Money Creation in Decentralized Finance: A Dynamic Model of Stablecoins and Crypto Shadow Banking *

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Abstract

We develop a dynamic model of stablecoins and crypto shadow banking, where the stablecoin issuer transforms risky assets, including cryptocurrencies, into digital tokens of stable values. Both the stablecoin issuer's reserve assets and users' collateral back the stablecoin. However, even under over-collateralization, a pledge of one-to-one convertibility to a reference currency can be fragile. The distribution of states is bimodal: A fixed exchange rate may persist, but once the stablecoin breaks the buck, the recovery is slow. When negative shocks drain the issuer's reserves, debasement allows the issuer to share risk with users, but it triggers a vicious cycle of depressed stablecoin demand, lower transaction volume and transaction fees, slow rebuild of reserves, and a persistent need for debasement. Stablecoin management requires the optimal combination of strategies commonly observed in practice, such as open market operations, dynamic requirement of users' collateral, transaction fees or subsidies, re-pegging, and issuances of "secondary units" that function as the stablecoin issuer's equity. Our model lends itself to an evaluation of regulatory proposals (e.g., capital requirement) and sheds light on the complex incentives behind the stablecoin initiatives led by the network companies (e.g., Facebook).

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1 Introduction

More than a decade ago, Bitcoin heralded a new era of digital payments. Cryptocurrencies challenge the bank-centric payment systems by offering fast and round-the-clock settlement, anonymity, low-cost international remittances, and programmable money through smart contracting (Brunnermeier, James, and Landau, 2019; Duffie, 2019). However, the substantial volatility exhibited by the first-generation cryptocurrencies limits their utility as a means of payment. Stablecoins aim to maintain a stable price against a reference currency or a basket of currencies by pledging to hold fiat money or other assets against which stablecoin holdings can be redeemed.[1]

This paper provides a dynamic model of stablecoins in continuous time that rationalizes a rich set of strategies that are common in practice (Bullmann, Klemm, and Pinna, 2019), such as open market operations, dynamic requirement of users' collateral, user transaction fees/subsidies, targeted price band, re-pegging, and the issuances of "secondary units" (governance tokens) that function as equity shares of the stablecoin issuer.[2] Our model lends itself to an evaluation of regulatory proposals. It can also be applied to analyze the complex incentives behind the stablecoin initiatives led by the well-established online networks (e.g., Facebook), such as stimulating transactions through the network effects and accumulating user-generated transaction data.

The creation of stablecoins resembles money creation in shadow banking—the unregulated creation of safe assets to meet agents' transactional demand (Moreira and Savov, 2017). The issuer transforms users' risky collateral assets, which are mainly cryptocurrencies in practice, into stablecoins with one-to-one convertibility to fiat currencies. Akin to shadow banking that relies on both collateral haircut and banks' implicit guarantee to buffer risk (Acharya, Schnabl, and Suarez, 2013), the creation of stablecoins often involves both a margin requirement on the stablecoin users who post collateral and the stablecoin issuer's dynamic management of reserves. In our model, the issuer's reserves is a key state variable in the Markov equilibrium.

Our model addresses the fundamental questions on the credibility and sustainability of a fixed exchange rate. Different from Routledge and Zetlin-Jones (2018) who study speculative attacks on under-collateralized stablecoins, we show that in line with evidence (Lyons and Viswanath-Natraj, 2020), stablecoins can break the buck even under over-collateralization when the issuer's reserves fall below a critical threshold. What drives debasement is the stablecoin issuer's trade-off between sustaining a stable value to stimulate demand and sharing risk with users to avoid costly liquidation.

[1] Balvers and McDonald (2021) study stablecoins with stable purchasing power rather than stable exchange rates.
[2] An alternative to collateralization is to use algorithmic supply rules to stabilize price but success has been limited.

The system exhibits a bimodal distribution of states. In states of high reserves, the stablecoin issuer maintains a fixed exchange rate, so stablecoin demand is strong and transaction volume is high. Through open market operations and transaction fees, the issuer collects revenues that further grow its reserves. The level of reserves is capped by an endogenous upper bound, beyond which the stablecoin issuer pays out dividends to equity (governance token) investors. In states of low reserves, the issuer off-loads risk to users. The depressed stablecoin demand and transaction volume imply low revenues and slow accumulation of reserves. Therefore, a fixed exchange rate can last for a long time without any hint of instability, but once debasement happens, recovery is slow.

From a theoretical perspective, the stablecoin issuer's reserve management reminisces the dynamic corporate cash management (Bolton, Chen, and Wang, 2011; Décamps, Mariotti, Rochet, and Villeneuve, 2011; Hugonnier, Malamud, and Morellec, 2015; Malamud and Zucchi, 2019), but different from a corporation, the stablecoin issuer can depreciate its liabilities (the outstanding stablecoins) through debasement, similar to a country monetizing its debt through inflation or debt write-off in restructuring. Stablecoins share similarities with contingent convertible bonds (CoCos) that automatically share risk between equity investors and debt (stablecoin) holders (Glasserman and Nouri, 2012; Chen, Glasserman, Nouri, and Pelger, 2017). Unlike CoCos, stablecoins do not pre-specify conversion rates and trigger events, which are both under the discretion of the issuer.

Next, we provide more details on the basic model setup, our main results, and extensions. The model is built to be technology-neutral so that it applies to stablecoins built on different blockchain protocols.[3] In a continuous-time economy, a digital platform issues stablecoins ("tokens"). Users derive a flow utility from token holdings, which captures the transactional benefits, and network effect of tokens as means of payment is modelled by embedding the aggregate holdings in individuals' flow utility (Cong, Li, and Wang, 2021).[4] Following Moreira and Savov (2017), we assume that users' transactional demand for tokens declines in the volatility of token price.[5]

Users can redeem token holdings for numeraire goods ("dollars") and trade tokens amongst themselves. The platform can continuously trade tokens against dollar reserves, directly influencing the token price. Thus, the token price is at any time optimal from the platform's perspective. On the platform's balance sheet, the liability side has tokens and equity. On the asset side, the platform

[3]At the current stage of stablecoin developments, policy makers take a technology-neutral approach that emphasizes economic insights over technological aspects of implementation ECB Crypto-Assets Task Force (2019).

[4]This money-in-utility approach follows the macroeconomics literature (Ljungqvist and Sargent, 2004). The modelling of network effect is in the traditional of social interaction (Glaeser, Sacerdote, and Scheinkman, 1996).

[5]To be liquid and circulate as a transaction medium, a security must be designed in a way that deters private information acquisition and thus avoids asymmetric information between trade counterparties (Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999; Dang, Gorton, Holmström, and Ordoñez, 2014).

holds dollar reserves that earn a constant interest rate and load on Brownian shocks. The shocks capture operational risk and unexpected fluctuations of reserve value. When the platform requires the stablecoin users to post collateral, the shocks originate from the fluctuation of users' collateral value, and the size of the shock can be controlled by through the margin requirement on the users.

In the platform's dynamic optimization program, the amount of excess reserves (i.e., equity) is the state variable. The recursive formulation through the Hamilton-Jacobi-Bellman (HJB) equation significantly simplifies the problem. First, the optimality condition on payouts to shareholders implies an endogenous upper bound on the state variable (the payout boundary). The platform pays out reserves to equity investors when it has accumulated a sufficient amount of reserves as risk buffer. Beyond the interests earned on reserves, the platform earn transaction fees charged to users. We specify the lower (liquidation) bound on the excess reserves to be zero, which is never reached in equilibrium, as the platform optimally stays away from costly and irreversible liquidation. Therefore, the equilibrium path always features over-collateralization.

The platform's choice of token price process boils down to choosing instantaneous drift and diffusion, and an optimal stochastic process of token price can be implemented through trading reserves against tokens. Beyond the transactional flow utility, users also care about the *net* appreciation or depreciation, i.e., drift minus fees that are proportional to token holdings. We show that the optimal fees, token-price drift, and token-price diffusion can be implemented through directly setting users' token holdings and token-price diffusion. We then recover the implied equilibrium process of token price by solving a differential equation problem with the optimal token demand and token-price diffusion as inputs. The recovery delivers the optimal token-price drift and fees.

In spite of over-collateralization, the stablecoin issuer cannot always sustain one-to-one convertibility between tokens and dollars. To avoid costly liquidation, the platform opts for debasement whenever equity falls below a threshold. Debasement triggers a vicious cycle as the depressed token demand leads to a reduction in fee revenues, which causes a slow recovery of equity and persistent debasement. However, debasement is a valuable option, as it allows the platform to share risk with users. When negative shocks decrease equity, debasement causes token liabilities to shrink. Above the debasement threshold, the platform credibly sustains one-to-one convertibility. Then a strong token demand allows the platform to collect revenues to grow equity, which further strengthens the one-to-one convertibility to dollar. This virtuous cycle implies persistent expansion of platform equity until it reaches the payout boundary. The stationary distribution of platform equity is thus bimodal with two peaks near zero and the payout boundary, respectively.

Next, we allow the platform to raise equity. Under a fixed cost of equity issuance, the platform first resorts to debasement when equity falls below the threshold and only issues equity when equity falls to zero and when equity issuance leads to a higher shareholders' value than further debasement does. Once the fixed cost is paid, the platform raises equity to replenish reserves all the way up to the payout boundary, where the marginal value of equity falls to one. The jump in reserves implies an immediate restoration of token-price stability and, accordingly, a jump in the aggregate token demand. To preclude a predictable jump in token price level (i.e., an arbitrage opportunity), the platform must expand token supply at the payout boundary with the proceeds from selling tokens distributed to shareholders, an operation akin to issuing debts for share repurchase or dividend payout. After the equity issuance, the token price is stable at the pre-issuance level (as if the token is re-pegged) until future negative shocks deplete the reserves below the debasement threshold.

We evaluate two types of stablecoin regulations. The first is a standard capital requirement that stipulates the minimal degree of over-collateralization (equity). The capital requirement fails to eliminate debasement. As long as the threat of liquidation (or equity issuance costs) exists, whether it is due to reserve depletion or the violation of regulation, it is optimal for the stablecoin issuer and users to share risk through debasement. Therefore, the second type of regulation, which enforces a fixed exchange rate, only hurts welfare by destroying the economic surplus from risk sharing. In practice, stablecoin issuers cannot commit against debasement. We show that even when such commitment is available, it would not not optimal. Admittedly, our model omits several elements and thus can underestimate the value of a perfectly stable token. For example, debasement invites speculation that in turn amplifies price fluctuation and triggers a vicious cycle (Mayer, 2020).

Stablecoins became the subject of heated debate after the technology giant Facebook and its partners announced their own stablecoin, Libra (now "Diem"), in June 2019.[6] Leveraging on their existing customer networks, global technology or financial firms are able to rapidly scale the reach of their stablecoins.[7] To understand the advantages of well-established networks in the stablecoin space, we compare platforms with different degrees of network effects. A stronger network effect is indeed associated with stabler tokens. The frequency of debasement declines as a stronger network effect allows the platform to accumulate fee revenues faster when equity runs out and, through a higher continuation value, incentivizes the platform to maintain a larger equity risk buffer.

[6] The announcement triggered a globally-coordinated response under the umbrella of the G7. From then on, the G20, the Financial Stability Board (FSB), and central banks around the world have also embarked on efforts to address the potential risks while harnessing the potential of technological innovation.

[7] Another example is JPM Coin, a blockchain-based digital coin for fast payment settlement that is being developed by JP Morgan Chase and was announced in February 2019.

Stablecoin initiatives sponsored by companies with global customer networks attract attention from regulators for not only its potential of wide adoption but also concerns over monopoly power. In our model, two counteracting forces determine the share of economic surplus seized by the platform. Under a stronger network effect, the platform can extract more rents from its users through fees or risk sharing, but it is also more eager to stimulate token demand by lowering fees and stabilizing tokens given that individual users do not internalize the positive network externalities. In equilibrium, these two forces balance each other out. As a result, the split of welfare between the stablecoin issuer and users is rather insensitive to the degree degree of network effects.

The enormous amount of transaction data brought by a payment system offers a strong incentive for digital platforms to develop their own stablecoins. We further extend our model to incorporate data as a productive asset for the platform. Data improves the quality of the platform and thereby increases the users' flow utility from token holdings (the transactional benefits). A feedback loop emerges: Transactions generate more data, which improves the platform and leads to a stronger token demand and even more transactions. As a result, data accumulates exponentially over time. The platform has to balance between acquiring data and preserving reserves. The former requires lower fees and a more stable token while the latter calls for higher fees and risk-sharing with users through debasement. A key result is that the amount of reserves is no longer the key state variable driving the platform's decisions. The state variable is now the ratio of reserves to data stock. Data enters into the platform's decisions through a sufficient statistic, the data q, which is the marginal contribution of data to shareholders' value in analogy to Tobin's q of productive capital.

An increase of data productivity captures the revolutionary progress in big data technology.[8] In response, the platform maintains more reserves before payout because data allows shareholders' investment to grow faster in expectation. However, more reserves do not lead to stabler tokens. The platform becomes more aggressive in stimulating transactions for data acquisition. A larger transaction volume and a greater value of stablecoin liabilities amplifies the platform's risk exposure, i.e., the shock loading of excess reserves (equity). As a result, debasement is more likely to happen. Therefore, a paradox exists — stablecoins built primarily for the acquisition and utilization of transaction data can become increasingly unstable precisely when data becomes more valuable.

[8] Alternative payment-service providers also benefit from regulatory initiatives that facilitate data sharing. A new European Union directive, PSD2, requires banks to provide non-bank service providers with data that would allow those providers to offer payment and other services to the banks' customers (Duffie, 2019).

2 Background: Crypto Shadow Banking in Decentralized Finance

Blockchain technology supports peer-to-peer transfer of assets on distributed ledgers, potentially eliminating the need to transact through intermediaries (Raskin and Yermack, 2016; Abadi and Brunnermeier, 2019; Brainard, 2019). Decentralization avoids sizable intermediation costs (Philippon, 2015). Depending on the blockchain protocols, decentralization can enhance operational resilience by eliminating single point of failure while still achieve scalability (John, Rivera, and Saleh, 2020).[9] Decentralized finance ("DeFi") offers blockchain-based alternatives to traditional financial services, such as banking, brokerage, and exchanges (Lehar and Parlour, 2021). It also builds on smart contracts (coded enforcement via programmable money (Tinn, 2017; Cong and He, 2019; Goldstein, Gupta, and Sverchkov, 2019), a concept independent from blockchain (Halaburda, 2018).

This emerging financial architecture requires blockchain-based currencies. A viable means of payment must maintain a stable value at least within the settlement period (i.e., the time required for generating decentralized consensus on transaction records (Chiu and Koeppl, 2017)). However, most cryptocurrencies are highly volatile (Hu, Parlour, and Rajan, 2019; Stulz, 2019; Liu and Tsyvinski, 2020). They are platform-specific currencies (Catalini and Gans, 2018; Sockin and Xiong, 2018; Li and Mann, 2020; Bakos and Halaburda, 2019; Gryglewicz, Mayer, and Morellec, 2020; Cong, Li, and Wang, 2021) whose values are unbacked and fluctuate along the supply and demand dynamics native to the hosting platforms (Cong, Li, and Wang, 2019).[10]

Stablecoins are advertised as blockchain-based copies of fiat currencies. The total market value is above \$100 billion dollars. The issuer can be a corporate entity or a consortium (e.g., a consortium led by Facebook, the developer of Diem). It can also be an internet protocol (e.g., MakerDAO, the issuer of Dai) whose rules may be updated upon users' consensus.[11] A stablecoin is backed by the issuer's portfolio of reserve assets. The stability of value is sustained by the issuer conducting

[9] Decentralized ledger technology is nascent and faces many challenges. The finality of settlement can be compromised when the nodes of a distributed network disagree (Biais, Bisiere, Bouvard, and Casamatta, 2019; Ebrahimi, Routledge, and Zetlin-Jones, 2020). Law of one price can fail in segmented markets (Makarov and Schoar, 2020). Proof-of-work protocol has innate limits on adoption Hinzen, John, and Saleh (2019), system security risks (Budish, 2018; Pagnotta, 2021), and requires wasteful energy consumption that crowds out other users (Benetton, Compiani, and Morse, 2021). Researchers are active in studying alternative protocols, such as proof-of-stake (e.g., Saleh, 2020; Fanti, Kogan, and Viswanath, 2019). The cost of decentralization also depends on the market structure of decentralized ledger keepers (Huberman, Leshno, and Moallemi, 2019; Pagnotta and Buraschi, 2018; Easley, O'Hara, and Basu, 2019; Cong, He, and Li, 2020; John, Rivera, and Saleh, 2020; Lehar and Parlour, 2020).

[10] Unbacked cryptocurrencies are exposed to platform-specific risks (Liu, Sheng, and Wang, 2020; Shams, 2020), developers' moral hazard (Chod and Lyandres, 2019; Davydiuk, Gupta, and Rosen, 2019; Garratt and Van Oordt, 2019), self-fulfilling and speculative beliefs (Garratt and Wallace, 2018; Benetton and Compiani, 2020). Their returns exhibit a factor structure akin to those of other risky assets (Liu, Tsyvinski, and Wu, 2019).

[11] It is technologically feasible to hard-code certain aspects of a protocol. Kim and Zetlin-Jones (2019) propose an ethical framework for developers to determine which aspects should be immutable and which should not.

open market operations (i.e., trading reserves against stablecoins) and meeting redemption requests (Bullmann, Klemm, and Pinna, 2019). The distributed ledger records the ownership and transfer of stablecoins but verifying reserves still relies on traditional auditing (Calle and Zalles, 2019).

Stablecoins are the link between decentralized finance and the real economy. The volatility of the first-generation cryptocurrencies, such as Bitcoin and Ether, limits their adoption in real-world transactions. Stablecoins, designed to be have stable exchange rate with respect to the reference fiat currencies, have the potential to mediate transactions of goods, services, and real assets. Stablecoins are also important for the cryptocurrency community. Traders' activities heavily involve rebalancing between stablecoins and more volatile cryptocurrencies. Cryptocurrency has become an emerging asset class with the total market capitalization around \$ 1.5 trillion dollars (with roughly \$ 700 billion in Bitcoin).[12] It is estimated that 50 to 60% of Bitcoin trading volume is against USDT, the stablecoin issued by Tether (J.P. Morgan Global Research, 2021).

In spite of the importance of stablecoins, there does not exist clear legal and regulatory frameworks. Unlike depository institutions, a stablecoin issuer does not have any obligation to maintain redemption at par. Given that reserves are often invested in risky assets, many are concerned that a major stablecoin "breaks the buck", triggering financial turmoil (Massad, 2021). The concern is motivated by the relatively recent episode of money market funds switching from stable to floating NAV (net asset value) during the global financial crisis of 2007-2008. Indeed, the creation of stablecoins is essentially a new form of shadow banking—unregulated safety transformation.[13]

The reserve assets are often risky and offer attractive yields.[14] Panel A of Figure 1 illustrates stablecoin creation that features over-collateralization.[15] The issuer's excess reserves (equity) buffers the fluctuation of reserve value. The equity shares are called governance tokens (or "secondary units") that carry the rights to vote on changes of protocols (i.e., control rights) and pay out cash flows generated by transaction fees charged on the stablecoin users. Governance tokens can be issued to replenish reserves, just as traditional corporations can raise cash by issuing equity.

A stablecoin issuer essentially takes a leveraged bet on the value of reserve assets. The issuer can increase its leverage by issuing new stablecoins to finance the purchase of reserve assets, just

[12] Nearly half of millennial millionaires have at least 25% of their wealth in cryptocurrencies (CNBC Survey).

[13] A stable value is essential for a transaction medium because it reduces asymmetric information between transaction counterparties (Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999; Dang, Gorton, Holmström, and Ordoñez, 2014). Without informational frictions, stability may not be necessary (Schilling and Uhlig, 2019).

[14] According to Nikhilesh De and Marc Hochstein at Coindesk on May 13, 2021, USDT, the largest stablecoin by market value, is backed by dollar cash, cash equivalents, and commercial papers (75.85%), secured loans (12.55%), corporate bonds, funds, and precious metals (9.96%), and other investments including digital tokens (1.64%).

[15] Over-collateralization is a common practice among stablecoin issuers (Bullmann, Klemm, and Pinna, 2019).

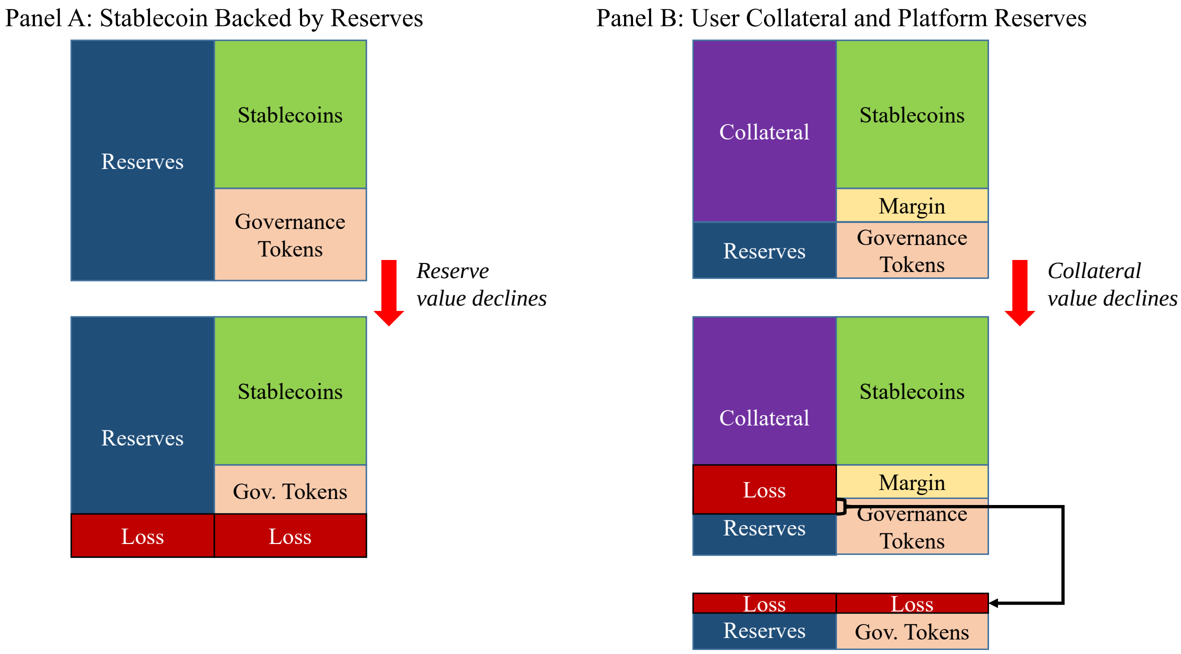

Figure 1: **Crypto Shadow Banking.** This figure illustrates the two structures of stablecoins. In Panel A, a platform issues stablecoins backed by its reserves. The excess reserves, i.e., the equity position, belong to the holders of governance tokens who have the control right (i.e., the control over platform policies). When reserves are invested in risky assets, a potential loss is absorbed by the equity position. As long as the stablecoins are over-collateralized, their value is intact. In Panel B, stablecoins are backed by both the user's collateral and the platform's reserves. When the collateral value declines and the user fails to meet the margin requirement, the platform sees the collateral and uses the proceeds (and its own reserves) to buy back stablecoins in the secondary market.

as banks finance their lending and security trading with newly issued deposits (i.e., inside money creation (Tobin, 1963; Bianchi and Bigio, 2014; Piazzesi and Schneider, 2016; Faure and Gersbach, 2017; Donaldson, Piacentino, and Thakor, 2018; Parlour, Rajan, and Walden, 2020)). Unlike banks that commit to redeem deposits at par, the stablecoin issuer can debase the stablecoins.

Panel B of Figure 1 illustrates a more complex structure that is similar the one adopted by MakerDAO, the issuer of Dai and an early decentralized autonomous organizations.[16] A user pledges her holdings of cryptocurrencies and other assets as collateral for newly created stablecoins, subject to a haircut (margin requirement). The user may transfer the stablecoins, which then circulate in the market, but she must maintain the margin requirement. If the collateral value declines and the user cannot maintain the margin, she loses her collateral to the stablecoin issuer, who then liquidates the collateral and uses the proceeds to buy back (and burn) the stablecoins created for this user.[17]. If the liquidation of collateral does not generate sufficient proceeds, the

[16]Decentralized autonomous organizations (DAOs) are organizations represented by rules encoded as computer programs and controlled by the organization members through various voting mechanisms on blockchains.

[17]Burning is to send the stablecoins to an irretrievable address

stablecoin issuer's reserves are used to supplement the expense of repurchasing the stablecoins.[18]

The structure in Panel B of Figure 1 is common in traditional shadow banking: A bank sets up a conduit (special purpose vehicle) that tranches risky investments into debt and equity, and at the same time, extends a guarantee to the debt investors so that when the conduit becomes insolvent, the bank internalizes the loss (Acharya, Schnabl, and Suarez, 2013). The stablecoin is like the debt (senior) tranche of the conduit, and the stablecoin issuer and the user in Panel B of Figure 1 correspond to the bank and the conduit, respectively. The stablecoin issuer's commitment to buy back stablecoins potentially with her own reserves is analogous to the bank's guarantee.

In the language of traditional shadow banking, the difference between the two structures in Figure 1 is that from Panel A to Panel B, the stablecoin issuer off loads risk to off-balance-sheet entities (i.e., the users) so that the stablecoins will be backed by both users' collateral and the issuer's reserves. Relative to the simple structure in Panel A of Figure 1, such double collateralization does not necessarily strengthen stability because both users' collateral and the issuer's reserve assets can be risky and highly correlated in value, especially when both are cryptocurrencies.

We set up our model in the next section following the structure in Panel A of Figure 1 and present the solution in Section 4, discussing the stablecoin issuer's strategies, welfare, and optimal regulations. In Section 5, we show that our model can be easily extended to incorporate double collateralization in Panel B of Figure 1, and we analyze the optimal margin requirement.

3 A Model of Stablecoins

Consider a continuous-time economy where a continuum of agents ("users") of unit measure conduct peer-to-peer transactions on a digital platform. The platform facilitates transactions by introducing a local currency ("token"). The generic consumption goods ("dollars") are the numeraire in this economy. The platform sets the exchange rate between tokens and dollars. Let P_t denote the token price in units of dollars (i.e., the exchange rate between tokens and dollars).

At time t, users can redeem their token holdings for dollars or buy more tokens from the platform at the dollar price P_t. By no arbitrage, users also trade tokens among themselves at the price P_t. In equilibrium, the dollar price of token has a law of motion, which the atomic users take as given:

$$\frac{dP_t}{P_t} = \mu_t^P dt + \sigma_t^P dZ_t \,, \tag{1}$$

[18]While the repurchase (and burn) of stablecoins is recorded on the blockchain, the liquidation of non-cryptocurrency collateral and reserves happens off-chain and still requires the traditional financial and legal systems.

where the standard Brownian shock, dZ_t, will be introduced below as a shock to the platform's reserves. We will show how μ_t^P and σ_t^P depend on how the platform's optimal strategies. Next, we first introduce users and then set up the platform's problem.

Users. We use $u_{i,t}$ to denote the numeraire value of user i's holdings of tokens, so user i holds $k_{i,t} = u_{i,t}/P_t$ units of tokens. The aggregate dollar value of token holdings is $N_t \equiv \int_{i\in[0,1]} u_{i,t}dt$.

A representative user i derives a flow utility from token holdings

$$\frac{1}{\beta}N_t^{\alpha}u_{i,t}^{\beta}A^{(1-\alpha-\beta)}dt - \eta u_{i,t}|\sigma_t^P|dt\,, \tag{2}$$

where $\alpha, \beta \in (0,1)$ with $\alpha + \beta < 1$, $A > 0$, and $\eta \geq 0$. We model the utility from holding means of payment following the classic models of monetary economics (e.g., Baumol, 1952; Tobin, 1956; Feenstra, 1986; Freeman and Kydland, 2000) and related empirical studies (e.g., Poterba and Rotemberg, 1986; Lucas and Nicolini, 2015; Nagel, 2016). In this literature, agents derive utility from the real value of holdings, i.e., $u_{i,t}$.[19] Following Rochet and Tirole (2003), we introduce network effect via N_t^{α}. As in Cong, Li, and Wang (2021), it captures the fact that when tokens are more widely used as means of payment, each user's utility is higher.[20] The quality of the payment system is captured by parameter A which we will endogenize in Section 6.2. Here we define tokens' transaction utility from an ex ante perspective and do not model the ex post circulation of tokens in line with the aforementioned literature on money-in-utility and cash-in-advance constraint.

The user's preference for stability is captured by the parameter η (> 0), and is defined on the absolute value of σ_t^P to capture the fact that users are averse to token price fluctuation no matter whether the price moves with ($\sigma_t^P > 0$) or against ($\sigma_t^P < 0$) the platform's reserve shock dZ_t. We motivate such preference for stability following Moreira and Savov (2017): To be liquid and circulate as a transaction medium, a security must be designed in a way that deters private information acquisition and thus avoids asymmetric information between trade counterparties (Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999; Dang, Gorton, Holmström, and Ordoñez, 2014).[21]

User i pays a proportional fee on her token holdings, $u_{i,t}f_t dt$, where f_t is set by the platform. In

[19] We refer readers to the textbook treatments (e.g., Galí, 2015; Ljungqvist and Sargent, 2004; Walsh, 2003). For the nominal value (i.e., $k_{i,t}$) to affects agents' decisions, additional frictions, such as nominal illusion (e.g., Shafir, Diamond, and Tversky, 1997) or sticky prices (e.g., Christiano, Eichenbaum, and Evans, 2005), have to be introduced.

[20] For instance, when there are more people use tokens, it becomes easier to find a transaction counterparty that accepts tokens, so token holders expect more token usage of means of payment.

[21] The disutility from token volatility can also be motivated by risk-averse preference or users' aversion to exchange-rate shocks that cause losses of net worth when assets and liabilities are denominated in different currencies (tokens and dollars) (Doepke and Schneider, 2017; Gopinath, Boz, Casas, Díez, Gourinchas, and Plagborg-Møller, 2020).

practice, fees are often charged on transactions, in that f_t can be interpreted as the transaction fee per dollar transaction.[22] Note that as long as the money (token) velocity is constant within a small time interval (dt), transaction volume is proportional to token holdings. There exists a technical upper bound on the volume of transactions that the platform can handle per unit of time. Without loss of generality, we model the bound as follows

$$N_t \leq \overline{N} \,. \tag{3}$$

Therefore, while the network effects generate positive externality of individual users' adoption on others, the bound on transaction volume implies negative externality through congestion.

Let $R_{i,t}$ denote user i's (undiscounted) *cumulative* payoff from platform activities. The instantaneous payoff depends on user i's choice of $u_{i,t} \geq 0$ and is given by

$$dR_{it} \equiv \frac{1}{\beta} N_t^{\alpha} u_{i,t}^{\beta} A^{(1-\alpha-\beta)} dt + u_{i,t} \Big(\frac{dP_t}{P_t} - r dt - f_t dt - \eta |\sigma_t^P| dt \Big) \,, \tag{4}$$

where the first term is the flow utility (2) and the second term includes the return from token price change net off the forgone interests and fees. A representative user i chooses $u_{i,t} \geq 0$ to maximize

$$\max_{u_{i,t} \geq 0} \mathbb{E}_t \left[dR_{it} \right] = \max_{u_{i,t}} \frac{1}{\beta} N_t^{\alpha} u_{i,t}^{\beta} A^{(1-\alpha-\beta)} dt + u_{i,t} \Big(\mu_t^P - \eta |\sigma_t^P| - r - f_t \Big) dt \,. \tag{5}$$

The Platform. Let S_t denote the total units of tokens outstanding. The token market clearing condition is given by

$$S_t = \int_{i \in [0,1]} \frac{u_{i,t}}{P_t} dt, \tag{6}$$

or equivalently, in the numeraire (dollar) value:

$$N_t = S_t P_t. \tag{7}$$

The platform decides on the fees and controls the dollar price of tokens, P_t, by adjusting the token supply. This is akin to central banks using open market operations to intervene in the foreign exchange markets (e.g., Calvo and Reinhart, 2002). When the platform issues more tokens ($dS_t > 0$), it collects dollar revenues as users buy tokens with dollars. When the platform retires

[22] Without loss of generality, we consider homogeneous users and the same fees applied to all users. In the presence of user heterogeneity as in Cong, Li, and Wang (2021), price discrimination in fees can be interesting but would still be difficult in practice as users can easily disperse their holdings into different addresses or wallets.

tokens ($dS_t < 0$), it loses dollars to users. Stablecoin platforms often claim a fixed exchange rate. However, we will show that optimal exchange rate depends on the platform's reserves.

Let M_t denote the platform's reserves (dollar holdings), which has a law of motion

$$dM_t = rM_t dt + (P_t + dP_t)dS_t + N_t f_t dt + N_t \sigma dZ_t - dDiv_t. \tag{8}$$

The first term is the interests earned on the reserves balance, and r is the constant interest rate. The second term is the revenues (losses) from issuing (buying back) tokens in dt from the secondary market. As the trade settles in dt, the amount of quantity adjustment, dS_t, is multiplied by the price in dt, $P_t + dP_t$. The third term is the fee revenues. The fourth term deserves more attention. Z_t is a standard Brownian motion, and its increment, dZ_t, captures the shocks to the net revenues, which can stem from operating expenses, risks in the reserve portfolio, and activities beyond fees and token management.[23] This shock is the only source of uncertainty in the model. Let Div_t denote the *cumulative* dividend process. The platform's reserves decrease when the platform pays its owners dividends, $dDiv_t$ (≥ 0 under limited liability).

The platform maximizes the expected discounted value of dividend payouts to its owners/shareholders:

$$V_0 \equiv \max_{\{f_t, dS_t, dDiv_t\}} \mathbb{E}\left[\int_0^\infty e^{-\rho t} dDiv_t\right] \quad \text{subject to} \quad (8) \text{ and } \quad dDiv_t \geq 0. \tag{9}$$

We assume that the platform's shareholders are impatient relative to other investors, $\rho > r$.[24]

Liquidation. The current discussion of stablecoins focuses on the bank runs (Brainard, 2019; G7 Working Group on Stablecoins, 2019; ECB Crypto-Assets Task Force, 2019; Massad, 2021). In our model, when $M_t < S_t P_t$ (i.e., the reserve value is below the value of stablecoins), a coordination failure can happen as users may redeem tokens en masse (Diamond and Dybvig, 1983).[25]

To distinguish our equilibrium analysis from the well-known mechanism of bank runs, we assume

[23] Examples include the revenues and costs associated with users' advertisement on the platform and loans to users, which are often enabled by the platform's possession of transaction data as will be discussed in Section 6.2. The risk of reserve portfolio may also arise from the platform holding different currencies and the fluctuation of these currencies' dollar exchange rates. Providers of global stablecoins (GSC) typically accept deposits in different currencies, and hold a portfolio of these currencies as backing (e.g., Libra Association, 2020).

[24] This impatience could be preference based or could arise indirectly because shareholders have other attractive investment opportunities. From a modeling perspective, impatience motivates the platform to pay out because, otherwise, the expected return on M_t is greater than r (due to revenues from the fees and token issuance), which then implies that the platform never pays out dividends.

[25] To avoid the run, the platform can potentially reset the redemption price P_t, but to make up the deficit, $S_t P_t - M_t$, the token price has to jump downward, causing the users to expect an infinite rate of change and, as a result, to liquid their token holdings immediately under our specification of the preference for stability.

that the platform is liquidated and its owners' value falls permanently to zero when $M_t < S_tP_t$ (i.e., reserves are insufficient to meet all users' redemption requests). Our focus on over-collateralized stablecoins also differentiates our analysis from earlier work of Routledge and Zetlin-Jones (2018) on the strategy of maintaining exchange rate stability for under-collateralized stablecoins. As we show below, the platform has a positive continuation value, so it optimally maintains $M_t \geq S_tP_t$ to avoid liquidation. Note that our model features (small) diffusive shocks, so the platform can maintain $M_t \geq S_tP_t$ through continuous adjustments.[26] Therefore, on the equilibrium path, $M_t < S_tP_t$ and a run never happen.[27] In other words, the platform always optimally over-collateralizes under our liquidation assumption. The conventional wisdom is that breaking the buck (or debasement) will not happen under over-collateralization. Our analysis below challenges this notion.

Discussion: Under-Collateralization. A run does not necessarily happen when M_t falls below S_tP_t. If we remove the assumption of liquidation upon $M_t < S_tP_t$, the equilibrium can feature under-collateralization and equilibrium multiplicity. When $M_t < S_tP_t$, one equilibrium path has users not withdrawing en masse and trading tokens at the unit price P_t, while the other equilibrium path features a run (closely related to the equilibrium dynamics in Donaldson and Piacentino (2020)). To pin down the equilibrium path, one may adopt the global game approach (Morris and Shin, 1998; Goldstein and Pauzner, 2005) and assume that users observe noisy signals of the stablecoin issuer's reserves, in line with the reality (Calle and Zalles, 2019; Duffie, 2019). Under the strategic uncertainty, a run (and liquidation) happens only if M_t, the value of reserves, falls sufficiently below S_tP_t, the value of stablecoins. As long as there exists a threshold of reserves below which liquidation happens, our analysis below carries through.

4 Equilibrium

In this section, we characterize the analytical properties of the dynamic equilibrium and, to sharpen the economic intuition, we also provide graphical illustrations based on the numerical solutions.

[26]In the presence of (large) jump shocks, always maintaining sustain $M_t \geq S_tP_t$ is infeasible.

[27]In the off-equilibrium event of liquidation, token holders receive the value of liquidated reserves pro rata.

4.1 Managing Stablecoin: Optimal Strategies

User Optimization. A representative user i solves a static problem in (5) and the solution is

$$u_{i,t} = \left(\frac{N_t^{\alpha} A^{(1-\alpha-\beta)}}{r + f_t - \mu_t^P + \eta |\sigma_t^P|} \right)^{\frac{1}{1-\beta}} . \tag{10}$$

Users' choices exhibit strategic complementarity as $u_{i,t}$ increases in the aggregate value N_t. In equilibrium $N_t = u_{i,t}$ under user homogeneity, which, through (10), implies

$$N_t = \frac{A}{\left(r + f_t - \mu_t^P + \eta |\sigma_t^P| \right)^{\frac{1}{1-\xi}}} , \tag{11}$$

where, to simplify the notations, we define $\xi \equiv \alpha + \beta$ (< 1). Aggregate token demand decreases in the fees charged by the platform, f_t, and depends on the token price dynamics, which the platform controls. The capacity constraint (3) on N_t has to be satisfied when the platform sets its strategy.

Platform Optimization. To solve the platform's optimal strategy, we first note that, given the token price dynamics (i.e., μ_t^P and σ_t^P), the platform can directly set N_t through the fees f_t. Rearranging (11), we can back out the fees implied by the platform's choice of N_t:

$$f_t = \left(\frac{A}{N_t} \right)^{1-\xi} - r + \mu_t^P - \eta |\sigma_t^P| . \tag{12}$$

Using (12), we substitute out f_t in the law of motion of reserves (8) and obtain

$$dM_t - (P_t + dP_t) dS_t = r M_t dt + N_t^{\xi} A^{1-\xi} dt - r N_t dt + N_t \left(\mu_t^P - \eta |\sigma_t^P| \right) dt + N_t \sigma dZ_t - dDiv_t . \tag{13}$$

Next, we show the state variable for the platform's dynamic optimization is the *excess reserves*,

$$C_t \equiv M_t - P_t S_t . \tag{14}$$

To derive the law of motion of C_t, we first note that

$$\begin{aligned} dC_t &= dM_t - d(S_t P_t) = dM_t - (P_t + dP_t) dS_t - S_t dP_t \\ &= dM_t - (P_t + dP_t) dS_t - N_t \left(\mu_t^P dt + \sigma_t^P dZ_t \right) . \end{aligned} \tag{15}$$

The second equality uses $d(S_tP_t) = dS_tP_t + S_tdP_t + dS_tdP_t$ (by Itô's lemma) and the last equality uses (1) and $N_t = S_tP_t$. From a balance-sheet perspective, the reserves, M_t, are the platform's assets and the outstanding tokens, P_tS_t, are the liabilities. The excess reserves constitute the equity. Thus, equation (15) is essentially the differential form of balance-sheet identity. Using (13), we substitute out $dM_t - P_tdS_t$ the right side of (15) and obtain the following law of motion of C_t:

$$dC_t = \left(rC_t + N_t^{\xi}A^{1-\xi} - N_t\eta|\sigma_t^P|\right)dt + N_t(\sigma - \sigma_t^P)dZ_t - dDiv_t\,. \tag{16}$$

Note that $N_t\mu_t^P$ disappears. As shown in (13), the platform receives more fee revenues (see (12)) when users expect tokens to appreciate ($N_t\mu_t^P$), but such revenues do not increase the platform's equity (excess reserves) as they are cancelled out by the appreciation of token liabilities. Thus, the drift term, $rC_t + N_t^{\xi}A^{1-\xi} - N_t\eta|\sigma_t^P|$, is the expected appreciation of the platform's equity position.

We characterize a Markov equilibrium with the platform's excess reserves, C_t, as the state variable. In the following, we solve the platform's control variables, $dDiv_t$, σ_t^P, and N_t, as functions of C_t, and thereby, show that (16) is an autonomous law of motion of the state variable.

The platform owners' value function at time t is given by

$$V_t = V(C_t) = \max_{\{N,\,\sigma^P,\,Div\}} \mathbb{E}\left[\int_{s=t}^{\infty} e^{-\rho(s-t)}dDiv_s\right]. \tag{17}$$

The platform pays dividends when the marginal value of excess reserves is equal to one, i.e., one dollar has the same value either held within the platform or paid out,

$$V'\left(\overline{C}\right) = 1\,. \tag{18}$$

The next proposition states that the value function is concave. The declining marginal value of excess reserves implies that $\overline{C}$ in (18) is an endogenous upper bound of the state variable C_t. At any $C_t \in \left(0, \overline{C}\right)$, the platform does not pay dividends to its owners because the marginal value of excess reserve, $V'\left(\overline{C}\right)$, is greater than one, i.e., the owners' value of dividend. And the optimality of payout at $\overline{C}$ also requires the following super-contact condition (Dumas, 1991),

$$V''\left(\overline{C}\right) = 0\,. \tag{19}$$

Proposition 1 (Value Function Concavity and Over-Collateralization). *There exists* $\overline{C} > 0$

such that $C_t \in \left(0, \overline{C}\right]$. For $C_t \in \left(0, \overline{C}\right)$, the value function is strictly concave, and $V'\left(C_t\right) > 1$ so the platform maintains excess reserves. At $C_t = \overline{C}$, $V'\left(\overline{C}\right) = 1$ and the platform pays dividends and $dDiv = \max\{C - \overline{C}, 0\}$.

The platform's token liabilities are over collateralized. The intuition can be understood through the wedge between the liquidation value (zero) and the strictly positive value of platform as an ongoing concern. At $C_t = 0$, even a tiny negative shock triggers a downward jump of platform value to zero. By holding excess reserves, even by a small amount, the platform can prevent this. Next we confirm that a platform as ongoing concern always has a positive value. In the interior region $C \in \left(0, \, \overline{C}\right)$, $dDiv_t = 0$ and we obtain the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V(C) = \max_{\{N \in [0, \overline{N}], \, \sigma^P\}} \left\{ V'(C) \left(rC + N^{\xi} A^{1-\xi} - \eta N |\sigma_t^P| \right) + \frac{1}{2} V''(C) N^2 (\sigma - \sigma^P)^2 \right\}, \tag{20}$$

Setting first $\sigma^P = \sigma$ and then $N = 0$ is feasible in the HJB equation, which implies

$$V(C) \geq \frac{V'(C)}{\rho} \left(rC + \max_{\{N \in [0, \overline{N}]\}} \left\{ N^{\xi} A^{1-\xi} - \eta N \sigma \right\} \right) \geq \frac{V'(C) rC}{\rho} > 0 \,. \tag{21}$$

Next, we analyze the platform's optimal choice of token exchange-rate process and transaction volume. Because μ_t^P disappears from (16), the platform's choice of redemption price, P_t (dollar exchange rate), boils down to the choice of $\sigma_t^P = \sigma^P\left(C_t\right)$. First, we consider how the choice of σ_t^P and N_t when C_t approaches zero. In the limit, σ_t^P must converge to σ to mute the shock exposure,

$$\lim_{C \to 0^+} \sigma^P(C) = \sigma \, ; \tag{22}$$

otherwise, dC_t's loading on dZ_t in (16) is positive and a negative shock can trigger liquidation. Equation (22) implies that, when taking the right-limit on both sides of (20), we obtain

$$\lim_{C \to 0^+} \frac{V(C)}{V'(C)} = \frac{1}{\rho} \max_{\{N \in [0, \overline{N}]\}} \left\{ N^{\xi} A^{1-\xi} - \eta N \sigma \right\} = \frac{A}{\rho} \left(\frac{\xi}{\eta \sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta \sigma \,, \tag{23}$$

where the second equality follows from plugging in the optimal N_t given by

$$\underline{N} \equiv \lim_{C \to 0^+} N(C) = \arg\max_{N \in [0, \overline{N}]} \left\{ N^{\xi} A^{1-\xi} - \eta N \sigma \right\} = A \left(\frac{\xi}{\eta \sigma} \right)^{\frac{1}{1-\xi}} \wedge \overline{N} \,. \tag{24}$$

Therefore, (22) and (24) characterize respectively the limiting behavior of σ_t^P and N_t as C_t

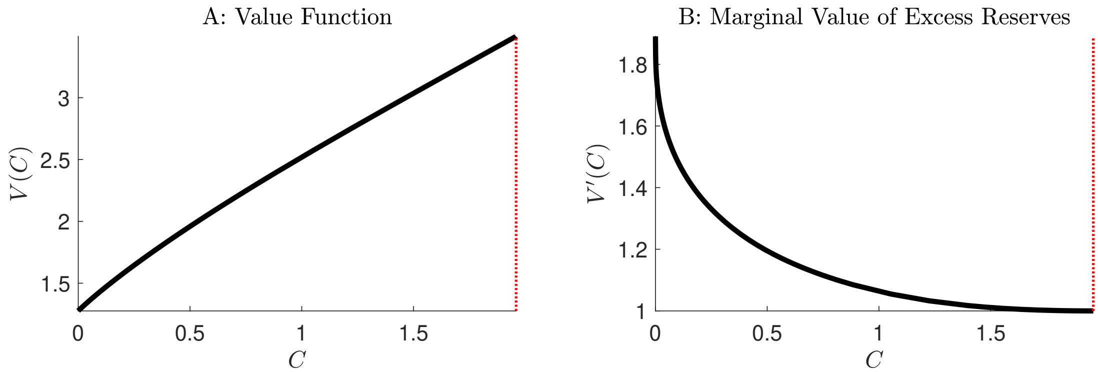

Figure 2: **Value Function.** This figure illustrates the level and first derivative of the platform's value function. The red dotted lines in both panels mark $\overline{C}$ (defined in Proposition 1). The parameters are $r = 0.05$, $\rho = 0.06$, $\sigma = 0.1$, $\overline{N} = 5$, $\eta = 0.15$, $\alpha = 0.45$, $\beta = 0.05$, and $A = 0.0025$.

approaches zero. In the process, we obtain a boundary condition (23) for the HJB equation. As an interim summary, the next proposition summarizes the value function solution as solution to an ordinary differential equation (ODE) problem with an endogenous boundary. Figure 2 plots the numerical solution of value function (Panel A) and the decreasing marginal value of excess reserves with the red dotted line marking the payout boundary $\overline{C}$.

Proposition 2 (**Value Function**). *The value function, $V(C)$, and the boundary $\overline{C}$ are solved by the ordinary differential equation (20) under the boundary conditions (18), (19), and (23).*

Next, we fully characterize the platform's optimal choices of σ_t^P and N_t as functions of the state variable, C_t (via the derivatives of $V(C)$). First, we define the platform's *effective risk aversion*:

$$\gamma(C) \equiv -\frac{V''(C)}{V'(C)}. \tag{25}$$

This definition is analogous to the classic measure of absolute risk aversion of consumers (Arrow, 1965; Pratt, 1964). From Proposition 1, $\gamma(C) \geq 0$ and, in $(0, \overline{C})$, $\gamma(C) > 0$. The *endogenous* risk aversion arises from the concavity of value function, which is in turn due to the gap between liquidation value and continuation value as previously discussed. The next proposition states the monotonicity of $\gamma(C)$ in C and summarizes the optimal $\sigma_t^P = \sigma^P(C_t)$ and $N = N(C_t)$.

Proposition 3 (**Risk Aversion, Token Volatility, and Transaction Volume**). *The platform's effective risk aversion, $\gamma(C)$, strictly decreases in the level of reserve holdings, C. There exists*

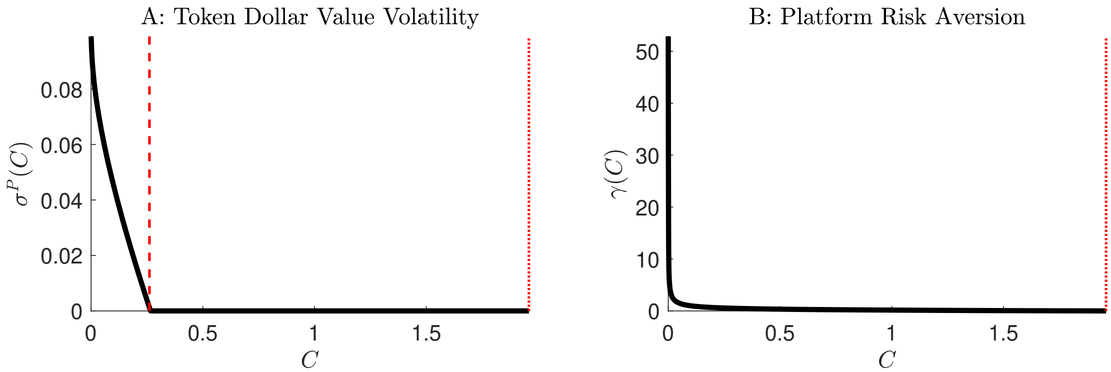

Figure 3: **Token Volatility and Platform Risk Aversion.** This figure plots token return volatility $\sigma^P(P)$ in Panel A and the platform's risk aversion $\gamma(C)$ in Panel B. In Panel A, the red dashed line marks $\widetilde{C}$ (in Proposition 3). The red dotted lines in both panels mark $\overline{C}$ (in Proposition 1). The parameterization follows Figure 2.

$\widetilde{C} \in \left(0, \overline{C}\right)$ *such that, at* $C \in \left(0, \widetilde{C}\right)$, $N(C) = \underline{N}$ *and* $\sigma^P(C)$ *strictly decreases in* C*, given by,*

$$\sigma^P(C) = \sigma - \frac{\eta}{\gamma(C)\underline{N}} \in (0, \sigma)\,, \tag{26}$$

and at $C \in \left[\widetilde{C}, \overline{C}\right]$, $\sigma^P(C) = 0$ *and* $N(C)$ *increases in* C*, given by*

$$N(C) = \min\left\{\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}}, \overline{N}\right\}. \tag{27}$$

When the platform's reserves are low, i.e., $C \in (0, \widetilde{C})$, it is the ratio of users' risk aversion to the platform's risk aversion that determines token volatility. Equation (26) shows that, in this region, when the platform accumulates more reserves and becomes less risk-averse, it absorbs risk from users by tuning down σ_t^P, and when the platform exhausts its reserves, it off-loads the risk in its dollar revenues to users.[28] The platform and its users engage actively in risk-sharing in $C \in (0, \widetilde{C})$. This is illustrated by the numerical solution in Panel A of figure 3 with $\widetilde{C}$ marked by the dashed line. In Panel B, we show that the platform's risk aversion declines in C. In this region of low reserves, the transaction volume, which is simply the dollar value of users' token holdings under our assumption of constant velocity, is pinned to the lowest level given by $\underline{N}$ in (24).

Once the platform's reserves surpass the critical threshold $\widetilde{C}$, its risk aversion becomes sufficiently low and it optimally absorbs all the risk in its dollar revenues, setting $\sigma^P(C)$ to zero which also implies that in this region $\mu^P(C) = 0$.[29] As a result, the transaction volume on the platform

[28]Equation (26) implies that the condition (23) is equivalent to $\gamma(C)$ (or $-V''(C)$) approaching infinity in the limit.
[29]This result arises because we express the equilibrium token price as a function of C, in that $P_t = P(C_t)$. Thus,

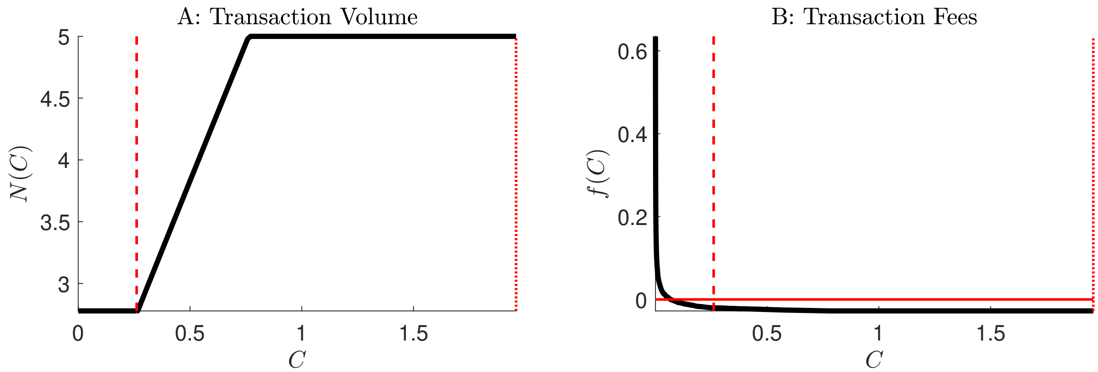

Figure 4: **Transaction Volume and Fees.** This figure plots transaction volume $N(C)$ in Panel A and fees per dollar of transaction $f(C)$ in Panel B. The red dotted lines mark $\overline{C}$ (in Proposition 1). In Panel A, the red dashed line marks $\widetilde{C}$ (in Proposition 3). In Panel B, the red solid line marks zero. The parameterization follows Figure 2.

starts to rise above the "hibernation level", $\underline{N}$, as illustrated by Panel A of Figure 4. Therefore, reserves are absolutely essential for stimulating economic activities on a stablecoin platform.

Interestingly, even though the platform shelters its users from risk at any $C > \widetilde{C}$, its risk aversion still shows up in N_t given by (27). As shown in (12), the platform's choice of N_t is implemented through fees. Therefore, the intuition can be more easily explained when we substitute (27), the optimal N_t, and the optimal $\sigma_t^P = 0$ (as well as $\mu_t^P = 0$) into (12) to solve f_t: when $\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}} < \overline{N}$,

$$f_t = \left(\frac{A\gamma(C)\sigma^2}{\xi}\right)^{\frac{1-\xi}{2-\xi}} - r\,, \tag{28}$$

and when $\left(\frac{\xi A^{1-\xi}}{\gamma(C)\sigma^2}\right)^{\frac{1}{2-\xi}} \geq \overline{N}$, i.e., C is sufficiently high such that $\gamma(C)$ falls below $\frac{\xi A^{1-\xi}}{\sigma^2 \overline{N}^{2-\xi}}$,

$$f_t = \left(\frac{A}{\overline{N}}\right)^{1-\xi} - r\,. \tag{29}$$

As a platform accumulates reserves, its risk aversion declines, which, through (28), implies low fees charged on users and in turn a larger token demand N_t (in (27)).

The platform faces a risk-return trade-off. The fees serve as a compensation for risk exposure but discourages users from participation. So when the platform's risk aversion rises, it charges users more per dollar of transaction at the expense of a smaller volume. When the platform's risk aversion declines, the fees per dollar of transaction decline while the total transaction volume increases. Once reserves are sufficiently high such that $\gamma(C) \leq \frac{\xi A^{1-\xi}}{\sigma^2 \overline{N}^{2-\xi}}$, the fees no longer declines

token volatility and token returns can be expressed as functions of C too, in that $\sigma_t^P = \sigma^P(C_t)$ and $\mu_t^P = \mu^P(C_t)$. Since $\sigma^P(C) = 0$ for $C > \widetilde{C}$, token price $P(C)$ must be constant for $C > \widetilde{C}$, implying $\mu^P(C) = 0$ for $C > \widetilde{C}$.

with the platform's risk aversion, as the platform has maxed out its transaction capacity, i.e., $N_t = \overline{N}$, and it becomes impossible to further stimulate user participation. Likewise, when the platform's reserves are below $\widetilde{C}$ and $\sigma^P(C) > 0$, $N_t = \underline{N}$, and the fees are given by

$$f_t = \left(\frac{A}{\underline{N}}\right)^{1-\xi} + \mu^P(C) - \eta\sigma^P(C) - r\,. \tag{30}$$

Even though the platform's risk aversion is high, it can no longer sacrifice transaction volume for higher fees because user participation already falls to the lowest level.

Panel B of Figure 4 plots the numerical solution of optimal fees that decrease in excess reserves. Depending on the parameters, fees can actually turn into user subsidies (i.e., fall below zero) when excess reserves are sufficiently high.[30] The next corollary summarizes the results on fees.

Corollary 1 (**Optimal Transaction Fees**). *Transaction fees, $f(C)$, decreases in excess reserves, C. At $C \in (0, \widetilde{C})$, where $\widetilde{C}$ is defined in Proposition 3, transaction fees are given by (30). At $C \in [\widetilde{C}, \widetilde{C}')$, where $\widetilde{C}'$ is defined by $\gamma(\widetilde{C}') = \frac{\xi A^{1-\xi}}{\sigma^2 \overline{N}^{2-\xi}}$, transaction fees are given by (28). At $C \in [\widetilde{C}', \overline{C})$, where $\overline{C}$ is defined in Proposition 1, transaction fees are given by (29).*

Another interesting implication of the optimal fees is that the platform charges (compensates) users the expected appreciation (depreciation) of tokens over risk-free rate, i.e., $\mu_t^P - r$ in f_t. To fully solve the fees, we need to know both $\gamma(C_t)$ and the function $\mu_t^P = \mu^P(C_t)$. In fact, the platform's choice of $\sigma_t^P = \sigma^P(C_t)$ already pins down the function of token price, $P_t = P(C_t)$, so the function $\mu^P(C_t)$ can be obtained from Itô's lemma. Next, we solve $P_t = P(C_t)$ from the function $\sigma^P(C_t)$. By Itô's lemma,

$$\sigma^P(C) = \frac{P'(C)}{P(C)} N(C) \left(\sigma - \sigma^P(C)\right), \tag{31}$$

where $N(C)\left(\sigma - \sigma^P(C)\right)$ is the diffusion of state variable C_t. Rearranging the equation, we solve

$$\frac{P'(C)}{P(C)} = \frac{1}{N(C)} \left(\frac{\sigma^P(C)}{\sigma - \sigma^P(C)}\right). \tag{32}$$

Using Proposition 2, we solve the value function $V(C)$ and obtain the function $\gamma(C)$. Then using Proposition 3, we obtain the functions $\sigma^P(C)$ and $N(C)$. Plugging $\sigma^P(C)$ and $N(C)$ into (32), we obtain a first-order ODE for the function of dollar price of token, $P(C)$.

[30] Specifically, under the particular parameterization, the condition is for fees to turn into subsidies near $\overline{C}$ is that $\frac{A^{1-\xi}}{\overline{N}^{1-\xi}} < r$ where we use (29) and the fact that $\mu^P(C) = 0$ for $C \in \left(\widetilde{C}, \overline{C}\right)$ (to be discussed later in this section).

To uniquely solve the function $P(C)$, we need to augment the ODE (32) with a boundary condition. In our model, both the platform and users are not concerned with the level of token price and only care about the expected token return, μ_t^P, and return volatility, σ_t^P. Therefore, we have the liberty to impose the following boundary condition:

$$P\left(\overline{C}\right) = 1\,. \tag{33}$$

i.e., the platform sets an exchange rate of one dollar for one token when C_t reaches $\overline{C}$. The next corollary states the solution of token price as solution to a first-order ODE problem.

Corollary 2 (**Solving Token Price**). *Given the solutions of $V(C)$ from Proposition 2 and $\sigma^P(C)$ and $N(C)$ from Proposition 3, the dollar price of token, $P(C)$, is solved by the ordinary differential equation (32) under the boundary condition (33).*

Proposition 3 states that, once C crosses above the critical threshold $\widetilde{C}$, $\sigma^P(C) = \mu^P(C) = 0$, which, by Itô's lemma (i.e., (31)), implies that $P'(C) = 0$. Therefore, if the platform's reserves are sufficiently high, it optimally fixes the dollar price (i.e., redemption value) of token at $P(C) = 1$. When C falls below $\widetilde{C}$, (31) implies that $P'(C) > 0$ (because $\sigma^P(C) \in (0, \sigma)$ in Proposition 3) so the token redemption value comoves with the platform's excess reserves.

The endogenous transition between redemption at par and redemption below par happens as the platform accumulates or depletes reserves through various activities laid out in (8) (and then (16)), including the platform's issuance of new tokens, users' token redemption, fee revenues, and shocks to the dollar reserve flow. The platform's choice of redemption price is optimally chosen and thus credible in the sense that the platform does not have incentive to deviate.

Proposition 4 (**Credible Redemption Value Regimes**). *At $C \in [\widetilde{C}, \overline{C}]$, where $\widetilde{C}$ is defined in Proposition 3, the platform credibly commits to redeem token at par, i.e., $P(C) = 1$. At $C \in (0, \widetilde{C})$, the redemption value of token comoves with the platform's excess reserves (i.e., $P'(C) > 0$).*

Proposition 4 states that redemption at par is credible if and only if the platform holds a sufficiently large amount of *excess* reserves ($C > \widetilde{C}$). When excess reserves fall below $\widetilde{C}$, the platform optimally debases its tokens. By allowing the redemption value to comove with its excess reserves, the platform off-loads the risk in its dollar revenues to users and thereby prevents liquidation.

Figure 5 plots the numerical solutions of aggregate token value, $N(C) = P(C)S(C)$ (Panel A), the redemption value of one token optimally set by the platform $P(C)$ (Panel B), and the total

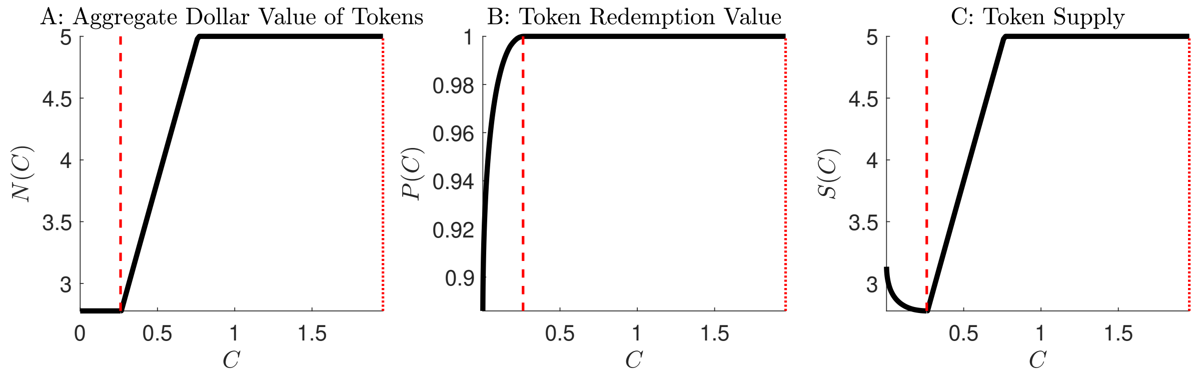

Figure 5: **Token Price and Quantity Dynamics.** This figure plots the aggregate dollar value of tokens $N(C)$ in Panel A, the token redemption value or dollar price $P(C)$ in Panel B, and token supply $S(C)$ in Panel C. The red dotted lines in all panels mark $\overline{C}$ (defined in Proposition 1). The red dashed lines in all panels mark $\widetilde{C}$ (defined in Proposition 3). The parameterization follows Figure 2.

quantity of tokens $S(C)$ implied by $N(C)$ and $P(C)$ (Panel C). The dashed line marks $\widetilde{C}$. The platform implements the optimal token redemption value through the manipulation of token supply. When the platform has enough reserves to credibly sustain redemption at par (i.e., $C > \widetilde{C}$), token supply comoves with demand so that $P(C)$ is fixed. Below $\widetilde{C}$, a decrease of excess reserves triggers the platform to supply more tokens in exchange for dollars that replenishes reserves. The users respond to token debasement by reducing demand to $\underline{N}$, which in turn reinforces the debasement.

In practice, stablecoin platforms often claim commitment to redemption at par and substantiate this commitment by holding reserves that just cover their token liabilities. However, our analysis so far has drawn two conclusions that challenge such conventional wisdom. First, as long as there exists uncertainty in a platform's dollar revenues, over-collateralization is not only necessary but also optimal from the platform's perspective. Second, redemption at par in every state of world is not credible or "incentive-compatible". As shown by Proposition 4, it is always in the platform's interest to debase its tokens once its excess reserves fall below the critical threshold. Therefore, redemption at par is only credible when reserves are sufficiently high.

Simulation and Long-Run Dynamics. Using the parameters in Figure 2 and the numerical solutions, we simulate in figure 6 a path of excess reserves C_t (Panel A), token redemption value P_t (Panel B), token supply S_t (Panel C), and transaction volume N_t (Panel D). The horizontal axis records the number of years. In the first three years, in spite of the volatility in C_t, the platform manages to sustain redemption at par, and with the transaction volume (or token demand) at the full capacity at $\overline{N}$, a fixed dollar price of token implies a fixed token supply. Following a sequence

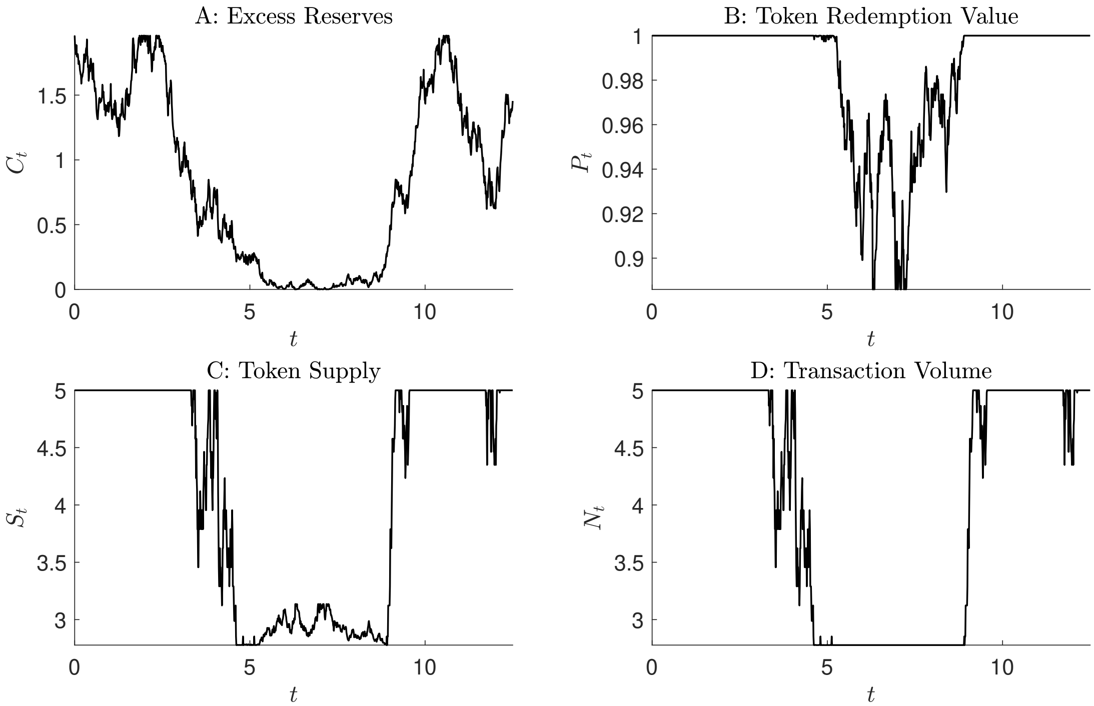

Figure 6: **Simulation.** Using the numerical solutions, we simulate a path of excess reserves (Panel A), token redemption value (Panel B), token supply (Panel C), and transaction volume on the platform (Panel D). The horizontal axis records the number of years. The parameterization follows Figure 2.

of negative shocks between the third and fourth years, the platform raises fees. Users respond by reducing their token demand N_t, so the platform reduces token supply, maintaining redemption at par. The platform optimally trades off replenishing dollars reserves by raising fees and using dollar reserves in token buy-back. As more negative shocks hit between the fourth and ninth years, the platform gives up the peg and off-loads risk to users through the fluctuation of token redemption price. Users' token demand hits $\underline{N}$, and the platform starts actively expanding token supply in exchange for dollar revenues. Then following a sequence of positive shocks, the recovery started in the ninth year, and by the tenth year, the platform restores redemption at par.

We demonstrate the long-run dynamics of the model in Figure 7. Panel A plots the stationary probability density of excess reserves. It shows how much time over the long run the platform spends in different regions of C. The distribution is bimodal. The concentration of probability mass near $C = 0$ is due to the fact that, when the transaction volume (or token demand) gets stuck at the hibernation level $\underline{N}$, the platform can only grow out of this region very slowly by accumulating reserves through fee revenues and proceeds from expanding token supply. The platform also spends a lot of time near the payout boundary $\overline{C}$ as this is a stable region where, given a sufficiently high

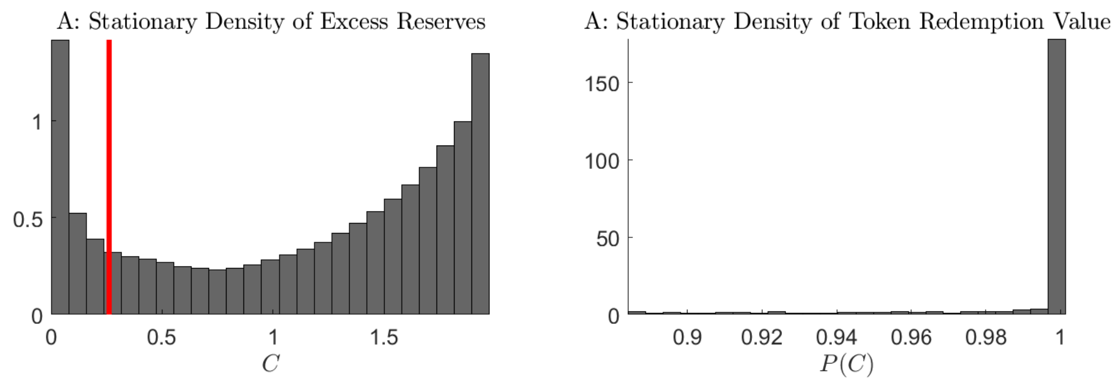

Figure 7: **Long-Run Dynamics and Stationary Density.** We plot stationary probability densities of excess reserves C (Panel A) and token value $P(C)$ (Panel B) in numerical solutions. The parameterization follows Figure 2.

level of reserve buffer, shocks' impact is limited. In Panel B, we show that, even though redemption at par seems to be the norm, the system exhibits significant risk of token debasement ($P(C) < 1$).

Discussion: Liquidation. In our model, the platform never liquidates. As C declines and approaches zero, the platform gradually off-loads risk to users (see (22)). As a result, the platform avoids liquidation and gradually accumulates reserves through the interests on reserve holdings, the fee revenues, and, in case users' token demand increase, through the issuance of new tokens. As C increases either through the various sources of revenues or positive shocks, the platform can escape the low-C region where user participation is the lowest at the hibernation level $\underline{N}$.

Our model can be easily extended to make liquidation a positive probability event. The platform's dollar revenues can be subject to both small shocks (i.e., the diffusive, Brownian shock dZ_t) and large negative jump shocks that arrive by a Poisson process. When the level of reserves is sufficiently low, a jump shock triggers liquidation. Before the jump shock, the behavior of the extended model is akin to that of the current model except that the expectation of liquidation increases the concavity of value function and thereby induces more precautionary reserve holdings.

4.2 The Optimal Issuance of Governance Tokens

In this subsection, we take an excursion to analyze the issuance of platform equity (or "governance tokens" in practice). So far, the platform recovers from the low-C region through the accumulation of internal funds. We now allow the platform to raise dollar funds by issuing equity shares to subject to a fixed financing cost, χ.[31] To characterize the optimal issuance policy, we first note that when issuing equity, the platform raises enough funds so that C jumps to $\overline{C}$ where $V'(\overline{C}) = 1$. Once the fixed cost χ is paid, raising one more dollar does not incur further costs, so as long as the marginal value of reserves, $V'(C)$, is greater than one, the platform keeps raising funds.[32]

The platform raises external funds only when C falls to zero. First, it is not optimal to issue equity at the payout boundary, $\overline{C}$, because newly raised funds will be paid out immediately and thus the issuance cost is incurred without any benefits. Therefore, let $\underline{C}$ denote the recapitalization boundary and we have $\underline{C} < \overline{C}$. Consider the *change* of existing shareholders' value after equity issuance: $[V(\overline{C}) - (\overline{C} - \underline{C}) - \chi] - V(\underline{C})$. To arrive at the existing shareholders' value, we deduct the issuance cost, χ, and competitive investors' equity value (equal to the funds raised), $(\overline{C} - \underline{C})$, from the total platform value post-issuance, $V(\overline{C})$. To calculate the change, we subtract $V(\underline{C})$, the value without issuance. Taking the derivative with respect to $\underline{C}$, we obtain $-V'(\underline{C}) + 1 < 0$ for $\underline{C} < \overline{C}$ because $V'(\underline{C}) > V'(\overline{C}) = 1$ under value function concavity.[33] Therefore, the platform prefers $\underline{C}$ to be as low as possible and thus optimally sets it to zero.

Finally, as in the baseline model, the platform can avoid liquidation by off-loading risk to users, as shown in (22), and obtain the value given by (23). Therefore, as C approaches zero, the platform only opts for recapitalization at $C = 0$ if recapitalization generates a higher value. Accordingly, the lower boundary condition (23) for the value function is modified to

$$\lim_{C \to 0^+} V(C) = \max \left\{ \lim_{C \to 0^+} V'(C) \frac{A}{\rho} \left(\frac{\xi}{\eta \sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta \sigma \, , \; V(\overline{C}) - \overline{C} - \chi \right\} , \tag{34}$$

The first term in the max operator is the value obtained from off-loading risk to users, given by (23). The second term is the post-issuance value for existing shareholders. The results in Proposition 1 to 2, 3 and Corollary 1 still hold except that the boundary condition (23) is replaced by (34).

Proposition 5 (Optimal Recapitalization). *The platform raises external funds through equity*

[31] Firms face significant financing costs due to asymmetric information and incentive issues. A large literature has sought to measure these costs, in particular, the costs arising from the negative stock price reaction in response to the announcement of a new issue. Lee, Lochhead, Ritter, and Zhao (1996) document that for initial public offerings (IPOs) of equity, the direct costs (underwriting, management, legal, auditing and registration fees) average 11.0% of the proceeds, and for seasoned equity offerings (SEOs), the direct costs average 7.1%. IPOs also incur a substantial indirect cost due to short-run underpricing. An early study by Asquith and Mullins (1986) found that the average stock price reaction to the announcement of a common stock issue was −3% and the loss in equity value as a percentage of the size of the new equity issue was as high as −31% (see Eckbo, Masulis, and Norli, 2007, for a survey).

[32] in Appendix A.6, we extend the model to incorporate a proportional cost of equity issuance.

[33] To prove the concavity of value function stated in Proposition 1, we only need the HJB equation (20) and the upper boundary conditions (18) and (19), so recapitalization does not affect value function concavity.

issuance only if $V(\overline{C}) - \overline{C} - \chi > \lim_{C \to 0^+} V'(C) \frac{A}{\rho} \left(\frac{\xi}{\eta\sigma}\right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi}\right) \eta\sigma$ *(see (34)), where* $\overline{C}$ *is given by Proposition 1, and only when excess reserves fall to zero. The amount of funds raised is* $\overline{C}$*.*

When recapitalization happens, C_t jumps from zero to $\overline{C}$. If the platform is sufficiently risk-averse and charges high transaction fees and/or shares risk with users by setting $\sigma^P(0)$, it follows $N(0) < N(\overline{N})$, so that the aggregate token demand N_t jumps up at recapitalization, reflecting that the platform lowers fees $f(C)$ when it holds more reserves and becomes less risk-averse. If the platform does not adjust the token supply, S_t, there will be an upward *predictable* jump in the dollar price of token at $C = 0$, which implies an arbitrage opportunity.

To preclude arbitrage, the platform must expand token supply when it adjusts fees right after recapitalization so that the dollar price of token stays at its pre-issuance level. Since C_t already reaches the payout boundary, $\overline{C}$, the dollar proceeds from supplying new tokens are immediately paid out to shareholders. From a balance-sheet perspective, total assets (reserves) stay at $\overline{C}$ while, on the liability side, token liability increases and equity decreases (through payout). This liability-structure adjustment is akin to corporations issuing debts for share repurchase.

Corollary 3 (**Post-Recapitalization Adjustment of Liability Structure**). *To preclude arbitrage in the token market, the platform adjusts its liability structure immediately after capitalization, supplying new tokens and paying out the dollar proceeds to the platform's shareholders.*

Finally, we revisit the results on the token price level in Corollary 2 and Proposition 4. Let $P_j(C)$ denote the token price function after the j-th recapitalization. According to Corollary (3), when recapitalization happens for the j-th time and C_t jumps from zero to $\overline{C}$, the platform sets

$$P_j(\overline{C}) = P_{j-1}(0)\,, \tag{35}$$

which replaces (33) as the boundary condition for the price-level ODE (32). Token price level before the first recapitalization, $P_0(C)$ is still solved under the boundary condition (33), i.e., $P_0(\overline{C}) = 1$.

Corollary 4 (**Recapitalization and Token Price Level**). *Token price level after the* j*-th recapitalization is solved by the ordinary differential equation (32) under the boundary condition (35).*

In the baseline model without recapitalization, the debasement of token is temporary: Token price level falls below 1 when C falls below $\widetilde{C}$ due to negative shocks and it recovers back to 1 when the platform accumulates sufficient amount of dollar revenues so that C crosses above $\widetilde{C}$

(Proposition 4). When recapitalization happens, the debasement is permanent. After the j-th recapitalization, token price level starts anew at a lower peg, $P_j(\overline{C}) = P_{j-1}(0)$, and if negative shocks deplete the platform's reserves and triggers another recapitalization, token price level declines along the process and, right after recapitalization, stabilizes at an even lower peg, $P_{j+1}(\overline{C}) = P_j(0)$.

Discussion: Financial Frictions. In our model, financial frictions play a key role. If costless recapitalization is possible, i.e., $\chi = 0$, then the platform will never allow the marginal value of reserves to exceed one because, when $V'(C) > 1$, it is profitable to raise funds from competitive investors that cost 1 per dollar and generates a value of $V'(C) > 1$. The constant marginal value of reserves implies that the platform is no longer risk-averse, i.e., $\gamma(C) = 0$, and thus, will absorb all risk, setting σ_t^P to zero. Tokens will always be redeemed at a fixed dollar value.

4.3 Regulating Stablecoins

We apply our model to analyze two types of stablecoin regulations. The first type, which is of our focus, stipulates a minimum level of excess reserves ("capital requirement"). The rationale behind is to generate a sufficient risk buffer so that the platform is unlikely to debase the tokens. The second type of regulation is more direct. It requires the platform to keep token volatility below a certain level ("volatility regulation"). Our conclusion is that capital requirement, if carefully designed, can achieve Pareto improvement for the platform and its users. Volatility regulation, in contrast, destroys the economic surplus from risk-sharing between the platform and its users.

Capital Requirement. The regulator requires $C \geq C_L$ and forces the platform to liquidate if the requirement is violated. Therefore, C_L replaces zero as the lower (liquidation) bound of excess reserves. In Figure 8, we plot the payout boundary $\overline{C}$ (Panel A), which is a measure of voluntary over-collateralization, and the welfare measures for different values of C_L. Not so surprisingly, when the capital requirement tightens, the whole region of excess reserves is pushed to the right, resulting in a higher payout boundary $\overline{C}$ in Panel A. Because reserves earn an interest rate r that is below the shareholders' discount rate ρ, the platform shareholders' value, V_0, declines in C_L, as shown in Panel B. Panel C shows that users' welfare is improved by the capital requirement but there exists a significant degree of decreasing return as the regulator pushes up C_L.

What is interesting is that, in Panel D of Figure 8, the total welfare is non-monotonic in C_L. When the regulator increases C_L from zero, the increase of users' welfare overwhelms the decrease

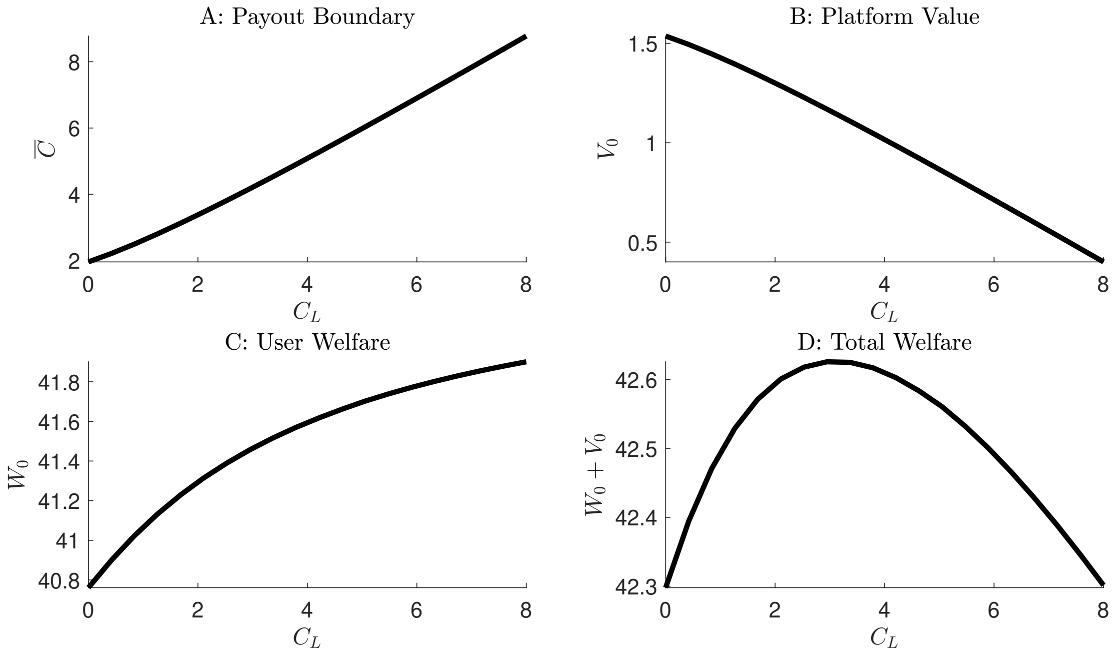

Figure 8: **Capital Requirement and Welfare.** We plot the numerical solutions of payout boundary $\overline{C}$ (Panel A), the platform shareholders' value at $t = 0$, V_0 (Panel B), users' welfare at $t = 0$, W_0 (Panel C), and total welfare $W_0 + V_0$ (Panel D). The parameterization follows Figure 2.

of platform value, but as the capital requirement is tightened, the loss of platform value eventually dominates. This suggests the existence of an optimal level of C_L that maximizes the total welfare.

As long as the users' welfare increases faster than the platform value decreases, the regulator can administer a transfer from users to the platform, making the regulation Pareto-improving. For example, the regulator can allow the platform to charge users a membership fees, i.e., a fixed cost of access, and imposes a cap on such fees. This type of access fees is commonly seen in the literature on regulation of utility networks (Laffont and Tirole, 1994; Armstrong, Doyle, and Vickers, 1996).

In Figure 9, we further demonstrate the stabilization effects of capital requirement. In Panel A, we plot the ratio of $\overline{C} - \widetilde{C}$ to $\overline{C} - C_L$ that measures the size of the stable subset of C where the platform maintains token redemption at par (i.e., $P(C) = 1$). As C_L increases, the stable region enlarges. In Panel B, we plot the probability of $C > \widetilde{C}$ (i.e., $\sigma^P(C) = 0$) based on the stationary distribution of C, which shows that over the long run the platform spends more time in the stable region when C_L increases. In Panel C, we plot the long-run average value of σ_t^P using the stationary probability distribution. A declining pattern emerges, indicating that capital requirement is indeed effective in reducing the token volatility. In Panel D, we plot the expected number of years it takes to reach $\widetilde{C}$ from C_L (denoted by $\tau(C_L)$). This recovery time decreases when the capital requirement

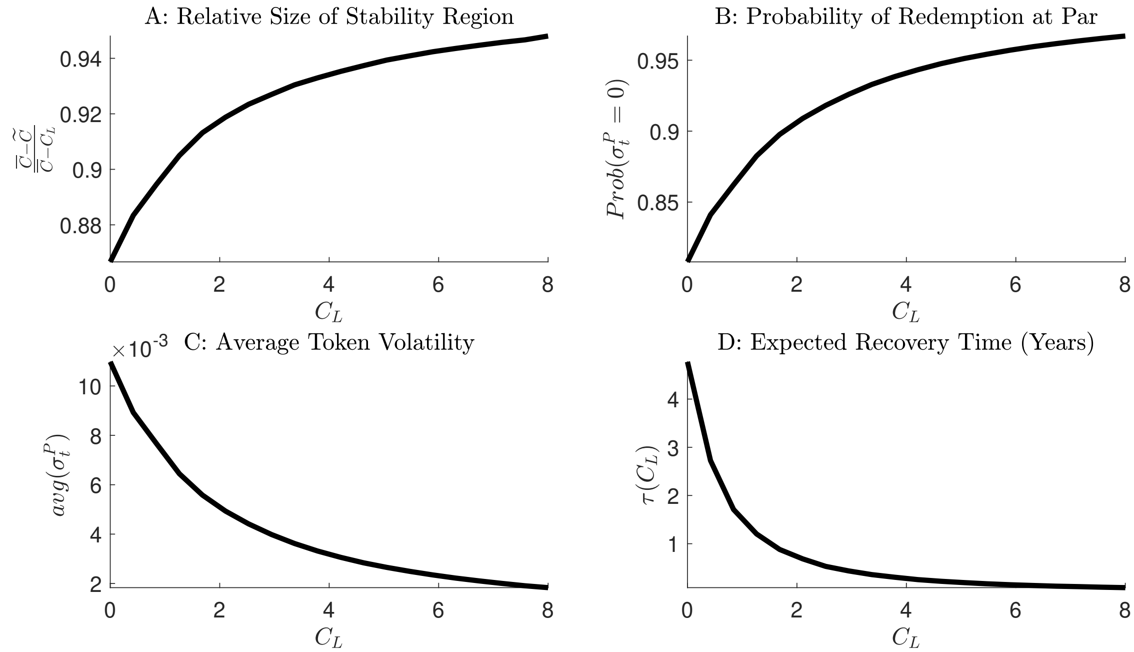

Figure 9: **Capital Requirement and Token Stability.** Using numeric solutions under different values of C_L, we plot the fraction of state space with redemption at par $\frac{\overline{C}-\widetilde{C}}{\overline{C}-C_L}$ (Panel A), stationary probability of zero token volatility (Panel B), the long-run average (based on stationary probability density) of token volatility (Panel C), and the expected time to reach $\widetilde{C}$ from C_L (Panel D). The parameterization follows Figure 2.

is tightened. The intuition is that, as C_L increases, the platform near C_L still has abundant cash that self-accumulates over time by earning the interest rate r.

Volatility Regulation. The concern over token volatility may motivate more direct regulatory intervention. A volatility regulation imposes a cap on σ_t^P and requires the platform to implement a rule of token supply to achieve this goal. In Figure 10, we compares the baseline model (solid line) and the model under regulation that forces $\sigma_t^P = 0$ (dotted line) over a range of values of users' risk aversion η.[34] As shown by the HJB equation (20), under $\sigma_t^P = 0$, the platform's value function and control variables no longer depends on η, so the (dotted) lines are flat in all panels.

In Panel A of Figure 10, we show that under the zero-volatility requirement, the platform has to maintain a higher level of excess reserves to reduce the likelihood of liquidation because the option of off-loading risk to users is no longer available. Holding more reserves with an interest rate below the shareholders' discount rate leads to a lower platform value, as shown in Panel B of Figure 10.

An interesting finding is that imposing the volatility regulation even decreases users' welfare

[34]Because the platform can no longer off-load risk to users as C approaches zero, liquidation can happen and $C = 0$ becomes an absorbing state. The boundary condition $V(0) = 0$ implies $-V''(C)$ approaches infinity, which is the same boundary condition that we use to solve the baseline model (see Footnote 28).

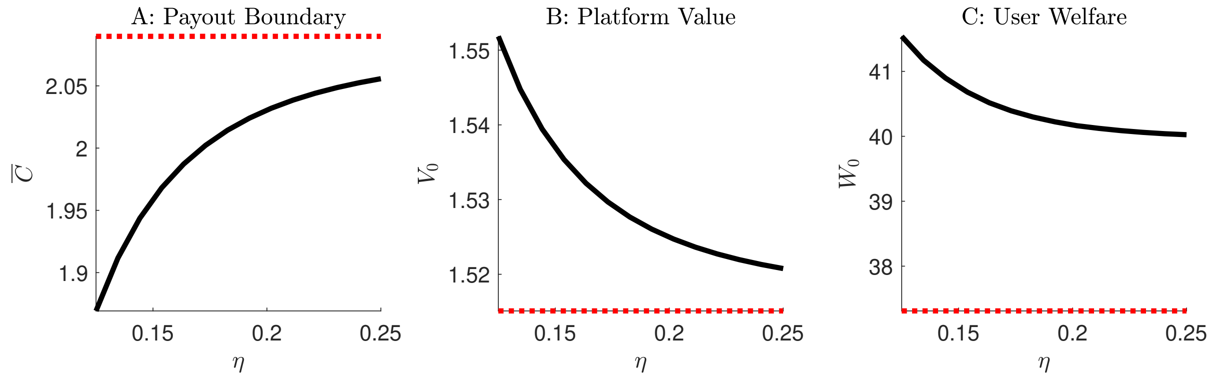

Figure 10: **Risk-Sharing, Volatility Regulation, and Welfare.** Using the numerical solutions, we calculate the payout boundary $\overline{C}$ (Panel A), the platform shareholders' value at $t = 0$, V_0 (Panel B), users' welfare at $t = 0$, W_0 (Panel C), and the long-run average fees based on stationary probability density (Panel D) over different values of users' risk aversion η for both the baseline model (solid line) and the model under zero-volatility regulation (red dotted line). The parameterization follows Figure 2.

(Panel C of Figure 10) across all values of η. This seems to contradict the intuition that, by forcing the platform to maintain a perfectly stable token value, users will benefit, especially when they are more risk-averse. However, the argument ignores that, unable to off-load risk to users, the platform can compensate its risk exposure with higher fees. Volatility regulation is counterproductive because it limits the risk-sharing between the platform and its users. When the platform is close to liquidation, its effective risk aversion can be higher than η, so there is economic surplus created from users' absorbing risk from the platform. Volatility regulation shuts down this insurance market. Our results on volatility regulation also reveals the fact that a commitment to perfectly stable token does not improve welfare. In practice, stablecoin issuers cannot commit against debasement, but even when such commitment is available, doing so would not not optimal.

5 Crypto Shadow Banking with User Collateral

The double-collateralization structure in Panel B of Figure 1 behind many stablecoins (e.g., Dai issued by MakerDAO) fits into our analytical framework. By requiring users to post collateral, the platform gains an additional degree of freedom (margin requirement). When setting the margin requirement, the platform faces the trade-off between reducing risk exposure and user participation.

For each dollar of stablecoins, the platform requires a user to post m_t dollars worth of collateral. In practice, many risky assets are eligible collateral, mainly cryptocurrencies such as Bitcoin and Ether, and thus are highly volatile. Let dZ_t denote a standard Brownian shock. Instead of interpreting it as a shock directly to reserves as in our baseline model, here we interpret the shock in the following way that is tied to the value of the user's collateral portfolio.

For simplicity, we do not model users' choice of collateral portfolio but rather assume that the collateral portfolio has a continuum of assets and, from t to $t + dt$, a fraction, $2(\delta dt - \sigma dZ_t)$, of these assets incur a percentage loss, denoted by θ_t, which is drawn independently across assets from a uniform distribution on $[0, 1]$.[35] At time t, the expected loss of the collateral portfolio is $\mathbb{E}\left[\frac{1}{2} \times 2(\delta dt - \sigma dZ_t)\right] = \delta dt$, where the expected loss per asset, $\frac{1}{2}$, is multiplied by the fraction of assets in losses. The collateral portfolio also generates an expected value appreciation, denoted by $\tilde{\mu}$. Therefore, for each dollar of stablecoins, a user posts m_t dollars worth of collateral, with an expected net return equal to $\tilde{\mu} - \delta - r$, where the last term represents the cost of giving up the outside option of return r by locking wealth in the collateral portfolio.[36]

Under the collateral requirement, a representative user i's problem of choosing the optimal dollar value of stablecoin holdings, $u_{i,t}$, given by (5) in the baseline model, is now described below

$$\max_{u_{i,t}} \frac{1}{\beta} N_t^{\alpha} u_{i,t}^{\beta} A^{(1-\alpha-\beta)} dt + u_{i,t}\Big(\mu_t^P - \eta|\sigma_t^P| - f_t\Big)dt + u_{i,t} m_t \Big(\tilde{\mu} - \delta - r\Big) , \tag{36}$$

where the last term reflects the fact that the user's wealth is being locked in a risky collateral backing the stablecoins worth $u_{i,t}$. As in the baseline model, to solve the platform's optimal strategies, we first note that, given the token price dynamics (i.e., μ_t^P and σ_t^P), the platform can directly set N_t through the fees f_t. Under the collateral requirement, users' optimal choice of $u_{i,t}$ implies the following equation that connects N_t (i.e., the aggregated $u_{i,t}$) and f_t:

$$f_t = \left(\frac{A}{N_t}\right)^{1-\xi} - m_t(r + \delta - \tilde{\mu}) + \mu_t^P - \eta|\sigma_t^P| \,. \tag{37}$$

Clearly, when $m_t = 1$, $\delta = 0$, and $\tilde{\mu} = 0$ (i.e., the platform does not impose a haircut and the collateral does not have expected losses or gains), equation (37) reduces to (12), the corresponding equation in the baseline model. Given f_t, a higher m_t leads to lower N_t according to (37), which reflects the fact that imposing a stricter collateral requirement leads to lower demand for stablecoins under the parameter restriction, $r + \delta - \tilde{\mu} > 0$ (i.e., it is costly for users to post collateral).

To derive the law of motion of the state variable C_t, the excess reserves, we first derive the

[35] Klimenko, Pfeil, Rochet, and Nicolo (2016) show that $2(\delta dt - \sigma dZ_t)$ is the $\Delta \to 0$ limit of a random variable whose value is $2(\delta\Delta - \sigma\sqrt{\Delta})$ or $2(\delta\Delta + \sigma\sqrt{\Delta})$ with equal probabilities. Before taking the limit, the parameters, δ and σ, can be chosen so that the random fraction is well-defined within $[0, 1]$. The convergence is akin to that shown by Cox, Ross, and Rubinstein (1979) in their Binomial model of option pricing.

[36] This expression is analogous to the user's cost of capital (Jorgenson, 1963) with the additional $\tilde{\mu}$.

platform's flow cost per dollar of stablecoins created:

$$
\begin{aligned}
&2(\delta dt - \sigma dZ_t)\mathbb{P}\left(\{m_t(1-\theta_a) < 1\}\right)\mathbb{E}\left[1 - m_t(1-\theta_a) | m_t(1-\theta_a) < 1\right] \\
&= \int_{1-\frac{1}{m_t}}^{1} (1 - m_t(1-\theta_a))d\theta_a = \frac{1}{m_t}(\delta dt - \sigma dZ_t)\,. \qquad (38)
\end{aligned}
$$

On the left side, the fraction of users' collateral assets that incur losses is multiplied by the probability of a sufficiently large loss that leads to the violation of the margin requirement, and the last component is the platform's loss upon receiving and liquidating the collateral (with a remaining value of $m_t(1-\theta_t)$) and repurchasing the one dollar worth of stablecoins out of circulation. Therefore, given N_t, the dollar value of all stablecoins issued, $-\frac{N_t}{m_t}(\delta dt - \sigma dZ_t)$ enters into the law of motion of reserves (8), replacing $N_t \sigma dZ_t$ (which is essentially the case where $m_t = 1$ and $\delta = 0$). This flow cost is essentially the consequence of the stablecoin issuer extending an guarantee of the stablecoins' value, which is a contingent liability akin to the guarantee that a bank extends to its off-balance-sheet conduits as discussed in Section 2.

Following the derivation in Section 4, we use (37) to substitute out f_t in the law of motion of reserves to obtain the law of motion of excess reserves, C_t:

$$
dC_t = \left(rC_t - r(m_t - 1)N_t + m_t(\tilde{\mu} - \delta)N_t + N_t^{\xi} A^{1-\xi} - N_t \eta |\sigma_t^P| - \frac{N_t \delta}{m_t} \right) dt + N_t \left(\frac{\sigma}{m_t} - \sigma_t^P \right) dZ_t \qquad (39)
$$

When $m_t = 1$, $\delta = 0$, and $\tilde{\mu} = 0$, equation (39) reduces to (16) in the baseline model. In Appendix A.7, we derive the HJB equation of the value function, $V(C_t)$ and the platform's optimal choices of fees (or equivalently, $N(C_t)$), token price dynamics (or equivalently, $\sigma^P(C_t)$), and the margin requirement $m(C_t)$. Figure 11 reports the numeric solutions.

In the model with user collateral, the shock to the platform's reserves, dZ_t, originates from the fluctuation of users' collateral value, and the platform's exposure is directly and inversely linked to the margin requirement, m_t, as shown in (39). Therefore, we expect the optimal margin requirement to be higher when the platform's excess reserves run down. This is shown in lower-left Panel of Figure 11. Introducing user collateral does not change the qualitative dynamics of the platform's franchise value, V, the transaction volume, N, and the stablecoin volatility, σ^P.

Discussion: Immediate Liquidation of Collateral. When users violate the margin requirement, the platform immediately liquidates users' collateral and repurchase stablecoins out of circula-

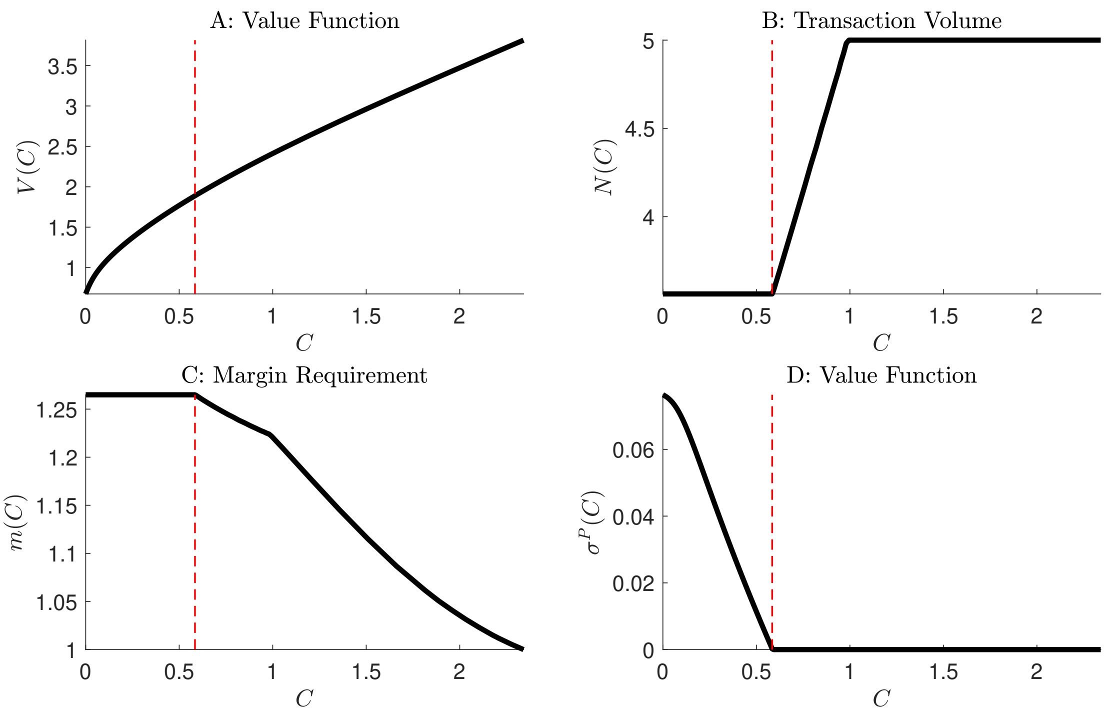

Figure 11: **Margin Requirement.** The parameterization follows Figure 2 with $\tilde{\mu} = 0.05$ and $\delta = 0.025$.

tion. A question naturally arises: Instead of liquidating the collateral and repurchasing stablecoins right away, why not incorporate the collateral assets into the platform's reserve portfolio? Doing so will create two types of stablecoins, one with the backing of both users' collateral and reserves (users of these stablecoins have not yet violated the margin requirements) and the other only backed by the platform's reserves (users of these stablecoins have violated the margin requirement). This is not done in practice, and analytically, it complicates the model by introducing a new stable variable, that is the fraction of stablecoins only backed by the platform's reserves.

6 Stablecoins, Online Social Network, and Big Data

The interest in stablecoins among practitioners and regulators skyrocketed after Facebook announced its stablecoin project Libra (recently renamed to Diem). Different from other stablecoin issuers, Facebook has the unique advantage of strong network effects. Its comprehensive infrastructure covers social network, social media, and e-commerce (Facebook Shop). For individual users, the benefit of adopting Diem is enormous if other users on Facebook adopts Diem, because a great variety of activities can be enabled by a universal and global means of payment.

The stablecoin project of Facebook attracted enormous attention also because of the big data

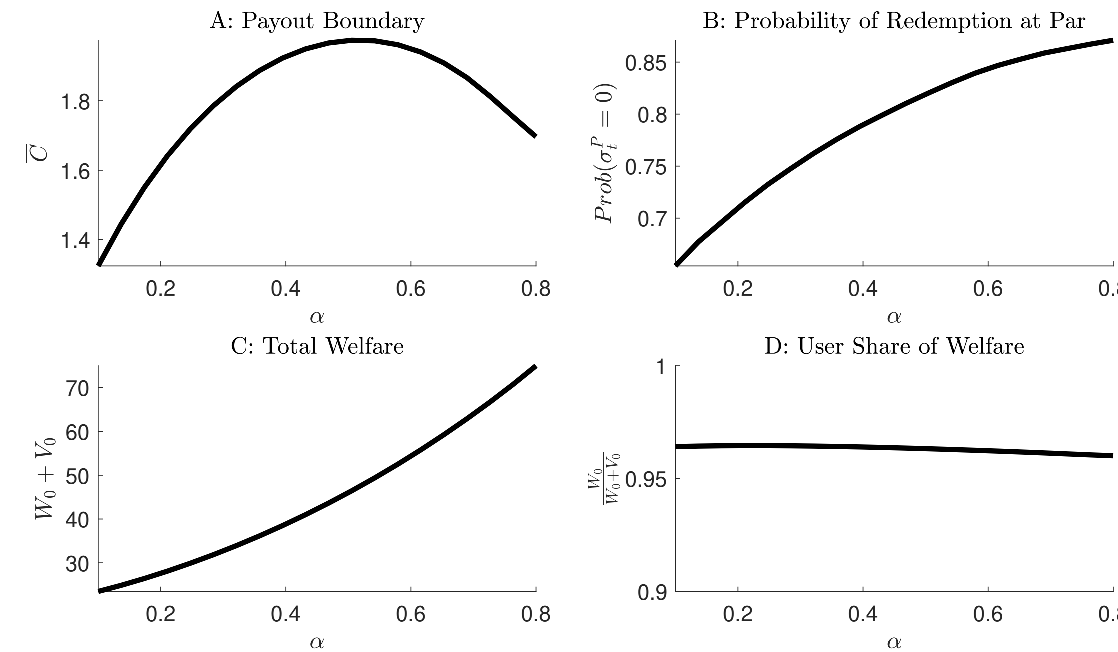

Figure 12: **Network Effects.** We plot the payout boundary (Panel A), the long-run probability of $C > \widetilde{C}$ based on stationary distribution (Panel B), the sum of platform value and users' welfare (Panel C), and users' share of total welfare (Panel D) over different values of α (degree of network externality). The parameterization follows Figure 2.

advantage of Facebook. Large platforms profits from user-generated data. A global payment system enabled by a stablecoin allows a platform to gather transaction data.

In this section, we analyze the role of network effects in stablecoin management and regulation. For simplicity, our analysis is based on the baseline model without users' collateral. After analyzing the network effects, we extend model to incorporate transaction data as a productive asset for the platform and explore how the incentive to accumulate transaction data affects a platform's stablecoin strategies and the optimal regulation.

6.1 The Role of Network Effects

In our model, strong network effects are captured by a large value of α (see (2)). In Figure 12, we compare stablecoin platforms with different degrees of network effects. Panel A of Figure 12 plots the payout boundary $\overline{C}$ as a measure of voluntary over-collateralization over different values of α. On the one hand, stronger network effects make the platform more profitable, which stimulates more precautionary savings to protect the franchise value. On the other hand, stronger network effects imply a higher level of user activities near $C = 0$, i.e., $\underline{N}$ in (24), which in turn implies a faster recovery out of the low-C region (through fee and token-issuance revenues) and thereby a weaker incentive to hold excess reserves. The two counteracting forces lead to the hump-shaped relationship between $\overline{C}$ and α in Panel A of Figure 12.

In Panel B of Figure 12, we show the long-run probability of $C > \widetilde{C}$ (based on stationary probability distribution of C) increases in α. Stronger network effects imply that, over the long run, the system spends more time in states with $P(C) = 1$. Under stronger network effects, recovery out of the low-C region is faster due to a higher level of user activities and the resultant faster replenishment of reserves via fees and token-issuance proceeds. Our paper sheds light on why stablecoins issued by Facebook and other technology giants with strong network infrastructure are regarded as more promising than those issued by start-up payment service providers. Stronger network effects lead to stabler tokens and a lower likelihood of token debasement.

Finally, we examine the impact of network effects on welfare. In Panel C of Figure 12, we show that total welfare of the platform and its users increases in the degree of network effects. This explains why it is particular beneficial for technology giants with strong network infrastructure to introduce stablecoins as common means of payment among their customers. Network infrastructure is not limited to social network and e-commerce. Financial network is another example. JPMorgan Chase introduces JPM Coin to facilitate transactions among institutional clients.

Interestingly, as we gradually increase the degree of network effects in Panel D of Figure 12, the split of total welfare between the platform and its users is rather stable. Under stronger network effects, the monopolistic platform can extract more rents from its users through fees or off-loading risk in distress. However, precisely due to the network effects, individual users do not internalize the positive effect of their adoption on other users, so the platform has incentive to internalize the network externality by stimulating user activities through fee reductions (or subsidies) and token stability. These two counteracting forces imply that, as network effects become stronger, the platform's share of total surplus does not necessarily increase. This result alleviates the concern over technology giants abusing network effects in stablecoin projects.

Lastly, Figure 13 demonstrates how network effects, as captured by α, drive the optimal capital requirement. Panel A plots the capital requirement C_L^* that maximizes total welfare. Panel B plots total welfare both with optimal capital requirements (solid line) and without capital requirements (dotted line). Panel C plots the welfare wedge between the optimally regulated equilibrium and laissez-faire equilibrium. Importantly, it is optimal to raise capital requirement as network effects strengthen. In fact, platforms with no network effects ($\alpha = 0$) do not benefit at all from the regulation. The key to this result is the positive externality of individual users' token holdings, which

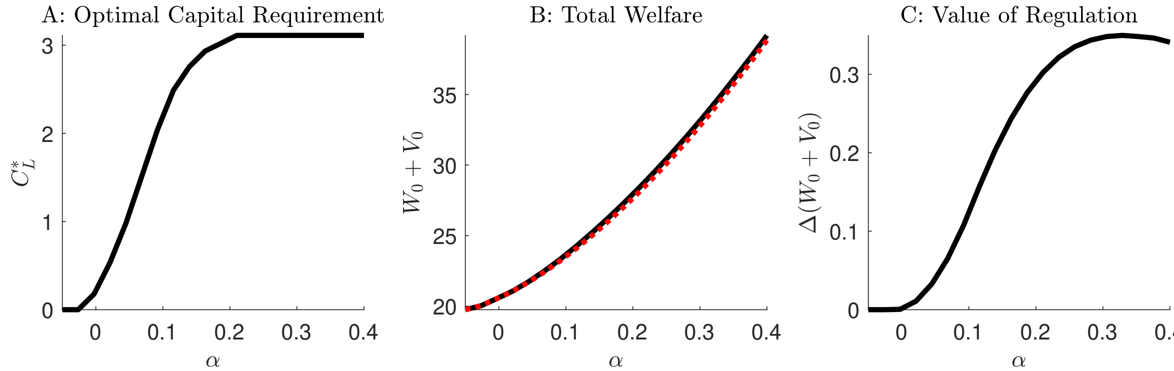

Figure 13: **Capital Requirement and Network Effects.** We calculate the optimal capital requirement C_L^* that maximizes total welfare (Panel A), total welfare both with capital requirement C_L^* (solid black line) and without capital requirement (dotted red line) (Panel B), and the welfare wedge between the optimally regulated equilibrium and the laissez-faire equilibrium (Panel C) over different values of α. Note that Panel C depicts the difference between the solid black line and the dotted red line from Panel B. The rest of parameterization follows Figure 2.

explains the deviation of laissez-faire equilibrium from social optimum. The platform internalizes such externality in its decision to preserve reserves and stabilize tokens, but the internalization is not perfect. As shown in Figure 12, the platform cannot seize the full surplus as its share of total welfare is rather stable in α and always below 100%. Therefore, as α increases, the total welfare increases together with the component that is not internalized by the platform. This calls for a tighter capital requirement that moves the overall level of reserves closer to social optimum.

6.2 Payment and Big Data

User-generated data is now a major asset of digital platforms. Social networks, such as Facebook and Twitter, utilize such data to target users for advertisement. Being able to utilize the enormous amount of transaction data has become a critical advantage of digital platforms relative to traditional payment service providers such as banks (Bank for International Settlements, 2019). Leading players, such as PayPal and Square, have become data centers and provide services beyond facilitating payments, for example, extending loans to consumers and businesses based on credit analysis that is enabled by transaction data. In this section, we follow Veldkamp (2005), Ordoñez (2013), Fajgelbaum, Schaal, and Taschereau-Dumouchel (2017), and Jones and Tonetti (2020) to model big data as a by-product of user activities.[37] Our analysis focuses how data as a productive asset affects the optimal strategies of stablecoin platforms and the efficacy of regulations.

[37] Veldkamp and Chung (2019) provide an excellent survey of the literature of data and aggregate economy.

6.2.1 Big Data as a Productive Asset.

In the baseline model, the quality parameter A is constant. We now interpret A as a measure of effective data units that enhance platform quality and assume the following law of motion

$$dA_t = \kappa A_t^{1-\xi} N_t^{\xi} dt\,. \tag{40}$$

Users' transactions generate a flow of raw data, $\kappa N_t^{\xi} dt$, where the parameter κ captures the technological efficiency of data processing and storage. To what extent the raw data contributes to the effective data units depends on the current amount of effective data via $A_t^{1-\xi}$. The complementarity between the old and new data captures the fact that the value of new data increases in the quality of statistical algorithms, which in turn depends on the amount of existing data that are needed to to select and train the algorithms.[38] The Cobb-Douglas form is chosen for analytical convenience.[39]

As the platform quality improves, we assume the transaction capacity increases accordingly, i.e., $\overline{N}_t = \overline{n} A_t$, where $\overline{n} > 0$ is constant. User optimization is static and follows the baseline model. As shown in (11), the dollar transaction volume (or token demand) $N_t \equiv n_t A_t$ where

$$n_t = \frac{1}{\left(r + f_t - \mu_t^P + \eta|\sigma_t^P|\right)^{\frac{1}{1-\xi}}} \wedge \overline{n}\,. \tag{41}$$

As in the baseline model, the platform sets n_t through the fees, f_t, and set the dynamics of token redemption price through its choice of σ_t^P. The model now has three natural state variables, reserves M_t, token supply S_t, and data stock A_t. Similar to the baseline model, $C_t = M_t - S_t P_t$ and A_t summarize payoff-relevant information, driving the platform value, $V_t = V(C_t, A_t)$, and the dollar value of token, $P_t = P(C_t, A_t)$. To simplify the notations, we will suppress the time subscripts.

We conjecture that the system is homogeneous in A, and in particular, the platform's value function and dollar value of token are given by $V(C, A) = v(c)A$ and $P(C, A) = P(c)$, respectively, where the excess reserves-to-data ratio is the key state variable for the platform's optimal strategies:

$$c \equiv \frac{C}{A}\,. \tag{42}$$

We will confirm the conjecture as we solve the platform's optimization problem in the following.

[38] Related, in Farboodi, Mihet, Philippon, and Veldkamp (2019), data have increasing return to scale.

[39] This data accumulation process is inspired by the specification of knowledge accumulation in Weitzman (1998).

First, to derive the law of motion of c_t, we follow the derivation of the baseline model to obtain

$$dC_t = \left(rC_t + A_t n_t^{\xi} - \eta A_t n_t |\sigma_t^P|\right) dt + A_t n_t(\sigma - \sigma_t^P) dZ_t - dDiv_t \,. \tag{43}$$

Given (40) and (43), the law of motion of c_t is given by

$$dc_t = \left(rc_t + n_t^{\xi} - \eta n_t |\sigma_t^P| - \kappa n_t^{\xi} c_t\right) dt + n_t(\sigma - \sigma_t^P) dZ_t - \frac{dDiv_t}{A_t} \,. \tag{44}$$

Under the value function conjecture, $V(C, A) = v(c)A$, and the laws of motion of A (40) and c (44), the HJB equation for $v(c)$ in the interior region (where $dDiv_t = 0$) is given by

$$\rho v(c) = \max_{n\in[0,\overline{n}],\sigma^P} \left\{ \left[v(c) - v'(c)c\right] \kappa n^{\xi} + v'(c) \left(rc + n^{\xi} - \eta n |\sigma^P|\right) + \frac{1}{2} v''(c) n^2 (\sigma - \sigma^P)^2 \right\} , \tag{45}$$

When the marginal value of reserves, $V_A(C, A) = v'(c)$, falls to one, the platform pays out dividends. We define the payout boundary as $\overline{c}$ through $v'(\overline{c}) = 1$. The optimality of $\overline{c}$ also implies $v''(\overline{c}) = 0$. Note that as in the baseline model, when C (or c) approaches zero, the platform can avoid liquidation by setting $\sigma^P(c) = \sigma$ to off-load risk to its users and gradually replenish reserves.[40] For simplicity, we do not consider recapitalization (equity issuance). In sum, the platform's excess reserves, C_t, move in $(0, \overline{c}A]$. As data grows, the platform accumulates more excess reserves.

Proposition 6 (**Data Growth and Over-Collateralization**). *The amount of excess reserves, C_t, varies in $(0, \overline{c}A_t]$ where the upper bound increases with A_t, the effective data units.*

The intuition behind Proposition 6 is that data growth provides another channel through which the continuation value appreciates, as shown on the right side of (45), which makes the platform more patient in distributing excess reserves to shareholders. The first term on the right side contains the marginal value of data (which we call "data q")

$$q(c) = \frac{\partial V(C, A)}{\partial A} = v(c) - v'(c)c \,. \tag{46}$$

Retaining more reserves allows the platform to sustain a wider region of c with credible token redemption at par. A more stable token in turn stimulates transactions and thereby allows the platform to accumulate more data and earn the data q, $q(c)$. Data as a productive asset and by-product of user activities enhances the platform's incentive to reserve for tokens.

[40] The boundary condition for $v(c)$ is that as c approaches zero, $-v''(c)$ approaches infinity (see footnote 28).

Next, we characterize the optimal transaction volume and token volatility. Following our analysis of the baseline model, we define the effective risk aversion based on $v(c)$:

$$\Gamma(c) = -\frac{v''(c)}{v'(c)}\,. \tag{47}$$

The following proposition summarizes the optimal choices of $n(c)$ and $\sigma^P(c)$.

Proposition 7 (Data q, Token Volatility, and Transaction Volume). *At c where the platform maintains the redemption of token at par, i.e., $\sigma^P(c) = 0$, the optimal transaction volume is*

$$N = n(c)A = \left[\frac{\xi}{\Gamma(c)\sigma^2}\left(1 + \frac{\kappa q(c)}{v'(c)}\right)\right]^{\frac{1}{2-\xi}} A \quad \wedge \overline{n}A\,; \tag{48}$$

otherwise, the optimal token volatility is

$$\sigma^P(c) = \sigma - \frac{\eta}{\Gamma(c)n(c)} \;\in (0,\sigma)\,, \tag{49}$$

and the optimal transaction volume is

$$N = n(c)A = \left[\frac{\xi}{\eta\sigma}\left(1 + \frac{\kappa q(c)}{v'(c)}\right)\right]^{\frac{1}{1-\xi}} A \quad \wedge \overline{n}A\,. \tag{50}$$

The optimal transaction volume is proportional to A, the effective data units. Therefore, as data grows following (40), the transaction volume grows too. With data as a productive asset, the platform faces a new trade-off. It can accumulate more reserves through higher fee revenues or, by reducing fees, boost the transaction volume to accumulate more data. Therefore, the ratio of marginal value of data (the data q) and marginal value of reserves, $q(c)/v'(c)$, emerges in both (48) and (50). When the data q is higher relative to the marginal value of reserves, the platform implements a higher transaction volume through lower fees. Note that given the token price dynamics, the monotonic relationship between transaction volume and fees is given by (41).

The optimal choice of token volatility resembles that in the baseline model. In the region where $\sigma^P(c) > 0$, it is the ratio of users' risk aversion to the platform's risk aversion that drives $\sigma^P(c)$. And in this region, the optimal transaction volume in (50), even scaled by A, is no longer the constant as in the baseline model but depends $q(c)/v'(c)$ instead, showing the trade-off between investing in data and accumulating reserves. Moreover, the optimal transaction volume depends on users' risk aversion η as η determines the cost of obtaining insurance from users (losing transaction volume

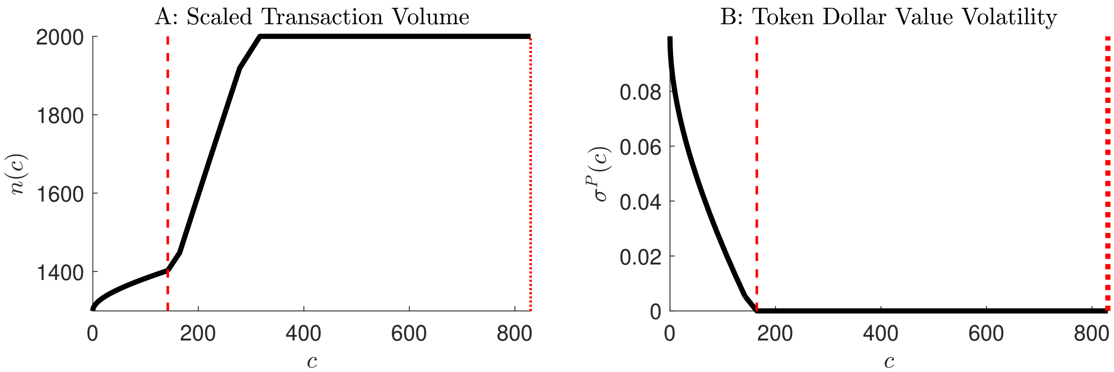

Figure 14: **Transaction Volume and Token Volatility.** This figure plots the A_t-scaled transaction volume $n(c)$ in Panel A and token return volatility $\sigma^P(c)$ in Panel B. In both panels, the red dotted lines mark the payout boundary $\overline{c}$, and the red dashed line marks $\widetilde{c}$, the threshold that separates the regions of volatile and constant token prices. The parameterization follows Figure 2 with the additional parameters $\overline{n} = 2000$ and $\kappa = 0.00025$. Note that $\overline{n} = 2000$ implies that for $A_0 = 0.0025$, $\overline{N}_t = \overline{n}A_t = 5$ as under the parameterization in the baseline (see Figure 2).

after off-loading risk to users). When the platform absorbs all risk (i.e., $\sigma^P(c) = 0$), the optimal transaction volume varies with its own risk aversion $\Gamma(c)$ (48) because $\Gamma(c)$ drives the required risk compensation through higher fees that causes the transaction volume to decline.

Panel A of Figure 14 reports the optimal transaction volume. In contrast to Panel A of Figure 4 where the transaction volume is constant in the region where $\sigma^P(c) > 0$, the A-scaled transaction volume now increases in c. The intuition is that as reserves become more abundant relative to data, the platform is more willing to lower fees, so it acquires more data through more active transactions at the expense of lower dollar revenues added to the reserve buffer. Panel B of Figure 14 shows a similar token volatility dynamics as Panel A of Figure 3 from the baseline model.

6.2.2 Data Technology Revolution and Stablecoin Platform Strategies

The last few decades have witnessed enormous progress in data science. In our model, such technological advance can be captured by an increase of the parameter κ. In Figure 15, we examine the impact of big data technology on the operation of stablecoin platforms. In Panel A, we show that in response to an increase in κ, the platform optimally raises the (A-scaled) payout boundary, $\overline{u}$, which suggests a greater degree of over-collateralization. The intuition of such response can be understood jointly with the platform's decision on token volatility and fees.

To accumulate transaction data, the platform would increase the transaction volume. This can be achieved through lower fees. As shown in Panel C and D of Figure 15, the average fees (calculated from the stationary distribution of c) decline and the transaction volume increases in κ.

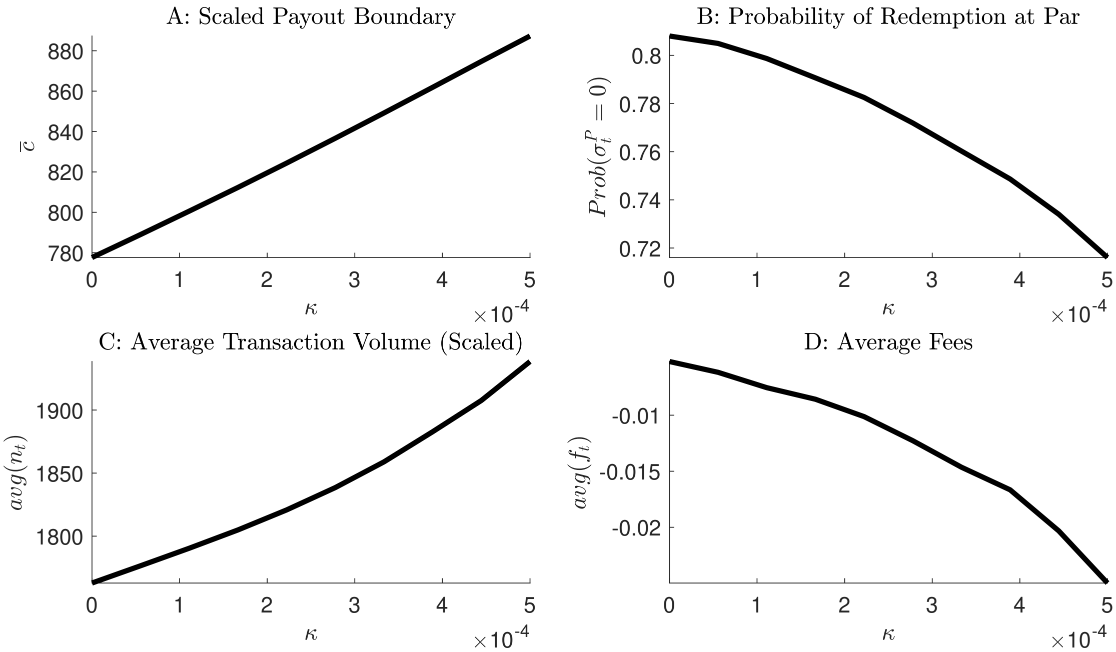

Figure 15: **Data Technology Progress and Platform Operation.** We plot the A-scaled payout boundary (Panel A), the probability of token redemption at par (Panel B), the average transaction volume (Panel C), and the average fees per dollar of transactions (Panel D) over κ (the efficiency of data technology). The moments in Panel B, C, and D are based on the stationary distribution of c. The parameterization follows Figure 2 with $\overline{n} = 2000$.

The average fees per dollar of transaction even dips into the negative territory, becoming subsidies to users. This prediction is consistent with the practice that large digital platforms offer subsidies and fee services to retain and grow their customer base (Rochet and Tirole, 2006; Rysman, 2009).

However, a higher n implies a large exposure to operation risk as shown in (44). The platform responds by delaying payout, i.e., raising the boundary $\overline{c}$, to increase the reserve buffer, which explains why the payout boundary $\overline{c}$ increases in κ in Panel A of Figure 15. The platform can also respond by off-loading more risk to users. As shown in Panel B, the stationary distribution of c implies a smaller probability of $\sigma^P(c) = 0$ and a higher average $\sigma^P(c)$ when κ increases.

In sum, when transaction data can be better utilized, the platform becomes more aggressive in raising transaction volume through fee reduction (or subsidy). Accordingly, the platform maintains more reserves to buffer the resultant increase in operation risk. Part of the increased risk is shared with users through token price fluctuation. In Figure 16, we show that the improving efficiency of data technology increases the total welfare (Panel A) while the platform's share is rather stable and always below 100%. Therefore, even though the platform has monopolistic power as a unique marketplace where users transact with each other using tokens, the platform cannot possess the full economic surplus created by big data technology. Data originates from user activities, so to obtain

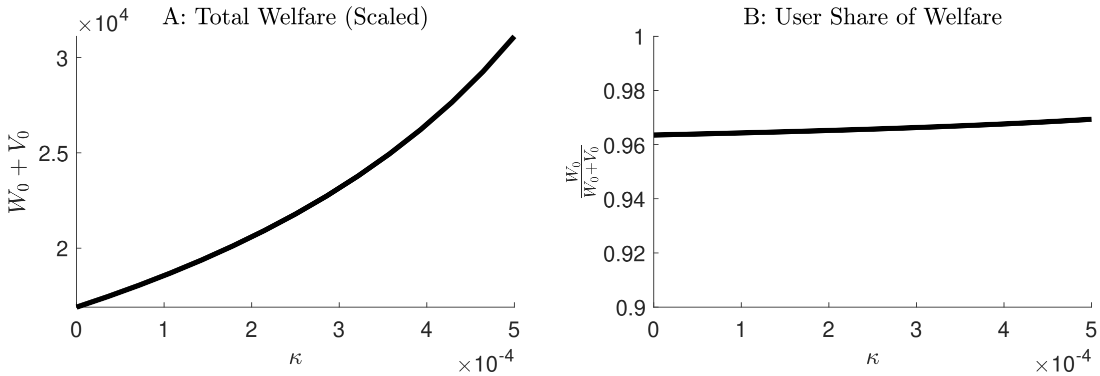

Figure 16: **Data Technology Progress and Welfare.** We plot the sum of platform value and users' welfare (Panel A), and users' share of total welfare (Panel B) against κ (the efficiency of data technology). The parameterization follows Figure 2. The parameterization follows follows Figure 2 with $\overline{n} = 2000$ and all quantities are scaled by data units A.

data, the platform must share the economic surplus with users. These outcomes also suggest that regulations targeting and limiting the use of transaction data undermine the platform's incentives to accumulate liquidity reserves and are detrimental for both user and total welfare.

6.2.3 Data Technology Revolution and Stablecoin Regulation

Because the transaction volume is proportional to A, equation (40) implies an exponential growth of effective data units that scales up the platform value and users' welfare. The improving efficiency of data technology causes the exponential growth to be increasingly steeper. In such an environment, how should the optimal capital requirement adjust? In this subsection, we address this question.

As previously discussed, a larger transaction volume N amplifies the shock exposure of reserves, and to achieve a larger transaction volume, the platform has to lower fees, sacrificing the growth of dollar reserves. Therefore, there exists tension between precautionary management of reserves and data acquisition through users' transactions. Capital requirement favors preserving reserves over stimulating transaction volume for data acquisition. Therefore, as data becomes more productive, i.e., κ increases, capital requirement becomes more burdensome.

Panel A of Figure 17 confirms the intuition. The optimal requirement of excess reserves (scaled by A) declines in κ. We study the scaled capital requirement to preserve the homogeneity property of the system and keep the solution in one-dimensional space of $c = C/A$. Indeed, tightening capital requirement causes the platform to build up reserves at the expense of data acquisition. However, given the self-reinforcing growth of data in (40), such regulatory measure hurts the long-run exponential growth of both platform value and users' welfare. In Panel B of Figure 17,

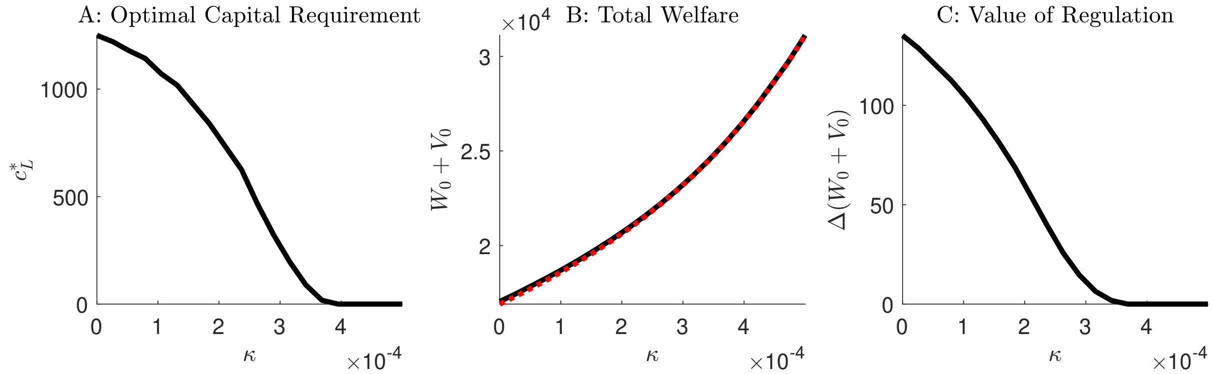

Figure 17: **Data Technology Progress and Capital Requirements.** We calculate the optimal scaled capital requirement $c_L^* \equiv C_L^*/A$ that maximizes total welfare (Panel A), scaled total welfare both with scaled capital requirement c_L^* (solid black line) and without capital requirement (dotted red line) (Panel B), and the welfare wedge between the optimally regulated equilibrium and laissez-faire equilibrium (Panel C) over different values of κ. Note that Panel C depicts the difference between the solid black line and the dotted red line from Panel B. The parameterization follows follows Figure 2 with $\overline{n} = 2000$ and all quantities are scaled by data units A.

we compare the total welfare under *optimal* capital requirement with that from the laissez-faire equilibrium, and in Panel C we plot the wedge. The increase of welfare in κ is not surprising. What is interesting is that the benefit of capital requirement dwindles as κ increases in Panel C.

Moreover, as we show in Figure 15, data as a self-accumulating productive asset offers a new opportunity for shareholders' equity to growth over time. This effectively makes the platform more patient in paying out dividends. Therefore, the voluntary build-up of reserves is strengthened as κ increases. As a result, capital requirement is less needed for the internalization of user-network effects. In sum, as data becomes more productive, the role of capital requirement weakens.

7 Conclusion

The first-generation cryptocurrencies, such as Bitcoin and Ethereum, were built to serve as transaction medium, but the price volatility compromises such functionality. As decentralized finance develops rapidly, various stablecoin initiatives arise to meet the demand for stable means of payment that are based on blockchains. Stablecoins are issued by private entities who promise to maintain price stability by holding collateral assets against which stablecoin holdings can be redeemed. However, as these issuers maximize their own payoffs rather than the total welfare, conflicts of interests between the issuers and stablecoin users naturally arise, making room for welfare-enhancing regulations. Moreover, well-established network companies (e.g., Facebook) plan to introduce their own stablecoins. Behind such initiatives, the incentives are even more complex, especially given the fact that operating a payment system allows the platform to gather and profit from transaction data.

In spite of the enormous attention from both regulators and practitioners, to this date, there has not been a unified framework to address these issues. In this paper, we fill this gap and develop a dynamic model of optimal stablecoin management. The equilibrium features two regimes. When the issuer's reserves are sufficiently high, the stablecoin price is fixed. When the reserves fall below a critical threshold, the stablecoin price comoves with the issuer's reserves, allowing the issuer to share risk with the stablecoin users and thus avoid costly liquidation. The distribution of states is bimodal. Above the debasement threshold, the issuer credibly sustains a fixed token price, which induces a strong token demand that allows the issuer to collect fee revenues and further grow reserve holdings. This virtuous cycle turns into a vicious cycle when negative shocks deplete the issuer's reserves below the debasement threshold. As the token price becomes volatile, the users' token demand declines, so the issuer's fee revenues and revenues from selling tokens in open market operations decline, which then slows down the rebuild of reserves, generating persistent debasement. The vicious cycle can be broken by issuing equity (governance tokens) to replenish reserves. Equity issuance must be done with an simultaneous expansion of token supply to eliminate arbitrage.

Our model provides a framework to evaluate regulatory proposals. We show that capital requirement can potentially improve the total welfare, but imposing a legally binding commitment to perfect price stability destroys welfare. Our framework can also be applied to analyze the incentives behind the stablecoin initiatives led by the well-established platform companies. We show that strong network connections can indeed lead to stability of token value, which makes these platform companies natural issuers of stablecoins. Moreover, a stablecoin may be introduced to stimulate transactions and the transaction data that can be used to improve the platform's profitability. However, an increase in the productivity of data destabilizes the stablecoin, as the platform becomes more eager to stimulate transactions, issuing more stablecoins per unit of reserves.

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A Derivations and Proofs

A.1 Value Function Concavity

We prove the concavity of value function in Proposition 1. Recall the HJB equation (20), that is,

$$\rho V(C) = \max_{\{N \in [0,\overline{N}],\, \sigma^P\}} \left\{ V'(C)\left(rC + N^{\xi}A^{1-\xi} - \eta N|\sigma^P|\right) + \frac{1}{2}V''(C)N^2(\sigma - \sigma^P)^2 \right\}.$$

Using the envelope theorem, we differentiate both sides of the HJB equation (evaluated under the optimal controls) with respect to C to obtain

$$\rho V'(C) = rV'(V) + V''(V)\left(rC + N^{\xi}A^{1-\xi} - \eta N|\sigma_t^P|\right) + \frac{1}{2}V'''(C)N^2(\sigma - \sigma^P)^2.$$

We can solve for

$$V'''(C) = \frac{2}{N^2(\sigma - \sigma^P)^2}\left[(\rho - r)V'(C) - V''(V)\left(rC + N^{\xi}A^{1-\xi} - \eta N|\sigma_t^P|\right)\right]$$

Using the smooth pasting condition, $V'(\overline{C}) = 1$, and the super-contact condition, $V''(\overline{C}) = 0-$, it follows that $V'''(\overline{C}) > 0$. As $V''(\overline{C}) = 0$, it follows that $V''(C) < 0$ in a left-neighbourhood of $\overline{C}$.

We show that $V''(C)$ for all $C \in [0, \overline{C})$. Suppose to the contrary that there exists $\hat{C} < \overline{C}$ with $V''(\hat{C}) \geq 0$ and set without loss of generality

$$\hat{C} = \sup\{C \geq 0 : V''(C) \geq 0\}. \tag{A.1}$$

As $V''(C) < 0$ in a left-neighbourhood of $\overline{C}$ and the value function is twice continuously differentiable, it follows that $V''(\hat{C}) = 0$ and therefore $\sigma^P(\hat{C}) < \sigma$. In addition, $V'(\hat{C}) \geq 1$, so that $V'''(\hat{C}) > 0$. Thus, there exists $C' > \hat{C}$ with $V''(C') \geq 0$, a contradiction. Therefore, the value function is strictly concave on $[0, \overline{C})$.

A.2 Optimal Control Variables

In this section, we characterize the optimization in (20) and solve for the optimal control variables $N = N(C)$ and $\sigma^P = \sigma^P(C)$ in Proposition 3. To start with, we define

$$\underline{N} = \arg\max_{N \leq \overline{N}} N^{\xi}A^{1-\xi} - \eta N\sigma,$$

which yields

$$\underline{N} = \min\left\{ \left(\frac{\xi A^{1-\xi}}{\eta\sigma}\right)^{\frac{1}{1-\xi}}, \overline{N} \right\}.$$

Now, first optimize the HJB equation over σ^P or equivalently over $N\sigma^P$.

If interior, the choice of σ^P satisfies the first order optimality condition. The first-order-condition in (20) with respect to σ^P is

$$-\eta V'(C) - V''(C)(N\sigma - N\sigma^P) = 0$$

Thus,

$$N\sigma^P = \max\left\{0, \frac{-\eta V'(C) - N\sigma V''(C)}{-V''(C)}\right\} = \max\left\{0, -\frac{\eta V'(C)}{-V''(C)} + N\sigma\right\}$$

We distinguish between two different cases

1. $\sigma^P > 0$, in which case we can insert σ^P into (20) to get

$$\rho V(C) = \max_{N\in[0,\overline{N}]}\left\{V'(C)\left[rC + N^\xi A^{1-\xi} - \eta N\sigma - \frac{\eta^2 V'(C)}{V''(C)}\right] + \frac{1}{V''(C)}\left[\frac{(\eta V'(C))^2}{2}\right]\right\}.$$

   Thus, $N = \underline{N}$ is the optimal choice of N, so that

$$\sigma^P = \max\left\{0, -\frac{\eta V'(C)}{-V''(C)\underline{N}} + \sigma\right\}$$

2. $\sigma^P = 0$, so the HJB becomes

$$\rho V(C) = \max_{N\in[0,\overline{N}]}\left\{V'(C)[rC + N^\xi A^{1-\xi}] + V''(C)\left[\frac{N^2\sigma^2}{2}\right]\right\}, \tag{A.2}$$

   and

$$N(C) = \min\left\{\left(\frac{A^{1-\xi}\xi V'(C)}{-V''(C)\sigma^2}\right)^{\frac{1}{2-\xi}}, \overline{N}\right\} \tag{A.3}$$

   is the optimal choice of N.

It follows that

$$N(C) = \underline{N} \iff -\frac{\eta V'(C)}{-V''(C)\underline{N}} + \sigma > 0,$$

as desired.

A.3 Effective Risk Aversion

We prove $\gamma'(C) < 0$, i.e., $\frac{d(-V''(C)/V'(C))}{dC} < 0$, in Proposition 3. Consider the following two cases:

1. $\sigma^P > 0$, $N = \underline{N}$. The HJB equation can be simplified to

$$\rho\frac{V(C)}{V'(C)} = rC + \underline{N}^\xi A^{1-\xi} - \eta\underline{N}\sigma - \frac{\eta^2}{2}\frac{V'(C)}{V''(C)}. \tag{A.4}$$

Differentiating the equation above with respect to C we obtain that in $(0, \overline{C})$

$$\rho\left(1-\frac{V''(C)V(C)}{V'(C)^2}\right)=r-\frac{\eta^2}{2}\frac{d(V'(C)/V''(C))}{dC}\,. \tag{A.5}$$

which implies $\frac{d(V'(C)/V''(C))}{dC}<0$ (because $V''(C)<0$ and $\rho>r$), i.e., $\frac{d(-V''(C)/V'(C))}{dC}<0$.

2. $\sigma^P=0$, so the HJB becomes

$$\rho V(C)=\max_{N\in[0,\overline{N}]}\left\{V'(C)[rC+N^{\xi}A^{1-\xi}]+V''(C)\left[\frac{N^2\sigma^2}{2}\right]\right\}, \tag{A.6}$$

In this case, we further consider two cases:

a. $N(C)<\overline{N}$ and $N=\left(\frac{A^{1-\xi}\xi V'(C)}{-V''(C)\sigma^2}\right)^{\frac{1}{2-\xi}}$. In this case, the HJB can be simplified to

$$\rho\frac{V(C)}{V'(C)}=rC+\frac{1}{2}\left(\frac{\xi A^{1-\xi}}{\sigma^{\xi}}\right)^{\frac{2}{2-\xi}}\left(\frac{2-\xi}{\xi}\right)\left(\frac{V'(C)}{-V''(C)}\right)^{\frac{\xi}{2-\xi}}\,. \tag{A.7}$$

Differentiating the equation above with respect to C, we obtain

$$\rho\left(1-\frac{V''(C)V(C)}{V'(C)^2}\right)=r-\frac{1}{2}\left(\frac{\xi A^{1-\xi}}{\sigma^{\xi}}\right)^{\frac{2}{2-\xi}}\left(\frac{V'(C)}{-V''(C)}\right)^{\frac{2\xi-2}{2-\xi}}\frac{d(-V'(C)/V''(C))}{dC}\,, \tag{A.8}$$

implying $\frac{d(V'(C)/V''(C))}{dC}<0$ (because $V''(C)<0$ and $\rho>r$), i.e., $\frac{d(-V''(C)/V'(C))}{dC}<0$.

b. $N(C)=\overline{N}$. In this case, the HJB can be simplified to

$$\rho\frac{V(C)}{V'(C)}=rC+\overline{N}^{\xi}A^{1-\xi}+\frac{\overline{N}^2\sigma^2}{2}\frac{V''(C)}{V'(C)}\,. \tag{A.9}$$

Differentiating the equation above with respect to C, we obtain

$$\rho\left(1-\frac{V''(C)V(C)}{V'(C)^2}\right)=r-\frac{\overline{N}^2\sigma^2}{2}\frac{d(-V''(C)/V'(C))}{dC}\,, \tag{A.10}$$

which implies $\frac{d(-V''(C)/V'(C))}{dC}<0$ (because $V''(C)<0$ and $\rho>r$).

A.4 Calculating User Welfare

To start with, recall that any users' utility flow is

$$d\hat{R}_{it}\equiv N_t^{\alpha}A^{1-\xi}\frac{u_{it}^{\beta}}{\beta}dt+u_{it}\Big(\frac{dP_t}{P_t}-rdt-f_tdt-\eta|\sigma_t^P|\Big)$$

As such,

$$\mathbb{E}d\hat{R}_{it} = N_t^{\alpha} A^{1-\xi} \frac{u_{it}^{\beta}}{\beta} dt + u_{it}\Big(\mu_t^P - rdt - f_t dt - \eta|\sigma_t^P|\Big).$$

Inserting $u_{it} = N_t$ and (12) and using $\xi = \alpha + \beta$ yields

$$\begin{aligned}\mathbb{E}d\hat{R}_{it} &= \frac{N_t^{\xi} A^{1-\xi}}{\beta} dt + N_t\Big(\mu_t^P - rdt - (N_t^{\xi-1}A^{1-\xi} + \mu_t^P - r - \eta|\sigma_t^P|)dt - \eta|\sigma_t^P|\Big) \\ &= \frac{N_t^{\xi} A^{1-\xi}}{\beta} dt - N_t^{\xi} A^{1-\xi} dt = \frac{(1-\beta)A^{1-\xi}}{\beta} N_t^{\xi} dt.\end{aligned}$$

As a next step, define the user welfare from time t onward, i.e.,

$$W_t := \mathbb{E}\left[\int_t^{\infty} e^{-r(s-t)}\left(dR_{is} - \eta u_{is}|\sigma_s^P|ds\right)\right]. \tag{A.11}$$

As C is the payoff-relevant state variable, we can express user welfare as function of C, in that $W_t = W(C_t)$. The dynamic programming principle implies that user welfare solves on $[0, \overline{C}]$ the ODE

$$rW(C_t)dt = \mathbb{E}d\hat{R}_{it} + \mathbb{E}dW(C_t).$$

We can rewrite the ODE as

$$rW(C) = \frac{(1-\beta)A^{1-\xi}}{\beta} N(C)^{\xi} + W'(C)\mu_C(C) + \frac{W''(C)\sigma_C(C)^2}{2}, \tag{A.12}$$

whereby

$$\begin{aligned}\mu_C(C) &= rC + N(C)^{\xi} A^{1-\xi} - \eta N(C)|\sigma^P(C)| \\ \sigma_C(C) &= N(C)(\sigma - \sigma^P(C))\end{aligned}$$

are drift and volatility of net liquidity C respectively.

The ODE (A.12) is solved subject to the boundary conditions

$$W'(\overline{C}) = 0$$

and

$$\lim_{C\to 0} W(C) = \frac{1}{r}\lim_{C\to 0}\left(\frac{(1-\beta)A^{1-\xi}}{\beta} N(C)^{\xi} + W'(C)\mu_C(C)\right).$$

A.5 Calculating the Expected Arrival Time

Note that there exists $\tilde{C} \in (0, \overline{C})$ such that $\sigma^P(C) = 0$. Given $C_t = C$ at time t, we calculate

$$\tau(C_t) = \mathbb{E}[\tau^* - t | C_t = C] \quad \text{with} \quad \tau^* = \inf\{s \geq t : C_s \geq \tilde{C}\},$$

which is the expected time until net liquidity reaches $\tilde{C}$ and token price volatility vanishes.

We can rewrite $\tau(C_t)$ as

$$\tau(C_t) = \mathbb{E}_t \left[\int_t^{t^*} 1 dt \right]. \tag{A.13}$$

By definition, it holds that when $C_t = C \geq \tilde{C}$, then $t^* = t$ and

$$\tau(C_t) = \tau(C) = 0.$$

By the integral expression (A.13) and the dynamic programming principle, it follows that For $C \leq \tau(C)$, the function $\tau(C)$ solves the ODE

$$0 = 1 + \tau'(C)\mu_C(C) + \frac{\sigma_C(C)^2 \tau''(C)}{2}, \tag{A.14}$$

where

$$\mu_C(C) = rC + N(C)^{\xi} A^{1-\xi} - \eta N(C)|\sigma^P(C)|$$
$$\sigma_C(C) = N(C)(\sigma - \sigma^P(C))$$

are drift and volatility of net liquidity C respectively. The ODE (A.14) is solved subject to the boundary condition

$$\tau(\tilde{C}) = 0 \tag{A.15}$$

at $C = \tilde{C}$. At $C = C_L$, the lower boundary of the state space, the boundary condition

$$\lim_{C \to C_L} [1 + \tau'(C)\mu_C(C)] = 0.$$

A.6 Proportional recapitalization costs

Suppose that the platform's refinancing/recapitalization costs are as follows. The platform must issue $1 + \omega$ dollars of equity (or secondary units) to raise one dollar of liquidity reserves. That is, recapitalization entails proportional costs ω. Note that in this specification, the platform's value function solves the HJB equation (20) whenever there is no refinancing event. As in Section 4.2, the platform would like to avoid incurring the costs ω and thus refinance only when $C = \underline{C} = 0$. As the costs are proportional to the amount of financing raised when $C = 0$, the platform refinances just enough to avoid liquidation when $C = \underline{C} = 0$. Thus, $C = \underline{C}$ becomes a reflecting boundary with boundary condition and

$$V'(0) = \min \left\{ 1 + \omega, V(0) \left(\frac{A}{\rho} \left(\frac{\xi}{\eta\sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta\sigma \right)^{-1} \right\} \tag{A.16}$$

hold. Note that the platform has two options at $C = 0$: i) debasement of token price and setting $\sigma^P(0) = 1$ or ii) refinancing. When

$$V'(0) = V(0) \left(\frac{A}{\rho} \left(\frac{\xi}{\eta\sigma} \right)^{\frac{1}{1-\xi}} \left(\frac{1-\xi}{\xi} \right) \eta\sigma \right)^{-1}, \tag{A.17}$$

then $\sigma^P(0) = 1$ and $C = 0$ is never reached. Note that (A.17) follows after rearranging (23). Then, if (A.17) holds, the platform never refinances.

On the other hand, when $V'(0) = 1 + \omega$, the platform refinances at $C = 0$ while $\sigma^P(0) < 1$, so refinancing prevents that C falls below zero. Overall, the total amount of funds raised F_t after $t = 0$ follows

$$dF_t = \max\{0 - C_t, 0\},$$

so $dF_t \geq 0$. The HJB equation determines the optimal choice of $\sigma^P(C)$ and $N(C)$ for all $C \geq 0$. When $\sigma^P(0) = 0$ and $V'(0) = 1 + \omega$, the token price is constant around $C = 0$, so there is no arbitrage around the recapitalization event. Suppose that $1 > \sigma^P(0) > 0$ and $V'(0) = 1 + \omega$, so the platform uses both debasement of token price and refinancing to avert liquidation and $C = 0$ is reached with positive probability. As $\sigma^P(0) > 0$, the token price increases upon a positive shock to liquidity reserves, $dZ_t > 0$. In addition, to preclude arbitrage at the recapitalization event, is must be that the token price adjusts downward upon a negative shock to liquidity reserves, $dZ_t < 0$, triggering a refinancing event. Thus, the token price can be written as

$$P_t = \hat{P}_t + L_t,$$

where $dL_t \leq 0$ (i.e., L_t is decreasing) and $\hat{P}_t = \hat{P}(C)$ solves the ODE

$$\sigma^P(C) = \frac{P'(C)}{P(C)} N(C) \left(\sigma - \sigma^P(C)\right),$$

subject to $\hat{P}(\overline{C}) = 1$. That is, after each refinancing event, there is repegging and the price is adjusted downward. This mechanism is analogous to the one presented in Section 4.2. Note that

$$L_t = -\sigma^P(0) F_t$$

so that $dL_t \leq 0$ and at time t with $C_t = 0$:

$$vol(dP_t) = \sigma^P(0).$$

A.7 Details on the Model with User Collateral

We define

$$\mu_C(C) = rC - r(m-1)N(C) + m(\tilde{\mu} - \delta)N(C) + N(C)^{\xi}A^{1-\xi} - N(C)\eta|\sigma^P(C)| - \frac{N(C)\delta}{m}$$
$$\sigma_C(C) = N(C)\left(\frac{\sigma}{m} - \sigma_t^P\right),$$

in that $\sigma_t^P = \sigma^P(C_t)$ and $N_t = N(C_t)$. Then, the HJB equation can be written as

$$\rho V(C) = \max_{N\in[0,\overline{N}],m\geq 1,\sigma^P} V'(C)\mu_C(C) + \frac{\sigma_C(C)^2 V''(C)}{2}.$$

To solve for the optimal controls, we distinguish between the cases 1) $\sigma^P(C) > 0$ and 2) $\sigma^P(C) = 0$:

1. Suppose that $\sigma^P(C) > 0$. Then, the FOC with respect to σ^P yields

$$-\eta V'(C) - V''(C)\left(\frac{N\sigma}{m} - N\sigma^P\right) = 0$$

so that

$$N\sigma^P = \max\left\{0, \frac{-\eta V'(C) - V''(C)\frac{N\sigma}{m}}{-V''(C)}\right\} = \max\left\{0, -\frac{\eta V'(C)}{-V''(C)} + \frac{N\sigma}{m}\right\}.$$

We can insert this expression for σ^P into (20) to get

$$\rho V(C) = \max_{N\in[0,\overline{N}],m} \Bigg\{V'(C)\Big[rC + N^{\xi}A^{1-\xi} - \frac{\eta N\sigma}{m} - r(m-1)N + m(\tilde{\mu} - \delta)N - \frac{N\delta}{m} - \frac{\eta^2 V'(C)}{V''(C)}\Big] + \frac{1}{V''(C)}\left[\frac{(\eta V'(C))^2}{2}\right]\Bigg\}.$$

The choice of m is independent of N. One can calculate that optimal m solves the first-order condition

$$\tilde{\mu} - \delta - r + \frac{\delta}{m^2} + \frac{\eta\sigma}{m^2} = 0,$$

so that

$$\frac{1}{m^2} = \frac{r + \delta - \tilde{\mu}}{\delta + \eta\sigma} \iff m = \overline{m} = \sqrt{\frac{\delta + \eta\sigma}{r + \delta - \tilde{\mu}}}.$$

Next, we can take the first-order condition with respect to N to obtain

$$\xi N^{\xi-1}A^{1-\xi} - \frac{\eta\sigma}{m} - r(m-1) + m(\tilde{\mu} - \delta) - \frac{\delta}{m} = 0.$$

Thus,

$$N(C) = \underline{N} = A \left(\frac{\xi}{\frac{\eta\sigma}{\overline{m}} + r(\overline{m} - 1) - \overline{m}(\tilde{\mu} - \delta) + \frac{\delta}{\overline{m}}} \right)^{\frac{1}{1-\xi}} \tag{A.18}$$

2. Suppose that $\sigma^P = 0$. Then, taking the derivative with respect to N yields

$$\frac{\partial V(C)}{\partial N} = \frac{1}{\rho} \left(V'(C) \left[\xi N^{\xi - 1} A^{1-\xi} - r(m-1) + m(\tilde{\mu} - \delta) - \frac{\delta}{m} \right] + N \left(\frac{\sigma}{m} \right)^2 V''(C) \right).$$

Taking the first-order condition with respect to m yields

$$\frac{\partial V(C)}{\partial m} = 0 \iff V'(C) N \left[\tilde{\mu} - \delta - r + \frac{\delta}{m^2} \right] - N^2 V''(C) \frac{\sigma^2}{m^3} = 0.$$

Thus,

$$N = N(C) = \frac{-V'(C)}{V''(C)} \left(\frac{r + \delta - \delta/m^2 - \tilde{\mu}}{\sigma^2/m^3} \right) = \frac{-V'(C)}{V''(C)} \left(\frac{(r + \delta - \tilde{\mu}) m^3 - \delta m}{\sigma^2} \right)$$

Finally, we discuss the value function at the payout boundary $\overline{C}$ where $V'(\overline{C}) - 1 = V''(\overline{C}) = 0$. At $C = \overline{C}$, we have

$$\begin{aligned} \sigma^P(\overline{C}) &= 0 \\ N(\overline{C}) &= \overline{N} \\ m(\overline{C}) &= \sqrt{\frac{\delta}{r + \delta - \tilde{\mu}}} \end{aligned}$$