poisson shocks hit, entrepreneurs use deposits to buy goods as investment inputs. In contrast to the existing macroeconomic models with financial intermediation that emphasize bankers' expertise on lending, this model emphasizes the liability side of bank balance sheets—banks add value to the economy because their debts are held by entrepreneurs as liquidity buffers.

Preferences. There is a unit mass of bankers. Let $\mathbb{B} = [0,1]$ denote the set of bankers. A representative banker maximizes the life-time, risk-neutral expected utility with discount rate ρ:

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t} dc_t^B\right], \tag{7}$$

where c_t^B denotes a banker's *cumulative* consumption up to time t.

Balance sheet. A banker incurs interest expenses $r_t dt$ on debt liabilities and earns risky return dr_t^T on her holdings of tangible capital, where r_t^T denotes the *cumulative* return that loads on shocks. To characterize dr_t^T, let k_t^{TB} denote a banker's holdings of tangible capital, with "T" and "B" indicating "tangible" and "banker" respectively. Capital stock depreciates stochastically, so

$$dk_t^{TB} = -k_t^{TB}\left(\delta dt - \sigma dZ_t\right) - k_t^{TB}\lambda dt\,. \tag{8}$$

The last term is from the λdt firms that lose capital due to the Poisson shocks. Through diversification, the banker faces a constant rate of capital destruction.

[24] Macro-finance models that are built upon diffusion processes typically do not feature bank default (e.g., Brunnermeier and Sannikov, 2014). Default may be introduced through aggregate Poisson shocks.

By Itô's lemma, equations (3) and (8) imply the tangible capital return:

$$dr_t^T = \frac{k_t^{TB}dt}{q_t^T k_t^{TB}} + \frac{d\left(q_t^T k_t^{TB}\right)}{q_t^T k_t^{TB}} = \left(\frac{1}{q_t^T} + \mu_t^T - \delta - \lambda + \sigma_t^T \sigma\right) dt + \left(\sigma_t^T + \sigma\right) dZ_t \tag{9}$$

$1dt/q_t^T$, is dividend yield—production flow, $1dt$, divided by the unit value, q_t^T. $\left(\mu_t^T - \delta - \lambda\right) dt$, account for the expected unit value change, quantity depreciation, and measure of firms hit by the Poisson shocks. $\sigma_t^T \sigma$, is Itô's quadratic covariation. Shock loading consists of σ_t^T, the endogenous return volatility of q_t^T (price risk), and σ, the exogenous volatility of depreciation shock (quantity risk).

Let n_t^B denote a representative banker's wealth with the following law of motion,

$$dn_t^B = x_t^B n_t^B dr_t^T - (x_t^B - 1)n_t^B r_t dt - dc_t^B \,, \tag{10}$$

where $x_t^B \equiv q_t^T k_t^{TB}/n_t^B$ is the asset-to-wealth ratio and debt value is $(x_t^B - 1)n_t^B$.

As shown by (10), intermediation involves risk-taking. Bankers issue safe deposits while holding risky tangible capital. Equity capital buffers risk. An undercapitalized banking sector cannot adequately fulfill its role as liquidity supplier. To capture this idea, I assume that banks cannot issue outside equity, i.e., $dc_t^B \geq 0$ as in Brunnermeier and Sannikov (2014).[25] This can be motivated by agency frictions. As a result, bankers' wealth drives the *intermediation capacity*. In this model, entrepreneurs' liquidity demand from Holmström and Tirole (1998) meets banks' limited balance-sheet capacity from Holmström and Tirole (1997).

[25]By inspecting equation (9), we can see that negative consumption is equivalent to issuing equity to replenish net worth. See also Phelan (2016) and Klimenko, Pfeil, Rochet, and Nicolo (2016) for similar specifications. Note that negative consumption is allowed for entrepreneurs except when liquidity shocks hit. In other words, entrepreneurs are only financially constrained at such Poisson times. Allowing negative consumption is equivalent to assuming large endowments of goods – if goods are non-durable, entrepreneurs always consume to clear the goods market, indifferent between consuming and saving. This fixes their marginal value of wealth at one and required return at ρ.

3.3 The Main Mechanism: Trends and Cycles

The main results are in two categories: (1) the economy's response to the increase of κ_t^I over time (i.e. the trends) and (2) the economy's response to the aggregate shock, dZ_t (i.e., the cycles). First, I explain the trends as I characterize the entrepreneurs' intangible-driven liquidity demand.

When hit by the Poisson shock, an entrepreneur maximizes investment profits given by (4) facing the liquidity constraint (6). Let π_t denote the marginal value of liquidity, i.e., the Lagrange multiplier of constraint (6). The Lagrange function summarizes the entrepreneur's problem:

$$\mathcal{L} = \max_{\{i_t, \theta_t\}} \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T (1 - \theta_t) - F(\theta_t) \right] i_t - i_t + \pi_t \left[m_t^E + q_t^T \kappa^T i_t (1 - \theta_t) - i_t \right] . \quad (11)$$

It is assumed that κ^T or κ_t^I is sufficiently high so the constraint (6) binds. The entrepreneur can pledge the value of tangible capital and lever up one unit of liquidity to $1/\left[1 - q_t^T \kappa^T (1 - \theta_t)\right]$:

$$i_t = \left(\frac{1}{1 - q_t^T \kappa^T (1 - \theta_t)} \right) m_t^E \quad (12)$$

The funds are raised against tangible capital at a fair price so the entrepreneur captures all surplus per unit of investment, i.e., $\left[q_t^I \kappa_t^I \theta_t + q_t^T \kappa^T (1 - \theta_t) - F(\theta_t)\right] - 1$.[26] Therefore, the marginal value of liquidity, π_t, is the marginal profit of investment multiplied by the leverage on liquidity:

$$\pi_t = \underbrace{\left\{ \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T (1 - \theta_t) - F(\theta_t) \right] - 1 \right\}}_{\textit{marginal profit of investment}} \underbrace{\left(\frac{1}{1 - q_t^T \kappa^T (1 - \theta_t)} \right)}_{\textit{leverage on liquidity}} \quad (13)$$

The entrepreneur's choice of θ_t is characterized by the first-order condition that equates the marginal values of intangible and tangible investments:

$$q^I \kappa_t^I - F'(\theta_t) = (1 + \pi_t) q_t^T \kappa^T . \quad (14)$$

Note that on the right side of (14), the marginal value of tangible capital, $q_t^T \kappa^T$, is amplified by π_t,

[26]The repayment for funds raised against tangible capital is in the ownership of the tangible capital. The entrepreneur is assumed to dutifully pass the production flows generated by the capital to its owners.

because investing more in tangible capital not only creates more production units but also relaxes the funding constraint (6). The next proposition summarizes the entrepreneur's liquidity-holding and investment decisions with a focus on the value of liquidity. Appendix A provides the proof.

Proposition 1 *Entrepreneurs' investment has the following properties:*
(1) The optimal intangible share of investment, θ_t, in (14) is increasing in κ_t^I;
(2) The marginal value of liquidity, π_t, given by (13), is increasing in κ_t^I and q_t^T,
and entrepreneurs accept a deposit rate below ρ:

$$r_t = \rho - \lambda \pi_t \,. \tag{15}$$

Proposition 1 implies several trends in equilibrium. As κ_t^I increases over time, intangible investment creates increasingly more production capacity than tangible investment, so the entrepreneurs optimally choose to tilt investment towards intangibles, i.e., to increase θ_t. As the intangible share increases, the entrepreneurs face a tighter liquidity constraint, so the marginal value of liquidity, π_t, increases, driving down the deposit rate r_t. The entrepreneurs accept $r_t < \rho$. The wedge, $\lambda\pi_t$, depends on the probability of liquidity needs and marginal value of liquidity.

The decline of r_t triggers a feedback mechanism. It lowers bankers' cost of financing and allows them to bid up the price of tangible capital, q_t^T. A higher value of tangible capital enlarges the financing capacity of investment projects, allowing liquidity to be leveraged to larger investments. A higher q_t^T also means investments are more profitable. Therefore, π_t, the marginal value of liquidity holdings, increases further, and r_t drops even lower. The downward trend in r_t and upward trend in q_t^T reinforce each other, generating a corporate savings glut. This savings glut arises endogenously in a closed-economy, distinct from an exogenous savings glut in open economies that has been shown to affect interest rates and asset prices (Caballero, Farhi, and Gourinchas, 2008).

Tangible capital has two sources of value. It produces goods and provides liquidity by backing deposits. The bankers transmit the entrepreneurs' liquidity premium to the value of tangible capital. To fully solve q_t^T, we need a complete characterization of bankers' discount rate, r_t + *risk premium*. For the risk-premium component, we obtain bankers' price of risk from the dynamics of marginal value of wealth. The homogeneity property of bankers' problem implies a linear value function $q_t^B n_t^B$. The marginal value of wealth, q_t^B, evolves in equilibrium

$$\frac{dq_t^B}{q_t^B} = \mu_t^B dt - \gamma_t^B dZ_t, \tag{16}$$

where μ_t^B and $\gamma_t^B d$ will be solved in equilibrium.

Proposition 2 *The equilibrium expected return on tangible capital is*

$$\mathbb{E}_t \left[dr_t^T \right] = r_t dt + \gamma_t^B \left(\sigma_t^T + \sigma \right) dt \,. \tag{17}$$

The equilibrium value of tangible capital satisfies the following equation

$$q_t^T = \frac{1}{\left[r_t + \gamma_t^B \left(\sigma_t^T + \sigma \right) \right] - \left[\mu_t^T + \sigma_t^T \sigma - \delta - \lambda \right]} \,. \tag{18}$$

Appendix A provides the proof. Intuitively, $dZ_t < 0$ reduces bankers' wealth and increases their marginal value of wealth, so the bankers require a risk premium, $\gamma_t^B \left(\sigma_t^T + \sigma \right) dt$, in the expected return on tangible capital.[27] This is a standard asset-pricing result: γ_t^B is the price of risk and $\left(\sigma_t^T + \sigma \right)$ is the quantity of risk, a sum of exogenous risk, σ, and endogenous price risk, σ_t^T (see (3)). In equilibrium, $r_t + \gamma_t^B \left(\sigma_t^T + \sigma \right) \leq \rho$. When both the entrepreneurs, whose discount rate is ρ, and bankers own tangible capital, the expected return is ρ; when only the bankers own tangible capital, the expected return must not be greater than ρ, the entrepreneurs' required return. Being able to issue deposits at interest rate r_t gives the bankers a discount-rate advantage.

Equation (18) resembles the Gordon growth formula. The numerator is cash flow (production). In the denominator, the first component is discount rate and the second is expected growth.[28] As κ_t^I drives up θ_t, the intangible share of investment, and π_t, the marginal value of liquidity, entrepreneurs accept an increasingly low deposit rate $r_t = \rho - \lambda \pi_t$ (see (15)), which drives down

[27]Like Tobin's Q, q_t^B, is a forward looking measure of profits per unit of equity. This offers an alternative view. Due to the negative shocks and their persistent effects under the equity issuance constraint, the whole banking sector becomes undercapitalized and shrinks for a sustained period of time. To clear the markets of tangible capital and deposits, the spread between the expected return on tangible capital and deposit rate will have to widen so that banks would hold tangible capital and issue deposits. As the expected future profits rise, q_t^B increases.

[28]Investment creates new capital instead of grows the existing capital, so it's not in the growth rate.

the discount rate in (18) and pushes up q_t^T. The bankers transmit the rising liquidity premium on deposits to q_t^T. The transmission is incomplete due to the risk-premium component of their discount rate. The risk premium can be shut down if the bankers were allowed to freely issue equity and thus have unlimited balance-sheet capacity.[29] Comparing (18) and the valuation of illiquid intangible capital (2), we can see that the source of variation in q_t^T is the liquidity value rather than production value. And, the liquidity value varies with the bankers' intermediation capacity.

While the increase in κ_t^I generates the self-enforcing trends in r_t and q_t^T, the endogenous variation of γ_t^B generates the fluctuations along the trends (i.e., the cycles). After positive shocks ($dZ_t > 0$),bankers become wealthier and their price of risk γ_t^B declines, so they bid up q_t^T, which in turn leads to a higher value of liquidity holdings for entrepreneurs (π_t) and a lower r_t. As the bankers' funding cost r_t declines, they push up q_t^T further. As the bankers expand balance sheet and entrepreneurs hold more deposits, investment booms because the entrepreneurs hold more liquidity and can lever up through a higher value of tangible capital.

Endogenous risk accumulates in booms of liquidity creation and investment. As r_t declines, the wedge between the bankers' discount rate, $r_t + \gamma_t^B \left(\sigma_t^T + \sigma\right)$, and entrepreneurs' discount rate, ρ, widens, which makes q_t^T increasingly sensitive to shocks that cause reallocation of tangible capital between the bankers and entrepreneurs. When negative shocks hit, the bankers sell tangible capital back to entrepreneurs who have a higher discount rate. The reallocation causes a decline in asset price, q_t^T. Endogenous asset-price volatility has impact on the real economy. Economic growth is directly tied to q_t^T through the leverage on liquidity and scale of investment (see (12)). A vicious cycle ensues. A lower q_t^T reduces investment profits and π_t, discouraging the entrepreneurs from saving for investments. This causes the rise of r_t, the bankers' funding cost, so the bankers' discount rate increases further, causing q_t^T to continue falling. Moreover, the decline of q_t^T erodes the bankers' wealth, further increasing their price of risk, γ_t^B. The risk premium channel and interest rate channel reinforce each other, generating a powerful response to negative shocks.

The accumulation of endogenous risk is asymmetric. Positive shocks trigger the reallocation of tangible capital to the bankers with low discount rates but eventually cause bankers to consume

[29] Under frictionless equity issuance, $q_t^B = 1$ (i.e., no incentive to retain equity), so $\gamma_t^B = 0$.

their wealth as q_t^B, the marginal value of wealth, falls to one (when the bankers become indifferent between retaining wealth and consumption). However, negative shocks cause a continuing reallocation of tangible capital to those with high discount rates. Such asymmetry sheds light on the findings that longer booms precede more severe crises.[30] The mechanism differs from the existing models on asymmetric cycles (e.g., Ordoñez, 2013).

The model is built on two frictions. The first is the illiquidity of intangible capital. This leads to demand for liquid assets and links the rising productivity of intangible investment to the decline of interest rate and other trends. The second friction is that the bankers cannot raise external equity frictionlessly.[31] This generates the response of risk price to shocks (He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014; Di Tella, 2017) and endogenous financial cycles. Removing the second friction eliminates the amplification of fluctuations along the trends but does not eliminate the trends. If bankers could raise equity freely to replenish net worth, their marginal value of wealth would be pinned to one and price of risk pinned to zero.[32]

This paper continues the tradition of incorporating financial frictions into macroeconomic models. The financial accelerators amplify both trends (driven by κ_t^I) and cycles (triggered by dZ_t). At the core is firms' savings, which is in contrast to the literature that focuses on firms' borrowing (Kiyotaki and Moore, 1997; Bernanke, Gertler, and Gilchrist, 1999; Gertler and Kiyotaki, 2010). Key to the financial cycle is the procyclical wedge in discount rate between bankers, who supply liquidity and are "natural buyers" of tangible capital, and the rest of the economy. The longer a boom lasts, the sharper asset price falls when negative shocks reduce bankers' wealth. This procyclical discount-rate wedge is distinct from the constant cash-flow wedge between intermediaries and households as asset owners in Brunnermeier and Sannikov (2014). Endogenous risk accumulation via discount-rate wedge also differs from recent studies that emphasize belief heterogeneity (Caballero and Simsek, 2020, 2021).

[30]Please refer to Baron and Xiong (2017), Jordà, Schularick, and Taylor (2013), Krishnamurthy and Muir (2016), and López-Salido, Stein, and Zakrajšek (2017) among others. The mechanism is consistent with banks' procyclical payout in data (Baron, 2014; Adrian, Boyarchenko, and Shin, 2016).

[31]Allowing limited equity issuance (e.g., He and Krishnamurthy, 2013) changes quantitative performances and causes calibration to deliver different parameter values, but will not change the mechanism.

[32]Without equity issuance friction, the equilibrium of intermediated liquidity supply is the same as the mutual-fund equilibrium that features constant asset price and zero endogenous risk.

Discussion: Intangible risk. A potential limitation of the model is that intangible capital valuation in (2) does not reflect risk premium. In Eisfeldt and Papanikolaou (2013), the risk of organizational capital from the cyclical variation in key personnel's outside option. Such risk premium may reduce capital valuation and thus discourages firms from intangible investment, counteracting the rise of κ_t^I. However, other forms of intangibles may serve as a hedge and their negative risk premia have a counterbalancing effect. An important type of intangible capital is technology.[33] Technological innovation displaces firms and workers that operate with old technologies and have difficulty to adapt (Kogan, Papanikolaou, Schmidt, and Song, 2020). Displacement risk makes technological innovation a hedge against systematic technological changes (Gârleanu, Kogan, and Panageas, 2012; Bena and Garlappi, 2019; Kogan, Papanikolaou, and Stoffman, 2020).

3.4 Aggregation and the Markov Equilibrium

Households. In reality, households also hold intermediaries' debts. Households' demand is not essential for the main mechanism but it is important to incorporate it for calibration and quantitative analysis. The literature takes a money-in-utility approach, motivated by the role of intermediaries' debts (e.g., deposits) as means of payment (Sidrauski, 1967; Stein, 2012; Van den Heuvel, 2018). Holdings of monetary assets generate utility flows separable from consumption (Poterba and Rotemberg, 1986; Nagel, 2016; Begenau and Landvoigt, 2018) and are complementary to income levels (Begenau, 2019; Krishnamurthy and Vissing-Jørgensen, 2015). Consider a unit mass of households, $\mathbb{H} = [0, 1]$. A representative household has labor that produces w_t^H units of goods. Let W_t^H ($= \int_{i \in \mathbb{H}} w_t^H(i)\, di$) denote the aggregate labor output, so the total output of the economy is $(K_t^I + K_t^T + W_t^H)dt$. The utility function is specified as

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho t}\left(dc_t^H + \frac{\left(w_t^H \beta_t\right)^{\xi}\left(m_t^H\right)^{1-\xi}}{1-\xi} dt\right)\right], \tag{19}$$

[33] Technology sector is the most relevant as corporate cash holdings mainly reside in "growth sectors" (Begenau and Palazzo, 2021; Graham and Leary, 2018; Pinkowitz, Stulz, and Williamson, 2015).

where c_t^H is the cumulative consumption process and m_t^H denotes deposit holdings.[34] The scaling variable is a function of time, $\beta_t = \beta(t)$.

The utility function in (19) implies the following optimality condition for m_t^H:

$$\left(\frac{m_t^H}{\beta_t w_t^H}\right)^{-\xi} = \rho - r_t\,, \tag{20}$$

which equates the marginal utility of holding deposits and marginal cost, i.e., the spread $\rho - r_t$. Rearranging (20) and aggregating over households, we obtain

$$M_t^H = W_t^H \beta_t (\rho - r_t)^{-\frac{1}{\xi}}\,. \tag{21}$$

To avoid introducing a new state variable, it is assumed that labor output is proportional to that of tangible capital, i.e., $W_t^H = \alpha K_t^T$. In other words, between labor and tangible capital, the labor share of output is a constant, $\alpha/(\alpha+1)$. This is consistent with the finding in Koh, Santaeulàlia-Llopis, and Zheng (2020) that labor share is stable without accounting for output associated with intangibles.[35] Under this assumption, households' deposits demand is given by

$$M_t^H = \alpha K_t^T \beta_t (\rho - r_t)^{-\frac{1}{\xi}}\,, \tag{22}$$

α only has a scaling effect, so only the calibration of $\beta_t = \beta(t)$ is necessary.

The real-financial linkage. Figure 4 summarizes the model. The economy has three markets to clear (goods, the ownership of tangible capital, and deposits). The output is generated by intangible capital, tangible capital, and labor. The λdt entrepreneurs who are hit by the Poisson shocks acquire goods to create new capital, and the remaining goods are consumed by the rest of the economy.[36] The entrepreneurs, bankers, and households can trade the ownership of tangible capital at

[34] Appendix B discusses the implications of incorporating risk-averse preferences and finite EIS.

[35] Intangibles include research and development, software, and entertainment, literary, and artistic originals (U.S. Bureau of Economic Analysis). Analyzing the decline of labor share (e.g., Karabarbounis and Neiman, 2013), Koh, Santaeulàlia-Llopis, and Zheng (2020) show that it is attributed to the incorporation of output related to intangibles.

[36] Under risk-neutral utility, the demand for consumption goods is perfectly elastic.

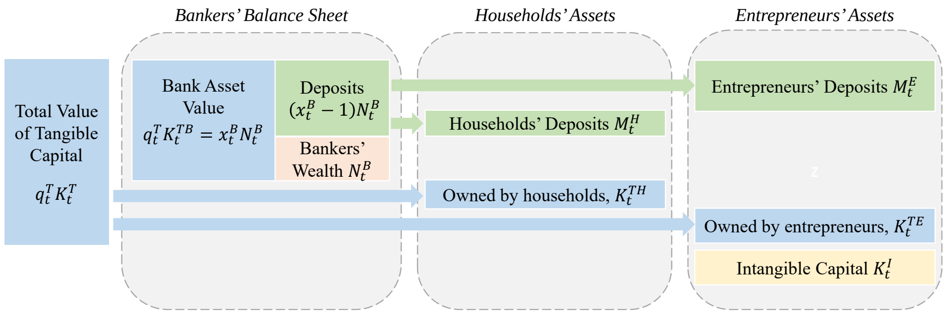

Figure 4: Model Structure

competitive price q_t^T given the stock K_t^T. In the deposit market, the bankers' supply is equal to the demand from the entrepreneurs and households. As in Caballero, Farhi, and Gourinchas (2008), only a fraction of output is capitalizable—tangible capital output—and the key inefficiency is a shortage of liquid assets. Depending on the bankers' risk-taking capacity (wealth), the bankers create liquidity by backing deposits with tangible capital. Entrepreneurs' deposits relax the liquidity constraint (6) on investment. Therefore, economic growth depends on the *intermediated liquidity supply*.

As shown in (12), one unit of liquidity is leveraged up to $1/\left[1 - q_t^T \kappa^T \left(1 - \theta_t\right)\right]$ units of goods invested. Given the entrepreneurs' aggregate deposits, M_t^E, the aggregate investment comes from the λdt entrepreneurs (hit by the Poisson shocks):

$$\left(\frac{1}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)}\right) M_t^E \lambda dt \,. \tag{23}$$

The deposit-market clearing condition links the entrepreneurs' liquidity to bankers' wealth:

$$M_t^E = \left(x_t^B - 1\right) N_t^B - M_t^H \,, \tag{24}$$

where the right side is the total deposits minus the households' holdings.

The law of motion of intangible capital is

$$dK_t^I = \underbrace{\left(\frac{1}{1-q_t^T\kappa^T(1-\theta_t)}\right)}_{leverage}\underbrace{\left[\left(x_t^B-1\right)N_t^B-M_t^H\right]}_{entrepreneurs' \; liquidity}\theta_t\kappa_t^I\lambda dt-\underbrace{\left(\delta dt-\sigma dZ_t+\lambda dt\right)K_t^I}_{depreciation, \; Poisson \; destruction}, \qquad (25)$$

and the law of motion of tangible capital is

$$dK_t^T = \left(\frac{1}{1-q_t^T\kappa^T(1-\theta_t)}\right)\left[\left(x_t^B-1\right)N_t^B-M_t^H\right](1-\theta_t)\kappa^T\lambda dt-\left(\delta dt-\sigma dZ_t+\lambda dt\right)K_t^T. \qquad (26)$$

Total investment in (23) is split into the tangible and intangible parts by entrepreneurs' choice of intangible share, θ_t. Then investments are multiplied by the productivities, κ_t^I and κ^T.[37].

Equations (25) and (26) highlight the link between intermediation capacity and growth. When bankers are well-capitalized, more deposits are issued. Liquidity can be leveraged up to create capital. Equations (25) and (26) also show how the financial conditions drive economic fluctuations. Entrepreneurs' leverage on liquidity increases in the value of tangible capital, q_t^T. Therefore, the endogenous asset-price volatility, i.e., σ_t^T in (3), feeds into investment dynamics and has a direct impact on the real economy. Moreover, the variation of q_t^T has a levered impact on the bankers' wealth and their capacity of liquidity creation.

State variables. The Markov equilibrium has four state variables, time, which drives κ_t^I and β_t, and the three stock variables, $N_t^B \equiv \int_{i\in\mathbb{B}} n_{i,t}^B di$ (the bankers' aggregate wealth), K_t^I, and K_t^T.[38] These four state variables have a convenient hierarchical property. First, apparently, time progresses linearly and has an autonomous law of motion. Second, $\left(N_t^B, K_t^I, K_t^T\right)$ can be equivalently represented by $\left(\eta_t, K_t^I, K_t^T\right)$, where η_t, the intermediation intensity, is defined by

$$\textit{Intermediation Intensity}: \; \eta_t \equiv \frac{N_t^B}{K_t^T}. \qquad (27)$$

[37]The shocked entrepreneurs' lost capital is evenly endowed to other entrepreneurs, so the λdt measure of lost capital lost is not in (25) and (26). One interpretation is that the λdt entrepreneurs' customer base is seized by the others through creative destruction (Aghion, Akcigit, and Howitt, 2014)

[38]Capital composition is a key state variable in Eberly and Wang (2008) who study agents' trade-off between diversification benefits and reallocation costs when two sectors are available for investment.

It is a ratio of the bankers' wealth to the amount of assets to be intermediated. The next proposition states that its evolution only depends on itself and time, and that the market prices, such as q_t^T and r_t, and the K_t^T-scaled quantities are functions of η_t and time only. To solve the equilibrium, I first focus on the sub-system where η_t and time are the two state variables and solve the market prices and the K_t^T-scaled aggregate quantities, which requires solving a system of differential equations. The solutions of these variables are then fed into the laws of motion of K_t^I and K_t^T (see (25) and (26)) for a complete characterization of equilibrium dynamics. Appendix A provides the proof.

Proposition 3 (Financial System) *The equilibrium law of motion of intermediation intensity is*

$$\frac{d\eta_t}{\eta_t} = \mu^\eta\left(\eta_t, t\right) dt + \sigma^\eta\left(\eta_t, t\right) dZ_t \,, \tag{28}$$

for $\eta_t \in (0, \overline{\eta}(t)]$. $\mu^\eta(\eta_t, t)$ and $\sigma^\eta(\eta_t, t)$ are defined in Appendix A, and $\overline{\eta}(t)$ is a reflecting boundary where the bankers consume. Prices and K_t^T-scaled quantities are functions of η_t and t: (1) the value of tangible capital, $q_t^T = q^T(\eta_t, t)$; (2) the deposit rate, $r_t = r(\eta_t, t)$; (3) the K_t^T-scaled households' deposits, $\widetilde{M}_t^H = \widetilde{M}^H(\eta_t, t)$; (4) the K_t^T-scaled entrepreneurs' deposits, $\widetilde{M}_t^E = \widetilde{M}^E(\eta_t, t)$; (5) the optimal intangible share of investment, $\theta_t = \theta(\eta_t, t)$; (6) bankers' asset-to-wealth ratio, $x_t^B = x^B(\eta_t, t)$; (7) the bankers' marginal value of wealth, $q_t^B = q^B(\eta_t, t)$.[39]

4 Quantitative Analysis

This section starts with calibration and presents the results on trends and cyclical variations due to endogenous financial risk. It ends with counterfactual analysis that demonstrates the quantitative importance of the rise of intangibles.

[39] $q_t^B \in [1, +\infty)$. At $\eta_t = \overline{\eta}(t)$, $q_t^B = 1$ and bankers consume. Consumption reduces N_t^B, but once q_t^B is above one, consumption stops (retaining wealth is worth $q_t^B > 1$). Thus, $\overline{\eta}(t)$ is a reflecting boundary. Bankers' HJB equation and Eq. (18) imply a system of differential equations for $q^B(\eta_t, t)$ and $q^T(\eta_t, t)$. Once they are solved, the other variables are solved analytically. See Appendix A.

4.1 Parameter Calibration

Calibration takes five steps. The guiding principles are explained first. The first step is to calibrate the investment technology to match the trends in intangible and tangible investments and volatilities along those trends. The productivity of intangible investment, $\kappa^I(t)$, is parameterized as $\kappa_t^I = \kappa_0^I + \kappa_1^I t$, and the cost of adjusting investment portfolio is specified as $F(\theta_t) = \frac{\phi}{2}\theta_t^2$. Thus the investment technology is summarized by four parameters, κ_0^I, κ_1^I, κ^T, and ϕ. As will be shown shortly, these specifications generate realistic investment dynamics.

The choice of intangible share, θ_t, drives firms' liquidity needs. After matching investment dynamics, the second step is to calibrate λ, the arrival rate of investment and liquidity needs, so the response of firms' liquidity holdings to changes in θ_t matches the estimate in Section 2.

Third, parameters in households' liquidity utility are calibrated to match the dynamics of household liquidity holdings. This is important for counterfactual analysis where investment technology is adjusted to create scenarios with and without the rise of intangibles while households' liquidity utility is fixed. Fourth, the shock size, σ, is calibrated to generate a volatility of bank asset return in the baseline model that matches the estimate in the literature.

So far, the calibration has been guided by the estimate in Table 1 and data displayed in Figure 3 in Section 2. The fifth and last step is to calibrate ρ, discount rate, and δ, capital depreciation rate. The calibration targets have been quantity variables, such as investment and liquidity holdings. Now the focus shifts to the two price variables, interest rate and tangible capital value. However, with only two parameters left, the calibration exercise cannot target different aspects of equilibrium dynamics (the level, trends, volatilities along the trends, etc.) but instead matches the interest rate and capital valuation at the beginning of sample period 1980 to 2019. This leaves the price variables' paths over time completely to the equilibrium forces. Therefore, when examining model performances, whether the dynamics of price variables match data is a stricter criterion than the match of quantity variables which benefits from more degrees of freedom in parameters.

Next, I provide more details on calibration. One unit of time in the model is set to one year. For calibration and later comparing the endogenous variables with empirical counterparts, I extract trends in data through 20-year rolling averages from 1980 to 2019 (the sample period in Section

Table 2: Parameter Calibration

Parameters	Symbol	Value	Moment	Model	Data
(1) Intangible investment productivity: Intercept	κ_0^I	1.075	Average $\mathbb{E}^\eta\left[\theta\left(\eta,t\right)\right]$	63.9%	61.6%
(2) Intangible investment productivity: Time coefficient	κ_1^I	0.018	Average annual change of $\mathbb{E}^\eta\left[\theta\left(\eta,t\right)\widetilde{I}\left(\eta,t\right)\right]$	1.6%	1.4%
(3) Tangible investment productivity	κ^T	0.011	Average annual change of $\mathbb{E}\left[\left(1-\theta\left(\eta,t\right)\right)\widetilde{I}\left(\eta,t\right)\right]$	0.0%	-0.1%
(4) Investment cost $F\left(\theta\right)=\frac{\phi}{2}\theta_t^2$	ϕ	9.540	Average $\frac{\text{Vol.}^\eta\left[\theta(\eta,t)\widetilde{I}(\eta,t)\right]}{\text{Vol.}^\eta\left[(1-\theta(\eta,t))\widetilde{I}(\eta,t)\right]}$	1.84	2.06
(5) Investment project arrival rate	λ	0.050	$\frac{\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,20)}{q^T(\eta,20)}\right]-\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,0)}{q^T(\eta,0)}\right]}{\mathbb{E}^\eta\left[\theta\left(\eta,20\right)\right]-\mathbb{E}^\eta\left[\theta\left(\eta,0\right)\right]}$	0.162	0.170
(6) Household deposit demand elasticity to deposit rate	ξ	1.100	Average annual change of $\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,t)+\widetilde{M}^H(\eta,t)}{q^T(\eta,t)}\right]$	0.0%	0.3%
(7) Household deposit utility scale: Intercept	β_0	0.196	$\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,t)}{\widetilde{M}^H(\eta,t)}\right],t=0$	9.8%	9.6%
(8) Household deposit utility scale: Time coefficient ($\leq$ 1992)	β_1	0.019	Average annual change of $\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,t)}{\widetilde{M}^H(\eta,t)}\right],t\leq 2$	0.32%	0.29%
(9) Household deposit utility scale: Time coeff. increase ($>$ 1992)	β_2	0.003	Average annual change of $\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,t)}{\widetilde{M}^H(\eta,t)}\right],t>2$	0.19%	0.20%
(10) Capital depreciation rate: Vol.	σ	0.020	Vol. of bank asset return	2.9%	2.6%
(11) Agents' discount rate	ρ	0.062	$\mathbb{E}^\eta\left[r\left(\eta,t\right)\right],t=0$	3.2%	3.5%
(12) Capital depreciation rate: Mean	δ	0.088	$\mathbb{E}^\eta\left[q^T\left(\eta,t\right)\right],t=0$	6.6	6.8

2).[40] In the model, the variation in η_t generates fluctuation along the trends. To extract trends from the solution, I average out η_t at every t.[41] For example, $\mathbb{E}^\eta[r(\eta,t=0)]$ is mapped to the first rolling average of interest rates in data, which centers around 1990. The same logic applies to all prices and K_t^T-scaled quantities, which will be used in calibration and, according to Proposition 3, are

[40] Before the 1980s, Regulation Q imposed various restrictions on deposit rates. For example, it prohibited banks from paying interest on demand deposits. This practice is inconsistent with the model specification that the deposit rate, r_t, is the price variable that clears the deposit market.

[41] Instead of averaging over the simulated paths, the η-averages can be calculated using the t-conditional stationary distribution of η_t, implied by (28), and the solved functions of endogenous variables, for example, $q_t^T=q^T\left(\eta_t,t\right)$. Appendix A solves the t-conditional stationary distribution of η_t.

also functions of η_t and t. The model is solved for $t \in [0, 20]$ because the last moving average in data centers around 2010 (which maps to $t = 20$) and ends in 2019 (the sample end).

The productivity of intangible investment has two parameters, κ_0^I that determines the base rate, and κ_1^I that determines the time trend. κ_0^I is calibrated so the average θ_t matches the sample average of *Intan./Investment* in Section 2. κ_1^I is calibrated so the average annual change in the trend of intangible investment/tangible capital, i.e., $\mathbb{E}^{\eta}[\theta_t I_t/K_t] = \mathbb{E}^{\eta}[\theta_t \widetilde{I}_t]$, matches data.[42] The productivity of tangible investment, κ^T, is calibrated so the average annual change in the trend of tangible investment/tangible capital, i.e., $\mathbb{E}^{\eta_t}[(1-\theta_t)I_t/K_t] = \mathbb{E}^{\eta_t}[(1-\theta_t)\widetilde{I}_t]$, matches data. The parameter ϕ in $F(\theta_t)$ governs the cost of adjusting investment composition and its calibration targets the relative volatilities of intangible and tangible investments. At time t, the conditional distribution of η_t (implied by (28)) is used to calculate the volatility ratio of intangible to tangible investment (both scaled by K_t^T), $\frac{\mathrm{Vol}^{\eta_t}\left[\theta\widetilde{I}\right]}{\mathrm{Vol}^{\eta_t}\left[(1-\theta)\widetilde{I}\right]}$, and the ratio is averaged over time to match the volatility ratio of detrended intangible to tangible investment.[43]

Firms' liquidity needs is driven by the random arrival of projects that require intangible investment. The arrival rate λ is calibrated so that the model-implied response of firms' liquidity holdings to the increase of intangible investment matches the estimate in Section 2 (Table 1, Column 8), the change in cash/assets for one unit of change of *Intan./Investment* (θ_t in the model). The model counterpart is $\left(\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^E(\eta,20)}{q^T(\eta,20)}\right] - \mathbb{E}^{\eta}\left[\frac{\widetilde{M}^E(\eta,0)}{q^T(\eta,0)}\right]\right) / \left(\mathbb{E}^{\eta}\left[\theta(\eta,20)\right] - \mathbb{E}^{\eta}\left[\theta(\eta,0)\right]\right)$, where the η_t-averages are used as the match focuses on disciplining the trend rather cyclical fluctuations and $\frac{\widetilde{M}_t^E}{q_t^T} = \frac{M_t^E}{q_t^T K_t^T}$ is the ratio of firms' liquidity scaled by tangible capital value that corresponds to the accounting asset value mostly excluding intangibles (e.g., Peters and Taylor, 2017).

Next, I calibrate the households' liquidity utility. The only goal of incorporating the households' liquidity utility is to generate realistic liquidity demand, especially relative to firms', for the purpose counterfactual analysis where the rise of intangibles and associated liquidity demand of firms will be shut down to examine how interest rate, asset valuation, and other variables respond. The value of ξ, households' liquidity demand elasticity, is chosen so that the model generates a

[42]Each year, I calculate cross-section total asset-weighted average of ratio of intangible investment to tangible capital (PPE) and calculate the twenty-year rolling averages.

[43]Each year, I take the ratio of intangible investment (scaled by PPE) and tangible investment (scaled by PPE) (see also footnote 42). The resulting time series exhibits a linear trend.

stable path over time of the ratio of safe assets (households' and firms' holdings of deposits) to capitalizable assets (tangible capital value), i.e., $\mathbb{E}^{\eta}\left[\frac{M^E(\eta,t)+M^H(\eta,t)}{q^T(\eta,t)K_t^T}\right] = \mathbb{E}^{\eta}\left[\frac{\widetilde{M}^E(\eta,t)+\widetilde{M}^H(\eta,t)}{q^T(\eta,t)}\right]$ in line with the stability in safe asset share (Gorton, Lewellen, and Metrick, 2012).[44] $\xi = 1.1$, close to households' deposit-demand elasticity in other banking models, e.g., 1.4 from Begenau (2019).

The scaling function, $\beta(t)$, in households' liquidity utility is calibrated to match the trends of households' liquidity holdings relative to firms'. $\beta(t)$ is specified as

$$\beta_t = \beta_0 + \beta_1 t + \beta_2 t\, \mathbb{I}_{\{t>2\}}\,. \tag{29}$$

In data, the logarithm of households' holdings of intermediary debts has a structural break in its time trend at 1992 ($t = 2$ in the model), detected by supremum Wald test and LR test with p-values below 0.0001 (Andrews, 1993; Perron, 2006). I take logarithm because households' deposits grow exponentially along with capital stock (see (22)) and empirically households' holdings of intermediary debts also exhibit exponential growth.[45] It is important to include the structural break, as, without it, the match of households' liquidity holdings deteriorates significantly. The value of β_0 is chosen so that $\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^E(\eta,0)}{\widetilde{M}^H(\eta,0)}\right]$, i.e., the initial η-average ratio of entrepreneurs' to households' holdings of deposits matches the rolling average of data centering at 1990.[46] The value of β_1 is chosen so the average annual change of $\left\{\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^E(\eta,t)}{\widetilde{M}^H(\eta,t)}\right]\right\}_{n\leq 2}$ matches its empirical counterpart, and β_2 is set so the average annual change of $\left\{\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^E(\eta,t)}{\widetilde{M}^H(\eta,t)}\right]\right\}_{n>2}$ matches data.

The shock size, σ, is chosen so the model generates a volatility of bankers' return that matches data (Gornall and Strebulaev, 2018). Later, when conducting counterfactual analysis by shutting down the rise of intangibles, I will fix the exogenous risk, σ, and show how endogenous risk responds. The discount factor, ρ, is chosen so $\mathbb{E}^{\eta}[r(\eta,0)]$ matches the average rate of intermediary debts in 1990.[47] The capital depreciation rate, δ, is chosen so $\mathbb{E}^{\eta}\left[q^T(\eta,0)\right]$ matches the

[44] The empirical counterpart is the ratio of nonfinancial firms' and households' holdings of intermediary debts (listed in Figure 2) to nonfinancial firms' fixed assets from the Bureau of Economic Analysis (current-cost net stock). I subtract the value of intellectual properties to obtain tangible asset value.

[45] I also use supremum Wald and LR tests on the ratio of households' holdings of intermediary debts to total assets and detect a break in the level at 1992. Figure D.4 in Appendix C reports the data.

[46] Data is from Panel C of Figure 3. Figure 2 list the securities that map to deposits in the model.

[47] The short-term interest rates are the real rates with the consumer price index as deflator. The debt instruments

Table 3: Long-Term Trends and Endogenous Financial Risk

Time	Intangible Inv. Share $\mathbb{E}^{\eta}\left[\theta(\eta,t)\right]$	$\frac{\text{Firm Deposits}}{\text{Capital Value}}$ $\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^{E}(\eta,t)}{q^{T}(\eta,t)}\right]$	$\frac{\text{Firm Deposits}}{\text{HH Deposits}}$ $\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^{E}(\eta,t)}{\widetilde{M}^{H}(\eta,t)}\right]$	Interest Rate $\mathbb{E}^{\eta}\left[r\left(\eta,t\right)\right]$	Capital Valuation $\mathbb{E}^{\eta}\left[q^{T}\left(\eta,t\right)\right]$	Financial Risk Multiplier $\max_{\eta}\left\{\frac{\sigma^{T}(\eta,t)+\sigma}{\sigma}\right\}$
$t=0$	55.2%	7.6%	9.8%	3.24%	6.6	2.7
Data '90	54.4%	6.3%	9.6%	3.45%	6.8	
$t=4$	58.7%	8.5%	11.1%	2.11%	6.9	3.2
Data '94	58.2%	7.1%	10.8%	2.59%	6.9	
$t=8$	62.2%	8.2%	10.7%	0.95%	7.3	3.6
Data '98	61.9%	7.9%	12.2%	1.77%	7.3	
$t=12$	65.7%	9.2%	12.2%	-0.20%	7.6	4.0
Data '02	66.0%	8.4%	13.1%	0.97%	7.5	
$t=16$	69.1%	10.2%	13.7%	-1.50%	7.8	4.4
Data '06	69.1%	9.1%	13.8%	0.46%	7.7	
$t=20$	72.6%	10.4%	14.1%	-2.88%	7.9	4.7
Data '10	72.7%	9.7%	14.0%	-0.36%	8.0	

average EV/EBITDA ratio in 1990.[48] Capital generates one unit of goods per year, so $q_t^T = q_t^T/1$ is the ratio of capital value to its annual output. Because tangible capital produces all capitalizable output, its value maps to firms' enterprise value (EV), which is the present value of cash flows reflected in debt and equity markets. The calibration of ρ and δ fixes the starting points of interest rate and capital valuation but leaves their paths over time to be determined by equilibrium forces.

4.2 The Rise of Intangibles and Long-Run Trends

The results are in two categories, the economy's response to a rising κ_t^I over time (trends) and response to shocks, dZ_t (cycles). This subsection focuses on the trends. Table 3 reports how

correspond to the list in Figure 2, which include: (1) jumbo and non-jumbo checking deposits, savings deposits, and certificate of deposits; (2) 1-, 2-, and 3-month AA-rated financial commercial papers; (3) 3- and 6-month bankers acceptance; (4) 1-, 2-, and 3-month AA-rated asset-backed commercial papers; (5) GCF repo rate with Treasury securities, mortgage-backed securities, and agency- and GSE-backed securities as collateral; (6) Fed fund (source: FRED and Federal Reserve Bank of New York).

[48]The average is taken over median EV/EBITDA of 11 Fama-French nonfinanical sectors (Compustat).

the economy evolves over time. According to the calibration of κ_0^I and κ_1^I, the productivity of intangible investment, $\kappa^I(t)$, increases by around 1.6% per year. Firms tilt investment towards intangibles gradually over time, increasing θ_t from 55.2% to 72.7% over twenty years. Column 1 shows the model generates a trend in intangible share of investment that matches data closely in every year. Note that the calibration of κ_1^I targets the average rate of change but does not guarantee the match with data every year. The year-by-year match suggests that the model has a proper specification of intangible investment productivity and a proper mapping from investment productivity to the intangible share through the setup of firms' investment problem.

As θ_t increases, firms face a tighter financial constraint and hold more liquidity. The calibration of λ, the arrival rate of investment needs, targets the response of firms' liquidity-to-tangible asset ratio to variation in θ_t. In Column 2 of Table 3, the trend in $\mathbb{E}^\eta\left[\frac{\widetilde{M}^E(\eta,t)}{q^T(\eta,t)}\right] = \mathbb{E}^\eta\left[\frac{M_t^E}{q_t^T K_t^T}\right]$ captures the well documented rise in firms' cash-to-asset ratio before 2010s. The ratio increased from 6.3% by more than 50% to 9.7% in data. In the model, it started at a higher level, 7.6%, and increased to 10.4%.[49] Later in the counterfactual analysis, I will examine how the economy responds when the rise of intangibles is shut down and the trend in firms' liquidity demand is muted. In this scenario, households' liquidity utility becomes the sole driver behind trends in liquidity demand. Therefore, it is important to match the relative dynamics of firms' vs. households' liquidity holdings in the baseline model. It is done through the calibration of households' liquidity utility as explained in Section 4.1. The results are reported in Column 3 of Table 3.

The rising intangible share of investment, θ_t, drives up the marginal value of liquidity, π_t, by tightening firms' financial constraint. The upward trend in π_t in turn leads to a downward trend in r_t, the yield on liquid assets in Column 4 of Table 3. The bankers take advantage of a lower funding cost and push up tangible capital value, q_t^T. Column 5 reports an upward trend in capital valuation that closely matches the data.[50] Importantly, these trends reinforce each other. A rising q_t^T further increases π_t and thereby lowers r_t (see Proposition 1). Multiplicity may arise due to

[49] The discrepancy in level is due to the omission of other determinants of firms' liquidity holdings that, unlike intangibles, do not exhibit trends over time. This paper focuses on intangible-induced trends.

[50] Tangible capital represents capitalizable production capacity. The ratio of q_t^T to one unit of goods produced per unit of time (one year) maps to EV-to-EBITDA ratio, since, by definition, enterprise value (EV) is the present value of capitalizable output of a firm, reflected in the debt and equity markets.

the feedback effects: A solution has low r_t, high q_t^T, high π_t, and high θ_t while the other has high r_t, low q_t^T, low π_t, and low θ_t. The solution with θ_t closest to data is chosen.[51] Multiplicity helps explain why the rise of intangibles and related trends are largely a U.S. phenomenon.

As κ_t^I increases and the economy becomes more intangible-intensive, it also becomes increasingly fragile. By Itô's lemma, the total value of capitalizable output, $q_t^T K_t^T$, evolves as

$$\frac{d\left(q_t^T K_t^T\right)}{q_t^T K_t^T} = \left(\mu_t^T - \delta - \lambda + \sigma_t^T \sigma\right) dt + \left(\sigma_t^T + \sigma\right) dZ_t\,, \tag{30}$$

A measure of endogenous risk is the ratio of total shock exposure of $q_t^T K_t^T$ (including σ_t^T, the endogenous volatility of q_t^T) to exogenous shock exposure, σ:

$$\textit{Financial Risk Multiplier}: \quad \frac{\sigma_t^T + \sigma}{\sigma}\,. \tag{31}$$

This ratio is a function of t and η_t (see Proposition 3). The last column of Table 3 reports the maximum (over η_t) at $t = 0, 4, ..., 20$. It also reports the corresponding years in data to show the model-implied accumulation of endogenous risk in real time. Over twenty years, the endogenous risk multiplier almost doubled as the economy became increasingly intangible-intensive.

Overall the solution matches data reasonably well except for a lower and more negative r_t in the 2000s. This may be explained by the omission of zero lower bound (ZLB) on nominal rates that binds in reality and, under nominal price rigidity, translates into a lower bound on real rates (Eggertsson and Woodford, 2003; Fischer, 2016; Korinek and Simsek, 2016; Caballero and Simsek, 2020). In fact, the model suggests that the rise of intangibles leads to a strong liquidity demand and thereby widens the wedge between the natural rate without nominal rigidity and the actual rate, exacerbating the liquidity trap at ZLB (Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson and Krugman, 2012; Caballero and Farhi, 2017; Guerrieri and Lorenzoni, 2017). While the rise of intangibles is largely a U.S. phenomenon, the resultant liquidity trap may spread globally (Caballero, Farhi, and Gourinchas, 2021). Appendix C discusses the model mechanism under ZLB

[51] Note that θ_t is still endogenous and optimally chosen by firms. If the firms' investment and liquidity management problems have not been properly specified, none of the solutions is likely to match data.

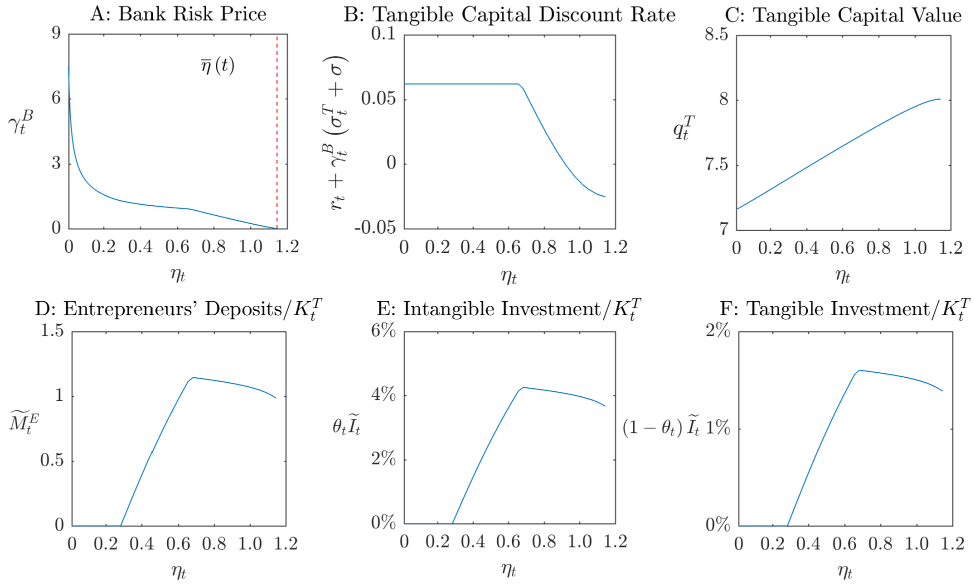

Figure 5: Financial Cycle

and its interactions with the forces in New Keynesian models.

4.3 Endogenous Financial Risk and Economic Fluctuation

This subsection focuses on economic fluctuations along the trend, driven by the intermediation intensity, η_t. Figure 5 plots six endogenous variables against η_t. The plots are for $t = 20$ (which maps to 2010 in data) and end at $\overline{\eta}(t)$, the endogenous upper boundary of η_t beyond which the bankers optimally consume (see Proposition 3). To understand the economy's response to shocks, first consider positive shocks that move η_t to the right. Panel A of Figure 5 plots the bankers' price of risk (or required Sharpe ratio) for holding tangible capital:

$$\gamma_t^B = \frac{\mathbb{E}_t\left[dr_t^T\right] - r_t}{\sigma_t^T + \sigma}, \tag{32}$$

which declines as η_t increases and eventually reaches zero at $\overline{\eta}(t)$. This implies a procyclical intermediation capacity. In Panel B, the discount rate for tangible capital, i.e., the expected return $\mathbb{E}\left[dr_t^T\right]$, is at ρ when η_t is low to clear the market by attracting demand from entrepreneurs and households whose discount rate is ρ. However, as η_t increases, bankers eventually hold all tangible capital and the discount rate falls below ρ. Recall that the cash flow of tangible capital is constant, so what drives the variation of q_t^T is the discount rate. Therefore, as the discount rate declines following positive shocks that increase η_t, the value of tangible capital, q_t^T, increases as shown in Panel C. Note that the increase of q_t^T in η_t is smooth even though the decrease of discount rate in η_t is not. Under rational expectation, q_t^T is forward-looking, so any increase of η_t raises the conditional probability of low discount-rate regions, and therefore, increases q_t^T.

As q_t^T increases, a feedback mechanism emerges. Investment becomes more profitable, and the leverage on liquidity is higher, so holding liquidity is more profitable. Therefore, entrepreneurs accept a lower r_t (Proposition 1), holding more deposits as shown in Panel D of Figure 5.[52] A lower r_t further reduces the bankers' discount rate, leading to an even higher q_t^T. In the process, the entrepreneurs hold more liquidity and invest more as shown in Panels E and F. Note that when scaled by K_t^T, the run-up of entrepreneurs' deposits and investments stops when the growth of the bankers' wealth outpaces that of the tangible capital value (bank asset value). When this occurs, bank equity crowds out debt on the balance sheet, causing a reduction in deposits/tangible capital.

The upward spiral triggered by positive shocks, $dZ_t > 0$, seems benign, featuring a boom of liquidity creation and investment. However, endogenous risk accumulates. Consider a value of η_t near zero in Panel A of Figure 6 (reproducing Panel B of Figure 5). The discount rate stays at ρ with a large probability. However, as we move to the right, η_t approaches the cutoff point where the discount rate falls below ρ. As a result, even small shocks can cause a large discount-rate change and variation of q_t^T. Therefore, q_t^T becomes more sensitive to shocks (i.e., higher σ_t^T) as η_t moves to the right. This explains why in Panel B of Figure 6, the risk multiplier, $(\sigma_t^T + \sigma)/\sigma$, is increasing in η_t. The amplification becomes stronger as booms prolong, so negative shocks trigger vicious downward spiral.[53] The mechanism eventually subdues as η_t approaches its upper bound where bankers are sufficiently rich and the sensitivity of discount rate to η_t diminishes.

The accumulation of endogenous risk in booms is asymmetric. Positive shocks trigger the

[52] When $\eta_t < 0.28$ (1.7% probability), $M_t^E = 0$ and $r_t < \rho - \lambda\pi_t$ (i.e., (15) no longer holds). r_t is solved by equating households' demand and bankers' supply. See Appendix A.2

[53] This mechanism offers a new explanation of the findings that long periods of banking expansion often precede severe crises (e.g., Jordà, Schularick, and Taylor, 2013; Baron and Xiong, 2017).

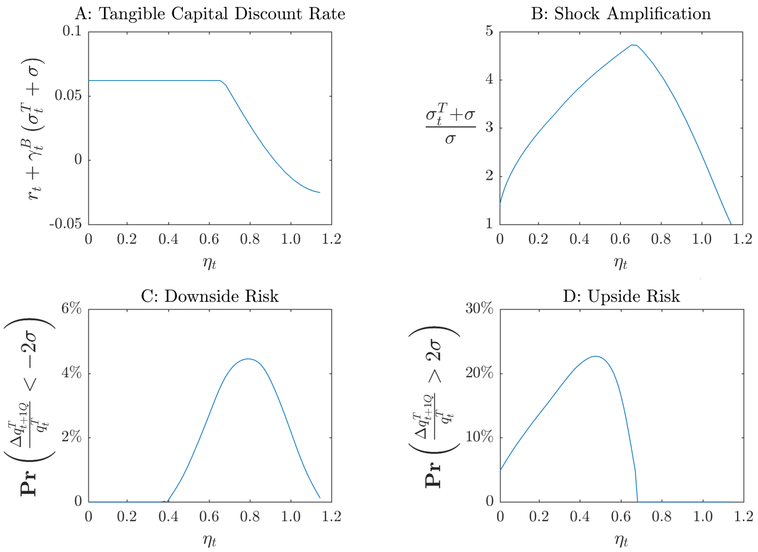

Figure 6: Endogenous Financial Risk

reallocation of tangible capital to bankers with low discount rates but eventually cause them to consume wealth at $\overline{\eta}(t)$; in contrast, negative shocks cause a continuing reallocation of tangible capital away from bankers. Panels C and D plot respectively the probabilities of a 2σ decrease and a 2σ increase of q_t^T in one quarter.[54] Note that at sufficiently low (high) values of η_t, a further decrease (increase) by 2σ is impossible as it goes beyond the equilibrium range of q_t^T. Following positive shocks, the probability of a drop in q_t^T increases as η_t increases. It eventually declines as shock amplification weakens (Panel B). The probability of an increase in q_t^T also rises but declines earlier, suggesting that risk accumulation is downward biased. Following negative shocks, the economy moves leftward. The downside risk in q_t^T rises in Panel C, while the upside risk is relatively insensitive in Panel D. This offers a new explanation of why downside risks rise faster than upside risks as financial conditions deteriorate (Adrian, Boyarchenko, and Giannone, 2019).

4.4 Counterfactual Analysis

I construct two hypothetical scenarios to examine the quantitative importance of the rise of intangibles. In *Intan. Trend*, the increase of intangible investment productivity is kept while the trend

[54]Given the model solution, these probabilities can be calculated using the Feynman–Kac PDEs.

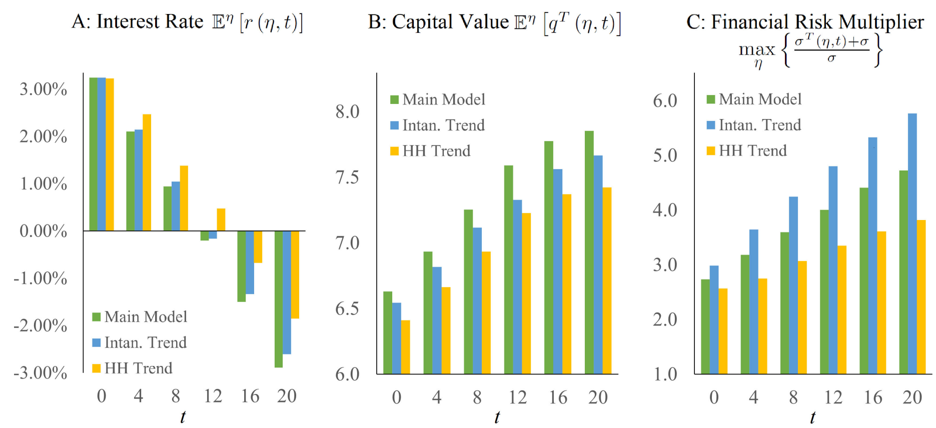

Figure 7: Counterfactual Analysis

in households' liquidity demand is muted (i.e., $\beta_1 = 0$ and $\beta_2 = 0$). In *HH Trend*, the increase of intangible investment productivity is muted (i.e., $\kappa_1^I = 0$) while the trend in households' liquidity demand remains. *HH Trend* sets a benchmark of the literature on households' liquidity demand and its implications on interest rate, asset price, and financial instability (Kiyotaki and Moore, 2000; Krishnamurthy and Vissing-Jørgensen, 2015; Piazzesi and Schneider, 2016; Moreira and Savov, 2017; Begenau and Landvoigt, 2018; Van den Heuvel, 2018; Begenau, 2019; Egan, Lewellen, and Sunderam, 2021).

Panel A and B of Figure 7 show respectively the trends of interest rate, $\mathbb{E}^{\eta}[r(\eta,t)]$, and asset price (tangible capital value), $\mathbb{E}^{\eta}\left[q^{T}(\eta,t)\right]$ for the three scenarios. A common pattern emerges: Removing the trend in intangibles (*HH Trend*) moderates the downward trend in interest rate and upward trend asset price more than removing the trend in households' liquidity demand (*Intan. Trend*) does. This suggests the trend in firms' demand for liquid assets driven by the rise of intangibles is a more potent force than households' liquidity demand.

The greater quantitative importance of firms' liquidity demand seems puzzling given the fact that firms' liquidity holdings are only 1/7 that of households by $t = 20$ and 1/10 at $t = 0$ both in the baseline model and data. This observation ignores the fact that once the trend in households' liquidity needs is removed, the firms' liquidity holdings will increase in equilibrium and rise faster over time in the absence of households' competition. The counterfactual, *Intan. Trend*, does not and should not fix the equilibrium level of firms' liquidity holdings to that of the main model. What should be fixed are the parameters underlying firms' liquidity management problems; likewise, in *HH Trend*, households' liquidity holdings increase in the absence of firms' competition as the rise of intangibles is shut down while parameters in households' liquidity utility are fixed.

In Panel C of Figure 7, *Intan. Trend* generates the most endogenous risk, the main model the second highest, and *HH Trend* the lowest, and the wedges widen over time as the different trends in the three models unfold. This finding is particularly interesting because one would have expected the main model to generate the most endogenous risk by having both firms' and households' liquidity needs trending up over time and feeding leverage to bankers. The key to understanding this result is the distinct cyclical property of firms' and households' liquidity demand. Consider positive shocks. The subsequent increase in q_t^T encourages the firms to save more as investment becomes more profitable and the leverage on liquidity holdings, backed by tangible capital, increases. The increase of liquidity value, π_t, drives down r_t. As r_t declines, the households' liquidity holdings decrease, counteracting the increase in firms' liquidity demand (see (22)). Following negative shocks, the opposite happens: q_t^T and π_t decline, resulting in a higher r_t that induces households to hold more liquidity, counteracting the decrease in firms' demand. In sum, firms' liquidity demand exhibits procyclicality, while the households' demand features countercyclicality.[55] In the main model, the two forces act against each other, while in *Intan. Trend*, there is only an upward trend in firms' demand for liquid assets so its procyclicality is fully unleashed.

Section 2 provides evidence that firms' liquidity demand increases in asset valuation, i.e., the procyclicality key to the quantitative importance of intangible-driven liquidity needs. Next, I show that empirically, households' demand for liquid assets decreases in measures of asset valuation, counteracting the procyclicality in firms' liquidity demand as in the model. For time-series regressions in Panel A of Table 4, the dependent variable is quarterly household holdings of intermediary debts scaled by GDP from 1980 to 2019.[56] The explanatory variables are measures of capital valuation (see Section 2) and housing price-to-rent ratio. Summary statistics are reported

[55]The countercycality is in line with flight to safety in crises (Caballero and Krishnamurthy, 2008).

[56]The data source is the Financial Accounts of the U.S. Intermediary debts are listed in Figure 2 and indirect holdings via money-market funds and mutual funds are attributed to underlying securities.

Table 4: Asset Valuations and Household Holdings of Intermediaries' Debts

***Panel A**: Regression Analysis of Aggregate Data*

LHS: HH Holdings of Intermediary Debts scaled by GDP	(1)	(2)	(3)	(4)	(5)	(6)
RHS: Financial-Market Valuation Metrics =	Tangible EV/EBITDA	Average EV/EBITDA	Tangible Tobin's Q	Average Tobin's Q		Tangible EV/EBITDA
Financial-Market Valuation	-0.017***	-0.010***	-0.190***	-0.095***		-0.016***
	(0.002)	(0.002)	(0.019)	(0.012)		(0.002)
Housing-Market Valuation (Price/Rent)					-0.142***	-0.060**
					(0.024)	(0.024)
Observations	160	160	160	160	160	160
Adjusted R^2	0.3015	0.1953	0.2880	0.2456	0.0771	0.3138

Heteroscedasticity-consistent standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

***Panel B**: Regression Analysis of Micro Data*

LHS: HH Cash Holdings scaled by Income	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta \ln$ (Housing Price Index)	-0.059	-0.119***	-0.114***	-0.046	-0.086***	-0.081**
	(0.045)	(0.040)	(0.040)	(0.037)	(0.031)	(0.039)
Controls	No	No	No	Yes	Yes	Yes
Household FE	No	Yes	Yes	No	Yes	Yes
State FE	No	No	Yes	No	No	Yes
Year FE	No	No	Yes	No	No	Yes
Observations	70,442	70,032	70,032	65,280	65,215	65,215
Adjusted R^2	0.0001	0.2389	0.2495	0.1370	0.3438	0.3510

State-time clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

in Appendix D. Column (6) shows that financial-market and housing valuations together explain 31% of variation.[57]

The analysis of aggregate data has a small sample size and does not utilize cross-sectional variations. Next, I use household-level micro data. The financial-market valuation metrics do not have regional variation and thus excluded. The Panel Study of Income Dynamics (PSID) reports

[57]This is the ratio of two time series in FRED: (1) All-Transactions House Price Index for the U.S.; (2) Consumer Price Index for Urban Consumers: Rent of Primary Residence in U.S. City Average.

biannual information on households' financials from 1999 to 2017.[58] The dependent variable is the liquidity holdings normalized by household income. The explanatory variable of interest is the log difference of state-level home price index from the Federal Housing Finance Agency (FHFA). Rent data are unavailable so the log difference is taken to address apparent non-stationarities in these house prices. Panel B of Table 4 reports a statistically significant negative response of households' liquidity holdings to an increase in house prices, robust to different combinations of control variables and fixed effects (FE).[59] Including control variables and fixed effects increases the adjusted R^2 to above 34% (in Columns (5) and (6)) by reducing noise, allowing the correlation to emerge between households' liquidity holdings and housing price variation. The evidence suggests that in line with the model setup, households' liquidity holdings respond negatively to asset-price increase, opposite to the positive response in firms' liquidity holdings (see Section 2).

5 Extension: Intangible Capital of Limited Pledgeability

The model is built upon the limited pledgeability of intangibles. Panel regressions in Column (1) and (4) of Table 5 show that more intangible firms borrow less, which indicates tighter credit constraints. The sample is from Section 2.[60] Credit relies on collateral or creditors' contractual rights to cash flows (Lian and Ma, 2020). For different measures of intangibility, Columns (2) and

[58] Liquidity holdings include checking/savings deposits, money market funds, certificates of deposit, Treasury securities (not including I.R.A.). A breakdown into instruments issued by intermediaries and the government is unavailable, but as shown in Figure D.3 in Appendix C, Treasury securities account for less than 15%. Related, to analyze households' mortgage refinancing behavior, Chen, Michaux, and Roussanov (2020) use data from Financial Accounts of the U.S. for time-series analysis and PSID (including households' liquidity holdings) for panel-data analysis. The regression samples starts in 2001 because the calculation of log difference requires housing price.

[59] Following studies on household consumption-savings decisions and portfolio allocation (Bergstresser and Poterba, 2004; Campbell and Cocco, 2007; Bogan, 2015; Chetty, Sándor, and Sziedl, 2017; Stroebel and Vavra, 2019), I construct the following control variables using PSID data: the log difference of total household income, the log difference of total household wealth, the number of people in a household, the age of household head, the education level of household head, a homeowner dummy, and a couple dummy (equal to one if the household head lives with a partner). I consider household, state, and year fixed effects. Note that the number of observations decline after household FE is added because 65 households only appear once in the panel. Appendix C provides summary statistics.

[60] Control variables are included following Lian and Ma (2020) who share their loan categorization data: Size (log total assets in 2005 dollars); market-to-book ratio; cash-to-asset ratio; EBITDA-to-asset ratio ([sale – cogs – xsga]/at); net cash receipts-to-asset ratio ([oancf + xint]/at); inventory-to-asset ratio (invt/at). Time fixed effects are added to absorb common variations, such as tax and regulatory changes.

Table 5: Intangible Capital and Credit Constraint

	Intangibility = Intan./Assets (decile)			Intangiblity = – PPE/Assets (decile)		
	(1)	(2)	(3)	(4)	(5)	(6)
Leverage = $\frac{\text{Debts}}{\text{Assets}}$	Total Debts	Asset-Based Loans	Cash Flow-Based Loans	Total Debts	Asset-Based Loans	Cash Flow-Based Loans
Intangibility	-1.219***	-0.745***	-0.715***	-0.914***	-0.728***	-0.158
	(0.092)	(0.083)	(0.197)	(0.090)	(0.076)	(0.118)
Controls	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	114,626	39,750	39,819	114,608	39,749	39,818
Adjusted R^2	0.2159	0.0891	0.1298	0.2116	0.0934	0.1263

Firm-year clustered standard errors in parentheses
$*\ p < 0.1$ $**\ p < 0.05$ $***\ p < 0.01$

(5) and Columns (3) and (6) show that intangible firms are disadvantaged on both fronts.

As the U.S. economy becomes more intangible-intensive, the legal system develops to improve the pledgeability of intangibles. This section presents an extension: When hit by the Poisson shock, an entrepreneur may raise funds from households against χ fraction of intangible capital as collateral.[61] The repayment is in the form of intangible capital ownership.[62] Equivalently, the entrepreneur may sell intangible capital rather than pledge it as collateral. It is assumed that bankers do not lend against intangibles or own intangibles.[63] In practice, intangibles are mainly financed by non-bank intermediaries (e.g., venture capital funds).

The improved pledgeability of intangibles relax the funding constraint:

$$i_t \leq m_t^E + q_t^T \kappa^T (1 - \theta_t) i_t + \chi \left(q^I \kappa_t^I \theta_t i_t \right). \tag{33}$$

The calibration of χ is based on the percentage of marketable intangibles. Among different cate-

[61]Financing for intangibles often comes from venture capital funds (VC). Akcigit, Dinlersoz, Greenwood, and Penciakova (2019) examine the role of VC in creating endogenous growth.

[62]Alternatively, the entrepreneur can promise to repay all the future goods produced by the intangible, which have the same present value as the intangible capital itself. Given risk-neutral preference, households are indifferent between owning intangible capital now or owning the steam of goods.

[63]Intangible capital is still less liquid than tangible capital due to search friction in patent trading (Akcigit, Celik, and Greenwood, 2016): (1) the market is specialized (often involving lawyers as middlemen); (2) the sensitivity of intellectual property makes potential participants reluctant to reveal information.

Table 6: Pledgeable Intangibles and the Reinforcing Trends

Time	Intangible Inv. Share $\mathbb{E}^{\eta}\left[\theta(\eta,t)\right]$	$\frac{\text{Firm Deposits}}{\text{Capital Value}}$ $\mathbb{E}^{\eta}\left[\frac{\widetilde{M}^{E}(\eta,t)}{q^{T}(\eta,t)}\right]$	Interest Rate $\mathbb{E}^{\eta}\left[r\left(\eta,t\right)\right]$	Capital Valuation $\mathbb{E}^{\eta}\left[q^{T}\left(\eta,t\right)\right]$	Financial Risk Multiplier $\max_{\eta}\left\{\frac{\sigma^{T}(\eta,t)+\sigma}{\sigma}\right\}$
Model $t=0$	55.2%	7.6%	3.24%	6.6	2.7
Pledgeable Intan.	57.3%	27.3%	2.58%	7.8	3.6
Model $t=4$	58.7%	8.5%	2.11%	6.9	3.2
Pledgeable Intan.	61.8%	30.4%	1.06%	8.5	4.4
Model $t=8$	62.2%	8.2%	0.95%	7.3	3.6
Pledgeable Intan.	66.6%	33.2%	-0.64%	9.2	5.2
Model $t=12$	65.7%	9.2%	-0.20%	7.6	4.0
Pledgeable Intan.	71.6%	36.2%	-2.59%	9.9	6.0
Model $t=16$	69.1%	10.2%	-1.50%	7.8	4.4
Pledgeable Intan.	76.9%	38.8%	-4.81%	10.4	6.8
Model $t=20$	72.6%	10.4%	-2.88%	7.9	4.7
Pledgeable Intan.	82.7%	42.1%	-7.32%	10.7	7.6

gories of intangible capital, intellectual properties have relatively clear market value (around 16% of patents according to Akcigit, Celik, and Greenwood (2016)). Intellectual properties accounted for 37.7% of intangible investment in the U.S. (Corrado et al., 2016). Therefore, χ is calibrated to be $6.0\% = 37.7\% \times 16\%$. This value is in the same magnitude as the value implied by the findings in Mann (2018): 38% of US patenting firms had previously pledged patents as collateral for financing, and these firms account for 20% of R&D expense and patenting in Compustat, so $\chi = 38\% \times 20\% = 7.6\%$.

Table 6 shows that the improved pledgeability of intangibles amplifies the mechanism. The intangible share of investment is higher, and its increase over time becomes convex. In contrast, the main model produces a linear trend. The difference widens from 2.1% at $t = 0$ to 10.1% by $t = 20$. The improved pledgeability of intangibles increases the leverage on liquidity holdings and the marginal value of liquidity. The feedback mechanism is strengthened, resulting in a much higher level and faster growth of entrepreneurs' liquidity holdings, a sharper decline of the interest rate, and a stronger upward trend in the value of tangible capital. The financial risk multiplier is higher than that of the main model as shown in the last column of Table 6. A lower level of the interest rate widens the discount-rate wedge between bankers and the rest of the economy, making the value of tangible capital more sensitive to shocks that trigger reallocation between the two groups. The concave upward trend in financial risk multiplier in the main model becomes a linear trend once intangibles become more pledgeable. A more volatile tangible capital value translates into more volatile liquidity creation and investment.

6 Conclusion

The transition towards an intangible-intensive economy has a profound impact on financial system. This paper provides a coherent account of several trends in the U.S. that emerged from the rise of intangibles, such as the accumulation of corporate cash holdings, growth of financial intermediation sector, declining interest rate, and rising valuation of risky assets. At the core of the model is the endogenous supply and demand for liquid assets. To finance intangible investment, firms hold cash in the form of financial intermediaries' debts. Firms' growing demand for liquid assets, driven by intangible investment needs, pushes down the interest rate and feeds cheap leverage to intermediaries, allowing intermediaries to bid up the market value of collateral assets that back debt issuances. The model characterizes a self-enforcing mechanism that connects these trends, and the feedback mechanism also amplifies economic fluctuations along the trends.

An interesting direction for future research is to incorporate nominal frictions. The savings glut leads to a negative real rate in the model that, under low inflation, implies a binding lower bound on the nominal rates. Therefore, the rise of intangibles exacerbates the liquidity trap (Eggertsson and Woodford, 2003; Fischer, 2016; Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson and Krugman, 2012; Caballero and Farhi, 2017; Guerrieri and Lorenzoni, 2017; Caballero and Simsek, 2020). A liquidity trap in one country can spread to the rest of the world (Caballero, Farhi, and Gourinchas, 2021), so the U.S. economy becoming more intangible-intensive has broader implications on the global financial system.

The interaction between industrial structure and financial system deserves more attention in future research. While this paper focuses on how the transition to an intangible-intensive economy affects financial system, a growing financial sector may also affect industrial structure. When financial intermediaries becomes more productive, it extends more credit (asset side of balance sheets) and issues more money-like securities (liability side of balance sheets). The former facilitates tangible investment, which is often credit-financed, while the latter facilitates intangible investment by providing firms with storage of internal funds. A bias in productivity improvement towards the liability side contributes to faster growth of intangible capital. This seems to be the case in the run-up to the global financial crisis, as the development of shadow banking allows more effective creation of near-money assets that firms hold via money market funds and other vehicles.

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Online Appendices

A A. Proofs and Solution Algorithm

A.1 Proofs

Ruling out self-financing. If entrepreneurs' investment projects can be self-financed, entrepreneurs do not need to hold liquidity for investment and the liquidity premium is zero. The equilibrium value of tangible capital is the production value, i.e., $1/(\rho+\delta+\lambda)$. If Assumption 1 holds, then even if entrepreneurs set the intangible share of investment, θ_t, to zero, the external financing capacity, $\kappa^T q_t^T = \kappa^T \left(\frac{1}{\rho+\delta+\lambda}\right)$ is still below 1, which is the cost of investment. This contradicts that investment is self-financed. Therefore, under Assumption 1, the investment project cannot be self-financed.

Proof of Proposition 3. First, I show that there exists an upper bound $\overline{\eta}(t)$ such that $\eta_t \leq \overline{\eta}(t)$. Note that $q_t^B \geq 1$ in equilibrium because if $q_t^B < 1$, bankers are better off consuming (worth 1) than retaining wealth (worth q_t^B). As will be shown later, q_t^B is a bivariate function, $q_t^B = q^B(\eta_t, t)$. Fixing t, let $\overline{\eta}(t)$ denote that lowest value of η_t where bankers consume. Therefore, $q^B(\overline{\eta}(t), t) = 1$ and $q_t^B > 1$ at $\eta_t < \overline{\eta}(t)$. Suppose there exists $\eta' > \overline{\eta}(t)$ such that η_t reaches η'. This leads to a contradiction – it is no longer optimal for bankers to consume at $\overline{\eta}(t)$ because their marginal value of wealth will surely increase: at $\overline{\eta}(t)$, if η_t increases, q_t^B will not decline because $q_t^B \geq 1$, and if η_t decreases, q_t^B will surely increase because, by definition of $\overline{\eta}(t)$, $q_t^B > 1$ for $\eta_t < \overline{\eta}(t)$. Therefore, η_t cannot increase beyond $\overline{\eta}(t)$, the upper boundary given by bankers' consumption optimality.

Next, I derive the law of motion of η_t in $(0, \overline{\eta}(t))$. According to (10), bankers' wealth satisfies the following law of motion in the region where bankers' consumption is zero, i.e., $\eta_t \in (0, \overline{\eta}(t))$:

$$\frac{dN_t^B}{N_t^B} = \mu_t^N dt + \sigma_t^N dZ_t \,, \tag{A.1}$$

where

$$\mu_t^N = r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T\right] - r_t\right) \,, \tag{A.2}$$

and

$$\sigma_t^N = x_t^B \left(\sigma_t^T + \sigma\right) \,. \tag{A.3}$$

The expression of expected return of tangible capital holdings, $\mathbb{E}_t \left[dr_t^T\right]$, can be obtained from (9). By Itô's lemma, the law of motion of η_t is given by

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dZ \,, \tag{A.4}$$

where

$$\mu_t^{\eta} = \mu_t^N - \mu_t^{KT} - \sigma_t^N \sigma + \sigma^2 , \tag{A.5}$$

(where μ_t^{KT} is the expected instantaneous growth rate of K_t^T) and

$$\sigma_t^{\eta} = x_t^B \left(\sigma_t^T + \sigma \right) - \sigma . \tag{A.6}$$

According to (26), the expected instantaneous growth rate of K_t^T is given by

$$\begin{aligned} \mu^{KT} &= \frac{\left(\frac{1}{1 - q_t^T \kappa^T (1-\theta_t)} \right) \left[\left(x_t^B - 1 \right) N_t^B - M_t^H \right] (1-\theta_t) \kappa^T \lambda}{K_t^T} - \delta \\ &= \left(\frac{1}{1 - q_t^T \kappa^T (1-\theta_t)} \right) \left[\left(x_t^B - 1 \right) \eta_t - \alpha \left(\frac{\rho - r_t}{\beta(t)} \right)^{-\frac{1}{\xi}} \right] (1-\theta_t) \kappa^T \lambda - \delta , \end{aligned} \tag{A.7}$$

where the second equation uses the definition of η_t and households' aggregate deposit demand given by (22). In A.2, q_t^T, r_t, x_t^B, θ_t, $\mathbb{E}_t \left[dr_t^T \right]$, σ_t^T, and the rest of variables in Proposition 3 are shown to be bivariate functions of η_t and t.

Proof of Proposition 1. First, I solve the investment problem of entrepreneurs who are hit by the Poisson shocks, and then embed the solution to the entrepreneurs' dynamic optimization. An investing entrepreneur solves the problem summarized by the Lagrange function (11):

$$\mathcal{L} = \max_{\{i_t, \theta_t\}} \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T (1-\theta_t) - F(\theta_t) \right] i_t - i_t + \pi_t \left[m_t^E + q_t^T \kappa^T i_t (1-\theta_t) - i_t \right] . \tag{A.8}$$

Given κ_t, q_t^T, m_t^E, q^I and κ^T, the entrepreneur chooses θ_t and i_t. The first-order condition (F.O.C.) for θ_t is

$$q^I \kappa_t^I - q_t^T \kappa^T (1+\pi_t) - F'(\theta_t) = 0 , \tag{A.9}$$

and the F.O.C. for i_t is (i.e., (13) in the main text)

$$\pi_t = \left\{ \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T (1-\theta_t) - F(\theta_t) \right] - 1 \right\} \left(\frac{1}{1 - q_t^T \kappa^T (1-\theta_t)} \right) \tag{A.10}$$

The F.O.C. for θ_t equates the marginal value of investing in intangibles and the marginal value of investing in tangibles (which includes both the value of tangible capital and the shadow value from relaxing the liquidity constraint). The F.O.C. for i_t solves the marginal value of liquidity as equal to the net profits of investment multiplied by the leverage on liquidity holdings. The liquidity constraint binds so the total investment is given by

$$i_t = \frac{m_t^E}{1 - (1-\theta_t) \kappa^T q_t^T} . \tag{A.11}$$

Next, I prove that θ_t is increasing in κ^I. First, note that, from (A.10),

$$\begin{aligned}\frac{\partial \pi_t}{\partial \theta_t} &= \left[q^I \kappa_t^I - q_t^T \kappa^T - F'\left(\theta_t\right)\right] \left(\frac{1}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)}\right) \\ &\quad - \left\{\left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T \left(1 - \theta_t\right) - F\left(\theta_t\right)\right] - 1\right\} \frac{q_t^T \kappa^T}{\left[1 - q_t^T \kappa^T \left(1 - \theta_t\right)\right]^2} \\ &= \frac{q_t^T \kappa^T \pi_t}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} - \frac{q_t^T \kappa^T \pi_t}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} = 0\,,\end{aligned} \tag{A.12}$$

where the second equation follows from (A.9) and (A.10). Differentiating (A.9) with respect to (w.r.t.) κ_t^I, I obtain

$$q^I - q_t^T \kappa^T \frac{\partial \pi_t}{\partial \theta_t} \frac{\partial \theta_t}{\partial \kappa_t^I} - q_t^T \kappa^T \frac{\partial \pi_t}{\partial \kappa_t^I} - F''\left(\theta_t\right) \frac{\partial \theta_t}{\partial \kappa_t^I} = 0\,. \tag{A.13}$$

Rearranging the equation and using (A.12), I solve

$$\frac{\partial \theta_t}{\partial \kappa_t^I} = \frac{q^I - q_t^T \kappa^T \frac{\partial \pi_t}{\partial \kappa_t^I}}{F''\left(\theta_t\right)}\,. \tag{A.14}$$

According to (A.10), the partial derivative of π_t w.r.t. κ_t^I is

$$\frac{\partial \pi_t}{\partial \kappa_t^I} = \frac{q^I \theta_t}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)}\,. \tag{A.15}$$

Using this equation to substitute out $\frac{\partial \pi_t}{\partial \kappa_t^I}$ in (A.14), I obtain

$$\frac{\partial \theta_t}{\partial \kappa_t^I} = \frac{1}{F''\left(\theta_t\right)} \left[q^I - q_t^T \kappa^T \frac{q^I \theta_t}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)}\right] = \frac{q^I \left(1 - q_t^T \kappa^T\right)}{F''\left(\theta_t\right) \left[1 - q_t^T \kappa^T \left(1 - \theta_t\right)\right]}\,. \tag{A.16}$$

In equilibrium, $q_t^T \kappa^T$ must be smaller than 1, because otherwise the entrepreneur sets $\theta_t = 0$ (i.e., investing all in tangible capital) and self-finances the project to achieve infinite profits. Therefore, the right side of (A.16) is positive, i.e., θ_t is increasing in κ_t^I.

The right side of (A.15) is positive, so π_t is increasing in κ_t^I. Finally, I prove that π_t is increasing in q_t^T. Differentiating (A.10) w.r.t. q_t^T, I obtain

$$\begin{aligned}\frac{\partial \pi_t}{\partial q_t^T} =& \frac{\kappa^T \left(1 - \theta_t\right)}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} - \left\{\left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T \left(1 - \theta_t\right) - F\left(\theta_t\right)\right] - 1\right\} \frac{\left[-\kappa^T \left(1 - \theta_t\right)\right]}{\left[1 - q_t^T \kappa^T \left(1 - \theta_t\right)\right]^2} \\ =& \frac{\kappa^T \left(1 - \theta_t\right)}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} + \pi_t \frac{\kappa^T \left(1 - \theta_t\right)}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} = \frac{\kappa^T \left(1 - \theta_t\right)}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} \left(1 + \pi_t\right) > 0\,.\end{aligned} \tag{A.17}$$

Next, I solve (15), i.e., the optimality condition for entrepreneurs' optimal liquidity holdings. Entrepreneurs maximize the life-time utility, $\mathbb{E}\left[\int_{t=0}^{+\infty} e^{-\rho t} dc_t^E\right]$ given the following law of motion of wealth:

$$dw_t^E = -dc_t^E + \mu_t^w w_t^E dt + \sigma_t^w w_t^E dZ_t + \left(\widehat{w}_t^E - w_t^E\right) dN_t,$$

$\mu_t^w w_t^E$ and $\sigma_t^w w_t^E$ are the drift and diffusion terms that depend on choices of tangible capital and deposit holdings and will be elaborated later. dN_t is the increment of the idiosyncratic counting (Poisson) process. At the Poisson time, an entrepreneur's wealth jumps by the total profits from investment minus the value of lost tangible capital holdings (denoted by k^{TE}),

$$\begin{aligned}\widehat{w}_t^E - w_t^E =& \left\{\left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T \left(1-\theta_t\right) - F\left(\theta_t\right)\right] - 1\right\} \left(\frac{1}{1 - q_t^T \kappa^T \left(1-\theta_t\right)}\right) m_t^E - q_t^T k_t^{TE} \\ =& \pi_t m_t^E - q_t^T k_t^{TE} .\end{aligned} \tag{A.18}$$

Note that w_t^E does not contain the existing stock of intangible capital, because when analyzing entrepreneurs' decisions, the production flows from intangible capital can be treated as goods that are directly consumed, given entrepreneurs' indifference in the timing of consumption.

I conjecture that the value function is linear in wealth w_t^E: $v_t^E = \zeta_t^E w_t^E + v^I$, where ζ_t^E is the marginal value of liquid wealth (i.e., without counting the value of intangible capital), and v^I is the present value of consumption from intangible capital. Consider a generic equilibrium diffusion process for ζ_t^E:

$$d\zeta_t^E = \zeta_t^E \mu_t^\zeta dt + \zeta_t^E \sigma_t^\zeta dZ_t,$$

where $\zeta_t^E \mu_t^\zeta$ and $\zeta_t^E \sigma_t^\zeta$ are the drift and diffusion terms, respectively. Entrepreneurs' marginal value of wealth, ζ_t^E, is a summary statistic of their investment opportunity set, which depends on the overall industry dynamics, so it does not jump when an individual is hit by the Poisson shocks.

Under this conjecture, the Hamilton-Jacobi-Bellman (HJB) equation is

$$\rho \zeta_t^E w_t^E dt = \max_{dc^E, k_t^{TE}, m_t^E} dc_t^E - \zeta_t^E dc_t^E + \{w_t^E \zeta_t^E \mu_t^\zeta + w_t^E \zeta_t^E \mu_t^w + w_t \zeta_t^E \sigma_t^\zeta \sigma_t^w + \lambda \zeta_t^E \left[\widehat{w}_t - w_t\right]\} dt .$$

Note that the consumption flow from intangible capital and $\rho v^I dt$ cancel each other out, because, by definition, v^I is the ρ-discounted present value of consumption flow.

Entrepreneurs can choose any $dc_t^E \in \mathbb{R}$, so ζ_t^E must be equal to one, and thus, I have also confirmed the value function conjecture. Since ζ_t^E is a constant equal to one, μ_t^ζ and σ_t^ζ are both zero. The HJB equation can be simplified:

$$\rho \zeta_t^E w_t^E dt = \max_{k_t^{TE} \geq 0, m_t^E \geq 0} \mu_t^w w_t^E dt + \lambda dt \left(\pi_t m_t^E - q_t^T k_t^{TE}\right) . \tag{A.19}$$

Wealth drift includes production, the value change of tangible capital holdings, and the deposit

return:
$$\mu_t^w w_t^E dt = \underbrace{k_t^{TE} dt + \mathbb{E}_t \left(q_{t+dt}^T k_{t+dt}^{TE} - q_t^T k_t^{TE} \right)}_{\mathbb{E}_t \left[dr_t^T \right] q_t^T k_t^{TE}} + r_t m_t^E dt.$$

Let $d\psi_t^E$ denote the Lagrange multiplier of the budget constraint, $q_t^T k_t^{TE} + m_t^E \leq w_t^E$. The first-order condition (F.O.C.) for optimal deposit holdings per unit of capital is: $m_t^E \geq 0$, and

$$m_t^E \left(r_t dt + \pi_t \lambda dt - d\psi_t^E \right) = 0.$$

The F.O.C. for optimal tangible capital holdings is : $k_t^{TE} \geq 0$, and

$$k_t^{TE} \left(-\mathbb{E}_t \left[dr_t^T \right] + d\psi_t^E \right) = 0.$$

Substituting these optimality conditions into the HJB equation, we have

$$\rho v_t^E dt = w_t^E d\psi_t^E .$$

Because $\zeta_t^E = 1$, $v_t^E = w_t^E$, and $d\psi_t^E = \rho dt$. Substituting $d\psi_t^E = \rho dt$ into the F.O.C. for m_t^E, we have

$$\rho - r_t = \lambda \pi_t .$$

Substituting $d\psi_t = \rho dt$ into the F.O.C. for k_t^{TE} and rearranging the equation, we have

$$\mathbb{E}_t \left[dr_t^T \right] = \rho dt \, ,$$

that is, when entrepreneurs hold tangible capital, they require an expected return of ρ.

Binding liquidity constraint. Consider the following inequalities:

$$\max_{\theta_t} \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T \left(1 - \theta_t\right) - F\left(\theta_t\right) \right] \geq q^I \kappa_t^I - F\left(1\right) \geq q^I \kappa_0^I - F\left(1\right) \; ,$$

where the first step follows $q_t^T \geq q^I$ (due to the additional liquidity value of tangible capital) and the optimality of θ_t, and the second step follows from $\kappa_t^I \geq \kappa_0^I$. Therefore, as long as

$$q^I \kappa_0^I - F\left(1\right) > 1 \, , \tag{A.20}$$

we have

$$\begin{aligned} \pi_t &= \max_{\theta_t} \left\{ \left[q^I \kappa_t^I \theta_t + q_t^T \kappa^T \left(1 - \theta_t\right) - F\left(\theta_t\right) \right] - 1 \right\} \left(\frac{1}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} \right) \\ &\geq \left[q^I \kappa_0^I - F\left(1\right) - 1 \right] \left(\frac{1}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} \right) > 0 \end{aligned}$$

and the liquidity constraint binds. Note that $\left(\frac{1}{1-q_t^T\kappa^T(1-\theta_t)}\right) > 0$ from Assumption 1. The calibrated parameter values satisfy the condition given by (A.20).

Proof of Proposition 2. Conjecture that the bank's value function takes the linear form: $v_t^B = q_t^B n_t^B$. Consider the following generic equilibrium diffusion process for q_t^B,

$$dq_t^B = q_t^B \mu_t^B dt - q_t^B \gamma_t^B dZ_t.$$

Define $dy_t^B = dc_t^B / n_t^B$, the consumption-to-wealth ratio of bankers. Under the conjectured functional form, the HJB equation is

$$\begin{aligned} \rho v_t^B dt &= \max_{dy_t^B} \left\{ \left(1 - q_t^B\right) \mathbb{I}_{\{dy_t^B > 0\}} n_t^B dy_t^B \right\} + \mu_t^B q_t^B n_t^B + \\ &\quad \max_{x_t^B} \left\{ r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T \right] - r_t \right) - x_t^B \gamma_t^B \left(\sigma_t^T + \sigma \right) \right\} q_t^B n_t^B, \end{aligned}$$

Dividing both sides by $q_t^B n_t^B$, n_t^B is eliminated, which confirms the homogeneity property,

$$\rho = \max_{dy_t^B} \left\{ \frac{\left(1 - q_t^B\right)}{q_t^B} \mathbb{I}_{\{dy_t^B > 0\}} dy_t^B \right\} + \mu_t^B + \max_{x_t^B} \left\{ r_t + x_t^B \left(\mathbb{E}_t \left[dr_t^T \right] - r_t \right) - x_t^B \gamma_t^B \left(\sigma_t^T + \sigma \right) \right\}, \tag{A.21}$$

and the conjecture of linear value function. The indifference condition for x_t^B is

$$\mathbb{E}_t \left[dr_t^T \right] = r_t + \gamma_t^B \left(\sigma_t^T + \sigma \right) . \tag{A.22}$$

Substituting the expression of $\mathbb{E}_t \left[dr_t^T \right]$ given by (9) and using (15), I obtain (18).

Substituting the optimality conditions into the HJB equation, I obtain

$$\mu_t^B = \rho - r_t . \tag{A.23}$$

The result that $\gamma_t^B = 0$ when bankers consume is given by the smooth-pasting condition, $\partial q^B (\eta_t, t) / \partial \eta_t = 0$ (so by Itô's lemma, $\gamma_t^B = 0$), which is discussed in more details in A.2. The upper boundary $\overline{\eta}(t)$ is given by the value-matching condition of bankers' consumption, $q^B(\overline{\eta}(t), t) = 1$, and is jointly determined with the function $q_t^B = q^B(\eta_t, t)$ in the solution of PDEs of $q^B(\eta_t, t)$ and $q^T(\eta_t, t)$ in A.2.

Conditional stationary distribution of η_t. Following Brunnermeier and Sannikov (2014), I derive the conditional stationary probability density of η_t. Fixing κ_t^I and β_t, the probability density of η_t at time t, $p(\eta, t)$, satisfies the Kolmogorov forward equation

$$\frac{\partial}{\partial t} p(\eta, t) = -\frac{\partial}{\partial \eta} \left(\eta \mu^\eta (\eta) p(\eta, t) \right) + \frac{1}{2} \frac{\partial^2}{\partial \eta^2} \left(\eta^2 \sigma^\eta (\eta)^2 p(\eta, t) \right).$$

Note that, fixing κ_t^I and β_t, μ_t^η and σ_t^η are functions of η_t as shown in A.2. A stationary density is a solution to the forward equation that does not vary with time (i.e. $\frac{\partial}{\partial t} p(\eta, t) = 0$). So I suppress the time variable, and denote stationary density as $p(\eta)$. Integrating the forward equation over η, $p(\eta)$ solves the following first-order ordinary differential equation within the reflecting boundary:

$$0 = C - \eta \mu^\eta(\eta) p(\eta) + \frac{1}{2} \frac{d}{d\eta} \left(\eta^2 \sigma^\eta(\eta)^2 p(\eta) \right), \quad \eta \in (0, \overline{\eta}].$$

The integration constant C is zero because of the reflecting boundary. The boundary condition for the equation is the requirement that probability density is integrated to one (i.e. $\int_{\underline{\eta}}^{\overline{\eta}} p(\eta) d\eta = 1$).

A.2 Solution Algorithm

The full solution of the model consists of two parts: first, the laws of motion of state variables, and, second, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$. The Markov equilibrium has four state variable: time, η_t, K_t^I, and K_t^T. As shown in the main text, time has an exogenous and autonomous law of motion, while the last three variables' laws of motions depend on the endogenous variables that are functions of these state variables. To simplify the notation, I suppress the time subscripts in the following.

I construct a mapping from η, t, $q^B(\eta, t)$, $q^T(\eta, t)$, $\partial q^B(\eta, t)/\partial \eta$, $\partial q^T(\eta, t)/\partial \eta$, $\partial q^B(\eta, t)/\partial t$ and $\partial q^T(\eta, t)/\partial t$ to the second-order derivatives with respect to η, $\partial^2 q^B(\eta, t)/\partial \eta^2$ and $\partial^2 q^T(\eta, t)/\partial \eta^2$, i.e., a system of second-order partial differential equations for $q^B(\eta, t)$ and $q^T(\eta, t)$. Once I solved these two functions, the rest of the price variables and K^T-scaled aggregate quantities can be solved as they will be shown to depend only on η, t, the levels and derivatives of $q^B(\eta, t)$ and $q^T(\eta, t)$. This confirms the statement in Proposition 3 that these variables are bivariate functions of η and t. After solving the price variables and K^T-scaled aggregate quantities, the laws of motion of K_t^I, K_t^T, and η_t are given by (25), (26), and (28), respectively.

Constructing PDEs for $q^B(\eta, t)$ and $q^T(\eta, t)$. Inputs are η, t (and thus, $\kappa^I(t)$ and $\beta(t)$), the levels and first derivatives of $q^B(\eta, t)$ and $q^T(\eta, t)$. Outputs are $\partial^2 q^B(\eta, t)/\partial \eta^2$ and $\partial^2 q^T(\eta, t)/\partial \eta^2$. It is convenient to define the following notations of elasticities:

$$\epsilon^T \equiv \frac{\partial q^T/q^T}{\partial \eta/\eta} \text{ and } \epsilon^B \equiv \frac{\partial q^B/q^B}{\partial \eta/\eta}.$$

Step 1: Calculate σ^η, σ^T, γ^B, x^B, and r.

Proposition 1 solves the optimal intangible share of investment, θ, and the marginal value of liquidity, π, that entrepreneurs assign to deposits, as functions of q^I (constant, see (2)), q^T, and $\kappa^I(t)$ and the parameters. Given $F(\theta_t) = \frac{\phi}{2}\theta^2$, (A.9) implies a quadratic equation for θ when π is is substituted out using (A.10). Once θ is solved, (A.10) solves π. In the following, I will discuss different cases, but the values of these variables will not change across different cases.

First, consider the case where entrepreneurs do not hold any deposits. With $M^E = 0$, the deposit-market clearing condition (24) is

$$\left(x^B - 1\right)\eta = M^H/K^T = \alpha\left(\frac{\rho - r}{\beta(t)}\right)^{-\frac{1}{\xi}}, \tag{A.24}$$

where the second equation is obtained from households' aggregate deposit demand (22). Within this case, there are two scenarios. First, bankers hold all tangible capital, so $q^T K^T = x^B N^B$, i.e.,

$$x^B = q^T/\eta\,, \tag{A.25}$$

and then from (A.24), r is calculated. If $r > \rho - \lambda\pi$, then entrepreneurs prefer to hold deposits, and I switch a different case where entrepreneurs hold deposits (to be discussed below). If $r \leq \rho - \lambda\pi$, I proceed to calculate σ^η, σ^T, and γ^B. Jointly using $\sigma^\eta = x^B\left(\sigma^T + \sigma\right) - \sigma$ from (A.6) and $\sigma^T = \epsilon^T\sigma^\eta$ from Itô's lemma, I obtain σ^η and σ^T. Using Itô's lemma again, I obtain $\gamma^B = -\epsilon^B\sigma^\eta$. Now the bankers' discount rate is given by $r + \gamma^B\left(\sigma^T + \sigma\right)$. If $\rho < r + \gamma^B\left(\sigma^T + \sigma\right)$, then the rest of the economy has a lower discount rate than bankers, so bankers cannot hold all tangible capital, and I switch to the scenario where entrepreneurs do not hold deposits and bankers do not hold all tangible capital (to be discussed in the next paragraph). If $\rho > r + \gamma^B\left(\sigma^T + \sigma\right)$, this scenario is internally consistent and I proceed to Step 2.

Now consider the scenario where entrepreneurs do not hold deposits and bankers do not hold all tangible capital. In this scenario, x^B is calculated as follows. Given that the rest of the economy holds tangible capital, the expected return on tangible capital is ρ, and from Proposition 2,

$$\rho = r + \gamma^B\left(\sigma^T + \sigma\right). \tag{A.26}$$

By Itô's lemma,

$$\sigma^T = \epsilon^T\sigma^\eta \text{ and } \gamma^B = -\epsilon^B\sigma^\eta. \tag{A.27}$$

I substitute these expressions of σ^T and γ^B into (A.26) to obtain a quadratic equation of σ^η, and the roots are

$$\sigma^\eta = \frac{-\epsilon^B\sigma \pm \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T(\rho - r)}}{2\epsilon^B\epsilon^T}.$$

I study a Markov equilibrium where $\epsilon^B \leq 0$ (i.e., bankers' marginal value of wealth declines in η_t), $\epsilon^T \geq 0$ (i.e., the value of tangible capital increases in η_t), and $\rho - r \geq 0$, so the only positive root is

$$\sigma^\eta = \frac{-\epsilon^B\sigma - \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T(\rho - r)}}{2\epsilon^B\epsilon^T}. \tag{A.28}$$

A positive root is selected because bankers have levered positions in tangible capital, so the shock impact is greater on N^B than on K^T, and thus, η responds positively to the Brownian shock. Using $\sigma^\eta = x^B \left(\sigma^T + \sigma\right) - \sigma$ from (A.6), I obtain

$$x^B = \frac{\sigma^\eta + \sigma}{\sigma^\eta \epsilon^T + \sigma} \,. \tag{A.29}$$

Using (A.29) to substitute out x^B in (A.24), I obtain

$$\left(\frac{\sigma^\eta + \sigma}{\sigma^\eta \epsilon^T + \sigma} - 1\right) \eta = \alpha \left(\frac{\rho - r}{\beta(t)}\right)^{-\frac{1}{\xi}} \,. \tag{A.30}$$

Using (A.28) to substitute out σ^η on the left side of (A.30), I obtain an equation for r. Once r is solved, I use (A.28) to solve σ^η, use (A.29) to solve x^B, and use (A.27) to solve σ^T and γ^B. Proceed to Step 2.

Finally, consider the case where entrepreneurs hold deposits. From Proposition 1, the equilibrium deposit rate is given by

$$r = \rho - \lambda \pi \,. \tag{A.31}$$

Given r, the deposit demand of households (scaled by K^T) is given by (22), and I obtain the aggregate deposit demand, $\left(M^E + M^H\right)/K^T$. Next, consider the scenario where bankers hold all tangible capital, i.e., $x^B = q^T/\eta$. From (A.29), I solve σ^η, and from (A.27), I solve σ^T and γ^B. Now the bankers' discount rate is given by $r + \gamma^B \left(\sigma^T + \sigma\right)$. If $\rho < r + \gamma^B \left(\sigma^T + \sigma\right)$, then the rest of economy have lower discount rate than bankers, so bankers cannot hold all tangible capital and I switch to the scenario where entrepreneurs hold deposits and bankers do not hold all tangible capital. If $\rho \geq r + \gamma^B \left(\sigma^T + \sigma\right)$, this scenario is internally consistent and I proceed to Step 2.

Now consider the scenario where entrepreneurs hold deposits and bankers do not hold all tangible capital. The expected return on tangible capital is ρ, so from Proposition 2,

$$\rho = r + \gamma^B \left(\sigma^T + \sigma\right) \,. \tag{A.32}$$

Using (A.31) to substitute r with $\rho - \lambda\pi$, I obtain

$$\lambda \pi = \gamma^B \left(\sigma^T + \sigma\right) \,. \tag{A.33}$$

Using Itô's lemma, i.e., (A.27), I substitute σ^T and γ^B out with $\epsilon^T \sigma^\eta$ and $-\epsilon^B \sigma^\eta$ respectively to obtain a quadratic equation of σ^η, and the roots are

$$\sigma^\eta = \frac{-\epsilon^B \sigma \pm \sqrt{\left(\epsilon^B \sigma\right)^2 - 4\epsilon^B \epsilon^T \lambda \pi}}{2\epsilon^B \epsilon^T}.$$

I study a Markov equilibrium where $\epsilon^B \leq 0$ (i.e., bankers' marginal value of wealth declines in η_t), $\epsilon^T \geq 0$ (i.e., the value of tangible capital increases in η_t), and, as the shadow price of funding constraint on investment, $\pi \geq 0$, so the only positive root is

$$\sigma^\eta = \frac{-\epsilon^B\sigma - \sqrt{\left(\epsilon^B\sigma\right)^2 - 4\epsilon^B\epsilon^T\lambda\pi}}{2\epsilon^B\epsilon^T}. \tag{A.34}$$

Using Itô's lemma again, i.e., (A.27), I solve σ^T and γ^B. Using $\sigma^\eta = x^B\left(\sigma^T + \sigma\right) - \sigma$ from (A.6), I solve x^B. Proceed to Step 2.

Step 2: Calculating the Second-Order Derivatives

The drift and diffusion of η are given in the proof of Proposition 3. Given q^T, π, γ^B, and σ^T, (18) solves μ^T. The following equation, obtained by Itô's lemma, solves $\frac{\partial^2 q^T}{\partial\eta^2}$:

$$\mu^T q^T = \frac{\partial q^T}{\partial t} + \frac{\partial q^T}{\partial \eta}\mu^\eta\eta + \frac{1}{2}\frac{\partial^2 q^T}{\partial\eta^2}\left(\sigma^\eta\eta\right)^2 . \tag{A.35}$$

According to (A.23), $\mu_t^B = \rho - r_t$, so the following equation, obtained by Itô's lemma, solves $\frac{\partial^2 q^B}{\partial\eta^2}$:

$$\mu^B q^B = \frac{\partial q^B}{\partial t} + \frac{\partial q^B}{\partial \eta}\mu^\eta\eta + \frac{1}{2}\frac{\partial^2 q^B}{\partial\eta^2}\left(\sigma^\eta\eta\right)^2 . \tag{A.36}$$

Boundary conditions for PDEs for $q^B(\eta, t)$ and $q^T(\eta, t)$. Tangible capital has constant cash flow, one unit of goods per unit of time, so what causes its price to vary is the discount-rate changes. Close to $\eta = 0$, an absorbing state, the banking sector is extremely small, so the discount rate (expected return) is fixed at ρ to induce the rest of economy to own tangible capital and clear the market. Thus, q^T should not vary as η approaches zero:

$$\lim_{\eta\to 0}\frac{\partial q^T(\eta, t)}{\partial\eta} = 0 . \tag{A.37}$$

Moreover, when bankers are extremely undercapitalized, their marginal value of wealth approaches infinity,

$$\lim_{\eta\to 0} q^B(\eta, t) = +\infty , \tag{A.38}$$

because q^B is the present value of one unit of equity, and it increases when the banking sector shrinks, widening the return spread between holding tangible capital and issuing deposits.

The upper boundary of η, $\overline{\eta}$, where bankers consume, is a reflecting boundary, so to rule out arbitrage (i.e., perfectly predictable variation of asset price),

$$\frac{\partial q^T(\overline{\eta},t)}{\partial \eta}=0\,. \tag{A.39}$$

For consumption to be optimal at $\overline{\eta}$, bankers' marginal value of wealth, q^B, satisfies the value-matching condition,

$$q^B\left(\overline{\eta},t\right)=1\,, \tag{A.40}$$

and the smooth-pasting condition

$$\frac{\partial q^B\left(\overline{\eta},t\right)}{\partial \eta}=0\,. \tag{A.41}$$

Finally, it is assumed that the linear trends of κ^I and β end at $t=\overline{t}$. When solving the model, I map $\overline{t}$ to 2010 in the data. When t reaches $\overline{t}$ and κ^I and β no longer vary, the economy converges to a time-homogeneous Markov equilibrium where the price variables and K^T-scaled quantities are functions of η_t only. Therefore, the boundary condition on the time dimension for $q^B\left(\eta,t\right)$ and $q^T\left(\eta,t\right)$ is the convergence to $\overline{q}^B\left(\eta\right)$ and $\overline{q}^T\left(\eta\right)$ of the time-homogeneous Markov equilibrium.

The functions, $\overline{q}^B\left(\eta\right)$ and $\overline{q}^T\left(\eta\right)$, of the time-homogeneous Markov equilibrium at $\overline{t}$ can be solved by a system of ordinary differential equations (ODEs) that are constructed following the same aforementioned procedure, except that at the very last step, by Itô's lemma, the second-order derivatives are solved by

$$\mu^B\overline{q}^B=\frac{d\overline{q}^B}{d\eta}\mu^\eta\eta+\frac{1}{2}\frac{d^2\overline{q}^B}{d\eta^2}\left(\sigma^\eta\eta\right)^2\,. \tag{A.42}$$

and

$$\mu^T\overline{q}^T=\frac{d\overline{q}^T}{d\eta}\mu^\eta\eta+\frac{1}{2}\frac{d^2\overline{q}^T}{d\eta^2}\left(\sigma^\eta\eta\right)^2\,. \tag{A.43}$$

The ODEs have the following conditions in analogy to (A.37) to (A.41):

- As η approaches zero: (1) $\lim_{\eta\to 0}\frac{d\overline{q}^T(\eta)}{d\eta}=0$; (2) $\lim_{\eta\to 0}\overline{q}^B\left(\eta\right)=+\infty$.
- At the upper reflecting boundary, $\overline{\eta}$: (3) $\frac{d\overline{q}^T(\eta)}{d\eta}=0$; (4) $\overline{q}^B\left(\overline{\eta}\right)=1$; (5) $\frac{d\overline{q}^B(\eta)}{d\eta}=0$.

Prices and K^T-scaled quantities in Proposition 3. The solution procedure has solved q_t^T, r_t, x_t^B, θ_t as bivariate functions of η_t and t because they only depend on η_t, t, q_t^T, ϵ^T, and ϵ^B. From (22), households' aggregate deposit holdings, M_t^H/K_t^T, is $\alpha\left(\frac{\rho-r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}$. Entrepreneurs' aggregate deposit holdings (scaled by K_t^T), M_t^E/K_t^T, is given by

$$\frac{M_t^E}{K_t^T}=\frac{\left(x_t^B-1\right)N_t^B-M_t^H}{K_t^T}=\left(x_t^B-1\right)\eta_t-\alpha\left(\frac{\rho-r_t}{\beta\left(t\right)}\right)^{-\frac{1}{\xi}}\,. \tag{A.44}$$

The aggregate intangible investment (scaled by K_t^T) is $\theta_t M_t^E / K_t^T$ and the aggregate tangible investment (scaled by K_t^T) is $(1-\theta_t)\, M_t^E / K_t^T$. Now it has been proven that the price variables and K_t^T-scaled aggregate quantities listed in Proposition 3 are bivariate functions of η_t and t.

The hierarchy of state variables. Time has its autonomous law of motion. The law of motion of η_t in the proof of Proposition 3 only depends on η_t and time t. The law of motion of K_t^T (i.e., (26) in the main text) only depends on η_t, time t, and K_t^T: using (A.7), I obtain

$$\frac{dK_t^T}{K_t^T} = \left[\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha\left(\frac{\rho - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T \left(1-\theta_t\right)}\right)\kappa^T\left(1-\theta_t\right)\lambda - \delta\right] dt + \sigma dZ_t\,, \tag{A.45}$$

where the drift is solved in the proof of Proposition 3 and the endogenous variables on the right side are bivariate functions of η_t and t. Finally, rewriting (25) from the main text, I obtain the law of motion of K_t^I, which depends on all four state variables,

$$\begin{aligned}\frac{dK_t^I}{K_t^I} &= \frac{K_t^T}{K_t^I}\left[\left(x_t^B - 1\right)\frac{N_t^B}{K_t^T} - \frac{M_t^H}{K_t^T}\right]\left(\frac{1}{1 - q_t^T\kappa^T\left(1-\theta_t\right)}\right)\theta_t\kappa^I\left(t\right)\lambda dt - \left(\delta dt - \sigma dZ_t\right)\\ &= \left[\frac{K_t^T}{K_t^I}\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha\left(\frac{\rho - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T \left(1-\theta_t\right)}\right)\kappa^I\left(t\right)\theta_t\lambda - \delta\right] dt + \sigma dZ_t\,.\end{aligned} \tag{A.46}$$

Solving the model with tradable intangibles. Allowing χ fraction of intangible capital to be tradable among entrepreneurs and households only change the optimality conditions for θ and i. The rest of solution algorithm is the same as that of the main model. The F.O.C. for θ_t is

$$q^I\kappa^I\left(1+\chi\pi\right) - q^T\kappa^T\left(1+\pi\right) - F'\left(\theta\right) = 0\,. \tag{A.47}$$

In contrast to (A.9), the marginal benefit of creating intangible capital has an additional component $q^I\kappa^I\chi\pi$ from relaxing the financial constraint. The F.O.C. for i is

$$\pi = \left\{\left[q^I\kappa^I\theta + q^T\kappa^T\left(1-\theta\right) - F\left(\theta\right)\right] - 1\right\}\left(\frac{1}{1 - q^T\kappa^T\left(1-\theta\right) - \chi q^I\kappa^I\theta}\right) \tag{A.48}$$

Given that $F\left(\theta\right) = \frac{\phi}{2}\theta^2$, (A.47) implies a quadratic equation for θ when π is substituted out by (A.48). Once θ is solved, (A.48) solves π.

B B. Risk Aversion and Finite EIS

In this section, I extend the model to incorporate risk-averse preferences and finite EIS (elasticity of intertemporal substitution) showing that the solution of the extended model can be achieved by making two modifications to the solution of the model in the main text. First, the functions of endogenous variables, such as $q^T(\eta_t, t)$, can be derived by the procedure in A.2 with the time discount rate ρ replaced by a function $\rho(\eta_t, t)$. The functional form of $\rho(\eta_t, t)$ depends on the risk-averse utility functions in the extended model.

Second, the laws of motion of state variables in the solution of the main model become the laws of motion under the risk-neutral measure in the extended model with risk aversion. To characterize the dynamics under the physical measure (probability measure of data generating process), a change of measure shall be performed using Girsanov's Theorem. The Markov equilibrium has four state variable: time, η_t, K_t^I, and K_t^T. A change of measure affects the laws of motion of the last three by adjusting their drifts. The adjustments depend on (1) the state variables' loadings of the aggregate shock (i.e., their diffusions) and (2) the consumers' price of risk implied by the risk-averse utility functions and the equilibrium process of aggregate consumption. This method of incorporating risk-averse preferences can be applied to other macro-finance models with risk-neutral preferences, (e.g., Brunnermeier and Sannikov, 2014).

After establishing these results, I characterize the conditions under which the equilibrium of the main (risk-neutral) model serves as an adequate approximation to the equilibrium of the extended model. The model solution has two parts, first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. I show that the first part of the solution is an adequate approximation if the expected growth rate of consumption is stable, which holds in the model and is consistent with the theories and evidence on long-run consumption risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008). I also show that ignoring risk aversion has little impact on the laws of motion of state variables under the standard risk aversion parameter in the asset pricing literature.

Incorporating preferences with risk aversion and finite EIS. Next, I introduce risk-averse preferences to the household sector. Entrepreneurs and bankers are reinterpreted as firms that maximize the present value of payouts to household shareholders, so households are the ultimate consumers in this economy. It is assumed that households face a complete market. For households, the relevant shock is the aggregate Brownian shock dZ_t. The market is complete if households can trade tangible capital and risk-free assets.[64] No arbitrage and complete markets imply the existence of a unique stochastic discount factor (SDF), denoted by Λ_t, which, in equilibrium, is determined by the households' marginal value of wealth (Duffie, 2001). The following analysis takes the equilibrium

[64] For risk-free assets, it is assumed that households can lend to and borrow from each other (in equilibrium, at the risk-free rate ρ_t), i.e., the negative drift of SDF in (B.1), and unlike deposits, such risk-free assets do not bring deposit-in-utility for households or relax entrepreneurs' liquidity constraints. They may represent personal IOUs.

process of Λ_t as given,

$$\frac{d\Lambda_t}{\Lambda_t} = -\rho_t dt - \gamma_t^H d\widehat{Z}_t\,, \tag{B.1}$$

where ρ_t is the households' time discount rate in equilibrium and γ_t^H is the households' price of risk. The endogenous discount rate ρ_t replaces the parameter ρ in the main text. After analyzing the entrepreneurs' and banks' problems, I specify the households' preferences and solve Λ_t. The stochastic process $\widehat{Z}_t$ is the cumulative aggregate shock under the physical measure.

By Girsanov's Theorem, we know the following connection between the aggregate shock to capital stock, dZ_t, under the risk-neutral measure and $d\widehat{Z}_t$, the shock under the physical measure,

$$dZ_t = d\widehat{Z}_t + \gamma_t^H dt\,. \tag{B.2}$$

The idiosyncratic Poisson shocks do not affect the change of measure because they are not priced in the SDF. Entrepreneurs' information filtration under the physical measure is generated by $\widehat{Z}_t$ and the idiosyncratic Poisson shocks that trigger investment needs. Their information filtration under the risk-neutral measure is generated by Z_t and the same idiosyncratic Poisson shocks. For bankers, idiosyncratic risks are diversified away, so the relevant information filtration is generated by Z_t under the risk-neutral measure and $\widehat{Z}_t$ under the physical measure.

Girsanov's Theorem implies a connection between objective functions under the physical and risk-neutral measures: a representative entrepreneur i maximize

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho_t t} dc_{i,t}^E\right] = \widehat{\mathbb{E}}\left[\int_0^{\infty} \frac{\Lambda_t}{\Lambda_0} dc_{i,t}^E\right]\,, \tag{B.3}$$

and

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\rho_t t} dc_{i,t}^B\right] = \widehat{\mathbb{E}}\left[\int_0^{\infty} \frac{\Lambda_t}{\Lambda_0} dc_{i,t}^B\right]\,, \tag{B.4}$$

where $\widehat{\mathbb{E}}\left[\cdot\right]$ is the rational-expectation operator under the physical measure, distinguished from $\mathbb{E}\left[\cdot\right]$, the rational-expectation operator under the risk-neutral measure, and, following the notations in Appendix A, $c_{i,t}^E$ and $c_{j,t}^B$ denotes the cumulative payout of non-financial firms and banks.

The full solution of the model consists of two parts: first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T(\eta_t, t)$, and, second, the laws of motion of state variables. The next proposition states the connection between an extended model with power utility and the model in the main text. The proof is at the end of this section. This method of incorporating risk-averse preferences into an originally risk-neutral model applies to any utility function. I use power (CRRA) utility as an example.

Proposition B.1 *Households have power utility over consumption and deposit-in-utility intro-*

duced in (19) and maximize

$$\mathbb{E}\left[\int_{t=0}^{\infty} e^{-\delta_H t}\left(\frac{\left(c_t^H\right)^{1-\underline{\gamma}_H}}{1-\underline{\gamma}_H}dt + \beta_t \frac{\left(m_t^H/w_t^H\right)^{1-\xi}}{1-\xi}\right)\right], \tag{B.5}$$

where δ_H and $\underline{\gamma}_H$ are the parameters for discount factor and relative risk aversion, respectively, and c_t^H denote the rate of consumption (instead of cumulative consumption).

The solutions of endogenous variables as functions of state variables can be obtained by the procedure in A.2 with the parameter ρ replaced by the following function:

$$\begin{aligned}\rho\left(\eta_t, t\right) = \underline{\delta}_H + \underline{\gamma}_H \mu^{KT}\left(\eta_t, t\right) + \underline{\gamma}_H \epsilon^{\widetilde{C}1}\left(\eta_t, t\right)\left[\mu^{\eta}\left(\eta_t, t\right) + \frac{1}{2}\epsilon^{\widetilde{C}2}\left(\eta_t, t\right)\sigma^{\eta}\left(\eta_t, t\right)^2 + \sigma^{\eta}\left(\eta_t, t\right)\sigma\right] \\ + \frac{1}{2}\left(\underline{\gamma}_H^2 - \underline{\gamma}_H\right)\left[\epsilon^{\widetilde{C}1}\left(\eta_t, t\right)\sigma^{\eta}\left(\eta_t, t\right) + \sigma\right]^2,\end{aligned} \tag{B.6}$$

where $\mu^{\eta}\left(\eta_t, t\right)$, $\sigma^{\eta}\left(\eta_t, t\right)$, and $\mu^{KT}\left(\eta_t, t\right)$ are given by (A.5), (A.6), and (A.7) respectively in A.1, and $\epsilon^{\widetilde{C}1}\left(\eta_t, t\right)$ is the elasticity of K_t^T-scaled aggregate consumption, $\widetilde{C}_t^H \equiv C_t^H/K_t^T$, to η_t,

$$\epsilon^{\widetilde{C}1}\left(\eta_t, t\right) \equiv \frac{\partial \widetilde{C}^H\left(\eta_t, t\right)}{\partial \eta_t}\frac{\eta_t}{\widetilde{C}^H\left(\eta_t, t\right)}, \tag{B.7}$$

and $\epsilon^{\widetilde{C}2}\left(\eta_t, t\right)$ is the elasticity of $\frac{\partial \widetilde{C}^H\left(\eta_t, t\right)}{\partial \eta_t}$ to η_t,

$$\epsilon^{\widetilde{C}2}\left(\eta_t, t\right) \equiv \frac{\partial^2 \widetilde{C}^H\left(\eta_t, t\right)}{\partial \eta_t^2}\frac{\eta_t}{\left(\frac{\partial \widetilde{C}^H\left(\eta_t, t\right)}{\partial \eta_t}\right)}. \tag{B.8}$$

By Girsanov's Theorem, the laws of motion of η_t, K_t^T, and K_t^I are given by (A.4), (A.45), and (A.46) respectively with dZ_t, the Brownian shock under the risk-neutral measure, replaced by

$$d\widehat{Z}_t + \gamma^H\left(\eta_t, t\right)dt \tag{B.9}$$

where $d\widehat{Z}_t$ is the Brownian shock under the physical measure, and $\gamma^H\left(\eta_t, t\right)$ is given by

$$\gamma^H\left(\eta_t, t\right) = \underline{\gamma}_H\left[\epsilon^{\widetilde{C}1}\left(\eta_t, t\right)\sigma^{\eta}\left(\eta_t, t\right) + \sigma\right], \tag{B.10}$$

which is the households' price of risk in equilibrium.

Comparing risk-neutral and risk-averse models. In the comparison between the risk-neutral and risk-averse models, a key object is $\epsilon^{\widetilde{C}1}\left(\eta_t, t\right)$, the the elasticity of K_t^T-scaled aggregate consumption, $\widetilde{C}_t^H \equiv C_t^H / K_t^T$, to η_t. Given $C_t^H = \widetilde{C}_t^H K_t^T$, by Itô's lemma, the volatility of consumption growth, σ_t^C is given by

$$\sigma_t^C = \epsilon^{\widetilde{C}1} (\eta_t, t) \, \sigma^\eta (\eta_t, t) + \sigma \, . \tag{B.11}$$

The constant return-to-scale technology implies that the volatility of capital growth, σ, is the volatility of output growth. Empirically, consumption growth is less volatile in data than output growth (e.g., Blanchard and Simon, 2001). Therefore, if the preference parameters are calibrated to match consumption volatility (as typically done in the asset-pricing literature (Cochrane, 2005a)), we have

$$\epsilon^{\widetilde{C}1} (\eta_t, t) < 0 \, . \tag{B.12}$$

The model solution has two parts: first, the endogenous variables as functions of state variables, for example, $q_t^T = q^T (\eta_t, t)$, and, second, the laws of motion of state variables. Therefore, according to Proposition B.1, a potential misspecification from ignoring risk aversion has two consequences. First, in the algorithm that solves the functions of endogenous variables in A.2, ρ should be replaced by $\rho (\eta_t, t)$. Second, the laws of motions of state variables are in fact risk-neutral dynamics. The dynamics under the physical measure require an adjustment of drifts by replacing dZ_t with $d\widehat{Z}_t + \gamma^H (\eta_t, t) \, dt$ (see B.2).

To analyze the impact of ignoring risk aversion on the functions of endogenous variables, I examine whether $\rho (\eta_t, t)$ can be approximated by a constant. The expression of $\rho (\eta_t, t)$ in (B.6) can be simplified with the consumption growth volatility in (B.11):

$$\begin{aligned} \rho (\eta_t, t) = \underline{\delta}_H + \underline{\gamma}_H \mu^{KT} (\eta_t, t) + \underline{\gamma}_H \epsilon^{\widetilde{C}1} (\eta_t, t) \left[\mu^\eta (\eta_t, t) + \frac{1}{2} \epsilon^{\widetilde{C}2} (\eta_t, t) \, \sigma^\eta (\eta_t, t)^2 + \sigma^\eta (\eta_t, t) \, \sigma \right] \\ + \frac{1}{2} \left(\underline{\gamma}_H^2 - \underline{\gamma}_H \right) \left(\sigma_t^C \right)^2 \, . \end{aligned} \tag{B.13}$$

Let $O(\sigma^2)$ denote the terms that involve the squared volatilities of growth rates (which all contain σ^2). Because volatilities and expectations of growth rates are of similar magnitudes in this model where aggregate quantities are driven by geometric Brownian motions, these volatility-squared terms tend to be small. Therefore, I use the following expression

$$\rho (\eta_t, t) = \underline{\delta}_H + \underline{\gamma}_H \mu^{KT} (\eta_t, t) + \underline{\gamma}_H \epsilon^{\widetilde{C}1} (\eta_t, t) \, \mu^\eta (\eta_t, t) + O\left(\sigma^2\right) \, . \tag{B.14}$$

The first and second terms are standard in asset pricing models. Given the constant return-to-scale technology, the capital growth rate, $\mu^{KT} (\eta_t, t)$, is the growth rate of aggregate output. In an endowment economy, the aggregate output (agents' endowments) is equal to the aggregate consumption in equilibrium, so, in these consumption-based models, the equilibrium risk-free rate only contains the first two terms on the right side of (B.14) (e.g., Lucas, 1978).

The third term is unique to this model. The drift of η_t, $\mu^\eta (\eta_t, t)$, is the expected growth rate of the ratio of bankers' wealth to tangible capital value. Because bankers hold a leveraged position in tangible capital, and the expected return on tangible capital is positive, bankers' wealth grows faster than tangible capital in expectation, and $\mu^{\eta}(\eta_t, t)$ is positive. Given that $\epsilon^{\widetilde{C}1}(\eta_t, t) < 0$ (see (B.12)), the third term on the right side of (B.14) is negative.

The economy becomes more intangible-intensive over time, and firms hold more cash, which leads to an upward trend in investment and output growth. The counteracting force is also getting stronger. As the economy becomes more intangible-intensive, the liquidity premium on deposits, $\rho_t - r_t$, becomes larger, which increases bankers' return on wealth, and thus, pushes up $\mu^{\eta}(\eta_t, t)$.

Assuming $\rho(\eta_t, t)$ is a constant in the main model is equivalent to assuming that these two forces, $\underline{\gamma}_H \mu^{KT}(\eta_t, t) > 0$ and $\underline{\gamma}_H \epsilon^{\widetilde{C}1}(\eta_t, t)\, \mu^{\eta}(\eta_t, t) < 0$, cancel each other out. The first force is from output growth. The second is from the fact that consumption is less volatile than output growth and, due to leverage, bankers' expected return on wealth is greater than tangible capital. Empirically, this assumption means that the *expected* consumption growth rate is stable. A stable consumption growth rate is consistent with the findings of highly persistent expected consumption growth in the literature on long-run risk (Bansal and Yaron, 2004; Hansen, Heaton, and Li, 2008). approximating $\rho(\eta_t, t)$ by a constant does not cause significant misspecification. When this approximation is adequate, the functions of endogenous variables, for example, $q_t^T = q^T(\eta_t, t)$, that are solved in A.2 and presented in Section 4.2, are adequate approximations to the functions from the risk-averse model.

Next, I examine the impact of ignoring risk-aversion on the laws of motion of state variables. According to Proposition B.1, the dynamics of capital stocks given by (A.45) and (A.46) should be adjusted by replacing the Brownian shock under the risk-neutral measure, dZ_t, by the Brownian shock under the physical measure (i.e., the real shock that drives the data generating processes), $d\widehat{Z}_t$, plus a drift adjustment $\gamma^H(\eta_t, t)\, dt$:

$$\frac{dK_t^T}{K_t^T} = \left[\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha\left(\frac{\rho_t - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T (1-\theta_t)}\right)\kappa^T(1-\theta_t)\lambda - \delta\right] dt + \underbrace{\sigma\gamma^H(\eta_t, t)\, dt}_{\textit{risk adjustment}} + \sigma d\widehat{Z}_t\,, \quad \text{(B.15)}$$

and

$$\frac{dK_t^I}{K_t^I} = \left[\frac{K_t^T}{K_t^I}\left(\frac{\left(x_t^B - 1\right)\eta_t - \alpha\left(\frac{\rho_t - r_t}{\beta(t)}\right)^{-\frac{1}{\xi}}}{1 - q_t^T \kappa^T (1-\theta_t)}\right)\kappa^I(t)\,\theta_t\lambda - \delta\right] dt + \underbrace{\sigma\gamma^H(\eta_t, t)\, dt}_{\textit{risk adjustment}} + \sigma dZ_t\,. \quad \text{(B.16)}$$

Consider a relative risk aversion $\underline{\gamma}_H = 5$, which is a common value in the asset pricing literature (Cochrane, 2005a). Given that $\epsilon^{\widetilde{C}1}(\eta_t, t) < 0$ and $\sigma^{\eta}(\eta_t, t) > 0$ (see (A.6)), I obtain the following upper bound on the households' price of risk: from (B.34),

$$\gamma^H(\eta_t, t) = \underline{\gamma}_H \left[\epsilon^{\widetilde{C}1}(\eta_t, t)\,\sigma^\eta(\eta_t, t) + \sigma\right] \leq \underline{\gamma}_H \sigma = 0.1\,, \tag{B.17}$$

where the last equation substitutes in the value of $\underline{\gamma}_H$ and σ. Given that $\sigma = 0.02$ and $\gamma^H(\eta_t, t) < 0.1$, the risk adjustment term is bounded above by 0.002. Therefore, ignoring risk aversion understates the expected growth rate of capital (and output), and the wedge is bounded above by 0.2%. The physical-measure dynamics feature higher growth rates than those of the risk-neutral dynamics because, when changing from the physical measure to the risk-neutral measure, probability mass shifts towards the relatively worse states of the world, i.e., the risk-averse attitude is reflected by probability re-weighting. A similar calculation can be applied to the law of motion of η_t. The risk adjustment increases the drift of η_t, and, averaging over time t and η_t on the simulated paths, such an increase is less than 6% of the drift, (i.e., $< 0.06 \times \mu_t^\eta$).

In the main text, I report the model's solutions in two ways: (1) the values of endogenous variables at different points in time, averaged over η_t (e.g., Section 4.2) and (2) endogenous variables as functions of η_t (e.g., Section 4.3). The impact of ignoring risk aversion on the laws of motion of state variable only affects (1), and (2) depends on whether replacing $\rho(\eta_t, t)$ with a constant ρ is an adequate approximation, as previously discussed.

This concludes the discussion on the consequences of model misspecification from ignoring risk aversion. Next, I derive the equations in Proposition B.1.

Proof of Proposition B.1. First, I solve the entrepreneurs' problem and the bankers' problem under the risk-neutral measure, taking as given the stochastic discount factor (SDF). After specifying the households'/consumers' risk-averse utility function, I solve the SDF and perform the change of measure to obtain the physical-measure dynamics of the extended model. This method of solving models under the risk-neutral measure and then analyzing the physical-measure dynamics by applying Girsanov's Theorem is often used in settings of complete markets (Duffie, 2001).

The entrepreneurs' investment problem stays intact as it is a static problem happening only at idiosyncratic Poisson times. Therefore, the Lagrange function defined by (11) still summarizes the investment problem, and the marginal value of liquidity for the investment projects, π_t, is given by (13). Because the time discount rate changes from ρ to ρ_t, (15) in Proposition 1 is now

$$r_t = \rho_t - \lambda \pi_t\,. \tag{B.18}$$

The rest of Proposition 1 hold.

Given the homogeneity property of the bankers' problem, their value function is still $q_t^B n_t^B$, where the marginal value of equity, $q_t^B = q^B(\eta_t, t)$, has the law of motion (16) under *the risk-neutral measure*. Proposition 2 can still be used to characterize the valuation of tangible capital and the bankers' required expected return on tangible capital holdings under the *risk-neutral* measure. Note that if bankers can also access complete markets as households can, their marginal value of wealth, q_t^B, will be pinned to one, and their price of risk, γ_t^B, to zero. Being able to freely trade the aggregate shock with households is equivalent to being able to freely raise equity from households (Di Tella, 2017). Therefore, it is assumed that bankers cannot hedge the aggregate shock.[65]

Bankers' required expected return under the risk-neutral measure is (17) in Proposition 2. Under the physical measure, by Girsanov's Theorem, it becomes

$$\widehat{\mathbb{E}}_t\left[dr_t^T\right] = r_t + \gamma_t^B\left(\sigma_t^T + \sigma\right) + \gamma_t^H\left(\sigma_t^T + \sigma\right)\,. \tag{B.19}$$

Under the physical measure, banks require risk compensations not only due to the equity issuance constraint, $\gamma_t^B\left(\sigma_t^T + \sigma\right)$, but also on behalf of the household shareholders, $\gamma_t^H\left(\sigma_t^T + \sigma\right)$.

The valuation equation (18) for tangible capital in Proposition 2 still holds. The derivation in Appendix A applies under the risk-neutral measure. Equation (18) can also be derived under the physical measure but the law of motion of q_t^T and the stochastic depreciation of capital holdings have to be adjusted by the change of measure. Under the risk-neutral measure:

$$\frac{dq_t^T}{q_t^T} = \mu_t^T dt + \sigma_t^T dZ_t\,, \tag{B.20}$$

and, given (B.2), under the physical measure

$$\frac{dq_t^T}{q_t^T} = \left(\mu_t^T + \gamma_t^H\sigma_t^T\right)dt + \sigma_t^T d\widehat{Z}_t\,, \tag{B.21}$$

where the price of risk γ_t^H is multiplied by the quantity of risk σ_t^T. When moving from (B.21) to (B.20), the drift is adjusted downward, reflecting a risk adjustment via the shift of probability mass towards relatively worse states of the model. Risk aversion is reflected in the adjustment of the probability mass. The stochastic depreciation rate of capital under the physical measure is

$$\left(\delta - \gamma_t^H\sigma\right) + \sigma d\widehat{Z}_t\,. \tag{B.22}$$

The expected depreciation rate is adjusted upward when moving from the physical measure, $\delta - \gamma_t^H\sigma$, to the risk-neutral measure, δ, as the probability mass is shifted towards relatively worse states of the world to reflect risk aversion encoded in the SDF. The expected return of tangible capital holdings consists of the dividend yield, $1/q_t^T$, the expected price appreciation, $\mu_t^T + \gamma_t^H\sigma_t^T$, the expected capital depreciation, $\left(\delta - \gamma_t^H\sigma\right) + \lambda$ (counting both the normal-time depreciation and

[65] Imperfect hedging can be easily incorporated. For example, bankers can only hedge a fraction χ^B of aggregate risk due to agency friction and the need to keep "skin in the gamme" (He and Krishnamurthy, 2013). Note that given that hedging is free and bankers are effectively risk averse, bankers will hedge as much as they can. Then bankers' risk exposure for one dollar of holdings of tangible capital is $\left(1-\chi^B\right)\left(\sigma_t^T+\sigma\right)$ in equilibrium, i.e., scaled down by χ^B fraction. Bankers' required expected return under the risk-neutral measure becomes $r_t + \gamma_t^B\left(1-\chi^B\right)\left(\sigma_t^T+\sigma\right)$. After the scaling, the same solution procedure still applies. Because the scaling reduces bankers' discount rate and increase the value of tangible capital and entrepreneurs' leverage on liquidity holdings, it amplifies the feedback mechanism.

idiosyncratic Poisson destruction), and the quadratic covariation $\sigma_t^T \sigma$ from Itô's calculus, which does not change due to the volatility-invariance property of change of measure Duffie (2001). In equilibrium, the expected return is equal to bankers' required expected return in (B.19):

$$r_t + \gamma_t^B \left(\sigma_t^T + \sigma\right) + \gamma_t^H \left(\sigma_t^T + \sigma\right) = \frac{1}{q_t^T} + \left(\mu_t^T + \gamma_t^H \sigma_t^T\right) - \left[\left(\delta - \gamma_t^H \sigma\right) + \lambda\right] + \sigma_t^T \sigma \, .$$

Note that $\gamma_t^H \left(\sigma_t^T + \sigma\right)$ shows up on both sides. Rearranging the equation, we obtain (18).

For any stochastic process, its dynamics under the risk-neutral measure can be adjusted to obtain the dynamics under the physical measure. For instance, under the risk-neutral measure,

$$\frac{dq_t^B}{q_t^B} = \mu_t^B dt - \gamma_t^B dZ_t \, , \tag{B.23}$$

so, (B.2) implies that the law of motion of q_t^B under the physical measure is given by

$$\frac{dq_t^B}{q_t^B} = \left(\mu_t^B - \gamma_t^B \gamma_t^H\right) dt - \gamma_t^B d\widehat{Z}_t \, , \tag{B.24}$$

where the diffusion stays the same (i.e., the standard diffusion-invariance result) and the drift of q_t^B is "risk-adjusted". Note that q_t^B is high in the relatively worse states of the world where banks are undercapitalized. The expected appreciation of q_t^B is adjusted upward when moving from (B.24) to (B.23) because, when changing from the physical measure to risk-neutral measure, more probability mass is shifted towards the relatively worse states of the world.

Given the function $\rho\left(\eta_t, t\right)$, the procedure in A.2 can be used to solve all the variables listed in Proposition 3, and then, the laws of motion of η_t, K_t^T, and K_t^I can be derived. These laws of motion are under the risk-neutral measure, so a change of measure needs to be performed to obtain the physical-measure laws of motion. As I have shown for q_t^T and q_t^B, change of measure simply entails substituting out the Brownian shock under the risk-neutral measure, dZ_t, using (B.2).

Using the procedure in A.2 to solve the model's dynamics under the risk-neutral measure only requires the function $\rho\left(\eta_t, t\right)$. It does not require the households' utility function. To perform the change of measure, I need to have the price of risk, γ_t^H, as a function of the state variables.

Next, I solve the SDF, linking ρ_t and γ_t^H to households' consumption (and wealth) dynamics. Specifically, I confirm that ρ_t only depends on η_t and time t, i.e., $\rho_t = \rho\left(\eta_t, t\right)$, and solve the functional form. I also solve the households' price of risk, γ_t^H, as a function of these state variables.

In the following, I consider the standard time-separable power utility as an example. In this case, the stochastic discount factor is the time-discounted marginal utility of consumption (Cochrane, 2005b):

$$\Lambda_t = e^{-\delta_H t} \left(c_t^H\right)^{-\gamma_H} \, . \tag{B.25}$$

In equilibrium, given that there is a unit mass of households, individual consumption is equal to the aggregate consumption, C_t^H. Denote the equilibrium law of motion of aggregate consumption under the physical measure by

$$\frac{dC_t^H}{C_t^H} = \mu_t^C dt + \sigma_t^C d\widehat{Z}_t \,. \tag{B.26}$$

By Itô's lemma, the law of motion of the SDF, Λ_t, is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -\left[\underline{\delta}_H + \underline{\gamma}_H \mu_t^C - \frac{1}{2}\underline{\gamma}_H \left(\underline{\gamma}_H + 1\right)\left(\sigma_t^C\right)^2\right] dt - \underline{\gamma}_H \sigma_t^C d\widehat{Z}_t \,. \tag{B.27}$$

To solve μ_t^C and σ_t^C, consider the goods market-clearing condition:

$$C_t^H dt + \frac{M_t^E}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} \lambda dt = \left(1 + \alpha\right) K_t^T dt \,. \tag{B.28}$$

The left side is the sum of households' consumption and the goods invested by the λdt measure of entrepreneurs who are hit by the Poisson shock. The right side is the goods produced by tangible capital and labor. For simplicity, the goods produced by intangibles are assumed to be consumed directly by the entrepreneurs, who run the firms, as compensation for their human capital (Hart and Moore, 1994; Bolton, Wang, and Yang, 2019). Adding intangibles' output to (B.29) expands the dimension of state variables in ρ_t from two (i.e., η_t and t) to four, because both K_t^T and K_t^I show up in (B.29), and, given their distinct laws of motion, they have to be tracked separately. Dividing both sides of (B.29) by $K_t^T dt$ and rearranging it, we have

$$\widetilde{C}_t^H = 1 + \alpha - \frac{\lambda \widetilde{M}_t^E}{1 - q_t^T \kappa^T \left(1 - \theta_t\right)} \,. \tag{B.29}$$

Following the notations in the main text, I denote K_t^T-scaled values by "$\tilde{\cdot}$". The procedure in A.2 solves the endogenous variables on the right side of (B.29) as functions of η_t and time t. Therefore, K_t^T-scaled aggregate consumption,

$$\widetilde{C}_t^H = \widetilde{C}^H \left(\eta_t, t\right) \,, \tag{B.30}$$

is a known function of η_t and t. so I can obtain $\epsilon^{\widetilde{C}1}\left(\eta_t, t\right)$ and $\epsilon^{\widetilde{C}2}\left(\eta_t, t\right)$. Note that under *the* *risk-neutral measure*, by Itô's lemma, I obtain

$$\frac{dC_t^H}{C_t^H} = \frac{d\widetilde{C}^H(\eta_t,t)}{\widetilde{C}^H(\eta_t,t)} + \frac{dK_t^T}{K_t^T} + \epsilon^{\widetilde{C}}(\eta_t,t)\,\sigma^\eta(\eta_t,t)\,\sigma dt\,, \tag{B.31}$$

$$= \left\{\epsilon^{\widetilde{C}1}(\eta_t,t)\left[\mu^\eta(\eta_t,t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t,t)\,\sigma^\eta(\eta_t,t)^2 + \sigma^\eta(\eta_t,t)\,\sigma\right] + \mu^{KT}(\eta_t,t)\right\}dt$$
$$+ \left[\epsilon^{\widetilde{C}1}(\eta_t,t)\,\sigma^\eta(\eta_t,t) + \sigma\right]dZ_t \tag{B.32}$$

where the risk-neutral measure dynamics, $\mu^\eta(\eta_t,t)$, $\sigma^\eta(\eta_t,t)$, and $\mu^{KT}(\eta_t,t)$ are given by (A.5), (A.6), and (A.7) respectively in A.1. To change the measure, using (B.2) to substitute dZ_t with $d\widehat{Z}_t + \gamma_t^H dt$, I obtain

$$\frac{dC_t^H}{C_t^H} = \left\{\epsilon^{\widetilde{C}1}(\eta_t,t)\left[\mu^\eta(\eta_t,t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t,t)\,\sigma^\eta(\eta_t,t)^2 + \sigma^\eta(\eta_t,t)\,\sigma\right] + \mu^{KT}(\eta_t,t)\right\}dt$$
$$+ \left[\epsilon^{\widetilde{C}1}(\eta_t,t)\,\sigma^\eta(\eta_t,t) + \sigma\right]\gamma_t^H dt + \left[\epsilon^{\widetilde{C}1}(\eta_t,t)\,\sigma^\eta(\eta_t,t) + \sigma\right]d\widehat{Z}_t \tag{B.33}$$

According the law of motion of Λ_t given by (B.27), the price of risk is

$$\gamma_t^H = \underline{\gamma}_H\sigma_t^C = \underline{\gamma}_H\left[\epsilon^{\widetilde{C}1}(\eta_t,t)\,\sigma^\eta(\eta_t,t) + \sigma\right]. \tag{B.34}$$

I substitute out γ_t^H in the drift term of (B.33) with the solution (B.34) and obtain

$$\mu_t^C = \mu^C(\eta_t,t) \tag{B.35}$$
$$= \epsilon^{\widetilde{C}1}(\eta_t,t)\left[\mu^\eta(\eta_t,t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t,t)\,\sigma^\eta(\eta_t,t)^2 + \sigma^\eta(\eta_t,t)\,\sigma\right] + \mu^{KT}(\eta_t,t)$$
$$+ \underline{\gamma}_H\left[\epsilon^{\widetilde{C}1}(\eta_t,t)\,\sigma^\eta(\eta_t,t) + \sigma\right]^2$$

Substituting the solutions of μ_t^C and σ_t^C into the drift term of (B.27), I obtain

$$\rho_t = \rho(\eta_t,t) \tag{B.36}$$
$$= \underline{\delta}_H + \underline{\gamma}_H\mu^{KT}(\eta_t,t) + \underline{\gamma}_H\epsilon^{\widetilde{C}1}(\eta_t,t)\left[\mu^\eta(\eta_t,t) + \frac{1}{2}\epsilon^{\widetilde{C}2}(\eta_t,t)\,\sigma^\eta(\eta_t,t)^2 + \sigma^\eta(\eta_t,t)\,\sigma\right]$$
$$+ \frac{1}{2}\left(\underline{\gamma}_H^2 - \underline{\gamma}_H\right)\left[\epsilon^{\widetilde{C}1}(\eta_t,t)\,\sigma^\eta(\eta_t,t) + \sigma\right]^2.$$

C C. The Implications of Zero Lower Bound

While the model does not feature any nominal variables, it is important to discuss the implications of zero lower bound and examine the robustness of results in the broader context of New Keynesian models. As shown in Table 3, the equilibrium outcome matches data well except a lower and more negative interest rate in the later sample periods. The rise of intangible capital over time increases firms' demand for liquid assets and thereby exerts downward pressure on the (natural) real rate. As discussed in Section 4, this trend widens the wedge between the natural real rate and real rate under nominal price rigidity and zero lower bound, exacerbating output loss due to the liquidity trap (Eggertsson and Woodford, 2003; Christiano, Eichenbaum, and Rebelo, 2011; Eggertsson and Krugman, 2012; Fischer, 2016; Korinek and Simsek, 2016; Caballero and Farhi, 2017; Guerrieri and Lorenzoni, 2017; Caballero and Simsek, 2020; Caballero, Farhi, and Gourinchas, 2021). In the following, I will discuss in more details how nominal frictions and zero lower bound interact in the shock amplification mechanism with a particular focus on the cyclical dynamics.

The feedback mechanism in my model emphasizes a discount rate channel of asset-price variation. The bankers have low discount rates (or cost of capital) because firms attach a liquidity premium to their debts as assets that hedge the intangible investment needs. Following positive shocks, the bankers become richer via a leveraged position in tangible capital, and as these low discount-rate agents acquire more assets, they push up asset prices. Higher asset prices imply stronger investment-driven liquidity needs, further reducing the bankers' discount rate and causing asset prices to rise more. An increasing liquidity premium means a widening discount-rate gap between the bankers and entrepreneurs. This implies that reallocation of assets away from the bankers, triggered by negative shocks, will cause a large decline in asset price.

There are two ways to think about how ZLB affects the mechanism. The first is to simply assume that r_t cannot be negative because there exists an exogenous supply of liquid assets that is perfectly elastic at $r_t = 0$. This certainly dampens the mechanism because the key to the mechanism is (liquid) asset shortage and the endogenous supply of assets by risk-taking financial intermediaries. However, where does the unlimited liquidity supply come from? This triggers the second way to think about ZLB, which is more in line with the New Keynesian tradition. I will argue that ZLB does not kill the discount-rate channel of financial instability but introduces a new asymmetric cash-flow channel at ZLB. Specifically, the mechanism in the model dampens the New Keynesian mechanism on the upside (i.e., in response to positive shocks) but does not necessarily interfere it on the downside, generating asymmetric output cycles.

Following Caballero and Simsek (2020), let us assume extremely sticky (constant) prices, so $r_t \geq 0$ because the nominal rate ($= r_t$) cannot be negative. And, let us adopt the AK technology and variable capital utilization as in Caballero and Simsek (2020). Moreover, to model the aggregate demand channel, we need to introduce a different preference and endogenize ρ_t, the agents' required return or discount rate which will depend on consumption growth in equilibrium. The wedge between ρ_t and the interest rate on liquid assets (bankers' debts in particular), r_t, is the liquidity premium. In equilibrium, $\rho_t - r_t$ is driven by households' liquidity demand and firms' liquidity demand that depends on the intangible investment productivity and asset price (present value of capitalizable output of tangible capital), just as in the main text.

Consider positive shocks at $r_t = 0$. The standard wealth effect drives up the aggregate demand and, through variable capital utilization, output increases. However, the mechanism in my model generates a counteracting force. As the bankers become richer through their leveraged position, these low discount-rate agents acquire more assets. The asset price rises and raises the liquidity premium, $\rho_t - r_t$ (see Proposition 1). Given that r_t cannot fall below zero, what has to adjust is ρ_t, the agents' required savings rate that depends on the consumption growth rate in equilibrium. ρ_t must increase and this weakens the aggregate demand. So the mechanism in my model counteracts the standard New Keynesian mechanism in response to positive shocks.

Next, consider negative shocks at $r_t = 0$. The aggregate demand and output decline through the wealth effect. The asset price decline reduces the liquidity premium $\rho_t - r_t$. Because r_t can rise above zero, a lower $\rho_t - r_t$ does not necessarily require a lower ρ_t (and a higher consumption growth), so my model does not generate a counteracting force against the standard New Keynesian mechanism in response to negative shocks. Therefore, incorporating my model into a New Keynesian setting with ZLB generates asymmetric cycles with dampened upside relative to a standard New Keynesian model but similar downside. What differs from the standard New Keynesian model is that here ZLB is applied to r_t, the interest rate on liquid assets, rather than ρ_t. Moreover, the wedge, $\rho_t - r_t$, depends on the endogenous variation in asset prices.

It is worth noting that the discount-rate channel of financial instability is still at work. The discount-rate gap between the bankers and the rest of economy still widens following positive shocks, and this implies an increasingly strong response to negative shocks that trigger asset reallocation from low discount-rate bankers to high discount-rate households/consumers. The existence of ZLB does not kill this mechanism. It simply infuses this mechanism into the aggregate demand channel of New Keynesian models through the endogenous ρ_t (consumers' discount rate or required savings return), and it does so in an asymmetric fashion by dampening the upside and but not necessarily interfering the downside. What the New Keynesian setup does to my model is to bring in a new cash-flow channel. Specifically, it makes the cash flow per unit of assets/capital (i.e., the output) variable through utilization, and, due to the asymmetry in output cycle, the shock amplification through the cash-flow channel is also asymmetric (stronger for negative shocks).

D D. Additional Tables and Figures

Table D.1: Summary Statistics for Firm Cash and Leverage Regressions

Variable	Below Median Intan./Asset			Above Median Intan./Asset		
	Mean	Median	Std.	Mean	Median	Std.
Cash/Assets (%)	12.395	5.759	16.758	24.521	15.597	24.682
Intangible Investment/Investment	0.434	0.427	0.302	0.802	0.840	0.156
Intangible Investment/Total Assets	0.043	0.042	0.029	0.238	0.178	0.270
PPE/Total Assets	0.364	0.315	0.261	0.194	0.156	0.152
Leverage (%)	29.388	27.127	22.415	18.078	12.170	20.327
Asset-backed Loans/Total Assets (%)	10.771	2.891	16.564	7.964	1.314	13.674
Cashflow-backed Loans/Total Assets (%)	20.288	15.972	23.055	12.639	0.591	24.402
Acquisitions/Total Assets	0.027	0.000	0.067	0.015	0.000	0.048
Cashflow/Total Assets	0.049	0.063	0.122	-0.056	0.051	0.287
Dividend Dummy	0.398	0.000	0.489	0.227	0.000	0.419
EBITDA/Total Assets	0.103	0.115	0.518	-0.034	0.087	0.524
Inventory/Total Assets	0.120	0.067	0.148	0.177	0.147	0.163
Net Cash Receipts/Total Assets	0.091	0.100	0.468	-0.036	0.061	0.557
Net Working Capital/Total Assets	0.071	0.053	0.182	0.107	0.113	0.229
Log Real Assets (Size)	5.827	5.789	2.109	4.538	4.407	1.967
Tobin's Q	1.452	1.241	0.746	1.961	1.595	1.180

Table D.2: Summary Statistics for Household Liquidity Holdings Regressions

***Panel A**: Summary Statistics for Time Series Regression*

Variable	Mean	Std.	p20	p40	p60	p80
Liquid Holdings/GDP	0.505	0.066	0.435	0.496	0.541	0.570
Average EV/EBITDA	10.364	2.793	7.479	9.652	11.170	13.027
Tangible EV/EBITDA	8.888	2.154	7.102	8.165	9.207	10.450
Average Tobin's Q	1.823	0.342	1.526	1.693	1.903	2.055
Tangible Tobin's Q	1.414	0.185	1.256	1.394	1.496	1.534
Price/Rent Ratio	1.27	0.129	1.177	1.201	1.250	1.333

***Panel B**: Summary Statistics for Panel Data Regression*

Variable	Mean	Std.	p20	p40	p60	p80
Cash/Income	0.205	0.584	0	0.011	0.052	0.180
Δ ln (Housing Price Index)	0.064	0.128	-0.038	0.059	0.098	0.139
Age	45.296	16.390	30	38	48	59
Couple Status	0.693	0.791	0	0	1	1
Education Level	13.124	2.657	12	12	14	16
Home ownership Status	0.551	0.497	0	0	1	1
Household Size	2.641	1.483	1	2	3	4
Δ ln (Household Income)	0.036	1.390	-1.073	-0.286	0.364	1.144
Δ ln (Wealth excluding Home Equity)	-0.034	6.808	-4.879	-0.870	0.862	4.623

Table D.3: Asset Tangibility, Capital Valuation, and Corporate Cash Holdings

***Panel A**: EV/EBITDA & Intangible-Driven Corporate Cash Holdings*

$\frac{\text{Cash}}{\text{Assets}}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PPE/Assets	-0.496	-0.223	-1.699***	-1.441***	-0.846**	-0.400	-1.909***	-1.476***
(decile)	(0.372)	(0.336)	(0.315)	(0.282)	(0.412)	(0.399)	(0.313)	(0.303)
Ave. EV/EBITDA	1.848***		1.020***					
	(0.275)		(0.244)					
PPE/Assets×	-0.259***	-0.281***	-0.136***	-0.153***				
Ave. EV/EBITDA	(0.034)	(0.032)	(0.027)	(0.025)				
Tan. EV/EBITDA					1.825***		0.882***	
					(0.331)		(0.273)	
PPE/Assets×					-0.267***	-0.307***	-0.141***	-0.174***
Tan. EV/EBITDA					(0.040)	(0.041)	(0.031)	(0.031)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,801	152,801	133,632	133,632	152,801	152,801	133,632	133,632
Adjusted R^2	0.1795	0.1859	0.3096	0.3164	0.1745	0.1838	0.3076	0.3159

***Panel B**: Tobin's Q & Intangible-Driven Corporate Cash Holdings*

$\frac{\text{Cash}}{\text{Assets}}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
PPE/Assets	-0.533	0.021	-1.752***	-1.280***	0.927	1.297*	-1.125**	-0.836
(decile)	(0.632)	(0.621)	(0.415)	(0.415)	(0.837)	(0.735)	(0.555)	(0.519)
Ave. Tobin's Q	9.908***		4.836**					
	(2.587)		(1.800)					
PPE/Assets×	-1.483***	-1.729***	-0.776***	-0.961***				
Ave. Tobin's Q	(0.327)	(0.332)	(0.211)	(0.214)				
Tan. Tobin's Q					20.213***		10.620***	
					(4.560)		(3.152)	
PPE/Assets×					-2.937***	-3.136***	-1.430***	-1.555***
Tan. Tobin'sQ					(0.577)	(0.513)	(0.373)	(0.347)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,801	152,801	133,632	133,632	152,801	152,801	133,632	133,632
Adjusted R^2	0.1726	0.1827	0.3072	0.3155	0.1732	0.1823	0.3077	0.3151

Firm-year clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

Table D.4: Intangible Investment, Tobin's Q, and Corporate Cash Holdings

$\frac{\text{Cash}}{\text{Assets}}$	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intan./Assets	-2.724	-3.372	-1.629	-2.272	-8.200***	-8.730***	-5.672**	-6.244***
(quintile)	(2.219)	(2.227)	(1.761)	(1.783)	(2.650)	(2.532)	(2.117)	(2.070)
Ave. Tobin's Q	-5.072***		-4.586***					
	(0.981)		(0.682)					
Intan./Assets×	4.993***	5.326***	3.729***	3.963***				
Ave. Tobin's Q	(1.215)	(1.235)	(0.958)	(0.983)				
Tan. Tobin's Q					-10.064***		-7.866***	
					(1.568)		(1.253)	
Intan./Assets×					10.317***	10.669***	7.648***	7.925***
Tan. Tobin's Q					(1.923)	(1.856)	(1.530)	(1.512)
Controls	No	No	Yes	Yes	No	No	Yes	Yes
Year FE	No	Yes	No	Yes	No	Yes	No	Yes
Observations	152,826	152,826	133,632	133,632	152,826	152,826	133,632	133,632
Adjusted R^2	0.1843	0.2038	0.2671	0.2831	0.1880	0.2057	0.2699	0.2842

Firm-year clustered standard errors in parentheses
* $p < 0.1$ ** $p < 0.05$ *** $p < 0.01$

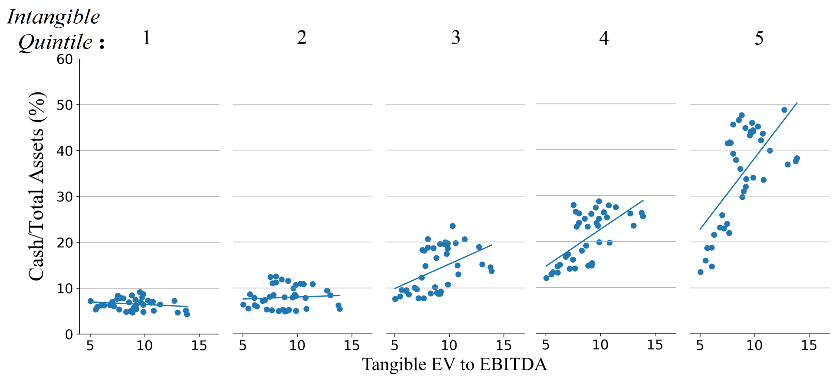

Figure D.1: Tangible Capital Valuation and Cash Holdings by Intangibility

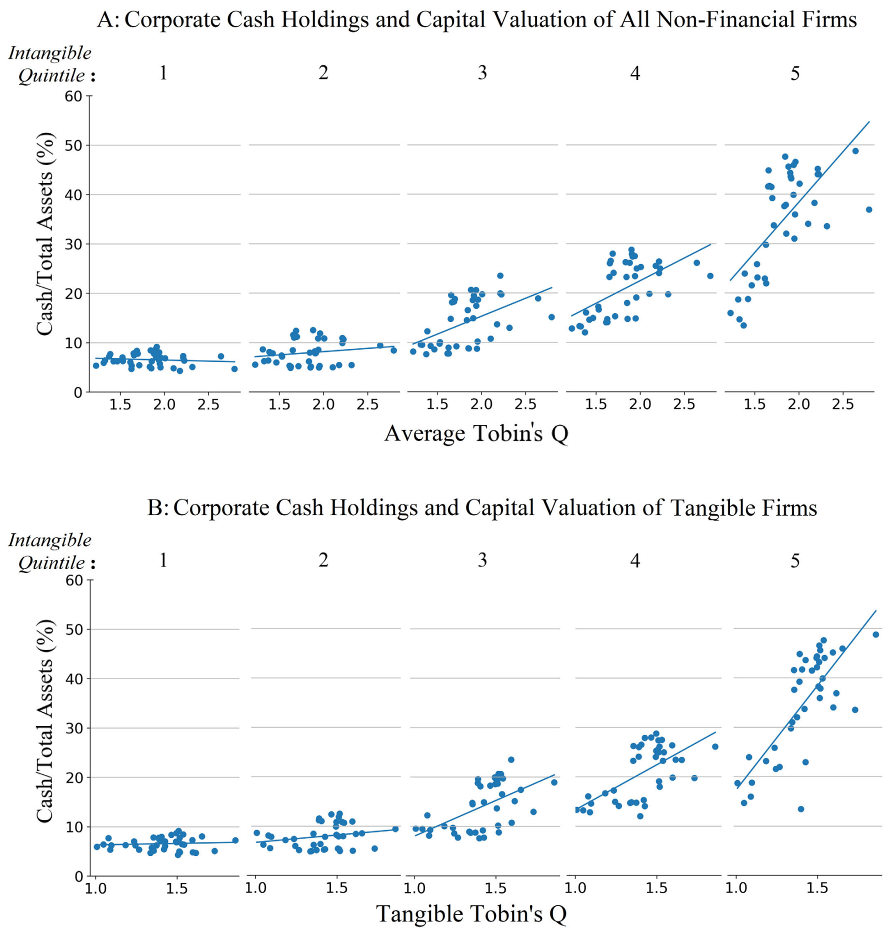

Figure D.2: Tobin's Q and Cash Holdings by Intangibility

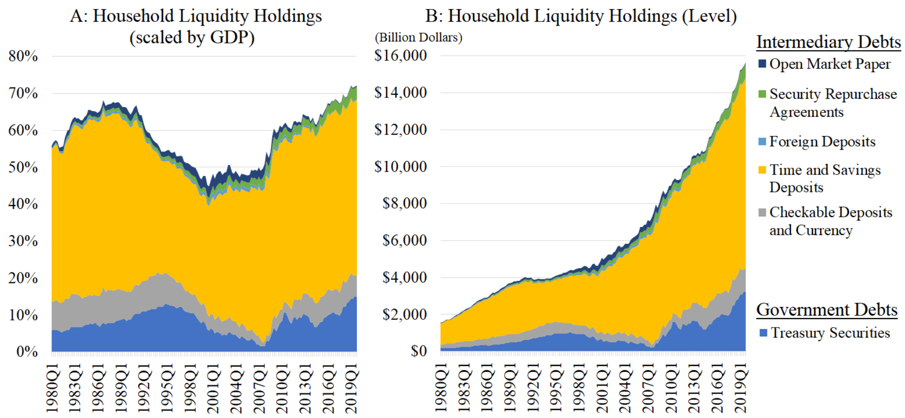

Figure D.3: Decomposing Households' Holdings of Liquid Securities

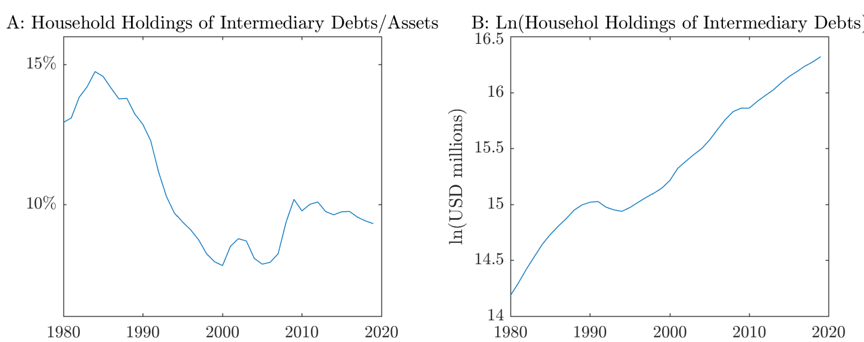

Figure D.4: Households' Holdings of Intermediary Debts